Partonic view of radiative corrections to elastic electron-proton scattering

Carl E. Carlson

Nuclear and Particle Theory Group

College of William and Mary

JLab, Newport News, 05 March 2004

Topics

- Review motivation
- Radiative corrections to elastic eN scattering—partonic calculation
- Applications to Rosenbluth determination of form factors
- Radiative corrections to polarizations and e^+p scattering

Collaborators

Andrei Afanasev (JLab)

Stan Brodsky (SLAC)

Yu-Chun Chen (NTU)

Marc Vanderhaeghen (JLab/W&M)

See it on the ArXiv: hep-ph/0403058

Introduction

Problem!

F_2, or G_E, for the proton, measured different ways gives different results.

Rosenbluth separation method

Cross section for one-photon exchange (w/ no polarization)

$$\frac{d\sigma}{d\Omega_{Lab}} = \frac{\sigma_{NS}}{\epsilon(1+\tau)}\Big(\tau|G_M(Q^2)|^2 + \epsilon|G_E(Q^2)|^2\Big)$$

where

$$\tau \equiv \frac{Q^2}{4M^2}\,, \qquad \frac{1}{\epsilon} \equiv 1 + 2(1+\tau)\tan^2\frac{\theta}{2}$$

(Note: forward direction, $\theta \to 0$, means $\epsilon \to 1$.)

Method: fix Q^2, vary angle (vary ϵ), adjusting incoming energy as needed,

and plot reduced cross section

$$|G_M|^2 + \frac{\epsilon}{\tau}|G_E|^2$$

vs. ϵ.

Get (one-photon theorist's view):

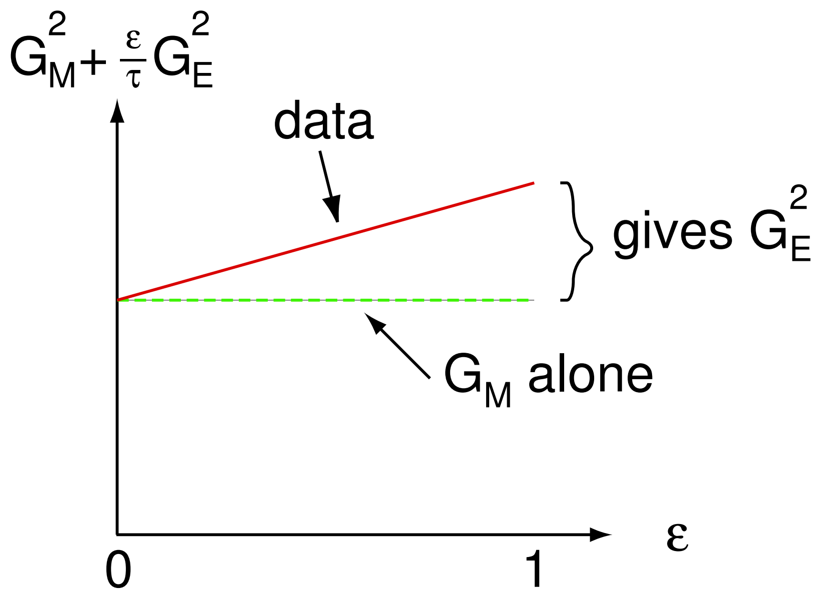


FIG. 1: Rosenbluth plot

Polarization transfer method

Polarized electron beam $\Rightarrow$ sideways and longitudinal proton polarization

