EXPERIMENTAL VERIFICATION OF THE GDH SUM RULE

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The GDH Sum Rule has not been investigated experimentally until recently. For the first time this fundamental sum rule is verified by the GDH-Collaboration with circularly polarized real photons and longitudinally polarized nucleons at the two accelerators Elsa and Mami. The investigation of the response of the proton as well as of the neutron allows an isospin decomposition. Data from the resonance region up to the onset of the Regge regime are shown. The experimental approach will be presented as well as systematic uncertainties. The level at which the GDH Sum Rule for the proton has been verified is presented and estimates for the GDH integral for the neutron and the iso-vector case are given based on our new data.

1. Introduction

The GDH Sum Rule has been derived in parallel by several authors in the second half of the 1960ies. Today mostly Gerasimov [1], Drell and Hearn [2] are credited. Both works are based on a dispersion theoretic derivation. Hosoda and Yamamoto [3] in 1966 used the current algebra formalism to derive the same sum rule.

Gerasimov [1] rated the sum rule mainly to be of academic interest, while Hosoda and Yamamoto [3] were convinced that it would be straightforward to experimentally test it. Drell and Hearn [2], however, took a test to be a formidable experimental challenge and call for it. In fact, the experimental test has been awaiting technical developments that have only recently been achieved.

Iddings [4] in 1965 on the other hand was already all the way there to write down the sum rule for $Q^2 = 0$ but falls short of an explicit mention. Nonetheless, his work already contains a version of what is called today a

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generalization of the integral of the GDH sum rule.

For most of the further discussion here we focus on the dispersion theoretic derivation used by Gerasimov, Drell and Hearn. Only fundamental constraints enter this derivation: Lorentz invariance and gauge invariance allow to write the Compton-forward amplitude in a simple form; unitarity provides the Optical Theorem; causality and the so-called *No-Subtraction-Hypothesis* lead to the Kramers-Kronig dispersion relation; the Low-Theorem [5,6] is based again on Lorentz and gauge invariance.

Among these constraints the *No-Subtraction-Hypothesis* is the only assumption which is open to reasonable questions. To reduce the dispersion relation for the spin-flip Compton forward amplitude $f_2(\nu)$ from a contour integral in the complex plane to an integration along the real axis one has to presume that $f_2(\nu)/\nu \to 0$ for $\nu \to \infty$. A violation of this hypothesis would lead to a weird behavior of this spin-flip amplitude and the corresponding differential forward cross sections, namely:

$$\lim_{\nu\to\infty} \frac{1}{d\Omega} \left(d\sigma_{3/2} - d\sigma_{1/2}\right)\big|_{\theta=0} = \infty \tag{1}$$

On the other hand for the total cross section Regge arguments related to the Froissart bound ensure the following behavior:

$$\lim_{\nu\to\infty} \left(\sigma_{3/2}^{\text{tot}} - \sigma_{1/2}^{\text{tot}}\right) = 0 \tag{2}$$

A possible failure of the GDH Sum Rule would be related to a violation of the No-Subtraction-Hypothesis. There have been several attempts in the past to find causes for such a failure. Here some of them are reviewed:
Based on the current algebra derivation by Hosoda and Yamamoto the authors Chang, Liang and Workman [8] have argued that an anomaly in the charge density commutator gives rise to a modification of the GDH Sum Rule. Pantförder [9] was able to show that the contribution from this anomaly cancels going to the infinite-momentum limit which ultimately reveals the GDH Sum Rule.
It was questioned if the Low-Theorem holds to all orders of electromagnetic coupling. While Low [5] showed the derivation only in the lowest order Gell-Mann and Goldberger stated in their original paper [6] that their derivation should be “exactly correct in any known theory”. Later Roy and Singh [10] established the low theorem to the order α^2. Any correction due to higher orders would be minuscule compared to experimental errors.
Haim Goldberg [11] suspected that the photoproduction of gravitons violates the GDH Sum Rule. Contributions at very high energies from photonuclear

reactions other than those of strong interactions may not be ignored a priori. On the other hand, the contribution of these effects at high energies to the sum rule will be largely suppressed due to the weighting with the inverse of the photon energy.

