WHAT ARE THESE SUM RULES GOOD FOR?

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Dispersive sum rules, of which the Drell-Hearn-Gerasimov sum rule is a special example, are a consequence of rather general principles. On the one hand, they can be used to make tests of these fundamental principles, allowing to probe deep mysteries of nature. On the other hand, they can be used to access experimental observables that are otherwise impossible to measure, and thus providing excellent opportunities to learn about the physics of strongly interacting systems, such as nucleons and nuclei. I will use examples to illustrate these points.

1. Introduction

The Drell-Hearn-Gerasimov (DHG) sum rule [1] belongs to a class of sum rules that are derived from dispersion relations, and thus is called a dispersive sum rule. There are other sum rules that can be derived from an algebra. For example, the famous Thomas-Reiche-Kuhn sum rule is derived from the Heisenberg algebra $[x, p] = i\hbar$.

Dispersion relations are derived from analyticity, which in turn is a consequence of causality, of scattering amplitudes. Analyticity implies Cauchy's theorem

$$f(\omega) = \int \frac{\mathrm{Im} f(\omega')}{\omega - \omega'} d\omega' \ , \tag{1}$$

where we have assumed that the only singularities are along the real axis of ω. At low energies, $f(\omega)$ can be calculated in low-energy effective theories as a power series in ω. For example, the leading-order terms might be determined by low-energy theorems, and/or calculable in chiral perturbation theory for the nucleon system and in nuclear effective field theory for a nucleus. On the other hand, unitarity relates the imaginary part of a forward scattering amplitude to a physical cross section. Together, a dispersion relation connects a physical cross section to a corresponding forward amplitude at low energies.

The above is the physical content of dispersive sum rules. What are possible uses of those sum rules? Here are some of the possibilities:

- They can be used to check fundamental assumptions going into the derivation of the sum rules. These include, for instance, causality and analyticity, low-energy effective theories, and asymptotic behavior of scattering amplitudes.
- They can be used to understand the physical content of the low-energy constants. For example, the deep-inelastic momentum sum rule tells us how the fraction of the nucleon momentum carried by quarks is distributed in Feynman momentum x.
- They can be used to determine the low-energy scattering amplitudes, some of which may not be available from direct scattering processes.

In this talk, I will mainly focus on the second and third points.

2. DHG sum rule for a target with an arbitrary spin

Let us consider the DHG sum rule for a target with an arbitrary spin S. The forward Compton amplitude can be expanded in terms of t-channel tensor structures,

$$f = f_0\hat{\epsilon}^* \cdot \hat{\epsilon} + f_1 i\hat{\epsilon}^* \times \hat{\epsilon} \cdot \vec{S} + f_2(\hat{k} \otimes \hat{k})^{(2)} \cdot (\vec{S} \otimes \vec{S})^{(2)} \hat{\epsilon}^* \cdot \hat{\epsilon} + \ldots , \quad (2)$$

where $\hat{\epsilon}$ and $\vec{k}$ are the photon polarization and momentum, respectively, $\vec{S}$ is the angular momentum operator of the target, and $\otimes$ indicates tensor coupling. The vector amplitude f_1 is related to the amplitude $f^{(m_s)}$ with the target in a good m-state by

$$f_1 = -\frac{3}{S(S+1)} \frac{1}{2S+1} \sum_{m_s} m_s f^{(m_s)} \ . \quad (3)$$

The low-energy expansion of the vector amplitude goes like

$$f_1 = -\frac{\alpha_{\rm em}\kappa^2}{4S^2M^2}\omega + 2\gamma\omega^2 \ , \quad (4)$$

where M is the mass, and the anomalous magnetic moment is [2]

$$\kappa = \mu - 2S \ , \quad (5)$$

where μ is the total magnetic moment in units of $e\hbar/2Mc$. The above relation implies that a point-like particle has $\mu_{\rm point} = e\hbar S/Mc$.

There has been much discussion in the literature about the magnetic moment of a point-like particle. In a paper by Belinfante in 1953, he conjectured that a point-like spin-S particle has a spin-independent magnetic moment $\mu = e\hbar/2Mc$. There are many "proofs" of the Belinfante conjecture in the literature. It is known, however, that once interactions are introduced, there is no renormalizable field theory for massive particles with spin greater than 1/2. Therefore, those "proofs" are based on a special version of interacting theories, which has no special significance if the theory is non-renormalizable. Quite often, additional ingredients must be introduced to form a physically sensible theory. For example, in the standard model, the W-boson has a magnetic moment $\mu_W = e\hbar/Mc$, consistent with the above low-energy theorem.