$$\overrightarrow{e} + p \rightarrow e + \overrightarrow{p}$$

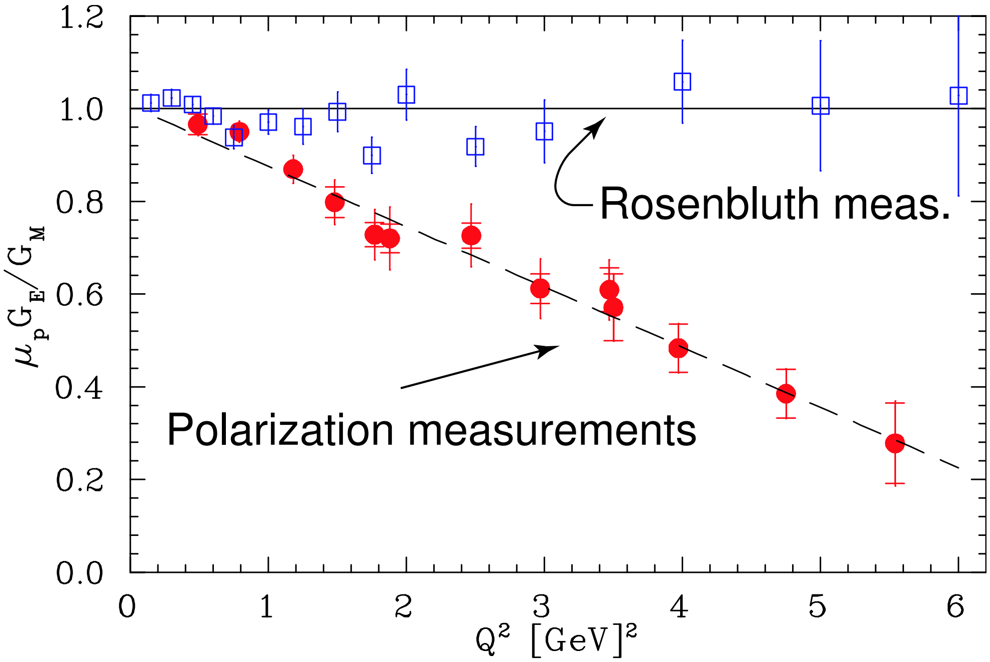
Measure sideways/longitudinal polarization ratio.

Get form factor ratio from,

$$\frac{\mathcal{P}_x}{\mathcal{P}_z} = -\sqrt{\frac{2\epsilon}{\tau(1+\epsilon)}}\,\frac{G_E}{G_M}\ ,$$

which follows from a lowest order (one-photon exchange) calculation.

Results:



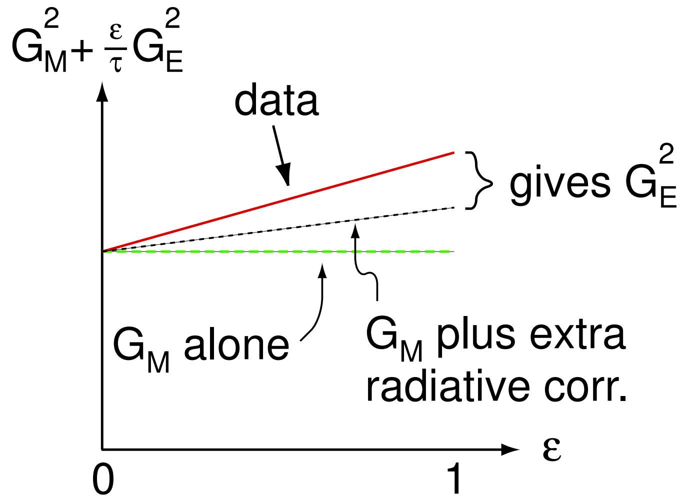
(figure from Arrington, PRC 2003)

Two methods, two results.

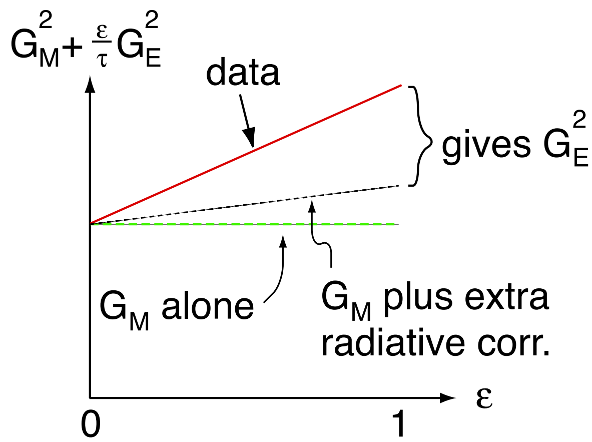
Dream solution

Dream: There are radiative corrections to the Rosenbluth experiment that are important and not yet included.

