# SINGLE-SPIN ASYMMETRIES FROM TWO-PHOTON EXCHANGE IN ELASTIC ELECTRON-PROTON SCATTERING*

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The parity-conserving single-spin beam asymmetry of elastic electron-proton scattering is induced by an absorptive part of the two-photon exchange amplitude. We demonstrate that this asymmetry has logarithmic and double-logarithmic enhancement due to contributions of hard collinear quasi-real photons. An optical theorem is used to evaluate the asymmetry in terms of the total photoproduction cross section on the proton.

It has been known for a long time [1,2,3] that the two photon exchange (TPE) mechanism can generate the single-spin normal asymmetry (SSNA) of electron scattering due to a nonzero imaginary part of the TPE amplitude. The first calculations of the beam SSNA on the proton [4] predicted the magnitude of beam SSNA at the level of a few parts per million (ppm). The predictions of Ref.[4] which include only the elastic intermediate proton state are in qualitative agreement with measurements from MIT/Bates [5].

However, the main theoretical problem in description of the TPE amplitude on the proton is a large uncertainty in the contribution of the inelastic hadronic intermediate states. In Ref.[6] the beam SSNA at large momentum transfers was estimated at the level of one ppm, using the partonic picture developed in Ref.[7] for TPE effects unrelated to the electron helicity flip. To

*This work was supported by the US Department of Energy under contract DE-AC05-84ER40150.

calculate the contribution of the nucleon resonance region in beam SSNA, the authors of Ref.[8] used a phenomenological model (MAID) for single-pion electroproduction.

Current experiments designed for parity-violating electron scattering allow to measure the beam asymmetry with a fraction of ppm accuracy [9,10,11] and may also provide data on the parity-conserving beam SSNA. In fact, such measurements are needed because beam SSNA is a source of systematic corrections in the measurements of parity-violating observables.

Preliminary data from MAMI [9] reported beam SSNA that is more than an order of magnitude larger than our earlier predictions [4] assuming no inelastic excitations of the intermediate hadronic states in TPE. Moreover, recent SLAC measurements [10] observe beam SSNA on the proton at a few ppm level, that exceed predictions of Ref.[4] by several orders (!) of magnitude.

The present report resolves this puzzle, demonstrating that the physics behind such an unexpected behavior is collinear photon exchange in the TPE amplitude, resulting in single- and double-logarithmic enhancement of the beam SSNA. Such enhancement does not take place for the target SSNA (with unpolarized electrons) and spin correlations caused by longitudinal polarization of the scattering electrons. For large electron energies and small scattering angles, we use an optical theorem to relate the nucleon Compton amplitude to the total photoproduction cross section and obtain a simple analytic formula for the beam SSNA in this kinematics.

First, we write the formula for SSNA in terms of rank-3 leptonic and hadronic tensors which appear in the interference between the Born and TPE amplitudes as shown in Fig.1.

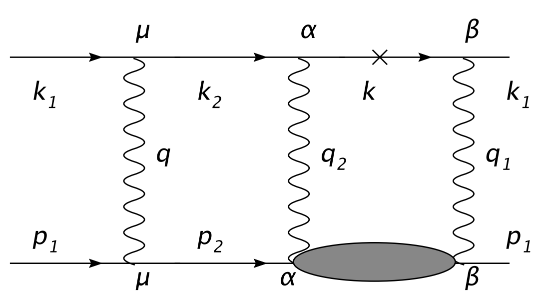


Figure 1. Interference between the Born and the TPE box diagrams in elastic e-p scattering that determines SSNA.

$$A_n = \frac{-i\alpha Q^2}{\pi^2 D(s,Q^2)} \int \frac{d^3k}{2E_k} \frac{L_{\mu\alpha\beta} H_{\mu\alpha\beta}}{q_1^2 q_2^2} \ , \tag{1}$$

where $Q^2 = -q^2$, $k(E_k)$ is the 3-momentum (energy) of the intermediate on-mass-shell electron in the TPE box diagram, $q_1$ and $q_2$ are the 4-momenta of the intermediate photons, $q_1 - q_2 = q$. The factor $Q^2/D(s,Q^2)$ in Eq.(1) is due to the squared Born amplitude, namely,

$$D(s,Q^2) = \frac{Q^4}{2}\big(F_1 + F_2\big)^2 + [(s-M^2)^2 - Q^2 s]\Big(F_1^2 + \frac{Q^2}{4M^2}F_2^2\Big), \tag{2}$$

where $F_1(F_2)$ is the Dirac (Pauli) proton form factor, $M$ is the proton mass and $s = (k_1 + p_1)^2$. Our sign convention for the beam asymmetry follows from the definition of the normal vector with respect to the electron scattering plane: $\mathbf{k_1} \times \mathbf{k_2}$.

