PROTON SPIN STRUCTURE AT HIGH PARTON DENSITIES

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This is a review of the experimental and phenomenological knowledge of the spin dependent structure function g_1 at low values of x and Q^2.

1. Introduction

The region of low values of the Bjorken scaling variable x corresponds to high parton densities, where new dynamical mechanisms may be revealed and where the knowledge of the spin dependent nucleon structure function $g_1(x, Q^2)$ is required to evaluate the spin sum rules necessary to understand the origin of the nucleon spin. The behaviour of g_1 at $x \lesssim 0.001$ and in the scaling region, $Q^2 \gtrsim 1$ GeV^2, is unknown due to the lack of colliders with polarised beams. Information about spin-averaged structure function $F_2(x, Q^2)$ in that region comes almost entirely from the experiments at HERA: F_2 rises with decreasing x, in agreement with QCD and the rise is weaker with decreasing Q^2, [1]. However even if such an inclusive quantity as F_2 can be described by the conventional DGLAP resummation, certain non-inclusive observables seem to be better described by the BFKL approach [2]. Thus non-inclusive reactions are crucial to understand the dynamics of high parton densities. Unfortunately in the case of spin, the longitudinal structure function, $g_1(x, Q^2)$, is presently the only observable which permits the study of low x spin dependent processes. Since it is being obtained exclusively from fixed-target experiments where low values of x are correlated with low values of Q^2, not only the measurements put very high demands on event triggering and reconstruction but also theoretical interpretations of the results require a suitable extrapolation of parton ideas to the low Q^2 region and inclusion of dynamical mechanisms, like the Vector Meson

Dominance (VMD), as it is the case for the low Q^2 spin-averaged electroproduction [3]. In the Q^2=0 limit g_1 should be a finite function of W^2, free from any kinematical singularities or zeros. For large Q^2 the VMD contribution to g_1 vanishes as $1/Q^4$ and can usually be neglected. The partonic contribution to g_1 which controls the structure functions in the deep inelastic domain and which scales there *modulo* logarithmic corrections, has to be suitably extended to the low Q^2 region.

2. Results of measurements

Experimental knowledge of the longitudinal spin dependent structure function $g_1(x, Q^2)$ comes entirely from the fixed-target setups: EMC, SMC and COMPASS at CERN, experiments at SLAC (E142, E143, E154, E155, E155X) and the HERMES experiment at the HERA *ep* collider.

In the past the lowest values of x were reached by the SMC due to a high energy of the muon beam and due to the demand of a final state hadron, imposed either in the off-line analysis [4] or in the dedicated low x trigger with a hadron signal in the calorimeter [5]. These requirements permitted measurements of muon scattering angles as low as 1 mrad and efficiently removed the dominant background of muons scattered elastically from target atomic electrons at x =0.000545, cf. [5]. Much lower values of x are presently being obtained by COMPASS [6].

Spin effects are weak, thus they are determined by measuring the cross section asymmetries in which spin-independent contributions cancel. Direct result of all measurements is the longitudinal cross section asymmetry, $A_\parallel$ which permits to extract the virtual photon – proton asymmetry, A_1 and finally, using F_2 and R, to get g_1.

The proton and deuteron g_1 was measured for $0.00006 < x < 0.8$, cf. Fig. 1 [7]. Direct measurements on the neutron are limited to $x \gtrsim 0.02$. No significant spin effects were observed at the lowest values of x, explored only by the SMC. Scaling violation in $g_1(x, Q^2)$ is weak: the average Q^2 is about 10 GeV^2 for the SMC and almost an order of magnitude less for the SLAC and HERMES experiments. For the SMC data [5], $\langle x \rangle = 0.0001$ corresponds to $\langle Q^2 \rangle = 0.02$ GeV^2; Q^2 becomes larger than 1 GeV^2 at $x \gtrsim 0.003$ (at $x \gtrsim 0.03$ for HERMES). At lowest x results on g_1 have very large errors but it seems that both g_1^p and g_1^d are positive there. Statistical errors dominate in that kinematic interval.

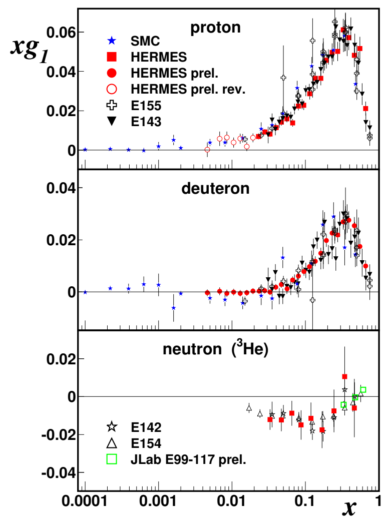

Figure 1. Compilation of data on $xg_1(x, Q^2)$. All the data are given at their quoted mean Q^2 values. Errors are total. Figure taken from [7].