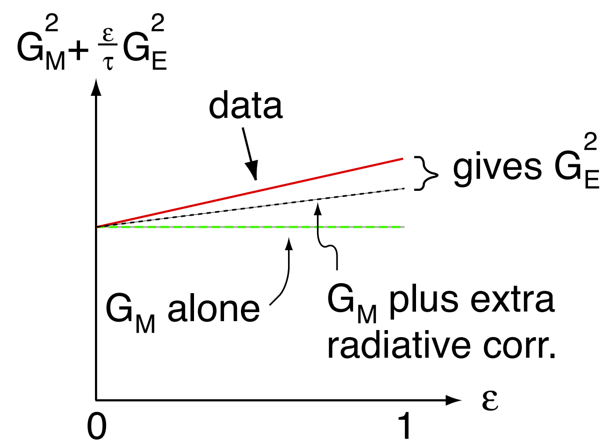
Further: the unincluded corrections are linear in ϵ, with positive slope.



Still further dreaming: The extra radiative corrections are not strongly Q^2 dependent. Since contributions from G_E^2 terms are smaller at high Q^2, have



Low τ (Low Q^2)



High τ (High Q^2)

I.e., the dream-solution Rosenbluth-extracted G_E shrinks more at high Q^2 than at low Q^2.

Numerical note:

Take $Q^2 = 6$ Gev^2, and find

$$\frac{G_E^2}{\tau G_M^2} = \frac{4M^2}{Q^2 \mu_p^2} = 7.6\%$$

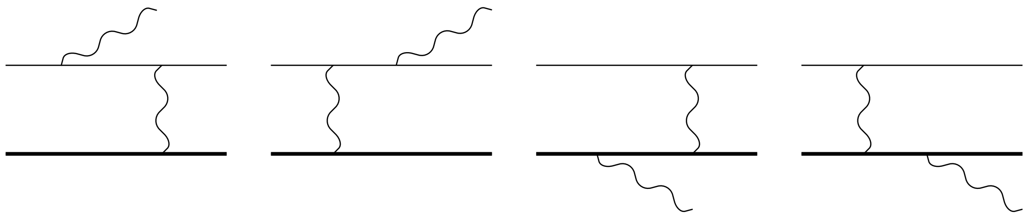
if G_E and G_M both scale the same way.

Typically, radiative corrections are a few percent (a few times α, in this case, not just α/π), and ϵ dependent. Thus, the radiative corrections are of the same size as the Rosenbluth measurement needed to determine G_E (at high Q^2).

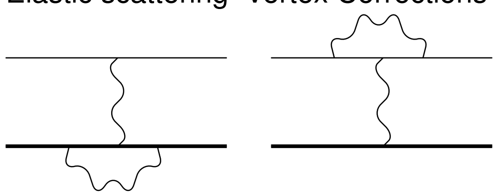
Where should we look?

Radiative Correction Diagrams:

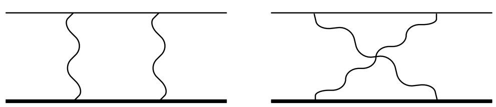
Bremsstrahlung



Elastic scattering–Vertex Corrections



Elastic Scattering–Box Diagrams



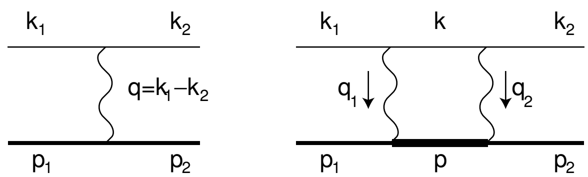
Comments

- Electrons are well understood. Corrections involving just electrons well done.
- Bremsstrahlung involves soft, long wavelength, photons. Compositeness of proton should not be an issue for bremsstrahlung.
- Box diagrams involve photons of all wavelengths. Contributions where one photon is soft are easy and give Coulomb phase correction times lowest order.
- Box contributions where both photons are hard require treating proton as structured, composite, system. Not done in "old days."

