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Simultaneous Bunch Length and Energy Spread Compression During Recirculation of Multiple Passes in the IR Demo

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Abstract

We show that appropriate choices of acceleration phase, energy recovery phase, and momentum compaction will simultaneously compress both the bunch length of the coasting beam at the first reinjection and the energy spread of the energy-recovered beam at the dump during three pass operation of the IR Demo. We find, moreover, that when run in multipass mode, the accelerator can be made to image the injected beam at the dump by a proper choice of accelerating phase.

Introduction

Three-pass operation of the IR Demo has been described in a previous note [1]. It was observed (both theoretically and empirically) that by accelerating, coasting, and energy recovering at phases as shown in Figure 1 and with an appropriate choice of momentum compaction, the beam momentum spread was limited throughout the system. For an arbitrary choice of accelerating phase ϕ_0, the following choice of recirculator momentum compaction (defined as the linear derivative of path length with respect to the relative energy offset) will compress the bunch length at the first reinjection – where after the beam coasts through the linac [2]:

$$M_{56} = \frac{\lambda_{RF}}{2\pi} \frac{\left(E_{inject} + E_{linac}\cos\phi_0\right)}{E_{linac}} \frac{1}{\sin\phi_0} \tag{1}$$

The short bunch then traverses the linac without generation of significant additional momentum spread; as noted in Ref. [1], the effect of the compaction on the beam during the third pass is to introduce a phase/energy correlation serving to compress the energy spread during energy recovery. At that time, it was not apparent to us that the compression would be exact; we rather vaguely wrote

> "Moreover, there is an ancillary benefit. If we accelerate ahead of crest (as during lasing), the phase offset to zero crossing is a bit over a quarter RF wavelength. Energy recovery then will occur on the trailing edge of the trough ("two bits" over a half wavelength), and the overall momentum compaction (now negative, transporting a short bunch from linac back to linac, in contrast to the *positive* value transporting a short bunch from *wiggler* to linac used during lasing operation with energy recovery at the leading edge of the trough) will serve to provide some energy compression during energy recovery."

It has since occurred to us that "some energy compression" in fact means that the bunch can be fully compressed, and that under the correct circumstances an image of the injected bunch can be formed, at the dump.

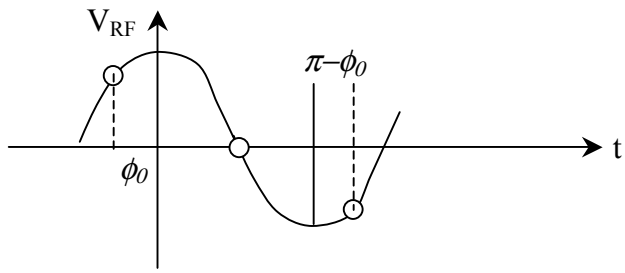

Figure 1: Phase arrangement giving correlated bunch length and momentum spread compression. The left point corresponds to an accelerating phase $\phi_0 < 0$, the center is for a beam coasting at zero crossing, and the right represents an energy recovered beam.

Compressional Trickery

Consider the injection and off-crest acceleration at phase ϕ_0 of a finite length, low momentum spread bunch. As noted above and previously documented [2], nonisochronous recirculation of the accelerated bunch can rotate it upright at some downstream point. If the recirculator compaction is as given in (1), and the recirculation path selected to make the reinjected bunch synchronous with the zero-crossing of the RF, the short bunch will coast down the linac with no induced momentum spread and exit the linac still upright. It will moreover traverse the recirculator transport with a time of flight leading to energy recovery of the centroid (Figure 1); we now show that the recirculator compaction specified above induces a phase energy correlation leading to energy compression during energy recovery.