Already in 1968 right after the discovery of the GDH Sum Rule Abarbanel and Goldberger [14] considered a $J = 1$ Regge fixed pole being a possible source for the failure of the No-Subtraction-Hypothesis. Despite such a fixed pole is forbidden by the Froissart theorem for purely hadronic processes and such a behavior has never been observed so far it cannot be ruled out completely for electro-weak processes. Nevertheless, it should be mentioned that it is not quite clear if such a fixed pole in the case of real Compton scattering would not violate the Landau-Yang theorem which forbids two photons to have a total angular momentum of $J = 1$.
Fairly recently however, Bass [12] has revisited the possibility of such a fixed pole. An observable effect of this would kick in only at very high energies. A connection of the fixed pole to the gluon topology is established. He conjectures a correction of the order of 10% to the GDH Sum Rule due to the fixed pole [13].

Further examples for possible failures of the sum rule that have been considered can be found in Ref. [7]. To summarize, today no compelling evidence for a modification of the GDH Sum Rule exists but also corrections at the level of 10% cannot be excluded *a priori*.

2. Experimental concept

The primary aim of the GDH-Collaboration[a] is to verify the Gerasimov-Drell-Hearn Sum Rule. The central issue of the experimental conception of the GDH-Collaboration is the reduction of systematic uncertainties in order to provide a setup compatible with the fundamental character of the sum rule.

2.1. *Region of integration*

The GDH integrand on the left hand side of the GDH Sum Rule is determined from the resonance region up to the onset of the Regge regime. This is achieved by the use of two electron accelerators with high duty cycle:

[a]For a list of participants of the GDH-Collaboration be referred to the author list of Ref. [19].

0.14 - 0.8 GeV	Mami (Mainz)
0.7 - 3.0 GeV	Elsa (Bonn)

At Elsa a completely new experimental area was setup for the GDH measurements while at Mami the existing tagging facility in the A2-Hall was available. At Mami two primary electron energy settings were to cover the energies from pion threshold up to 800 MeV. Five primary electron energy settings at Elsa allow to cover photon energies up to 3 GeV. The circular polarization of the photons is given by the helicity transfer of the longitudinal polarization of the electrons.

2.2. *Beam polarization*

At both accelerators the polarization of the electron beam is achieved by high intensity sources with strained super-lattice GaAs-crystals. The typical polarization of the delivered electron beam is $65 - 75\%$ [24,25].

The race-track of the electrons at Mami is deterministic. Hence, almost no polarization is lost on the way from the source to the experiment. Møller polarimetry is provided simultaneously to the photon tagging by a magnetic tagging spectrometer.

Elsa is a storage type accelerator with depolarizing resonances. The spin of the electrons has to be transported vertically in Elsa and rotated to the longitudinal direction in the external beam line for the experiment. Because of these more delicate circumstances of spin maintenance a dedicated 2-arm Møller spectrometer with large acceptance was built. It enables fast spin diagnostics in all 3 vector components [26,34].

2.3. *Frozen spin target*

A new solid state polarized frozen-spin target has been developed for the GDH measurements [23]. The central part of this new target consists of a ^{3}He/^{4}He dilution refrigerator that is installed horizontally along the beam axis. The refrigerator includes an internal superconducting holding coil to maintain the nucleon polarization in the frozen-spin mode longitudinally to the beam. The design of the dilution refrigerator and the use of an internal holding coil enabled for the first time the measurement of a spin-dependent total cross section in combination with a polarized solid state target. Due to the low fringe field of the holding coil and the horizontal alignment allows the detection of emitted particles with an angular acceptance of almost 4π (see below). Butanol provided polarized protons. In addition, ^{6}LiD was used at ELSA to obtain polarized deuterons which allows to extract the polarization dependent cross sections of the neutron. Instead, at the expense of the dilution factor, at MAMI deuterated butanol was used to obtain polarized deuterons to minimize the nuclear binding effects in ^{6}LiD.