3. Regge model predictions

The low x behaviour of g_1 for fixed Q^2 reflects the high energy behaviour of the virtual Compton scattering cross section with centre-of-mass energy squared, $s \equiv W^2 = M^2 + Q^2(1/x - 1)$; here M is the nucleon mass. This is the Regge limit of the (deep) inelastic scattering where the Regge pole exchange model should be applicable. It gives the following parametrisation of the (singlet and nonsinglet) spin dependent structure function at $x \to 0$ (i.e. $Q^2 \ll W^2$):

$$g_1^i(x, Q^2) \sim \beta(Q^2) x^{-\alpha_i(0)} \tag{1}$$

where $i = s, ns$: $g_1^s(x, Q^2) = g_1^p(x, Q^2) + g_1^n(x, Q^2)$ and $g_1^{ns}(x, Q^2) = g_1^p(x, Q^2) - g_1^n(x, Q^2)$ respectively. The intercepts, $\alpha_i(0)$, correspond to the axial vector mesons with I=0 (g_1^s; f_1 trajectory) and I=1 (g_1^{ns}; a_1 trajectory). It is expected that $\alpha_{s,ns}(0) \lesssim 0$ and that $\alpha_s(0) \approx \alpha_{ns}(0)$, [8]. A Regge type approach has been used in a global analysis of the g_1^p and g_1^n data in the range 0.3 GeV$^2 < Q^2 <$ 70 GeV^2 and 4 GeV$^2 < W^2 <$ 300 GeV^2, [9]; fits gave a smooth extrapolation of g_1 down to the photoproduction limit. At large Q^2 the Regge behaviour of $g_1(x, Q^2)$ is unstable against the DGLAP evolution and against resummation of the $\ln^2(1/x)$ terms which generate more singular x dependence than that implied by Eq.(1) for $\alpha_{s,ns}(0) \lesssim 0$, cf. Section 4.

Other considerations based on the Regge model give further isosinglet contributions to the g_1: a term proportional to $\ln x$, [10] and a term proportional to $2\ln(1/x)$–1, [11]; a perversely behaving term proportional to $1/(x\ln^2 x)$, recalled in [10], is not valid for g_1, [12].

Testing the Regge behaviour of g_1 through its x dependence should in principle be possible with the low x data of the SMC [5] which include the kinematic region where W^2 is high, $W^2 \gtrsim 100$ GeV^2, and $W^2 \gg Q^2$. Thus the Regge model should be applicable there. However for those data W^2 changes very little: from about 100 GeV^2 at $x = 0.1$ to about 220 GeV^2 at $x = 0.0001$, contrary to a strong change of Q^2: from about 20 GeV^2 to about 0.01 GeV^2 respectively. Thus those data cannot test the Regge behaviour of g_1.

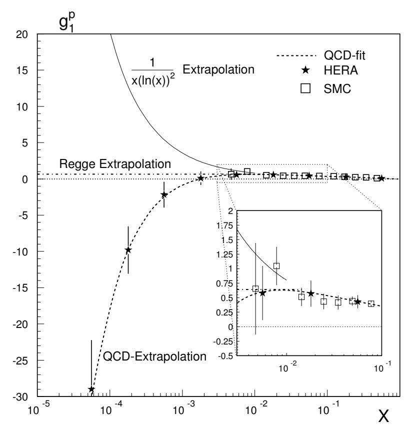

Figure 2. Three scenarios of the possible behaviour of g_1^p at low x [14].

Employing the Regge model prediction, $g_1 \sim x^0$ to obtain the $x \to 0$ extrapolation of g_1, often used in the past to extract the g_1 moments (cf.[13] and Fig.2) is not correct. The values of g_1 should be evolved to a common value of Q^2 before the extrapolation, cf. Eq.(1). Therefore other ways of extrapolating g_1 to low x were adopted in the analyses, see Sections 4.1 and 4.3.