The energy differential δE imparted during acceleration to a particle displaced by δl from the nominal phase is as follows.

$$\delta E(\delta l) = E_{linac}\left[\cos\left(\frac{2\pi}{\lambda_{RF}}\delta l + \phi_0\right) - \cos\phi_0\right]$$

In reference to Figure 1, we remark that $\delta l<0$ indicates a shorter path length – corresponding to earlier arrival time ($\delta t<0$) in the notation of the diagram. Expanding this in terms of the small parameter $\delta l/\lambda_{RF}$ and redistributing yields the following relation.

$$\delta E(\delta l) = -\frac{2\pi E_{linac}}{\lambda_{RF}}\sin\phi_0\ \delta l \qquad (2)$$

The energy differential generated while coasting is obtained from this expression by setting $\phi_0=\pi/2$:

$$\delta E(\delta l) = -\frac{2\pi E_{linac}}{\lambda_{RF}}\ \delta l \qquad (2')$$

The energy differential arising during energy recovery is similarly obtained by replacing ϕ_0 by π-ϕ_0 and is seen to be identical to that generated during acceleration and is described by (2) as well. This is obvious from Figure 1 – the slope of the waveform is the same at both the acceleration and the energy recovery phase. The linear response of the energy offset to a path length deviation is thus described as follows:

during acceleration and energy recovery:

$$\begin{pmatrix} \overline{\delta l} \\ \overline{\delta E} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\dfrac{2\pi E_{linac}}{\lambda_{RF}} \sin\phi_0 & 1 \end{pmatrix} \begin{pmatrix} \delta l \\ \delta E \end{pmatrix} \tag{3}$$

while coasting:

$$\begin{pmatrix} \overline{\delta l} \\ \overline{\delta E} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\dfrac{2\pi E_{linac}}{\lambda_{RF}} & 1 \end{pmatrix} \begin{pmatrix} \delta l \\ \delta E \end{pmatrix} \tag{3'}$$

We have included (3') to emphasize that if a zero length bunch is reinjected at zero crossing, the outgoing energy spread is unaffected – that is, $(0,\delta E)$ is mapped to $(0,\delta E)$.

Similarly, the linear response δl of path length through the recirculator to a differential energy offset δE is as follows, where $E_{total} = E_{inject} + E_{linac} \cos \phi_0$.

$$\begin{pmatrix} \overline{\delta l} \\ \overline{\delta E} \end{pmatrix} = \begin{pmatrix} 1 & M_{56}/E_{total} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \delta l \\ \delta E \end{pmatrix} \tag{4}$$

The linear behavior of the system from injection to reinjection (on the first pass through the linac and recirculator) is described by the product of the matrices in (3) and (4).

$$\begin{pmatrix} \overline{\delta l} \\ \overline{\delta E} \end{pmatrix} = \begin{pmatrix} 1 & M_{56}/E_{total} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\dfrac{2\pi E_{linac}}{\lambda_{RF}} \sin\phi_0 & 1 \end{pmatrix} \begin{pmatrix} \delta l \\ \delta E \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \dfrac{M_{56}}{E_{total}} \dfrac{2\pi}{\lambda_{RF}} E_{linac} \sin\phi_0 & \dfrac{M_{56}}{E_{total}} \\ -\dfrac{2\pi}{\lambda_{RF}} E_{linac} \sin\phi & 1 \end{pmatrix} \begin{pmatrix} \delta l \\ \delta E \end{pmatrix}$$

By appropriate selection of M_{56} we can ensure that a bunch with zero momentum spread and finite length at injection is transformed to a bunch with zero length and finite

momentum spread at reinjection. This is equivalent to an optical parallel-to-point focus, and is achieved by requiring the 1-1 matrix element to vanish.

$$1-\frac{M_{56}}{E_{total}}\frac{2\pi}{\lambda_{RF}}E_{linac}\sin\phi_0=0$$

Thus we recover (1): $M_{56} = (\lambda_{RF}/2\pi)(E_{total}/E_{linac})\sin\phi_0$. Note that with this choice the matrix collapses to the following.

$$\begin{pmatrix} 0 & M_{56}/E_{total} \\ -E_{total}/M_{56} & 1 \end{pmatrix} \tag{5}$$

Now consider the recirculation and energy recovery of the upright reinjected bunch. As indicated by (3'), it is unaffected by coasting through the linac (as the second pass beam). The subsequent recirculation and energy recovery (on the second pass through the recirculator and third pass through the linac) are described by the product of the matrices in (4) and (3) (in reversed order from the previous calculation).