Typical values for the polarization of the protons in the butanol that have been reached during data taking were 70-80%. The average deuteron polarization for the ^{6}LiD was about 27% and for d-butanol about 60%.

2.4. *Detector concepts*

Two detector concepts are used to meet the special requirements for the different energy ranges: The DAPHNE detector at MAMI and the GDH-Detector at ELSA.

DAPHNE [27] is well suited for charged particle detection and for the identification of low multiplicity states. It is essentially a charged particle tracking detector having a cylindrical symmetry. In addition it has a useful detection efficiency for neutral pions. In forward direction a silicon microstrip device called MIDAS [29] extends the acceptance for charged particles.

The GDH-Detector [28] has been specifically designed for measurements of total cross sections and is perfectly suited for situations where the contributing channels are not well known and extrapolations due to unobserved final states are not advisable. The concept of the GDH-Detector is to detect at least one reaction product from all possible hadronic processes with almost complete acceptance concerning solid angle and efficiency. This is achieved by an arrangement of scintillators and lead. The over all acceptance for any hadronic process is better than 99 %.

Both detection systems have similar components in forward direction. The electromagnetic background is suppressed by about 5 orders of magnitude by means of a threshold Čerenkov detector [28]. The Čerenkov detector is followed by the Far-Forward-Wall – a component similar to the central parts of the GDH-Detector – to complete the solid angle coverage [30].

3. Results

3.1. *Systematic studies*

Measurements of unpolarized total photoabsorption cross sections were performed [31,32,33] to ensure that both detection systems are operational even for measurements of differences of cross sections. An unprecedented data quality has been reached in unpolarized measurements on ^{1}H, ^{2}H and ^{3}He in the photon energy range from pion threshold to 800 MeV as well as on Carbon and Beryllium in the energy range from 250 MeV to 3100 MeV.

Systematic studies with respect to spin have been performed with an unpolarized butanol target in frozen spin mode with all possible holding field configurations. In any case the false asymmetry of $\sigma_{3/2} - \sigma_{1/2}$ turned out to be less than 2 μb [35].

3.2. *Polarized cross sections in the resonance region*

3.2.1. *Proton data*

Fig. 1 shows the final doubly polarized results for $\sigma_{3/2} - \sigma_{1/2}$ on the proton. For comparison also the unpolarized cross section is plotted. These proton data have already been published [15,16,17]. One observes that the data sets for the different energy settings at the two accelerators match each other very well. The three major resonances known from the unpolarized total cross section are present in the difference as well - they are even more pronounced.

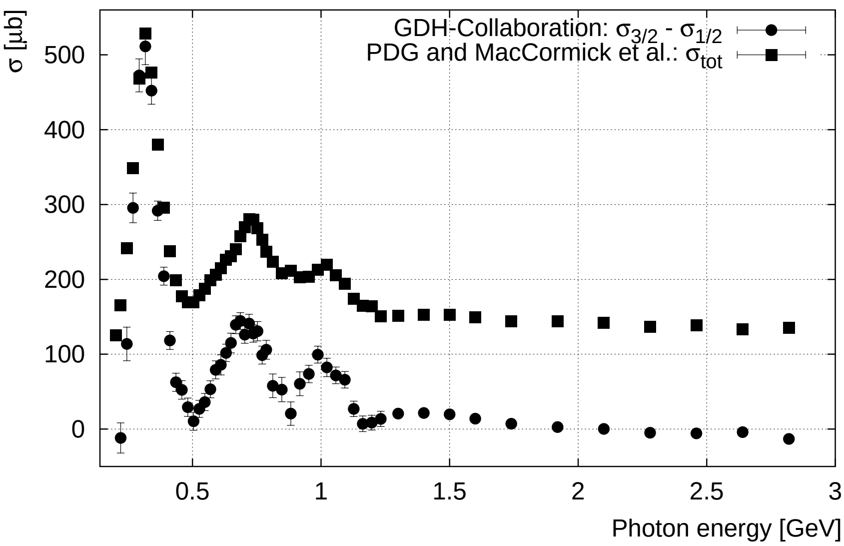

Figure 1. Difference of the polarized total photoabsorption of the proton in comparison to the unpolarized cross section

It is beyond the scope of this document to summarize the wealth of information obtained for the single resonances especially with respect to partial channels. A detailed review of our results can be found in [18] as well as in [19,20,21,22].

