Q^2 DEPENDENCE OF GENERALIZED BALDIN SUM RULE

YONGGUANG LIANG
Ohio University, Athens, OH 45701, USA
E-mail: yl0094a@jlab.org

The generalized Baldin sum rule for virtual photon, an unpolarized analog of the generalized Gerasimov-Drell-Hearn sum rule, provides an unique way to investigate the transition between the perturbative QCD and hadronic descriptions of nucleon structure. We report on new measurements in Hall C at Jefferson Lab of the generalized Baldin integral for the proton at Q^2 of 0.3-4.0 GeV^2.

1. Introduction

In 1960s, by applying a once-subtracted dispersion relation [1] and the low energy theorem [2,3] to real forward Compton scattering (momentum transfer $Q^2 = 0$), A.M. Baldin introduced a sum rule to connect the sum of the electric and magnetic polarizabilities of the nucleon $(\alpha + \beta)$ to the integral of the ν^2-weighted nucleon unpolarized photoabsorption cross section [4],

$$\alpha + \beta = \frac{1}{4\pi^2} \int_{\nu_0}^{\infty} \frac{\sigma_{\frac{1}{2}} + \sigma_{\frac{3}{2}}}{\nu^2} \, d\nu, \tag{1}$$

where $\sigma_{\frac{1}{2}}$ and $\sigma_{\frac{3}{2}}$ are the photoproduction cross sections of the 1/2 and 3/2 helicity state, respectively. ν is the energy carried by the photon, and ν_0 is the pion photoproduction threshold. $\alpha + \beta$ is the helicity non-flip electromagnetic polarizability. The Baldin sum rule establishes a relation between the low energy nucleon structure quantities (electric and magnetic polarizabilities) and the nucleon excitation spectrum, such that these polarizabilities can be extracted from the precision measurement of the photoabsorption cross sections of real Compton scattering. For proton, the recent measurement gives $(\alpha + \beta)_p = 13.69 \pm 0.14$ [5].

As an analogy to the generalized Gerasimov-Drell-Hearn sum rule [6], D. Drechsel *et.al* generalized the Baldin sum rule to virtual Compton scattering $(Q^2 > 0)$ [7]. This process includes the absorption of a virtual photon, relating it to inclusive electron-nucleon scattering. At finite Q^2, the generalized sum rule gives

$$\alpha(Q^2) + \beta(Q^2) = \frac{1}{4\pi^2} \int_{\nu_0}^{\infty} \frac{K}{\nu} \frac{\sigma_{\frac{1}{2}} + \sigma_{\frac{3}{2}}}{\nu^2} \, d\nu = \frac{e^2 M}{\pi Q^4} \int_0^{x_0} 2x F_1(x, Q^2) \, dx, \tag{2}$$

where the integral on the right hand side is the second Cornwall-Norton moment [8] of the nucleon structure function F_1. Here, M is the nucleon mass, $K = (W^2 - M^2)/2M$ is the equivalent real photon energy needed to excite the nucleon to mass W [9], $x = Q^2/2M\nu$ is the Bjorken scaling variable, and x_0 corresponds with pion photoproduction threshold.

At large Q^2, the coupling constant of QCD is very small, and perturbative QCD theory provides an excellent interpretation of the deep inelastic scattering (DIS) process. At large Q^2, according to Callan-Gross relation [10], the second moment of structure function F_1 is approximately equal to the first moment of structure function F_2 which is approximately a constant, and the generalized Baldin sum rule $\sim 1/Q^4$, and $\to 0$ as $Q^2 \to \infty$. At low energy, the coupling constant of QCD increases very fast, and the scattering process must be described in terms of hadronic degree of freedom using Chiral Perturbative Theory. The generalized sum rule recovers the Baldin sum rule of real Compton scattering at $Q^2 = 0$. Between these two regions is the so called resonance region ($M < W < 2$ GeV), where a descriptive theory is lacking at present. Most of our understanding of the resonance region is based on phenomenology. Measuring the generalized Baldin sum rule at low Q^2 (up to a few GeV) provides an unique window to understand the transition from the DIS incoherent process to the resonance dominated coherent process.

