

## DETERMINATION OF THE LOW $Q^2$ EVOLUTION OF THE BJORKEN INTEGRAL

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We report on an experimental determination of the  $Q^2$ -dependence of the Bjorken sum using data from Jefferson Lab Hall A and Hall B in the range  $0.16 < Q^2 < 1.1$  GeV<sup>2</sup>. A twist analysis is performed. Overall, the higher twist corrections are found to be small due to a cancellation between the twist 4 and 6 terms.

### 1. The GDH and Bjorken Sum Rules

A main reason to study the generalized GDH sum is to understand the transition from the hadronic to the partonic descriptions of the nucleon. The generalized GDH sum is in principle calculable at any  $Q^2$ , which makes it an ideal tool to study the hadronic to partonic transition. This topic was covered in the symposium<sup>1</sup>. However, the validity domains of the available chiral perturbation theory ( $\chi$ PT) and pQCD calculations do not overlap. Lattice QCD should provide the bridge between the two domains but no calculation is available yet.

The Bjorken sum rule<sup>2</sup> has been a cornerstone of polarized pQCD studies. At leading twist, it reads:

$$\int_0^{1^-} (g_1^p - g_1^n) dx = \frac{g_a}{6} \left[ 1 - \frac{\alpha_s}{\pi} - 3.58 \left( \frac{\alpha_s}{\pi} \right)^2 - 20.21 \left( \frac{\alpha_s}{\pi} \right)^3 + \dots \right] \quad (1)$$

where  $g_a$  is the nucleon axial charge. The connection between a generalized GDH sum and the Bjorken sum was made by M. Anselmino *et al.*<sup>3</sup>. More recently, X. Ji and J. Osborne made the reason for the connection clear: the GDH and the Bjorken sum rules are two particular cases of a more general sum rule<sup>4</sup>. The extended GDH sum rule, as generalized in Ref.<sup>4</sup>, links the first moment of the spin structure functions to the spin dependent forward Compton scattering amplitudes. Hence, the relation between the

generalized GDH and Bjorken sums is:

$$\int_0^1 g_1^p - g_1^n dx = \frac{Q^2}{16\pi^2\alpha} (\text{GDH}^p(Q^2) - \text{GDH}^n(Q^2)) \quad (2)$$

Considering this  $p - n$  flavor non-singlet quantity yields many advantages.

- (1) At large  $Q^2$ , we have a sum rule (the Bjorken sum rule) without hypothesis beyond QCD (as opposed to the Ellis-Jaffe sum rule<sup>5</sup> for singlet quantities).
- (2) The estimations of the unmeasured low- $x$  part of the integral are more reliable.
- (3) The pQCD evolution equations are simpler.
- (4) At low  $Q^2$ , the  $\chi$ PT calculations are more reliable due to the cancellation of the  $\Delta_{1232}$  contribution.

These advantages might help in extending the validity domains of pQCD and  $\chi$ PT. It is conceivable that the hadron-parton gap might be bridged, allowing one to fundamentally describe the nucleon structure at all scales<sup>6</sup> for the first time. Hence the Bjorken sum is arguably one of the most convenient quantities to measure in the resonance region to understand the hadron-parton transition.

Precise data are available from the Thomas Jefferson National accelerator facility (JLab). Results were published on the proton<sup>7</sup> and deuteron<sup>8</sup> from CLAS in Hall B and on the neutron extracted from polarized <sup>3</sup>He from Hall A<sup>9,10</sup>. We used these data to extract the Bjorken sum from  $Q^2 = 0.16$  to  $1.1 \text{ GeV}^2$ . To combine proton and neutron data, the <sup>3</sup>He data were reanalyzed at the same  $Q^2$  as those of Ref.<sup>7</sup>. For consistency, the unmeasured low- $x$  part of the integral was re-evaluated for the three data sets using a consistent prescription<sup>11</sup>. The part beyond the validity range of Ref.<sup>11</sup> was estimated using a Regge form and forcing the total integral as measured by the SLAC E155 experiment<sup>12</sup> at  $Q^2=5 \text{ GeV}^2$  and completed by the estimation<sup>11</sup> and our Regge form, to verify the Bjorken sum rule at  $Q^2=5 \text{ GeV}^2$ . The results are shown on Figure 1. The elastic contribution is not included. The negative horizontal bands give the systematic uncertainties on the two data sets. Also plotted are the SLAC E143 results in the resonance region<sup>13</sup>. Two  $\chi$ PT calculations, from Bernard *et al.*<sup>14</sup> and Ji *et al.*<sup>15</sup>, are shown at low  $Q^2$ . At  $Q^2 = 0$ , the Bjorken sum is constrained by the GDH sum rule (see eq. 2). At higher  $Q^2$  the leading twist calculation up to third order in  $\alpha_s$  is shown by the gray band. Its width is due to the