Fig. 2 shows the separate helicity contributions to the total cross section. The separated helicity states are obtained by adding resp. subtracting our polarized cross section difference from the unpolarized data. Clearly, most of the resonance strength of the first three resonances originates from the 3/2 helicity channel. This can be understood intuitively as all major resonances contributing to the cross section have $J \geq 3/2$. The situation is quite different for the 4th resonance. This structure has not been observed before in unpolarized total cross section data. Here the structure stems at least partially from the drop in the strength of the contribution from helicity 1/2. It might be due to the F_{35} and the F_{37} resonances.

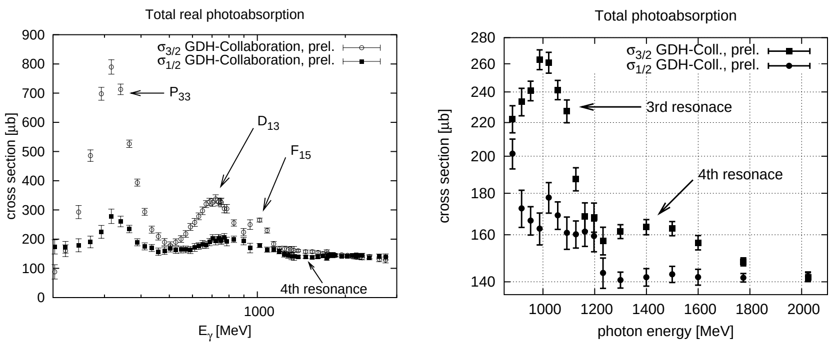

Figure 2. Left: Separate helicity state total cross sections $\sigma_{3/2}$ and $\sigma_{1/2}$ in the resonance region; Right: For the 3rd and the "4th" resonance only.

3.2.2. *Neutron data*

To compute the neutron cross sections from the ^{6}LiD data we have accounted for nuclear effects and chemical admixtures that modify the neutron polarization relative to the measured polarization of the free deuteron. Fig. 3 shows the response of the neutron to polarized real photons. Our data exhibit a structure in the 3rd resonance region similar to the proton data. Our proton data is well described by the single pion photoproduction prediction MAID [37]. This would indicated that this structure is dominantly generated by single pion production alone and not by double pion production.

However, the single pion photoproduction prediction does not describe the neutron. This could indicate the opposite i.e. dominance of multi pion production. Of course, also a complete failure of MAID in the 3rd resonance region cannot be excluded. This puzzle has to be resolved by future experiments with partial channel resolution.

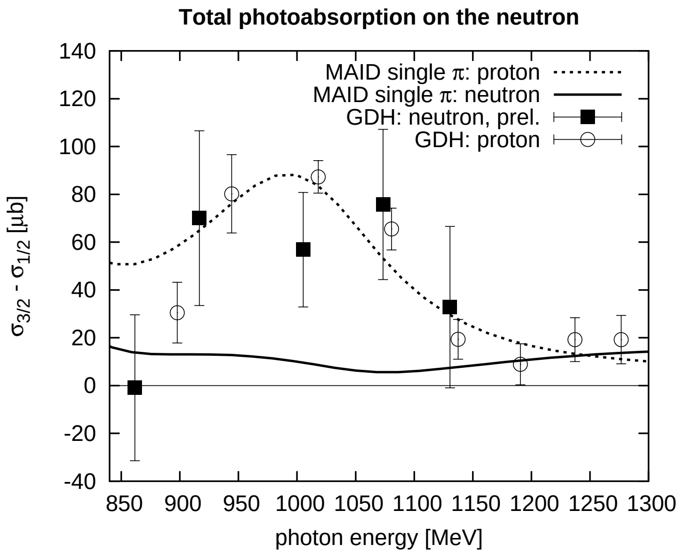

Figure 3. Comparison of the polarized cross sections in the region of the 3rd resonance for the proton and neutron with the MAID [37] predictions

