

SPIN ASYMMETRY AND GDH SUM RULE FOR REAL AND VIRTUAL PHOTONS FOR THE DEUTERON*

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An explicit evaluation of the spin asymmetry of the deuteron and the associated GDH sum rule is presented which includes disintegration, single and double pion and eta production. For the GDH integral a large cancellation is found between the disintegration channel and the meson production channels. Furthermore, first results for the contribution of the disintegration channel to the generalized GDH integral at constant four-momentum transfer reveal a dominance of the isovector M1 transition to the 1S_0 -state near threshold resulting in a negative contribution with a minimum around $Q^2 \approx 0.2 \text{ fm}^{-2}$ which is driven by the nucleon anomalous isovector magnetic moment.

1. Introduction

The GERASIMOV-DRELL-HEARN sum rule links the anomalous magnetic moment κ of a particle to the integral over the energy weighted spin asymmetry of the absorption cross section with respect to circularly polarized photons and a polarized target

$$I^{GDH} = \int_0^\infty \frac{d\omega}{\omega} \left(\sigma^P(\omega) - \sigma^A(\omega) \right) = 4 \pi^2 \kappa^2 \frac{e^2}{M^2} S, \quad (1)$$

where S denotes the spin of the particle and M its mass. Obviously, for $\kappa \neq 0$ the particle possesses an internal structure. However, the opposite is not in general true. A particle having a vanishing or very small κ need not be pointlike or nearly pointlike. The deuteron is an instructive example for such a case.

2. GDH Sum Rule for the Deuteron¹

The deuteron is an isoscalar with a very small $\kappa_d = -0.143 \text{ n.m.}$ and thus a very small sum rule value $I_d^{GDH} = 0.65 \mu\text{b.}$ On the other hand it is well known that the deuteron has quite an extended spatial structure due to its

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small binding energy. In fact, the small κ arises from an almost complete cancellation of proton and neutron anomalous magnetic moments in the deuteron because of their parallel spin orientation. Thus it is expected that also for the sum rule integral such a cancellation occurs. As absorptive processes one distinguishes (i) photodisintegration $\gamma + d \rightarrow n + p$ and (ii) meson production. The latter process is certainly dominated by quasi-free production on the nucleons, and a simple estimate of its contribution to the sum rule is obtained by summing incoherently neutron and proton GDH contributions $I_p^{GDH} + I_n^{GDH} = 438 \mu\text{b}$, neglecting interference and other binding effects. In view of the tiny deuteron GDH value a negative contribution of almost equal size has to come from the photodisintegration channel.

Indeed, near threshold the spin asymmetry of photodisintegration is dominated by the isovector M1 transition to the 1S_0 -state. Since it can only be reached for antiparallel photon and deuteron spins, a negative spin asymmetry arises which is quite huge as is shown in Fig. 1. Besides an earlier evaluation² with static interactions and exchange currents (MEC), isobar configurations (IC) in impulse approximation (IA) and relativistic contributions (RC), we show in addition the result of a recent calculation³ based on a retarded potential with retarded π -MEC, a coupled channel $N\Delta$ -dynamics, and RC. It leads to a significant change of the spin asymmetry and thus its GDH contribution.

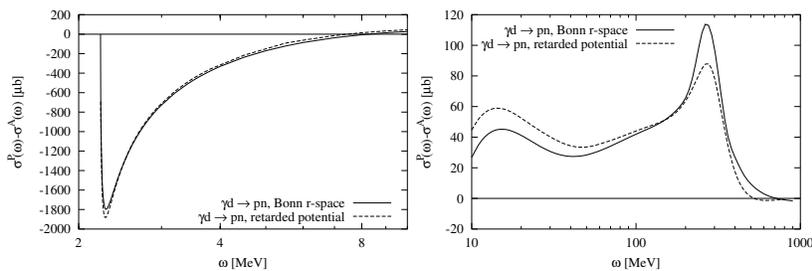


Figure 1. Spin asymmetry of deuteron photodisintegration using (a) Bonn r-space potential⁴+MEC+IC(IA)+RC and (b) retarded potential + retarded π -MEC, Δ -degrees in coupled channel, πd -channel + RC³. Left panel: low energy; right panel: high energy.