2. Experiment

We measured the inclusive scattering of unpolarized electrons from a hydrogen target in Hall C at Jefferson Lab (JLab) in summer of 1999 [11]. The data were accumulated in the nucleon resonance region at Q^2 between 0.3 to 5.0 GeV^2. The structure function $R = \sigma_L/\sigma_T$ (ratio of longitudinal to transverse cross sections) was extracted from the measured differential cross sections using two methods (Rosenbluth separation, and a global fitting procedure) [12]. After obtaining R, the unpolarized nucleon structure function F_2, F_1 (purely transverse), and F_L (purely longitudinal) were extracted from the cross sections. A completed description regarding the experiment, data analysis, and systematic uncertainty estimation may be found in reference [12,13].

3. Results and Conclusions

A sample of $2xF_1$ data ($2xF_1 \sim \sigma_T$) is shown in Fig. 1, as a function of x for various Q^2. The triangles represent our Rosenbluth separations, and the crosses are the data extracted from SLAC Rosenbluth data [14]. The dashed curve was calculated using the parameterizations extracted from our data set [12] at $W < 2$ GeV, and the SLAC DIS parameterizations [14,15] at $W > 2$ GeV. It is noticed that the dashed curve nicely reproduces the data in both the resonance and DIS regions. The solid curve was calculated using only the SLAC DIS parameterizations [14,15]. The second moment of F_1 was obtained by integrating the area below the dashed curve over the range $0 < x < x_0$. The area corresponding to $W < 2$ represents the resonance contribution to the moment, while the area corresponding to $W > 2$ is the DIS contribution.

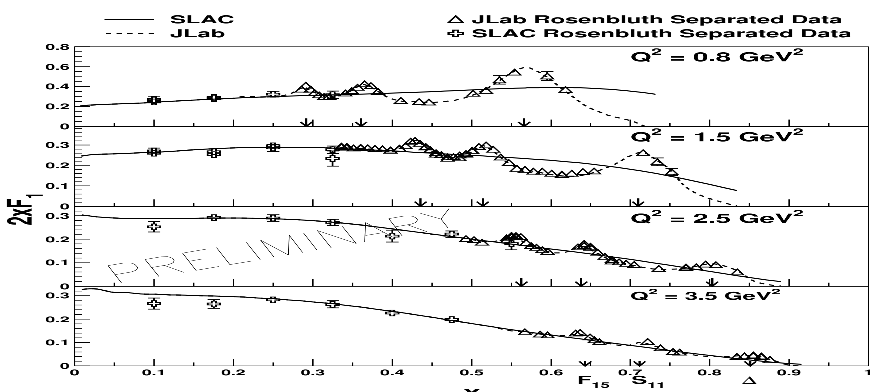

Figure 1. $2xF_1$ is plotted as a function of x, at four different Q^2. The three arrows indicate where the three primary resonances are located.

Similarly, we extracted the generalized Baldin integral by calculating the contributions from the resonance and DIS regions,

$$\alpha(Q^2) + \beta(Q^2) = \frac{e^2 M}{\pi Q^4} \int_{x_{res}}^{x_0} 2xF_1(x, Q^2)\, dx + \frac{e^2 M}{\pi Q^4} \int_0^{x_{res}} 2xF_1(x, Q^2)\, dx, \tag{3}$$

where x_{res} corresponds to $W = 2$ GeV. In Fig. 2 is plotted the generalized Baldin integral versus Q^2, along with two MAID estimates [16,17]. It shows that, unlike the generalized GDH sum rule, the generalized Baldin integral goes smoothly to $Q^2 = 0$. By comparison it is clear our data is more consistent with the three channel MAID estimate. The discrepancy between the data and the MAID estimate at $Q^2 > 1$ GeV^2 is largely due to the fact that the MAID estimates shown here only calculate the resonance contribution. As Q^2 increases, this is less significant compared to the DIS contribution to the extracted integral. A 3% uncertainty was assigned to the extracted data, which is dominated by the normalized systematic uncertainties of the measured cross sections, and the uncertainties of the fitting to the data [12,14,15].