Let us now consider the dispersion relation for the $J=1$ amplitude,

$$f_1(\omega) = \frac{2}{\pi}\omega \int_0^\infty d\omega' \frac{\mathrm{Im} f_1(\omega')}{\omega'^2 - \omega^2} \ . \tag{6}$$

Using the optical theorem, one has

$$f_1(\omega) = \frac{\omega}{2\pi^2} \int_0^\infty d\omega' \omega' \frac{\sigma_1(\omega')}{\omega'^2 - \omega^2} \ . \tag{7}$$

Substituting the low-energy expansion f_1 into Eq. (7), the first term yields the DHG sum rule, now extended to a target of any spin S,

$$\frac{\alpha_{\rm em}\kappa^2}{4S^2M^2} = \frac{1}{2\pi^2}\int_0^\infty d\omega' \frac{\sigma_1(\omega')}{\omega'} \ , \tag{8}$$

where $\sigma_1 = [3/S(S+1)(2S+1)]\sum_{m_s} m_S \sigma_{m_S}$.

3. GDH sum rule for the nucleon and deuteron

How is the low-energy theorem for the spin-1/2 nucleon reproduced in effective field theory? The amplitude of interest is of $\mathcal{O}(p^3)$ in power counting in chiral perturbation theory. There is a spin-dependent magnetic photon coupling at order $\mathcal{O}(p^2)$, which has an interaction vertex

$$\frac{2(Z+\kappa)}{2m} N^\dagger \vec{\sigma}\cdot(\vec{k}\times\vec{\epsilon})N \ , \tag{9}$$

where k is the photon momentum, and Z is the charge. There is also a seagull interaction coming from the spin-orbit type of relativistic corrections at $\mathcal{O}(p^3)$,

$$\frac{e^2}{4m^2} Z(Z+2\kappa) N^\dagger \vec{\sigma}\cdot(\vec{A}\times\vec{E})N \ , \tag{10}$$

where $\vec{E}$ is the electric field. The low-energy theorem is reproduced by the Feynman diagrams shown in Fig. 1

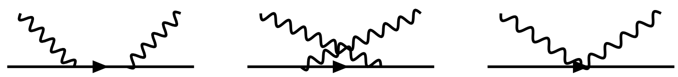

Figure 1. Feynman diagrams for the spin-dependent nucleon Compton scattering, which reproduce the low-energy theorem.

Consider the nucleon DHG sum rule in the large N_c limit. The isoscalar part of the left-hand side is proportional to κ_1^2/M^2, which scales like N_c^2. On the right-hand side, the only contribution which scales like N_c^2 comes from the delta resonance. Therefore, the isoscalar part of the DHG sum rule is entirely saturated by the photoproduction of the Δ resonance in the large N_c limit [3].

The low-energy theorem for the deuteron is slightly more involved. First of all, there is the contribution from individual nucleons, as shown in Fig. 2a. The contribution from the interference in Fig. 2b is crucial to understanding the small anomalous magnetic moment of the deuteron. The spin-orbit type relativistic corrections is needed to obtain the correct low-energy theorem.

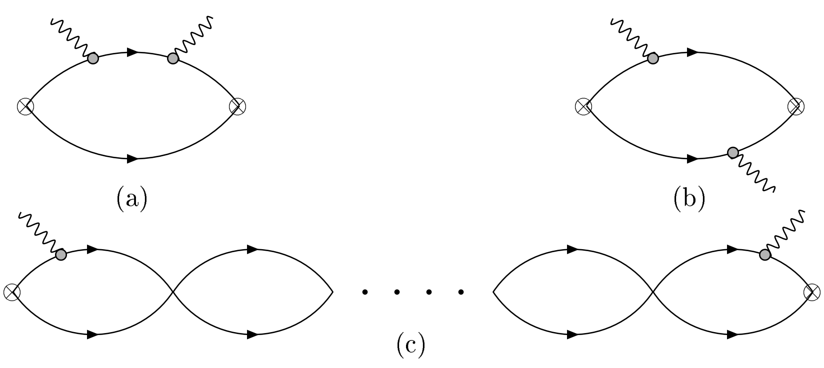

Figure 2. Feynman diagrams for spin-dependent Compton scattering on the deuteron.