An improved calculation of the spin asymmetries of single pion and eta production on the deuteron has been performed recently in which for the elementary production operator the MAID model⁵ has been used and final state interactions (FSI) are included completely in the NN- and π N-subsystems. The results are shown in Fig. 2. For charged pion production

the FSI effects are small, but quite sizeable for incoherent neutral pion production due to the non-orthogonality of the final state in impulse approximation. For comparison, the corresponding spin asymmetries for pion

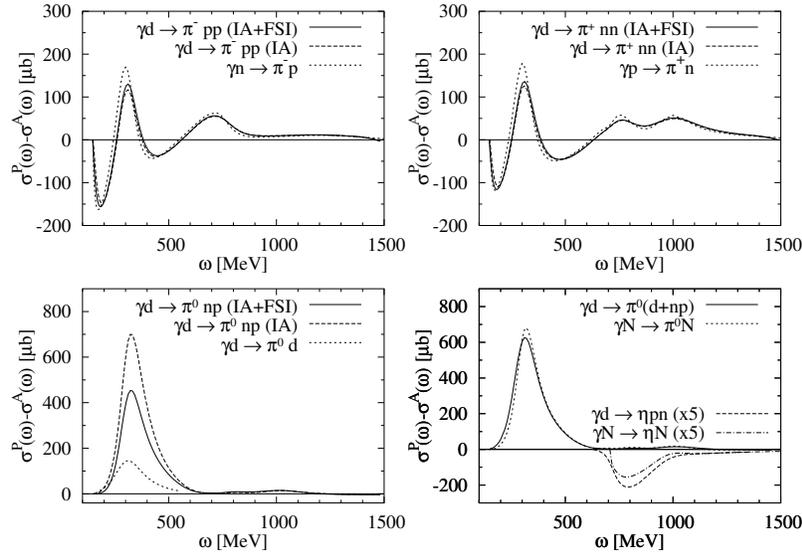


Figure 2. Spin asymmetries of single pion and eta production on nucleon and deuteron for various charge channels. The results for the deuteron are in IA and with inclusion of FSI in final NN- and πN -subsystems. Upper panels: charged single pion production; lower left panel: coherent and incoherent π^0 -production on deuteron; lower right panel: total π^0 - and η -production on nucleon and deuteron. For π^0 - and η -production on the nucleon the sum of spin asymmetries of neutron and proton is shown. The result for coherent π^0 -production on the deuteron is taken from Ref.².

and eta production on the nucleon are also shown in Fig. 2. Significant differences between the spin asymmetries of nucleon and deuteron are readily seen which certainly prevent one to extract in a simple manner the neutron spin asymmetries from deuteron data.

Table 1. Contributions of single π -production on nucleon and deuteron to finite GDH integral up to 1.5 GeV in μb .

nucleon	$\pi^0 p$	$\pi^0 n$	$\pi^0(n+p)$	$\pi^- p$	$\pi^+ n$
	159.10	147.34	306.44	-8.39	17.28
deuteron	$\pi^0 d$	$\pi^0 np$	$\pi^0(d+np)$	$\pi^- pp$	$\pi^+ nn$
	63.26	216.61	279.87	-18.94	2.51

