The Role of Color Neutrality in Nuclear Physics — Modifications of Nucleonic Wave Functions

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Abstract

We explore nucleon swelling in the nuclear medium and relate it to color transparency.

Here we explore one approach to modification of the structure of the nucleon due to being in a nuclear medium[1, 2]. We expect this might be significant since the potential depth in the center of a nucleus is about 50 MeV and a typical excitation energy is about 500 MeV (from the Roper mass). Thus we would expect an effect the order of 50/500 = 10%. Since this is small we treat the interaction with the medium as a perturbation.

We follow the approach of Ref. [1]. Consider a nucleon moving in nuclear matter. The Hamiltonian can be written as:

$$H = H_0 + V \tag{1}$$

where H_0 is the free Hamiltonian for the nucleon and V is its interaction with nuclear matter. Using perturbation theory the in-medium wave function can be written as:

$$|0>_m = |0> + \sum_i \frac{|i><i|V|0>}{E_0 - E_n} \tag{2}$$

where the subscript m means in medium. We will now calculate the in medium value of an arbitrary observable θ. We have:

$$_m<0|\theta|0>_m = <0|\theta|0> + 2\sum_i \frac{<0|\theta|i><i|V|0>}{E_0 - E_n} \tag{3}$$

Now we use closure on the sum over intermediate states, i. If we had used harmonic oscillator states and take $V \propto r^2$ then closure would be exact sine r^2 only connects the ground state to the $2\hbar\omega$ state. Closure gives:

$$_m<0|\theta|0>_m = <0|\theta|0> + 2\frac{<0|\theta V|0> - <0|\theta|0><0|V|0>}{E_0 - \bar{E}} \tag{4}$$

where $\bar{E}$ is the closure energy. Introducing the notation $\Delta\theta$ we have:

$$\Delta\theta = \frac{{}_m\!< 0|\theta|0 >_m - < 0|\theta|0 >}{< 0|\theta|0 >} \tag{5}$$

$$= 2\frac{< 0|V|0 >}{E_0 - \bar{E}}\left(\frac{< 0|\theta V|0 >}{< 0|\theta|0 >< 0|V|0 >} - 1\right) \tag{6}$$

The simplest operator to consider is when θ is a delta function restricting all the quarks to the origin. If we assume that the potential, V, vanishes when all the quarks are at zero separation we have:

$$\Delta\delta = 2\frac{< 0|V|0 >}{\bar{E} - E_0} \tag{7}$$

The quantity, $< 0|V|0 >$, is the expectation value of the full interaction of the nucleon with the nuclear medium. It is the depth of the nuclear potential and is the order of -50 MeV. The closure energy, $\bar{E}$, we take to be the Roper resonance mass, 1.44 GeV. This means that the wave function at the origin is reduced by about 20%. Only three assumptions were made in this estimate: 1) perturbation theory is valid, 2) the interaction with the medium vanishes when the quarks are close together, and 3) closure is valid with a closure energy of 1.44 GeV.

The reduction of the wave function at the origin suggests that the nucleon may be swelling. Let us test this by calculating r^2. Using this operator for θ and assuming that V is also proportional to r^2. This choice of potential satisfies the condition that it vanish when all the quarks are close together. With this choice we have:

$$\Delta r^2 = 2\frac{< 0|V|0 >}{E_0 - \bar{E}}\left(\frac{< 0|r^4|0 >}{< 0|r^2|0 >^2} - 1\right) \tag{8}$$

Since $V \propto r^2$ the proportionality constant cancels between the numerator and denominator. The quantity in large parentheses on the right is positive so the radius increases. The amount is however model dependent. Lets take an oscillator model. In this case the expression in large parenthesis reduces to $2/d$ where d is the dimension of the oscillator. For the six dimensional oscillator expected from the constituent quark model we get about a 7% increase in the radius squared.

If instead of using the oscillator model we related $< 0|r^2|0 >$ and $< 0|r^4|0 >$ to the form factor ($< 0|r^2|0 >= -6df/dq^2, < 0|r^4|0 >= 60d^2f/dq^4$). For the standard dipole form factor this will give a 15% reduction in $< r^2 >$. Twice the effect for the oscillator model. Thus we see that there is considerable model dependence in this quantity even after we have made the assumptions of the interaction depending on the nucleon size.

In this talk we are considering entirely low energy phenomena and normal low energy techniques should be relevant. In this regime the assumption that the coupling strengths goes like the size is quite unusual. It is far more common to assume that the mesons couple directly to the quarks independently[3]. Thus the only size effect comes in through the form factor which goes to one for zero momentum transfer. Examples of such models are cloudy bag model, meson quark coupling model, and non-relativistic quark models. Even in models that do not have explicit quark degrees of freedom the ω meson couples to the conserved baryon current. Thus again we do not have the interaction vanishing as the quarks become close together. The reduction of the wave function at the origin and the swelling we see will not be obtained with most of the commonly used models for the nucleon-nucleon interaction but depends on the perturbative QCD input.

The suppression of the wave function at the origin can be related to color transparency[4], the suppression of initial and final state interactions in the $(p, 2p)$ reaction and final state proton interactions in $(e, e'p)$ reaction. Both color transparency and the nuclear swelling rely on small objects interacting weakly. They also rely on closure. The condition for closure in color transparency is[5]:

$$\frac{{E_i}^2 - E_0^2}{q} \ll \frac{2\sqrt{6}}{R_A}.$$

For $E_i = 1.44$ GeV this gives:

$$q >> 1.1 \text{ GeV} A^{1/3}$$

For $E_i = 2.00$ GeV this gives

$$q >> 2.9 \text{ GeV} A^{1/3}$$

$\bar{E}$	0.5 GeV	1.0 GeV
$\Delta\delta$	20%	10%
E_j	1.5 GeV $A^{1/3}$	3.0 GeV $A^{1/3}$
E_j (^{27}Al)	4.5 GeV	9.0 GeV

In both the nuclear swelling calculation and color transparency it is V, the interaction with the medium, that cuts of the sum over excited states of the nucleon. Thus we expect similar energies to be important in both cases. In the Table we show for a given closure energy, $\bar{E}$, the change in the wave function at the origin and the energy for the ejectile, E_j, where we expect color transparency effects to set in. If color transparency is observed at energies available at Jefferson National Laboratory we expect the reduction of the wave function at the origin to be important.

References

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