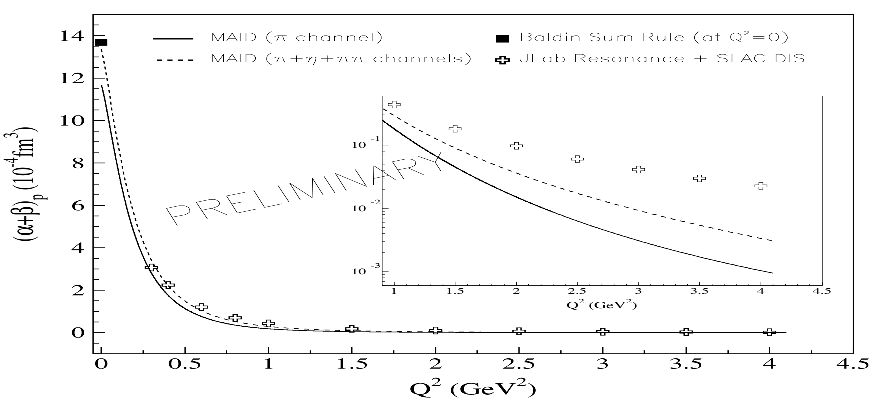

Figure 2. Generalized Baldin integral as a function of Q^2, along with two MAID estimates. Baldin sum rule at $Q^2 = 0$ is also shown.

The $Q^4/2M$-weighted generalized Baldin integral, as well as its resonance and DIS contributions, is plotted versus Q^2 in Fig. 3, along with two MAID estimates and one DIS estimate. The DIS estimate is extracted from SLAC parameterizations [14,15]. It shows that the resonance contribution extracted from our experiment is in excellent agreement with the three channel MAID estimate down to $Q^2 = 0.6$ GeV^2. The generalized sum rule is mainly saturated by the resonance contribution at $Q^2 \leq 1$ GeV^2, while the DIS part dominates at $Q^2 \geq 2$ GeV^2. At $1 < Q^2 < 2$ GeV^2, a transition from partonic incoherent processes to resonance dominated coherent processes occurs. Also, $\frac{Q^4}{2M}(\alpha + \beta)$, which is proportional to the second moment of F_1, is nearly flat at $Q^2 > 2.5$ GeV^2. This behavior has been predicted by the partonic description of the DIS process at large Q^2, and our data shows it is observed down to $Q^2 = 3$ GeV^2.

In summary, we have precisely measured the inclusive electron-proton scattering cross sections in the resonance region, and extracted the unpo-

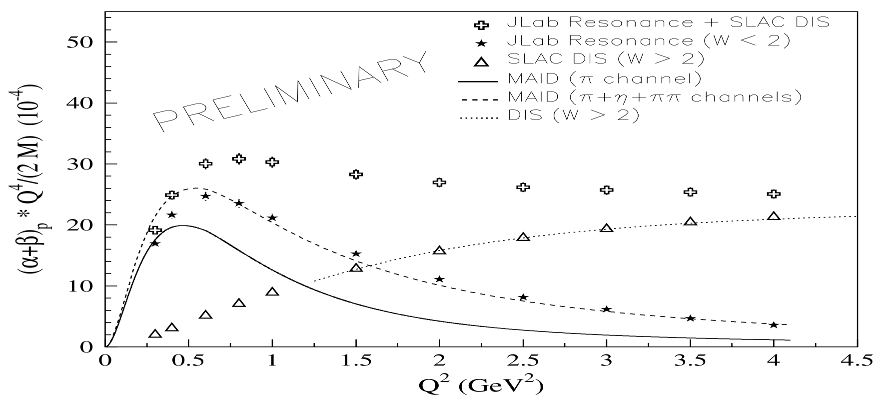

Figure 3. Q^2 evolution of $\frac{Q^4}{2M}$-weighted generalized Baldin integral.

larized structure functions R, F_2, F_1 and F_l. The F_1 data were used to calculate the generalized Baldin sum rule at Q^2 from 0.3 to 4.0 GeV^2. A transition from partonic incoherent process to resonance dominated coherent process was observed around Q^2 between 1 and 2 GeV^2. The generalized Baldin sum rule evolutes smoothly with Q^2 to the real photon point.

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