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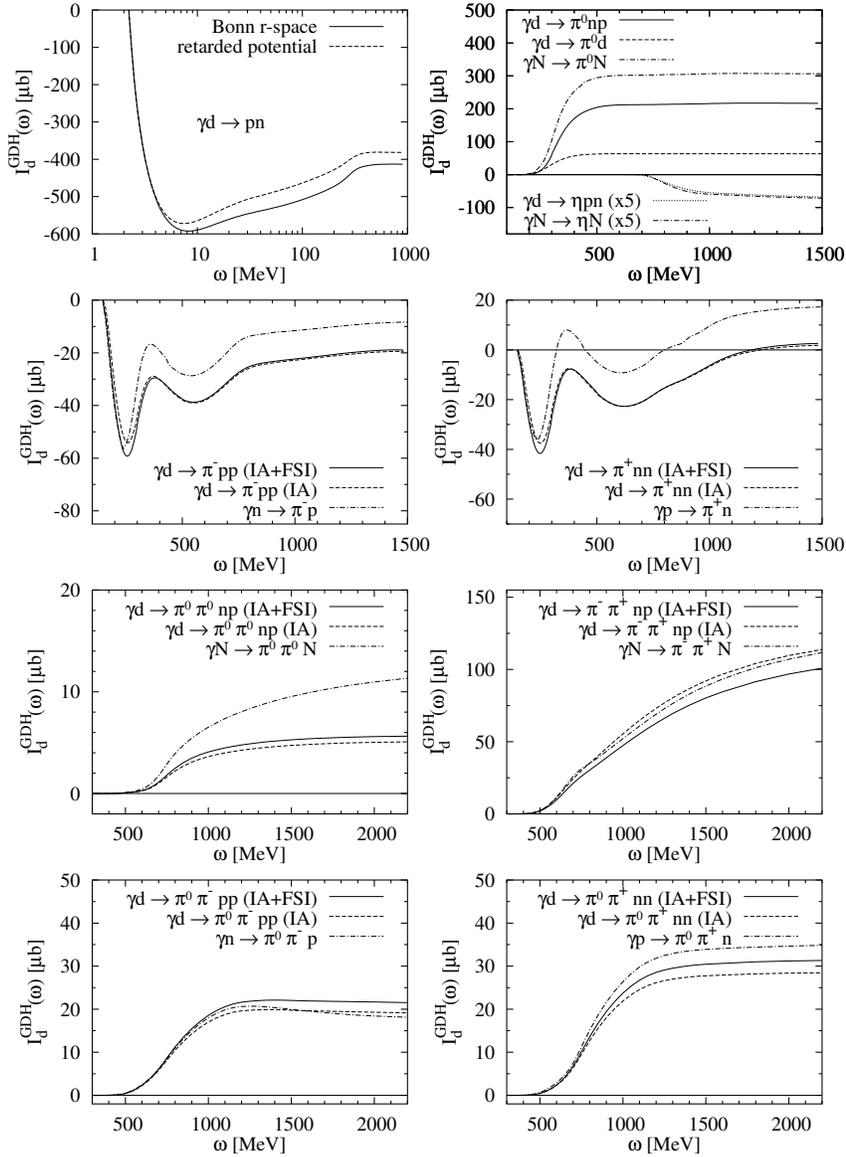


Figure 3. Contribution of various channels to the finite GDH integral as function of the upper integration limit for deuteron disintegration, single and double pion production on nucleon and deuteron. For the neutral charge channels π^0 , η , $\pi^0\pi^0$, and $\pi^-\pi^+$, the nucleon integrals are the sum of neutron and proton integrals.

Now we will turn to the explicit evaluation of the finite GDH integral

as defined by

$$I^{GDH}(\omega) = \int_0^\omega \frac{d\omega'}{\omega'} (\sigma^P(\omega') - \sigma^A(\omega')). \quad (2)$$

The results for photodisintegration, single and double pion and eta production with and without FSI are exhibited in Fig. 3 together with the corresponding nucleon values. The two-pion results are based on a recent evaluation using an effective Lagrangean approach analogous to Ref.⁶. It is obvious that convergence of the GDH integral is reached for $\gamma d \rightarrow np$ already at about 0.8 GeV and for neutral pion production at 1.5 GeV, however, not completely for charged π -production. Also two-pion production has not reached convergence at 2.2 GeV, in particular for $\pi^- \pi^+$ production. One furthermore notes a sizeable reduction of the np contribution from retardation in potential and π -MEC. For comparison, the finite GDH integrals for meson production on the nucleon are also shown in Fig. 3 where one sees quite clearly significant differences between nucleon and deuteron results because of (i) Fermi motion and (ii) final state interactions. The results for the GDH integrals for single pion production on nucleon and deuteron, are listed in Table 1. Also here one clearly notes significant differences between the nucleon and deuteron values.