So: There is an opening. Study two-photon exchange (box) contributions.

Preliminary: what has been done

♣ "Old days" E.g., Tsai [1961] or Mo & Tsai [1968] box diagram evaluation

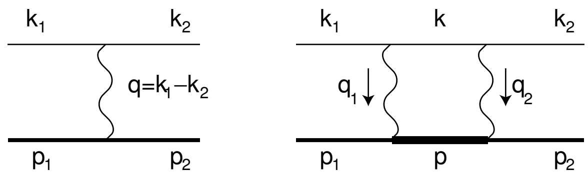


Did minimal calculation to give the IR divergent terms correctly. (Box diagram IR divergences needed to cancel bremsstrahlung IR divergences.)

- Intermediate hadron only proton.
- Note IR divergences come from $q_1 \approx 0$ or $q_2 \approx 0$. Hence, set (e.g.) $q_2 = 0$ everywhere "safe." Meaning: $q_2 = 0$ in q_1 propagator, and in numerator.
- Not accurate when both photons hard. Quote: "assume the noninfrared parts of these diagrams to be negligible." Honest and totally o.k. if true.

♡ Improved by Maximon and Tjon [2000] ($q_2 \to 0$ in fewer places).

♠ Better hadronic evaluation: Blunden, Melnitchouk, and Tjon [2003]



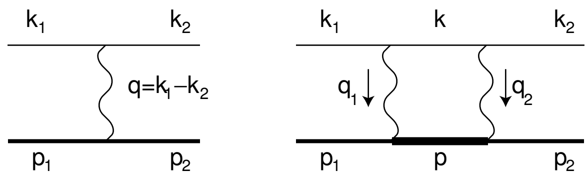
- Intermediate hadron only proton
- Include form factors for proton, within integral
- But need form factors when proton, not just photon, is off-shell.
- At higher momentum transfers, need resonances in intermediate state if pursuing hadronic calculation.

♢ Also 2003, Possibility proof for parton calc.: Guichon & Vanderhaeghen

- But not *ab initio* calculation.

Partonic Evaluation of Box Diagrams.

Start with notation for $ep \to ep$ amplitudes



Basic: extra structure from the multiple photon exchange,

$$\mathfrak{M}^N = \frac{Ze^2}{Q^2}\Big\{\bar{u}(k_2)\gamma_\mu u(k_1) \times \bar{u}(p_2)\left[\gamma^\mu G'_M - \frac{(p_1+p_2)^\mu}{2M}F'_2\right]u(p_1)$$

$$+\,\bar{u}(k_2)\gamma_\mu\gamma_5 u(k_1) \times \bar{u}(p_2)\left[\gamma^\mu\gamma^5 G'_A\right]u(p_1)\Big\}$$

Alternatively, equivalent to

$$\mathfrak{M}^N = \frac{Ze^2}{Q^2}\,\bar{u}(k_2)\gamma_\mu u(k_1) \times \bar{u}(p_2)\left[\gamma^\mu \tilde{G}_M - \frac{(p_1+p_2)^\mu}{2M}\tilde{F}_2 + \frac{(p_1+p_2)^\mu(\not{k}_1+\not{k}_2)}{4M^2}\tilde{F}_3\right]u(p_1)$$

Form factors above have contributions from 1γ and 2γ exchanges,

$$\begin{aligned} G'_M &= G_M^{(1\gamma)} + G_M'^{(2\gamma)} = G_M + \delta G'_M \\ G'_E &= G_E + \delta G'_E \\ G'_A &= \text{zero } + \delta G'_A \end{aligned}$$

$G_{M,E}$ are standard form factors defined from matrix element of e.m. current.