The deuteron has a magnetic moment $0.8574\mu_N$. A point-like deuteron would have a magnetic moment close to μ_N. Therefore, the anomalous part is $0.143\mu_N$, which contributes to the DHG integral $I = 0.65\mu b$. On the other hand, for a loosely-bound deuteron, its magnetic moment is

$$\mu_p + \mu_n = 0.880\mu_N \ . \tag{11}$$

Therefore the small anomalous magnetic moment does not reflect that the deuteron is close to a point-like particle. Rather it reflects a delicate cancellation of the physical effects at different scales, accurate to 0.1 to 0.2%!

How is the deuteron's DHG sum rule saturated? Arenhövel has made an estimate of various contributions to the integral up to 900 MeV for photodisintegration and 1.5 GeV for meson production [4]. According to him, the photodisintegration $\gamma + d \to p + n$ contribution is about $-383\mu b$; the coherent pion production $\gamma + d \to d + \pi$ about $99\mu b$; the quasielastic pion production contribution about $200\mu b$; the two-pion production about $82\mu b$, and the eta production about $-12\mu b$. The total sum is about $-13.7\mu b$. Although this is still more than an order-of-magnitude larger than the left-hand side, the strong cancellation is manifest in this estimate.

4. Nucleon Compton scattering and polarizabilities

Real Compton scattering on the nucleon can be described by nucleon polarizabilities, such as the electric polarizability α, the magnetic polarizability β, and various spin polarizabilities γ_i. Besides direct extractions of these polarizabilities from Compton cross sections, they can also be determined from photoabsorption data through dispersive sum rules. Sometimes the latter is the only way to obtain these quantities.

What can we learn about the nucleon physics from the polarizabilities? The nucleon structure in the low-energy processes is dominated by chiral dynamics. Indeed, because of the small pion mass, the pion cloud physics is separated from the physics at the hadron mass scale. One can calculate the non-analytic dependence on the pion mass. Moreover, the Δ resonance plays an important role, from which, one can learn some important features of the large N_c expansion.

The electric polarizability α measures the deformation of the nucleon in the presence of a static external field. It is related to the dipole excitation strength,

$$\alpha = 2\alpha_{\rm em} \sum_{n\neq 0} \frac{|\langle n|d_z|0\rangle|^2}{E_n - E_0} \ . \tag{12}$$

The dominant contribution comes from the p-wave pion-nucleon scattering states,

$$\alpha = \frac{5\alpha_{\rm em} g_A^2}{6(4\pi f_\pi)^2 m_\pi} = 13.6 \times 10^{-4}\ {\rm fm}^4\ . \tag{13}$$

Other theoretical calculations can be found in a recent review article [5].

The forward spin polarizability can be determined by GDH-like dispersion relation: $\gamma = -1.01 \pm 0.08 \pm 0.10 \times 10^{-4}\ {\rm fm}^4$. It goes like $1/m_\pi^2$ at leading order [6]. The complete order $1/m_\pi$ contribution has been obtained by Kao et al., and others [7]. Partial contribution at $\mathcal{O}(\ln m_\pi)$ has been obtained by Bernard et. al [8]. There is a question of convergence of the perturbation expansion. In fact, the first few orders appear as

$$\gamma^p = 4.5 - 8.3 + 6.0\ ({\rm partial}) + O\,(\Delta\ {\rm contribtuion})\ . \tag{14}$$

There is a large contribution from the Δ resonance which is analytic in pion mass. A similar problem exists for the magnetic polarizability β.