Table 2. Contributions of various channels to the finite GDH integral (in μb), integrated up to 0.8 GeV for $\gamma d \rightarrow np$, 1.5 GeV for single pion and eta production and 2.2 GeV for double pion production on nucleon and deuteron.

	np	π	$\pi\pi$	η	Σ	sum rule
n+p		315.33	175.95	-14.54	476.74	437.94
d	-381.52	263.44	159.34	-13.95	27.31	0.65

The GDH contributions of various channels to nucleon (sum of neutron and proton) and deuteron are listed in Table 2. The deuteron results include NN-FSI. Compared to our earlier evaluation one now finds indeed for the deuteron a large cancellation between the various channels contributing to the GDH sum rule. This strong cancellation between the regions at low and high energies is a fascinating feature clearly demonstrating the decisive role of the pion as a manifestation of chiral symmetry governing strong interaction dynamics in these two different energy regions. The cancellation constitutes also a challenge for any theoretical framework since it requires a unified consistent description of hadron and e.m. properties for both energy regions.

At the end of this section we would like to recall our concern with respect to attempts to extract the neutron spin asymmetry from the one of the deuteron by subtracting the proton asymmetry. We will not repeat the arguments which had been put forward by one of us⁷. We only would like to emphasize that these arguments are confirmed by the results in Fig. 3 and in Table 2.

We only would like to add the following consideration: Assuming the validity of the sum rule and assuming that indeed the contribution from meson production on the deuteron would result in the sum of neutron and proton GDH sume rules (438 μb) than it would mean that the contribution of the photodisintegration channel has to be almost equal the negative value of this sum except for the deuteron sum rule value, i.e. $-(437.94 - 0.65) \mu\text{b}$. But the best theory at present yields only a converged value of $-381.52 \mu\text{b}$ for this channel.

3. Generalized GDH Sum Rule for Virtual Photons⁸

The spin asymmetry for real photons corresponds to the beam-target vector asymmetry A_{ed}^V of the general inclusive cross section of deuteron electrodisintegration⁹

$$\sigma(h, P_1^d, P_2^d) = \sigma_0 (1 + P_1^d A_d^V + P_2^d A_d^T + h [A_e + P_1^d A_{ed}^V + P_2^d A_{ed}^T]), \quad (3)$$

where σ_0 denotes the unpolarized cross section, h the electron polarization, and P_1^d and P_2^d deuteron vector and tensor polarization, respectively. The various asymmetries depend on the deuteron orientation angles θ_d and ϕ_d . For deuteron orientation parallel to the momentum transfer \vec{q} the asymmetry A_{ed}^V is determined by the transverse form factor $F_T'^{10}$ which in turn at the photon point is related to the spin asymmetry for real photons⁹

$$\sigma_\gamma^P(\omega^{lab}) - \sigma_\gamma^A(\omega^{lab}) = \frac{\sqrt{6} M_d}{W_{np} q^{c.m.}} F_T'^{10}|_{Q^2=0}, \quad (4)$$

where W_{np} denotes the invariant mass of the np system. Therefore, we introduce as spin asymmetry for transverse virtual photons

$$\sigma_{T,\gamma^*}^P(\omega^{lab}, Q^2) - \sigma_{T,\gamma^*}^A(\omega^{lab}, Q^2) = \frac{\sqrt{6} M_d}{W_{np} q^{c.m.}} F_T'^{10}(\omega^{lab}, Q^2). \quad (5)$$

The transverse spin asymmetry has been evaluated for the electrodisintegration channel for various values of $Q^2 = \text{const}$ with inclusion of MEC, IC and RC⁸. The results are shown in Fig. 4. A detailed analysis⁸ has revealed that also for virtual photons the transverse spin asymmetry is