3.3. *High-energy behavior*

Regge fits are able to describe many unpolarized total cross sections simultaneously [38]. All data follow a simple power law, namely $\sigma_T \simeq c_1 \cdot s^{\alpha_R(0)-1} + c_2 \cdot s^{\alpha_P(0)-1}$ with $\alpha_R(0)$ being the ρ, ω trajectory intercept and $\alpha_P(0)$ being that of the Pomeron. For real photo absorption these fits are valid down to photon energies as low as 1.3 GeV. For a detailed discussion see [40]. In the polarized case Regge fits have recently been applied to deep inelastic scattering data [41,42,43]. The extrapolation of these fits to $Q^2 = 0$ indicate that the integrand of the GDH Sum Rule on the proton could be negative at higher energies. Our polarized proton data up to 3 GeV photon energy disagree with these Regge fits but indicate a sign change at the highest energies.

Bass and Brisudová [41] have argued that the polarized cross section difference for the absorption of *virtual* photons can be described by the following Regge behavior:

$$\sigma_{3/2} - \sigma_{1/2} = \left[c_1 \ s^{\alpha_{a_1}-1} \cdot I + c_2 \ s^{\alpha_{f_1}-1} + c_3 \frac{\ln s}{s} + \frac{c_4}{\ln^2 s} \right] F(s, Q^2)$$

where I denotes the isospin of the nucleon. The logarithmic terms are due to Regge cuts and can be neglected at $Q^2 = 0$ [44]. Also $F(s, Q^2)$ simplifies to a constant at the real photon point and can be absorbed into c_1 and c_2. α_{a_1} and α_{f_1} are the Regge intercepts of the respective trajectories. Hence in the case of real photons the expression for the Regge behavior simplifies considerably to

$$\sigma_{3/2} - \sigma_{1/2} = c_1 \ s^{\alpha_{a_1}-1} \cdot I + c_2 \ s^{\alpha_{f_1}-1} \tag{3}$$

The intercept of the f_1 trajectory is rather well defined by the deep inelastic scattering data to be about -0.5. The situation is less clear with α_{a_1} where the values from different fits range from about -0.2 to +0.5. For the further calculations here we adopt a value of +0.2.

Fig. 4 shows a simultaneous fit to both our proton and neutron data via c_1 and c_2. The result for the proton indicates a sign change at about 2 GeV photon energy as does the data. The proton data below 1.8 GeV significantly deviates from the fit which is a consequence of the 4th resonance structure previously discussed. A fit to the proton data alone does not exhibit this feature [40].

There is no polarized data for the neutron at energies above those where the proton shows the 4th resonance. Hence, the fit to the neutron might be impaired by a similar 4th resonance. The shown statistical error band of the fit for the neutron is of the order of 10 μb while the impact of the 4th resonance on the cross section difference for the proton is about 20 μb. The systematic error due to the ignorance of a possible 4th resonance in the neutron case is of the same order as the statistical uncertainty. Therefore, we will neglect this effect in the further discussion.

The fit to the proton data alone and the resulting prediction for the neutron's strength as shown in [40] was about 2 standard deviations higher than our data. This is mainly due to the inclusion of the 4th resonance in the fit for the proton. However, it shall be noted that the Regge fit prediction [42] based on DIS data including neutron data was off by more