4. Low x implications from perturbative QCD

4.1. *DGLAP fits to the g_1 measurements*

In standard QCD, the asymptotic, small x behaviour of g_1 is created by the "ladder" processes, see Fig. 1 in [15]. This behaviour is more singular than that implied by Eq.(1) for $\alpha_{s,ns}(0) \lesssim 0$: Regge behaviour of $g_1(x, Q^2)$ is unstable against the QCD evolution.

Several analyses of the Q^2 dependence of g_1 have been performed [13,16,17,18,19,20,21] on the world data in the framework of the NLO QCD but the present data do not permit to determine the shapes of parton distributions with sufficient accuracy, especially at small x. Thus extrapolations of the DGLAP fits to the unmeasured low x region give different g_1 behaviours in different analyses, e.g. g_1^p at $x \lesssim 0.001$ is positive and increasing with decreasing x in [20] and negative and decreasing in [13,17]. The results for x values below these of the data do not influence the results of the fit. Therefore there is no reason to expect that the partons at very low x behave as those in the measured (larger x) region. Nevertheless extrapolations of the QCD fit are being used to get the $x \to 0$ extrapolation of g_1 [13], necessary to evaluate its first moments. They strongly disagree with the Regge asymptotic form, cf. Fig. 2.

4.2. *Double logarithmic $ln^2(1/x)$ corrections to $g_1(x, Q^2)$*

The LO (and NLO) QCD evolution which sums the powers of $\ln(Q^2/Q_0^2)$ is incomplete at low x. Powers of another large logarithm, $\ln(1/x)$, have to be summed up there. In the spin-independent case this is accomplished by the BFKL evolution equation (see e.g. [22]) which gives the leading low x behaviour of the structure function, e.g. $F_1^s \sim x^{-\lambda_{BFKL}}$ where $\lambda_{BFKL} > 1$.

It has recently been pointed out that the small x behaviour of both singlet and non-singlet spin dependent structure function $g_1(x, Q^2)$ is controlled by the double logarithmic terms, i.e. by those terms of the perturbative expansion which correspond to powers of $\alpha_s \ln^2(1/x)$ at each order of the expansion [23]. The Regge behaviour of g_1 is unstable against the resummation of the $\ln^2(1/x)$ terms which generate more singular x dependence than that implied by Eq.(1) for $\alpha_{s,ns} \lesssim 0$, i.e. they generate the leading small x behaviour of the g_1.

The double logarithmic terms in the non-singlet part of $g_1(x, Q^2)$ are generated by ladder diagrams [26] as in Fig. 1 in [15]. Contributions of non-ladder diagrams [23] to g_1^{ns} are numerically small for N_c=3 but are non-negligible in the case of g_1^s; they are obtained from the ladder ones by adding to them soft bremsstrahlung gluons or soft quarks [27]. At low x, the singlet part, g_1^s, dominates over g_1^{ns}.

The double logarithmic $\ln^2(1/x)$ effects go beyond the standard LO (and NLO) QCD evolution of spin dependent parton densities. One of the ways to accommodate them into the QCD evolution formalism is based on unintegrated spin dependent parton distributions, $f_j(x', k^2)$ ($j = u_v, d_v, \bar{u}, \bar{d}, \bar{s}, g$) where k^2 is the transverse momentum squared of the parton j and x' the longitudinal momentum fraction of the parent nucleon carried by a parton [15,27,28]. This formalism is very suitable for extrapolating g_1 to the region of low Q^2 (including $Q^2 = 0$) at fixed W^2, [15].

The conventional (integrated) distributions $\Delta p_j(x, Q^2)$ (i.e. $\Delta q_u = \Delta p_{u_v} + \Delta p_{\bar{u}}$, $\Delta \bar{q}_u = \Delta p_{\bar{u}}$ etc. for quarks, antiquarks and gluons) are related in the following way to the unintegrated distributions $f_j(x', k^2)$:

$$\Delta p_j(x, Q^2) = \Delta p_j^0(x) + \int_{k_0^2}^{W^2} \frac{dk^2}{k^2} f_j(x' = x(1 + \frac{k^2}{Q^2}), k^2) \qquad (2)$$

Here $\Delta p_j^0(x)$ denote the nonperturbative parts of the of the distributions, corresponding to $k^2 < k_0^2$ and the parameter k_0^2 is the infrared cut-off ($k_0^2 \sim$1 GeV^2). In [15,28,27] they were treated semiphenomenologically and were parametrised as follows:

$$\Delta p_j^0(x) = C_j(1 - x)^{\eta_j} \qquad (3)$$

The unintegrated distributions $f_j(x', k^2)$ are the solutions of the integral equations [15,28,27] which embody both the LO Altarelli-Parisi evolution and the double $\ln^2(1/x')$ resummation at small x'. These equations combined with Eq.(2) and with a standard relation of g_1 to the polarised quark and antiquark distributions lead to approximate $x^{-\lambda}$ behaviour of the g_1 in the $x \to 0$ limit, with $\lambda \sim 0.4$ and $\lambda \sim 0.8$ for the nonsinglet and singlet parts respectively which is more singular at low x than that generated by the (nonperturbative) Regge pole exchanges.