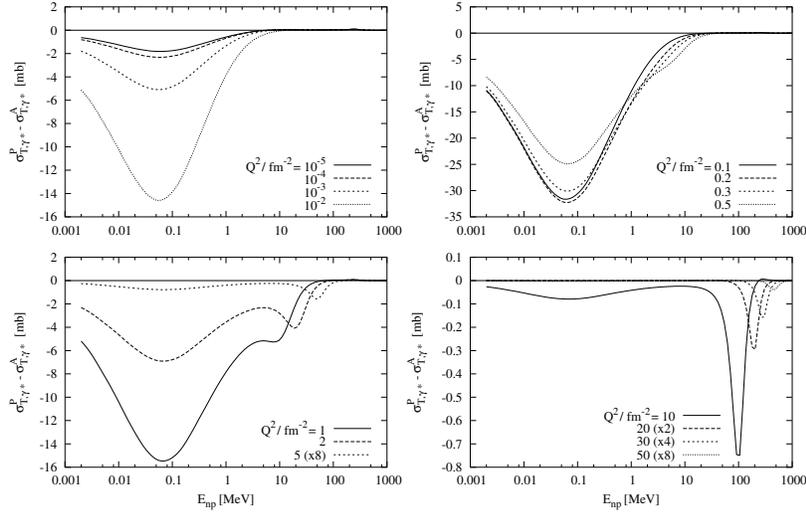


Figure 4. Transverse spin asymmetry $\sigma_{T,\gamma^*}^P - \sigma_{T,\gamma^*}^A$ of $d(e, e')np$ as function of E_{np} for various values of Q^2 . Calculation based on Argonne V_{18} potential¹⁰ including interaction and relativistic effects.

dominated near threshold by the isovector M1 transition to the “anti-bound” 1S_0 -state resulting in a large negative contribution which is deepest around $Q^2 \approx 0.2 \text{ fm}^{-2}$. The rapid fall-off with increasing E_{np} ensures a good convergence. For higher Q^2 a negative peak at quasi-free kinematics $E_{np}/\text{MeV} = 10 (q^{c.m.})^2/\text{fm}^{-2}$ emerges from quasi-free scattering off neutron and proton which can occur for antiparallel spin orientation only. However, its amplitude decreases rapidly with growing Q^2 . The dominance of the M1 transition into the 1S_0 state near threshold is demonstrated in Fig. 5.

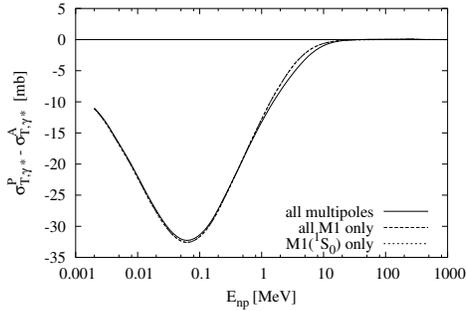


Figure 5. Comparison of the transverse spin asymmetry for the M1 transition $^3S_1 \rightarrow ^1S_0$ alone to all M1 transitions and to all multipoles for $Q^2 = 0.2 \text{ fm}^{-2}$.

We now turn to the definition of the generalized GDH integral

$$I_{\gamma^*d}^{GDH}(Q^2) = \sqrt{6} \int_{\omega_{th}^{lab}}^{\infty} \frac{d\omega^{lab}}{\omega^{lab}} \frac{M_d g(\omega^{lab}, Q^2)}{W_{np} q^{c.m.}} F_T'^{10}, \quad (6)$$

where $F_T'^{10} = F_T'^{10}(E_{np}, q^{c.m.})$ is an implicit function of ω^{lab} and Q^2 . The factor $g(\omega^{lab}, Q^2)$ appears because the generalization of the GDH integral is to a certain extent arbitrary. The only restrictions are (i) at the photon point $Q^2 = 0$ the condition $g(\omega^{lab}, 0) = 1$, and (ii)

$$\lim_{\omega^{lab} \rightarrow \infty} g(\omega^{lab}, Q^2)|_{Q^2=const.} < \infty. \quad (7)$$

As simplest extension we have chosen $g(\omega^{lab}, Q^2) \equiv 1$.