5. Deuteron photoproduction and polarizabilities

Denote the deuteron photoproduction cross section as $\sigma^{(m)}$ when the photon has helicity $+1$ and the deuteron target has polarization m. The scalar, tensor, and vector polarizabilities of the deuteron are related to these cross sections through the following dispersive sum rules,

$$\alpha_{E0} + \beta_{M0} = \frac{1}{6\pi^2}\int_0^\infty d\omega' \frac{\sigma^{(1)} + \sigma^{(0)} + \sigma^{(-1)}}{\omega'^2}\ , \tag{15}$$

$$\alpha_{E2} + \beta_{M2} = \frac{1}{4\pi^2}\int_0^\infty d\omega' \frac{\sigma^{(1)} + \sigma^{(-1)} - 2\sigma^{(0)}}{\omega'^2}\ , \tag{16}$$

$$\gamma = -\frac{1}{8\pi^2}\int_0^\infty d\omega' \frac{\sigma^{(1)} - \sigma^{(-1)}}{\omega'^3}\ , \tag{17}$$

where γ is a sum of four different spin polarizabilities, related to Faraday rotation and optical activity.

The above polarizabilities can in principle be determined through elastic scattering processes. One way is to scatter the deuteron off a heavy atom in a static Coulomb field. For example, Rodning et al. determined the scalar electric polarizability this way [9]. The second is direct Compton scattering. However, because the deuteron binding energy is about 2 MeV, one needs a beam of photons with energy much less than that to measure the polarizabilities. Finally, one can use the above dispersive sum rules by integrating over the relevant photoproduction data, just as in the case of the DHG sum rule. Because of the energy weighting, the integral is dominated by the coherent deuteron physics, and one expects much better convergence here.

To understand the size of the deuteron polarizabilities, one needs a theory to describe the deuteron structure physics. One such theory developed in the past several years is a nuclear effective theory by Kaplan, Savage, and Wise [10]. In this theory, there are low-energy scales (generically denoted as Q) determined by the binding energy of the deuteron. Because we are dealing with a nonrelativistic system, the internal momentum scale $\gamma = \sqrt{BM}$ is actually large. The high-energy scales involved (denoted as Λ) include the pion mass, the parameters involved in the nucleon-nucleon interactions such as the inverse of the effective range parameter r, and of course, the nucleon mass. In an effective field theory expansion, the two scales are assumed to be well separated, i.e., $Q/\Lambda \ll 1$. This is certainly true in the limit of loose binding. The physical observables then depend mostly on the long-range tail of the deuteron wave function, and can be calculated as Taylor expansions in powers of Q/Λ.

Let us consider the scalar polarizability α_{E0}. This is a term in the Compton amplitude proportional to ω^2. Since the Compton amplitude itself is of order 1, the leading term in α_{E0} goes like $\alpha_{\rm em}M/\gamma^4$ by power counting. This quantity has been calculated up to N^{3}LO [11]

$$\alpha_{E0} = \frac{\alpha_{\rm em} M_N}{32\gamma^4} Z_d \left[1 + \frac{2\gamma^2}{3M_N^2} + \frac{M_N\gamma^3}{3\pi} D_P\right] \ , \tag{18}$$

where $Z_d = 1/(1-\gamma\rho_d) = 1.69$ is the deuteron wave function renormalization; $\rho_d = 1.764$ fm; $D_p = -1.51$ fm^3. Numerically $\alpha_{E0} = 0.6339$ fm^3. Modern potential model calculation yields 0.6328 ± 0.0017 fm^3.

The deuteron has a small admixture of D wave which comes from one pion exchange in the nucleon-nucleon force. In effective field theory, this can be generated through a local interaction coupling the two channels. The interaction strength can be determined by D/S ratio $\eta_{sd} = 0.0254$. The D-wave supports a tensor polarizability, which appears as a coefficient of an ω^2 term in the tensor amplitude. Therefore, it has the same leading power as α_{E0}. A straightforward calculation yields [12]

$$\alpha_{E2} = -\frac{3\sqrt{2}\alpha_{\rm em} M}{32\gamma^4} Z_d \eta_{sd} = -2.7\eta_{sd} \ \ {\rm fm}^3 \ . \tag{19}$$

Finally, let us consider the vector polarizability γ which is the coefficient of the ω^3 (more precisely $q^2\omega$) term in the spin-dependent Compton amplitude. Thus the vector polarizability has the same Q counting as the scalar one. However, since the intermediate state must be an isovector state, there is a large enhancement from the isovector magnetic moment. The leading order result is [12]

$$\gamma^{\rm LO} = \frac{\alpha_{\rm em}(\mu^{(1)})^2}{16\gamma^4}\left[1+\frac{M_N\gamma}{2\pi}\mathcal{A}_{-1}^{(^1S_0)}(-B)\left(1+\frac{M_N\gamma}{4\pi}\mathcal{A}_{-1}^{(^1S_0)}(-B)\right)\right] + \frac{\alpha_{\rm em}(4\mu^{(1)}-1)}{128\gamma^4}\ , \quad (20)$$

where $\mathcal{A}$ is the scattering amplitude. The numerical value of γ at this order is 3.762 fm^4. The NLO correction is small after taking into account the wave function renormalization Z_d.