$$\begin{pmatrix} \overline{\delta l} \\ \overline{\delta E} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{2\pi E_{linac}}{\lambda_{RF}}\sin\phi_0 & 1 \end{pmatrix}\begin{pmatrix} 1 & M_{56}/E_{total} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \delta l \\ \delta E \end{pmatrix}$$
$$= \begin{pmatrix} 1 & M_{56}/E_{total} \\ -\frac{2\pi E_{linac}}{\lambda_{RF}}\sin\phi_0 & 1-\frac{2\pi E_{linac}}{\lambda_{RF}}\frac{M_{56}}{E_{total}}\sin\phi_0 \end{pmatrix}\begin{pmatrix} \delta l \\ \delta E \end{pmatrix}$$

With M_{56} as specified by (1), the 2-2 matrix element vanishes; the transport from linac to dump thus provides a point-to-parallel longitudinal focus. Hence, (0, δE) is transformed to ($\delta l=(M_{56}/E_{total})\delta E$, 0) – or, in other words, the recirculated energy spread is exchanged for bunch length and thereby compressed.

$$\begin{pmatrix} \overline{\delta l} \\ \overline{\delta E} \end{pmatrix} = \begin{pmatrix} 1 & M_{56}/E_{total} \\ -E_{total}/M_{56} & 0 \end{pmatrix}\begin{pmatrix} \delta l \\ \delta E \end{pmatrix} \tag{6}$$

We therefore find that the compaction producing minimum bunch length at reinjection also leads to energy compression during energy recovery.

Imaging Trickery

The linear behavior of the entire acceleration, recirculation, coasting, recirculation, and energy recovery cycle is described by the product of the matrices in (5), (3'), and (6).

$$\begin{pmatrix} \overline{\delta l} \\ \overline{\delta E} \end{pmatrix} = \begin{pmatrix} 1 & M_{56}/E_{total} \\ -E_{total}/M_{56} & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\dfrac{2\pi E_{linac}}{\lambda_{RF}} & 1 \end{pmatrix} \begin{pmatrix} 0 & M_{56}/E_{total} \\ -E_{total}/M_{56} & 1 \end{pmatrix} \begin{pmatrix} \delta l \\ \delta E \end{pmatrix}$$

$$= \begin{pmatrix} -1 & \dfrac{M_{56}}{E_{total}} \left(2 - \dfrac{2\pi}{\lambda_{RF}} \dfrac{E_{linac}}{E_{total}} M_{56} \right) \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \delta l \\ \delta E \end{pmatrix} = \begin{pmatrix} -1 & \dfrac{M_{56}}{E_{total}} \left(2 - \dfrac{1}{\sin\phi_0} \right) \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \delta l \\ \delta E \end{pmatrix}$$

This result shows that the choice ϕ_0=30° will reduce the transform to a negative identity. In this case, the injected beam will be (inversely) imaged at the dump.

"Practical" Applications

Though basically a novelty in the IR Demo, this type of manipulation may have some use in a broader context. Possible applications include:

- Imaging of an injected beam at the dump in an energy-recovered linac – either to "see" (as a diagnostic) the injected bunch or to characterize beam quality preservation throughout the acceleration, transport, and energy recovery cycle.

- Use of identical recirculation arcs in compact FEL drivers. Such devices typically will embed the FEL in the machine back-leg, opposite the linac. The "normal" acceleration/energy recovery phasing (with the recovery phase 180° out from the accelerating) requires compactions of opposite sign for bunch length compression at the wiggler and energy spread compression at the dump (as in the IR Demo). For a small, tightly coupled design, this implies somewhat different geometries for the two transport arcs to/from the wiggler. Use of phasing as in Figure 1 allows identical compactions for each portion of the operation cycle; the transport to and from the wiggler can thus be very similar in design if desirable. We note, however, that use of the Figure 1 geometry can (*will!?*) lead to incomplete cancellation of the RF phase vectors.

- Current doubling with energy recovery. As advocated by Hutton [3], this technique may require appropriate longitudinal manipulations to manage momentum spread during the coasting and energy recovered passes. The methods described above (and implemented, albeit inadvertently, in the IR Demo) will (at least in some limited range of application) meet such requirements.

References

[1] D. Douglas and C. Tennant, "Three-Pass Operation of the IR Demo Driver", JLAB-TN-01-043, 28 August 2001.

[2] Douglas, D., A. Hutton, C. Leemann, and D. Neuffer, "Analytic Modeling and Lattice Scaling Relations for FEL Driver Accelerators", CEBAF-TN-95-015, 21 March 1995.

[3] A. Hutton, to be published.