Results of a complete, unified formalism incorporating the LO Altarelli–Parisi evolution and the $\ln^2(1/x)$ resummation at low x for g_1^p show that resummation of $\ln^2(1/x)$ terms gives a steeper g_1 behaviour than that generated by the LO evolution alone and this effect is visible in g_1^{ns} already for $x \lesssim 10^{-2}$ (at large Q^2) [15]. The double $\ln^2(1/x)$ effects are not important in the W^2 range of the fixed target experiments.

The formalism including the $\ln^2(1/x)$ resummation and the LO Altarelli-Parisi evolution, [27], was used to calculate g_1 at x and Q^2 values of the SMC measurement and a reasonable description of the data on $g_1^{p,d}(x, Q^2)$ extending down to $x \sim$0.0001 at $Q^2 \sim$0.02 GeV^2 was obtained, cf. Fig.1 in [29]. Of course the (extrapolated) partonic contribution may not be the only one at low Q^2; the VMD part may play a non-negligible role as well, cf. Section 5.

4.3. *Low x contributions to g_1 moments*

The spin sum rules involve first moments of g_1, i.e. integrations of g_1 over the whole range of x values, from 0 to 1, including the experimentally unmeasured regions, $[0,x_{min})$ and $(x_{max},1]$. The latter is not critical but contribution from the former may significantly influence the moments. The value of x_{min} depends on the value of the maximal lepton energy loss, ν_{max}, accessed in an experiment at a given Q_0^2. For the CERN experiments, with muon beam energies about 200 GeV and at Q_0^2=1 GeV^2 it is about 180 GeV which corresponds to $x_{min} \approx 0.003$. Contribution to the g_1 moments from the unmeasured region, $0 \leq x < 0.003$, has thus to be estimated phenomenologically.

Unified system of equations including the double $\ln^2(1/x)$ resummation effects and the complete LO Altarelli-Parisi evolution, [27], was used to extrapolate the spin dependent parton distributions and the polarised nucleon structure functions down to very low values of x [30].

Results show that at Q^2=10 GeV^2, a contribution of 0.0080 from the unmeasured region, $0 \leq x < 0.003$, to the Bjorken integral was obtained while the contribution resulting from the pure LO Altarelli-Parisi evolution was 0.0057. These have to be compared with 0.004 obtained when g_1=const, consistent with Regge prediction was assumed and fitted to the lowest x data for proton and deuteron targets (see [15] and references therein).

Extrapolation to the unmeasured region ($0 \leq x < 0.003$) of the NLO DGLAP fits to the world data results in about 10% contribution of that low x region to the g_1^p moment [13]. The NLO DGLAP fit to the SMC data gave a contribution of 0.010 to the Bjorken integral at Q^2=10 GeV^2, i.e. about 6% of that integral [13]. These numbers rely on the validity of the assumption that the parton distributions behave as x^δ as $x \to 0$.

5. Nonperturbative effects in g_1

Data on polarized nucleon structure function $g_1(x, Q^2)$ extend to the region of low values of x, which are reached simultaneously with low values of Q^2, [4,5,7]. This latter region is of particular interest since nonperturbative mechanisms dominate the particle dynamics there and a transition from soft- to hard physics may be studied. The partonic contribution to g_1 which controls the structure function in the deep inelastic domain has thus to be suitably extended to the low Q^2 region and complemented by a nonperturbative component.