uncertainty on  $\alpha_s$ . The model of Soffer and Teryaev<sup>16</sup> overestimates the data at large  $Q^2$ . An improved version accounting for the pQCD radiations was presented<sup>17</sup> and seems to agree better with the data. The calculation from Burkert and Ioffe<sup>18</sup> agrees well with the data.

In the particular case of the  $\chi$ PT calculation done in the heavy Baryon approximation<sup>15</sup>, the comparison with the data may indicate that the  $\chi$ PT domain of validity is indeed extended since they agree, up to about  $Q^2 = 0.25 \text{ GeV}^2$  (to be compared to  $Q^2 = 0.1 \text{ GeV}^2$  typically for singlet quantities). However, the calculations from Bernard *et al.*<sup>14</sup>, that do not employ the heavy Baryon approximation, do not support this conclusion. In any case, a gap between the  $\chi$ PT calculations and the pQCD calculation clearly remains. More details can be found in Ref. <sup>19</sup>

## 2. Twist Analysis

It is remarkable that the data agree with the leading twist calculation down to quite low  $Q^2$ . This may indicate that higher twist terms are small or cancel each other. To quantitatively address this, we performed a twist analysis. The coefficient  $\mu_4^{p-n}$  of the  $1/Q^2$  correction to Eq. 1 is:

$$\mu_4^{p-n} = \frac{M^2}{9} (a_2^{p-n} + 4d_2^{p-n} + 4f_2^{p-n}), \quad (3)$$

where  $a_2^{p-n}$  is the target mass correction given by the second moment of  $g_1^{p-n}$ , and  $d_2^{p-n}$  is a twist-3 matrix element given by

$$d_2^{p-n} = \int_0^1 dx x^2 (2g_1^{p-n} + 3g_2^{p-n}). \quad (4)$$

The term  $a_2^{p-n}$  is computed using our data and  $d_2^{p-n}$  is obtained from SLAC and JLab data<sup>20</sup>. The twist 4 term  $f_2^{p-n}$  is extracted from a fit of the Bjorken sum including the elastic contribution. After re-estimating the low-x part of the world data using our method, we fit our data together with the world data in the 0.66-15  $\text{GeV}^2$   $Q^2$ -range. To verify that the twist series is convergent, the next twist term  $\mu_6^{p-n}$  was included. We obtain, for  $Q^2 = 1 \text{ GeV}^2$ ,  $f_2^{p-n} = -0.17 \pm 0.05(\text{uncor})_{-0.05}^{+0.04}(\text{cor})$  and  $\mu_6^{p-n} = 0.09 \pm 0.02(\text{uncor}) \pm 0.01(\text{cor}) \text{ GeV}^4$  where *uncor* (*cor*) specifies the uncertainties due to the point to point uncorrelated (*correlated*) uncertainty on the JLab data. Comparing  $\mu_6^{p-n}/Q^4 = 0.09 \pm 0.02$  to  $\mu_4^{p-n}/Q^2 \simeq -0.06 \pm 0.02$ , we find that the  $Q^{-2}$  and  $Q^{-4}$  terms have opposite sign and similar magnitude, making the overall twist correction small at  $Q^2 = 1 \text{ GeV}^2$ . This result may