6. Virtual Compton scattering on the nucleon and related sum rules

How do we extend the above sum rules away from the real photon point, $Q^2 = 0$? The starting point is the virtual-photon forward scattering amplitude. We denote the incoming and outgoing photon polarization indices α and β, space-like virtual mass Q^2 and energy ν, the nucleon momentum P^μ and helicity h. The forward Compton tensor is defined as

$$T^{\alpha\beta}(h) = i\int e^{iq\cdot\xi}d^4\xi\langle Ph|\mathrm{T}J^\alpha(\xi)J^\beta(0)|Ph\rangle\ . \quad (21)$$

We are mainly interested in the forward scattering amplitude $S_1(Q^2,\nu)$ which is the difference between scattering of a photon of helicity $+1$ off the nucleon target with helicities $\pm 1/2$. From general principles (causality and unitarity) as well as the assumption about the large-ν behavior of $S_1(Q^2,\nu)$, we can write down a dispersion relation,

$$S_1(\nu,Q^2) = 4\int_{Q^2/2M}^{\infty}\frac{\nu' d\nu' G_1(\nu',Q^2)}{\nu'^2-\nu^2}\ , \quad (22)$$

where $G_1(\nu',Q^2)$ is the spin-dependent nucleon structure function entering the deep-inelastic scattering cross section.

While $G_1(\nu,Q^2)$ is difficult to calculate, it can be measured experimentally. On the other hand, $S_1(\nu,Q^2)$ is hard to measure experimentally, but it can be calculated theoretically. The dispersion relation provides the missing link to test theoretical ideas against experimental data. If expanding at small ν,

$$S_1(\nu,Q^2) = \sum_{n=0,2,4,\ldots}\nu^n S_1^{(n)}(Q^2) \quad (23)$$

where $S_1^{(n)}(Q^2)$ may be regarded as Q^2-dependent forward polarizabilities. The above dispersion relation becomes [13],

$$S_1^{(n)} = 4\int_{Q^2/2M}^{\infty} \frac{d\nu}{\nu^{n+1}} G_1(\nu, Q^2) \qquad (n = 0, 2, 4, ...) \tag{24}$$

Let us consider the $n = 0$ case. As $Q^2 \to 0$, the low energy theorem, $\bar{S}_1(0,0) \to \kappa^2$, where $\bar{S}_1 = S_1 - S_1^{\rm el}$ with the elastic intermediate state removed and κ is the anomalous magnetic moment of the nucleon. At small but finite Q^2, chiral perturbation theory provides a sound theoretical method to calculate corrections to the low-energy theorem. Kao, Osborne and I have done such a calculation to fourth order in chiral perturbation theory, and we found [14]

$$\begin{aligned} \bar{S}_1(0, Q^2) = & -\frac{\kappa^2}{M^2} + \frac{g_A^2}{12(4\pi f_\pi)^2 M m_\pi}(1 + 3\kappa_V + 2(1 + 3\kappa_S)\tau^3)Q^2 + \cdots \\ & \sim -\kappa^2 + 24Q^2(\mathrm{GeV}^2) + \cdots \ . \end{aligned} \tag{25}$$

The result shows a rapid Q^2-dependence near $Q^2 \sim 0$ and expansion in Q^2 may be convergent for $Q^2 < 0.1$ GeV^2. The chiral perturbation theory prediction for $\bar{S}_1(0, Q^2)$ combining with the dispersion relation yields the generalized DHG sum rule which can be tested at Jefferson Lab [15].