Two attempts using (G)VMD methods have recently been made to describe g_1 in the low x, low Q^2 region. In the first one [29] the following representation of g_1 was assumed:

$$g_1(x,Q^2) = g_1^{VMD}(x,Q^2) + g_1^{part}(x,Q^2) = \frac{M\nu}{4\pi} \sum_{V=\rho,\omega,\phi} \frac{M_V^4 \Delta\sigma_V(W^2)}{\gamma_V^2 (Q^2+M_V^2)^2} + g_1^{part}(x,Q^2) \tag{4}$$

where γ_V^2 are determined from the leptonic widths of vector mesons V. The unknown cross sections $\Delta\sigma_V(W^2)$ are combinations of the total cross sections for the scattering of polarised vector mesons and nucleons. It was assumed that they are proportional (with a proportionality coefficient C) to the appropriate combinations of the nonperturbative contributions $\Delta p_j^0(x)$ to the polarised quark and antiquark distributions. As a result the cross sections $\Delta\sigma_V$ behave as $1/W^2$ at large W^2 which corresponds to zero intercepts of the appropriate Regge trajectories. The partonic contribution, g_1^{part} was parametrised as discussed in Section 4.2. The statistical accuracy of the SMC data is too poor to constrain the value of the coefficient C but the SLAC E143 data [31] seem to prefer a small negative value of C in g_1^p.

In the other attempt [32] the GVMD model was used together with the Drell-Hearn-Gerasimov-Hosoda-Yamamoto (DHGHY) sum rule [33] to constrain the coefficient C. The partonic contribution, g_1^{part}, was treated as an extrapolation of the QCD improved parton model structure function, $g_1(x,Q^2)$, to arbitrary values of Q^2. The value of C was then fixed in the photoproduction limit where the first moment of g_1 is related to the anomalous magnetic moment of the nucleon via the DHGHY sum rule, cf. [34,35],

$$I(0) = I_{res}(0) + M \int_{\nu_t(0)}^{\infty} \frac{d\nu}{\nu^2}\, g_1\left(x(\nu),Q^2\right) = -\kappa_{p(n)}^2/4. \tag{5}$$

where the DHGHY moment, $I(Q^2)$, before taking the Q^2=0 limit has been split into two parts, one corresponding to $W < W_t \sim 2$ GeV (baryonic resonances) and the other to $W > W_t$. Here $\nu_t(Q^2) = (W_t^2+Q^2-M^2)/2M$. Substituting $g_1\left(x(\nu),0\right)$ in Eq. (5) by Eq. (4) at $Q^2 = 0$ the value of C may be obtained from (5) if $I_{res}(0)$, is known e.g. from measurements (see e.g. [36]). As a result the constant C was found to be –0.24 or –0.30, for two different ways of parametrizing the polarised parton distributions.

The nonperturbative, Vector Meson Dominance contribution was obtained negative in both attempts [29,32] as well as from earlier phenomenological analyses of the sum rules [35,37].

6. Outlook

The longitudinal spin dependent structure function, $g_1(x, Q^2)$, is presently the only observable which permits an insight into the spin dependent low x physics. Contrary to spin-independent structure functions, it is sensitive to double logarithmic, $\ln^2(1/x)$ corrections, generating its leading small x behaviour. However its knowledge is limited by the statistical accuracy and by the kinematics of the fixed-target experiments. In the latter, the low values of x are reached simultaneously with low values of the four momentum transfer, Q^2. While the low Q^2 domain may be of great interest due to a transition from soft to hard physics, it also challenges theoretical predictions based on partonic ideas which have to be suitably extended to the nonperturbative region.

Until now, experimental data on $g_1(x, Q^2)$ at low x came mainly from the SMC at CERN. They do not permit to constrain the low x parton distributions, nor to test the Regge model but they seem to leave room for contributions other than (low Q^2 extrapolated) partonic mechanisms. They also permitted first quantitative studies of nonperturbative mechanisms; results consistently point towards large and negative contribution of the latter.

New low x data on $g_1(x, Q^2)$ will soon be available from COMPASS. Their statistics will be by far larger so that statistical errors should no longer be dominating. A crucial extension of the kinematic domain of the (deep) inelastic spin electroproduction will take place with the advent of the polarised Electron-Ion Collider, EIC, at BNL [38,39]. With its centre-of-mass energy only about 2 times lower than that at HERA, this machine will open a field of perturbative low x spin physics where also other, semi-inclusive and exclusive observables, will be accessible for testing the high parton density mechanisms.