Explicit evaluations of the generalized GDH integral for the electrodisintegration channel are exhibited in Fig. 6. The prominent feature is the pronounced minimum around $Q^2 \approx 0.2 \text{ fm}^{-2}$ reflecting the absolutely largest spin asymmetry in Fig. 4 for this value of Q^2 . The left panel shows the influence of various interaction effects from MEC, IC and RC. Near the minimum, the largest interaction effect arises from MEC, increasing the depth by about 10 %, and to a smaller extent from IC while their influences in other regions of Q^2 is quite small. Relativistic contributions are substantial near the photon point as has been noted already for photodisintegration². But at higher Q^2 they are quite tiny. The right panel of Fig. 6 shows a comparison for three realistic potential models, the Bonn r-space, the Bonn p-space (OBEPQ-B)⁴ and the Argonne V_{18} ¹⁰ models. Obviously, the potential model variation is quite small compared to the interaction effects. The fact that near threshold the spin asymmetry is essentially determined by the nucleon isovector anomalous magnetic moments, is demonstrated by evaluating $I_{\gamma^*d}^{GDH}(Q^2)$ for vanishing anomalous moments. The result is also shown in the right panel of Fig. 6 and is indeed quite tiny. Thus the disintegration contribution to the generalized GDH-integral is essentially driven by the nucleon's anomalous magnetic moments.

4. Conclusions and outlook

(i) Real photons:

- The spin asymmetry of the deuteron is a very interesting observable of its own value because of a strong anticorrelation between low energy photodisintegration and at high energy meson production channels.
- The spin asymmetry is very sensitive to relativistic effects at quite low energies which have never been tested in detail for this observable.

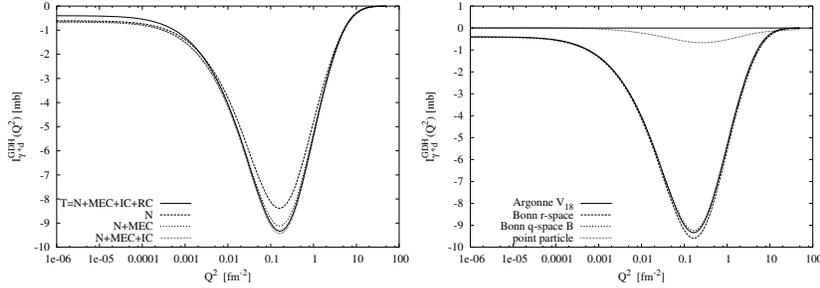


Figure 6. Generalized Gerasimov-Drell-Hearn integral as function of Q^2 for deuteron electrodisintegration $d(e, e')np$. Left panel: separate current contributions from normal nonrelativistic theory (N) and successively added meson exchange currents (MEC), isobar configurations (IC), and relativistic contributions (RC). Right panel: results of the complete calculation (T) for different potential models and for vanishing anomalous nucleon magnetic moments (labeled “point particle”).

- A direct access to the neutron spin asymmetry from the spin asymmetry of the deuteron is not possible.
- However, the deuteron spin asymmetry will provide a more detailed test for π -production on the neutron and thus in an indirect manner on the spin asymmetry of the neutron.

(ii) Virtual photons:

- The deuteron spin asymmetry of the electrodisintegration channel $d(e, e')np$ for $Q^2 = \text{const.}$ exhibits as function of the final state energy E_{np} a pronounced minimum around $E_{np} \approx 70$ KeV, the location of the “anti-bound” 1S_0 state, which is deepest for $Q^2 \approx 0.2 \text{ fm}^{-2}$.
- This minimum is dominated by a single M1 transition to this 1S_0 state and almost completely driven by the nucleon isovector anomalous magnetic moment.
- An experimental check of this feature would provide a significant test for our present theoretical understanding of the properties of few-body nuclei.
- The spin asymmetry falls off rapidly with increasing E_{np} so that the generalized GDH integral converges fast for this channel.
- An independent check in the framework of effective field theory would be very interesting.
- As future task remains the evaluation of the other channels, like coherent and incoherent single and double pion and eta electroproduction.

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