As $Q^2 \to \infty$, quarks inside the nucleon appear to be free and the single quark scattering process dominates the Compton amplitude. In particular, the current algebra method, originally motivated from the free quark model, can be applied. Indeed, Bjorken found [18]

$$S_1(Q^2, 0) \to \frac{1}{Q^2}\sum_i e_i^2 \langle P|A_i^\mu|P\rangle \ . \tag{26}$$

When plugged into the dispersion relation, the above result for the Compton amplitude translates immediately into the Bjorken sum rule. How do we extend this sum rule to large but finite Q^2? Perturbative QCD introduces two types of corrections. The first is the radiative corrections: gluons are radiated and absorbed by active quarks, etc. The second type is the higher twist corrections in which more than one parton from the target participate in scattering. With these corrections, we modify Bjorken's prediction for the Compton amplitude,

$$
\begin{aligned}
Q^2 S_1(0,Q^2) \sim & \left(1-\frac{\alpha_s}{\pi}-3.58\left(\frac{\alpha_s}{\pi}\right)^2-\cdots\right)\left(\pm\frac{g_A}{12}+\frac{g_8}{36}\right) \\
& +\left(1-0.33\frac{\alpha_s}{\pi}-0.55\left(\frac{\alpha_s}{\pi}\right)^2-\cdots\right)\frac{g_0}{8} \\
& +\frac{\mu_4(Q^2)}{Q^2}+\cdots\ , \qquad (27)
\end{aligned}
$$

where α_s is a strong interaction coupling constant evaluated at the scale Q^2; $\mu_4 \sim \langle PS|\bar\psi\tilde F^{\mu\nu}\gamma_\nu\psi|PS\rangle \sim (0.4)^2$ GeV^2 is a twist-four matrix element. Since the scale that controls the twist expansion is on the order of $0.1-0.2$ GeV^2, we believe the pQCD prediction for $S_1(0,Q^2)$ is good down to $Q^2 \sim 0.5$ GeV^2. Substituting this into the dispersion relation, we get the *generalized* Bjorken sum rule. Since it is difficult to do an experiment at $Q^2=\infty$, it is the generalized Bjorken sum rule at finite Q^2 that is commonly tested experimentally. In the region $Q^2\sim 2-10$ GeV^2, the generalized Bjorken sum rule has been checked at the level of 10% accuracy. At moderate Q^2, the resonance contribution must be averaged to the parton contribution because of the small scale of μ_4. This is what's called parton-hadron duality in the literature.

The best way to see the connection of the low and high Q^2 generalized sum rules is to consider the Q^2 dependence of

$$
\begin{aligned}
\Gamma(Q^2) &\equiv \frac{Q^2}{8}S_1(0,Q^2) \\
&= \frac{Q^2}{8}\bar S_1(0,Q^2)+\frac{1}{2}F_1(Q^2)(F_1(Q^2)+F_2(Q^2))\ , \qquad (28)
\end{aligned}
$$

where the second term is the elastic contribution which dominates at low Q^2[13]. $\Gamma(Q^2)$ starts with $1(0)+\kappa/2$ from the proton (neutron) at $Q^2=0$ and rapidly decreases to about 0.2 at Q^2=0.7 GeV^2 and remains essentially flat as $Q^2\to\infty$. [In this definition, the DHG sum rule result affects only the slope of $\Gamma(Q^2)$ at the origin.] There is a nice and simple physical interpretation for this Q^2 variation. The forward Compton amplitude is an amplitude for photon scattering off a nucleon target and remaining in the forward direction. This is very much like a diffraction process and $\Gamma(Q^2)$ is the "brightness" of the diffraction center. For low Q^2 photons, scattering from the different parts of the proton is coherent, and the scattered photons produce a large diffraction peak at the center. As Q^2 becomes larger, the photon sees some large scale fluctuations in the nucleon; the scattering becomes less coherent. The large scale fluctuation can largely be understood in terms of the dissociation of the nucleon into virtual hadrons. When $Q^2 > 0.5$ GeV^2, the photons see parton fluctuations at the scale of $1/Q$. As $Q^2 \to \infty$, photons see individual quarks inside the nucleon and the scattering is completely incoherent. The diffraction peak is just the sum of diffractions from individual quarks. In short, the Q^2 variation of the sum rules just reflects the change of the diffraction amplitude of the virtual photons as the virtual mass is varied.

There has been interesting progress recently in calculating the Q^2-dependence of the forward spin polarizability γ_0 and δ_{LT} and the d_2 matrix element [16,17]. I will not discuss them here in detail but refer you to the relevant talks at this conference.

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