References

1. ZEUS; S. Chekanov et al., Eur. Phys. J. **C21**, 443 (2001); H1; C. Adloff et al., Phys. Lett. **B520**,183 (2001).
2. Small x Collaboration, B. Andersson et al., Eur. Phys. J. **C25**, 77 (2002).
3. ZEUS; J. Breitweg et al., Phys. Lett. **B487**, 53 (2000) and references therein; B. Badełek and J. Kwieciński, Z. Phys. **C43**, 251 (1989); Phys. Lett. **B295**, 263 (1992).
4. SMC; B. Adeva et al., Phys. Rev. **D58**, 112001 (1998).
5. SMC; B. Adeva et al., Phys. Rev. **D60**, 072004 (1999) and erratum: *ibid.* **D62**, 079902 (2000).
6. COMPASS; B. Badełek, talk at the International Europhysics Conference on

High Energy Physics, July 2003, Aachen, Germany; to appear in the Proceedings.
7. U. Stösslein, Acta Phys. Pol. **B33**, 2813 (2002).
8. R.L. Heimann, Nucl. Phys. **B64**, 429 (1973); J. Ellis and M. Karliner, Phys. Lett. **B213**, 73 (1988).
9. N. Bianchi and E. Thomas, Phys. Lett. **B450**, 439 (1999).
10. F.E. Close and R.G. Roberts, Phys. Lett. **B336**, 257 (1994).
11. S.D. Bass and P.V. Landshoff, Phys. Lett. **B336**, 537 (1994).
12. J. Kuti in "The Spin Structure of the Nucleon", World Scientific, Singapore, 1996; M.G. Ryskin, private communication (Durham, 1998).
13. SMC; B. Adeva et al., Phys. Rev. **D58**, 112002 (1998).
14. A. De Roeck et al., Eur. Phys. J. **C6**, 121 (1999).
15. B. Badełek and J. Kwieciński, Phys. Lett. **B418**, 229 (1998).
16. For early QCD analyses see e.g. R. Windmolders, Nucl. Phys. B (Proc. Suppl.) **79**, 51 (1999).
17. M. Glück, E. Reya, M. Stratmann and W. Vogelsang, Phys. Rev. **D63**, 094005 (2001).
18. J. Blümlein and H. Böttcher, Nucl. Phys. **B636**, 225 (2002).
19. E. Leader, A.V. Sidorov and D.B. Stamenov, Eur. Phys. J. **C23**, 479 (2002).
20. C. Bourrely, J. Soffer and F. Buccella, Eur. Phys. J. **C23**, 487 (2002).
21. AAC; M. Hirai et al., hep-ph/0408024.
22. L.N. Lipatov, in "Perturbative QCD", edited by A.H. Mueller, World Scientific, 1989.
23. J. Bartels, B.I. Ermolaev and M.G. Ryskin, Z. Phys. **C70**, 273 (1996); *ibid.* **C72**,627 (1996).
24. B.I. Ermolaev, S.I. Manayenkov and M.G. Ryskin, Z. Phys. **C69**, 259 (1996); S.I. Manayenkov, Z. Phys. **C75**, 685 (1997).
25. J. Kwieciński, Acta Phys. Pol. **B27**, 893 (1996).
26. J. Kwieciński, Phys. Rev. **D26**, 3293 (1982) and references therein.
27. J. Kwieciński and B. Ziaja, Phys. Rev. **D60**, 054004 (1999).
28. J. Kwieciński, Acta Phys. Pol. **B29**, 1201 (1998).
29. B. Badełek, J. Kiryluk and J. Kwieciński, Phys. Rev. **D61**, 014009 (2000).
30. B. Ziaja, Acta Phys. Pol., **B32**, 2863 (2001).
31. SLAC, E143; K. Abe et al., Phys. Rev. **D58**, 112003 (1998).
32. B. Badełek, J. Kwieciński and B. Ziaja, Eur. Phys. J. **C26**, 45 (2002).
33. S.D. Drell and A.C. Hearn, Phys. Rev. Lett. **16**, 908 (1966); S.B. Gerasimov, Sov. J. Nucl. Phys. **2**, 430 (1966); M. Hosoda and K. Yamamoto, Prog. Theor. Phys. Lett. **36**, 425 (1966).
34. B.L. Ioffe, Surveys in High Energy Physics, **8**, 107 (1995).
35. B.L. Ioffe, Phys. At. Nucl. **60**, 1707 (1997).
36. GDH; K. Helbing, Nucl. Phys. (Proc. Suppl.) **105**, 113 (2002).
37. V. Burkert and B.L. Ioffe, Phys. Lett. **B296**, 223 (1992); V. Burkert and B.L. Ioffe, J. Exp. Th. Phys. **78**, 619 (1994).
38. See e.g. http://www.bnl.gov/eic.
39. S.D. Bass and A. De Roeck, Eur. Phys. J. **C18**, 531 (2001).