Using the above notation, we have

$$L_{\mu\alpha\beta} = \frac{1}{4} Tr(\hat{k}_2 + m_e)\gamma_\mu(\hat{k}_1 + m_e)(1 - \gamma_5 \hat{\xi^e})\gamma_\beta(\hat{k} + m_e)\gamma_\alpha \ , \tag{3}$$

and

$$H_{\mu\alpha\beta} = \frac{1}{4} Tr(\hat{p}_2 + M)\Gamma_\mu(\hat{p}_1 + M)(1 - \gamma_5 \hat{\xi^p})\Im T_{\beta\alpha} \ , \tag{4}$$

where $m_e$ is the electron mass, $\xi^e(\xi^p)$ is the polarization 4-vector of the electron beam (proton target), $\Gamma_\mu = \gamma_\mu(F_1+F_2) - (p_{1\mu}+p_{2\mu})F_2/(2M)$, and $T_{\beta\alpha}$ is in general a non-forward proton Compton tensor that describes any possible hadronic intermediate states in the TPE amplitude. In accordance with Eq.(2) the single-spin normal asymmetry probes the imaginary part of contraction of the leptonic and hadronic tensors defined by Eqs.(3) and (4), respectively. These tensors satisfy the conditions

$$\begin{aligned} q_\mu L_{\mu\alpha\beta} = q_{2\alpha} L_{\mu\alpha\beta} = q_{1\beta} L_{\mu\alpha\beta} = 0, \\ q_\mu H_{\mu\alpha\beta} = q_{2\alpha} H_{\mu\alpha\beta} = q_{1\beta} H_{\mu\alpha\beta} = 0, \end{aligned} \tag{5}$$

separately for spin–independent and spin–dependent parts, as follows from gauge invariance of electromagnetic interactions.

If one of the photons in the box diagram is collinear to its parent electron, for example,

$$q_1 = x k_1, \quad x = \frac{W^2 - M^2}{s - M^2} \ , \tag{6}$$

where $W^2$ is the squared invariant mass of the intermediate hadronic system, the leptonic tensor can be written as

$$L_{\mu\alpha\beta} = \frac{1-x}{x} q_{1\beta} L^B_{\mu\alpha} + i m_e x L^\xi_{\mu\alpha\beta} \ . \tag{7}$$

The tensor $L^B_{\mu\alpha}$ coincides with the Born one of elastic electron–proton scattering and

$$L^\xi_{\mu\alpha\beta} = -g_{\alpha\beta}(\mu q k_1 \xi^e) + \frac{q^2}{2}(\mu\alpha\beta\xi^e) + \xi^e_\mu(\alpha\beta q k_1) + \\ (\xi^e k_2)(\mu\alpha\beta k_1) + k_{2\mu}(\alpha\beta k_1 \xi^e) + k_{1\mu}(\alpha\beta k_2 \xi^e) \ . \tag{8}$$

In the case of longitudinal polarization of the electron beam ($\xi^e_{mu} = k_{1\mu}/m_e$) the tensor $L^\xi_{\mu\alpha\beta}$ is zero. It is also zero for elastic intermediate proton state ($W = M$). Therefore, we expect no contribution from considered kinematics to the target SSNA or to longitudinal-spin correlations because any gauge invariant hadronic tensor has to give zero after contracting with $q_{1\beta}$ (see Eq.(5)).

In the case of the normal polarized electron beam

$$\xi^e_\mu = \frac{2(\mu k_1 p_1 q)}{\sqrt{Q^2[(s-M^2)^2 - Q^2 s]}} \ . \tag{9}$$

tensor $L^\xi_{\mu\alpha\beta}$ is not zero and the considered collinear photon kinematics contributes with essential logarithmic enhancement.

Therefore, conservation of the electromagnetic current that follows from gauge invariance (Eq.(5)) is the reason why the collinear intermediate photons appear in the TPE contribution to the beam SSNA, but not to the target SSNA. By analogy, we do not anticipate contributions from collinear-photon exchange in unpolarized electron-proton scattering, parity-conserving and parity-violating asymmetries due to longitudinal electron polarization the normal polarization of leptons is not involved.

Let us consider the hadronic tensor. Using a general form of the Compton tensor $T_{\beta\alpha}$ in terms of 18 independent invariant amplitudes that are free from kinematical singularities and zeros [12] and keeping in mind that the main contribution to the beam SSNA arises from collinear photon kinematics, the hadronic tensor can be written in the following form

$$H_{\mu\alpha\beta} = 2\pi W_1 \Big(F_1 - \tau F_2\Big) p_{1\mu} \Big( - g_{\alpha\beta} - \frac{[p_1 q]_{\alpha\beta}}{W^2 - M^2}\Big) \ , \ \tau = \frac{Q^2}{4M^2} \tag{10}$$

where $W_1$, defines the total photoproduction cross section [13] as

$$W_1(W^2, 0) = \frac{W^2 - M^2}{8\pi^2\alpha} \sigma^{\gamma p}_{tot}(W^2) \ . \tag{11}$$