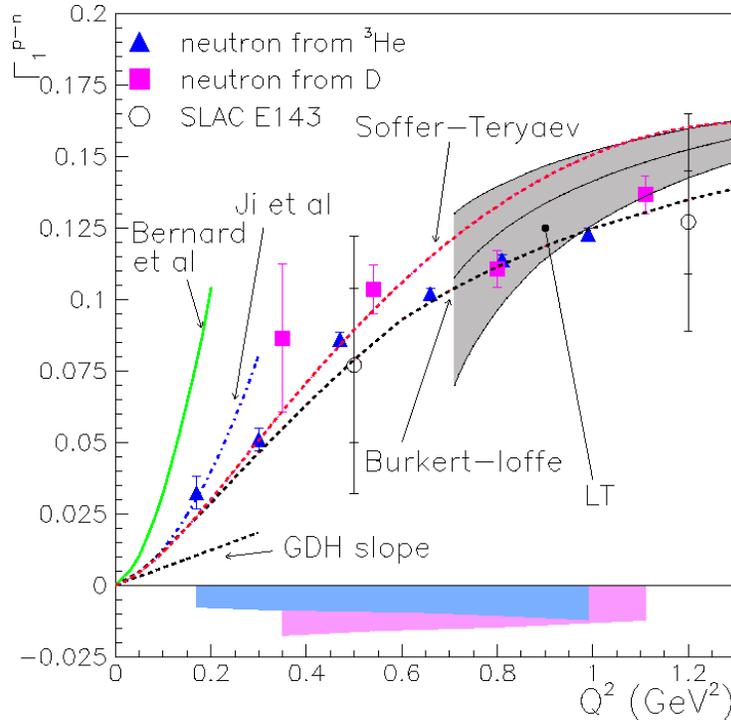


Figure 1.  $Q^2$ -evolution of the Bjorken sum. The Bjorken sum formed using neutron data extracted from  $^3\text{He}$  (D) data is shown by the triangles (squares) and the horizontal band spanning from  $Q^2 = 0.16$  to  $1.0 \text{ GeV}^2$  (from  $Q^2 = 0.35$  to  $1.1 \text{ GeV}^2$ ) is the corresponding systematic uncertainty. The resonance data from SLAC E143 are also shown (circles). The pQCD calculation at leading twist is represented by the gray band. Two  $\chi$ PT calculations (Ji *et al.* and Bernard *et al.*) can be seen at low  $Q^2$  as well as the GDH slope which constrains the Bjorken sum near the photon point. The dashed curves are the predictions of two phenomenological models (Burkert and Ioffe, bottom curve, and Soffer and Teryaev, top curve).

explain why many experimental JLab results tend to indicate that pQCD works down to surprisingly low  $Q^2$ .

### 3. Summary and outlook

We have extracted the Bjorken sum in the  $Q^2$  range of  $0.16$ - $1.1 \text{ GeV}^2$ . Compared to singlet quantities, the  $\chi$ PT calculation seems to agree with the data over a larger  $Q^2$  range only in the case of the Heavy Baryon approximation. Such possible improvement is not seen with the calculations of Ref.<sup>14</sup>. This last point is not unlike the conclusion reached in Ref.<sup>21</sup> where

$\chi$ PT calculations were compared to measurements of the generalized spin polarizability  $\delta_{LT}$ , a quantity in which the  $\Delta$  degrees of freedom are also suppressed. At any rate the parton to hadron gap, if smaller, is not bridged yet. The magnitudes of the higher twists were extracted, in particular  $f_2^{p-n}$ . The higher twist effects appear to be small, due to a cancellation of the  $1/Q^2$  and  $1/Q^4$  terms. The analysis of new proton and deuteron data from CLAS will be finalized soon. These cover a larger  $Q^2$  range (0.05 to 4 GeV<sup>2</sup>) with improved statistics<sup>22</sup>. Data at even lower  $Q^2$  are expected to be available in the upcoming years from both Jefferson Lab Hall A (<sup>3</sup>He and neutron<sup>23</sup>) and Hall B (proton)<sup>24</sup>.

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