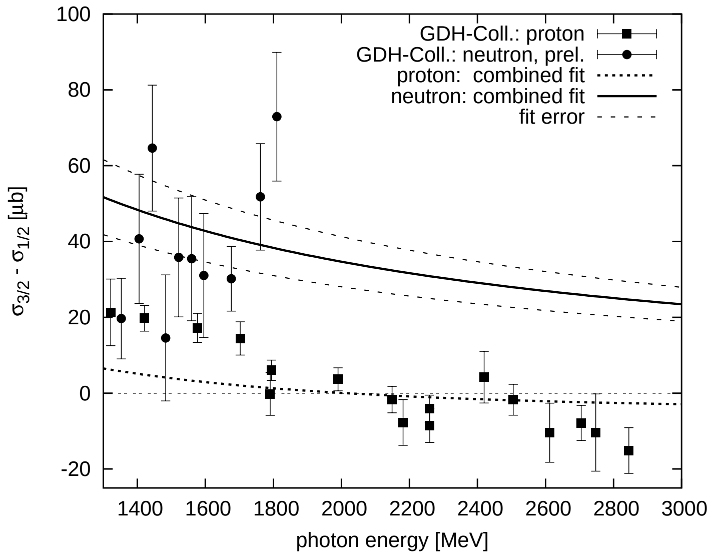

Figure 4. Simultaneous fit to our proton and neutron data

standard deviations. This underlines once more the general difficulty to draw conclusion for the real photon point based on data with non vanishing photon virtuality.

3.4. *Verification of the GDH Sum Rule*

3.4.1. *The proton*

The GDH Sum Rule prediction for the proton amounts to 205 μb. The experimental value of the running GDH integral up to 2.8 GeV clearly overshoots this prediction. The value of the GDH sum up to 2.9 GeV is $226{\pm}5_{\rm stat}{\pm}12_{\rm syst}$ μb [17]. This includes an unmeasured negative contribution at the pion threshold up to 200 MeV taken from Ref. [37]. Since at threshold only a simple E_{0+} amplitude contributes this can be regarded to be a reliable estimate.

The integrand $\sigma_{3/2} - \sigma_{1/2}$ remains positive from about 230 MeV on up to about 2 GeV as seen in Fig. 4. The sign change of the integrand at higher energies as indicated by the data and the Regge fit leads to a better agreement between the measurement and the GDH prediction. Indeed our Regge fit (see Sec. 3.3) gives a contribution of -15 μb to the GDH integral above 2.9 GeV. This fit result from out data almost coincides with those based on DIS data. The parameterization by Bianchi and Thomas [42] gives -14 μb and Simula *et al.* [43] give -13 μb.

Including this extrapolation to higher energies we obtain 215 μb for the value of the GDH integral. This is in good agreement with the GDH Sum Rule prediction. The over all level of precision obtained for the verification of the GDH Sum Rule for the proton is less than 10% including systematic uncertainties. This represents the first verification of the GDH Sum Rule ever.

3.5. *The neutron and the iso-vector Sum Rule*

The GDH Sum Rule prediction for the neutron is 233 μb which is almost 30 μb higher than the value for the proton. Moreover, the contribution below 200 MeV due to the E_{0+} amplitude is -50 μb [37] i.e. even 22μb lower than for the proton. The cross section difference in the Δ-resonance as predicted by MAID is very similar to that of the proton. Indeed, our preliminary results [45] for the cross section up to 800 MeV for the deuteron look like twice the proton cross section within the statistical uncertainties[b]. For our estimate for the neutron we account for the integral from 200 MeV to 800 MeV with the same value that we have obtained experimentally for the proton which is 226 μb.

At higher energies however, the situation is different with the neutron. The integral from 800 MeV up to 1820 MeV amounts to 34 μb as compared to only 29 μb for the proton. The major difference comes from the extrapolation of our data to account for the Regge behavior above 3 GeV. Here we obtain +45 μb as compared to -15 μb for the proton. In total, we obtain an estimate of 255 μb for the neutron GDH integral. This is in good agreement with the Sum Rule prediction of 233 μb.

Considering the iso-vector case the situation gets even more accentuated. The GDH integral is driven by the behavior at threshold and at energies above about 1 GeV. The +22 μb up to 200 MeV are more than compensated by -65 μb in the range above 800 MeV. Here -60 μb stem alone from the Regge fit to our data. The estimate for the total integral amounts to -43 μb. This is to be compared to the GDH Sum Rule prediction of -23 μb. Within the systematic uncertainties this again represents a good agreement. Also, this estimate shows that most of the strength in

[b]Given the large statistical uncertainties nuclear effects can be neglected for this discussion

the iso-vector case comes from high energies and not from the resonance regime. Since this part has been neglected in most previous estimates for the iso-vector GDH Integral even the sign of these predictions was wrong.

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