Contracting the leptonic and hadronic tensors, we write the beam SSNA at small values of $Q^2$ as

$$A_n^e = \frac{m_e\sqrt{Q^2}\sigma_{tot}^{\gamma p}}{4\pi^3}\frac{F_1 - \tau F_2}{F_1^2 + \tau F_2^2} I \ , \tag{12}$$

$$I = \int \frac{d^3k}{2E_k}\frac{(W^2 - M^2)^2}{(s - M^2)^2}\frac{Q^2}{q_1^2 q_2^2} \ .$$

When integrating with respect to $W^2$ we take $\sigma_T(W^2, q_1^2) \rightarrow \sigma_{tot}^{\gamma p}(W^2)$ and assume $\sigma_{tot}^{\gamma p}(W^2)$ to be constant with energy ($\approx$ 0.1 mb, according to Ref.[14]). The integration in Eq. (12) can be done analytically, resulting in the following master formula that defines the beam SSNA for small values of $Q^2$ and takes into account contributions from intermediate collinear photons in the TPE box diagrams

$$A_n^e = \frac{m_e\sqrt{Q^2}\sigma_{tot}^{\gamma p}}{16\pi^2}\frac{F_1 - \tau F_2}{F_1^2 + \tau F_2^2} \times$$

$$\left(\log^2\frac{Q^2}{m_e^2} - 6\log\frac{Q^2}{m_e^2} + \frac{4\pi^2}{3} + 4\right) \ . \tag{13}$$

One can see that at fixed values of $Q^2$ the beam SSNA does not depend on the beam energy if the total photoproduction cross section is energy-independent. This remarkable property of small-angle beam SSNA follows from unitarity of the scattering matrix and does not rely on a specific model of nucleon structure.

In numerical calculations, we introduce additional $Q^2$ dependence by an empirical form factor $\exp{(-BQ^2/2)}$ with $B$=8 GeV$^{-2}$, that was measured experimentally in the Compton scattering on the nucleon in the diffractive regime (see [15] for review). The results for beam SSNA are presented in Fig.2 as a function of $Q^2$ for different energies of incident electrons. It can be seen that at fixed $Q^2$ the magnitude of beam SSNA is predicted to be approximately constant, as follows from slow logarithmic energy dependence of the total photoproduction cross section. We also calculated the contribution of the elastic intermediate proton state to the beam SSNA for high energies and small electron scattering angles using the formalism of Ref.[4] and found it to be highly supressed compared to the inelastic excitations. For the kinematics of SLAC E158 [10], this suppression is a few orders of magnitude due to different angular and energy behavior of these contributions.

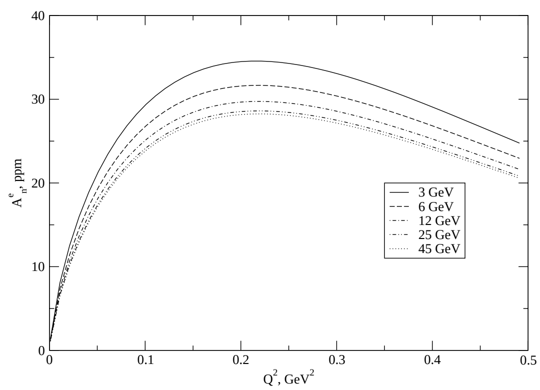


Figure 2. Beam SSNA as a function of $Q^2$ for different beam energies.

Fig.3 shows the calculated beam SSNA at fixed $Q^2$ in a wide energy range up to $\sqrt{s}$=500 GeV, where we used several parameterizations for the total photoproduction cross section on a proton from Refs.[16,17]. The physical reason for the almost constant photoproduction cross sections at high energies is believed to be soft Pomeron exchange [17], therefore the beam SSNA in the considered kinematics is sensitive to the physics of soft diffraction.

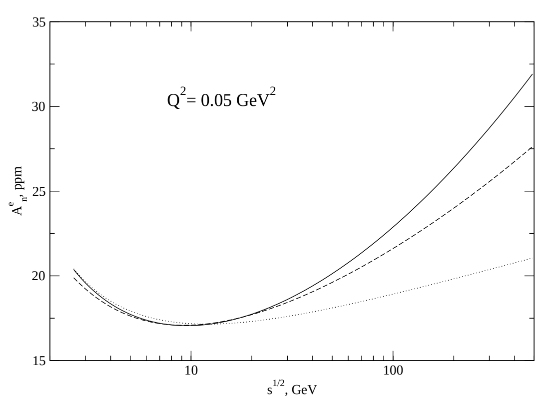


Figure 3. Beam SSNA as a function of c.m.s. energy for fixed $Q^2 = 0.05$ GeV$^2$ for different parameterizations of the total photoproduction cross section. A solid (dotted) line is a double-logarithmic fit 1 (single-logarithmic fit 3) from Block and Halzen [16] and a dashed line is an original Donnachie and Landshoff fit [17].

Because of the enhanced collinear-photon exchange contributions, experiments measuring normal SSNA are sensitive to the same nucleon Compton amplitudes (namely, their absorptive parts) that can be accessed in Compton scattering experiments in which at least one of the photons is real. In contrast, TPE effects for unpolarized electron scattering have contributions from the nucleon Compton amplitude with two space-like virtual photons.

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