Cross section (to LO and $\mathcal{O}(e^2)\times$ LO),

$$\frac{d\sigma}{d\Omega_{Lab}} = \frac{\sigma_{NS}}{\epsilon(1+\tau)}\left(\tau|G'_M|^2 + \epsilon|G'_E|^2 + 2\sqrt{\tau(1+\tau)(1-\epsilon^2)}\,\mathrm{Re}\,G'^*_M G'_A\right)$$

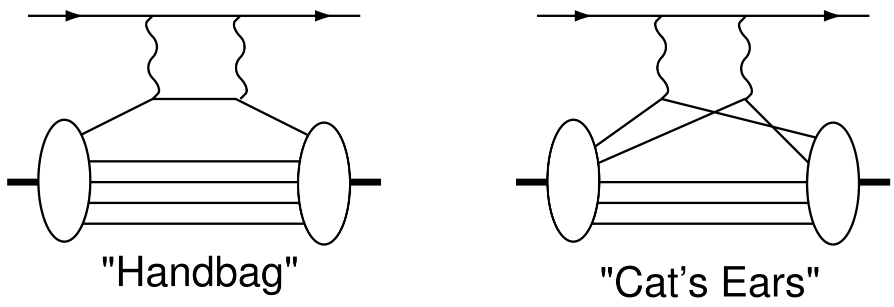
with

$$\sigma_{NS} = \frac{4\alpha^2\cos^2(\theta/2)}{Q^4}\,\frac{E_2^3}{E_1}$$

Partonic calculations

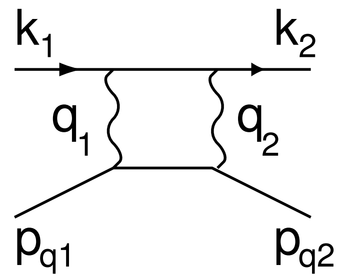
• Main contributions come from "handbag" diagrams (one active quark).

• "Cat's ears" diagrams, where photons interact with different quarks, important for getting overall IR divergence correct. However, contributions when both photons are hard is suppressed at higher Q^2.



• Calculate box (and crossed box) diagram at quark level, then embed in proton using generalized parton distribution (GPD).

Box diagrams for $eq \to eq$, with massless quarks



$$\mathfrak{M}^q = \mathfrak{M}^q_{LO} + \frac{e^2}{Q^2}\,\bar{u}(k_2)\gamma_\mu u(k_1) \times \bar{u}(p_{q2})\left[\gamma^\mu\, e_q^2 \tilde{f}_1 + P_q^\mu\ \not{K}\, e_q^2 \tilde{f}_3\right] u(p_{q1})$$

for $P_q \equiv (p_{q1} + p_{q2})/2$ and $K \equiv (k_1 + k_2)/2$.

Calculation same as $e\mu \to e\mu$, which has been done analytically, can be found in the literature (e.g., van Nieuwenhuizen [1971]), and has been verified locally.

- Boxes have IR divergence, which must cancel or disappear in end; control by temporarily putting in photon mass λ.
- Separate soft (IR divergent) and hard parts by criterion of Grammer and Yennie.

We have both real and imaginary parts of $\tilde{f}_1$ and $\tilde{f}_3$.

For here, just display imaginary parts,

$$\begin{aligned}
\mathrm{Im}\, \tilde{f}_1^{soft} &= \frac{e^2}{4\pi} \ln\left(\frac{\hat{s}}{\lambda^2}\right) \\
\mathrm{Im}\, \tilde{f}_1^{hard} &= \frac{e^2}{4\pi} \left\{ -\frac{Q^2}{2\hat{u}} \ln\left(\frac{\hat{s}}{Q^2}\right) - \frac{1}{2} \right\} \\
\mathrm{Im}\, \tilde{f}_3 &= -\frac{e^2}{4\pi} \frac{1}{\hat{u}} \left\{ \frac{\hat{s}-\hat{u}}{2\hat{u}} \ln\left(\frac{\hat{s}}{Q^2}\right) + 1 \right\}
\end{aligned}$$

($\hat{s}$ and $\hat{u}$ are Mandelstam variables for the subprocess $eq \to eq$).

And also

$$\mathrm{Re}\, \tilde{f}_1^{soft} = \frac{e^2}{4\pi^2} \left\{ \ln\left(\frac{\lambda^2}{\sqrt{-\hat{s}\hat{u}}}\right) \ln\frac{\hat{s}}{-\hat{u}} + \frac{\pi^2}{2} \right\}$$

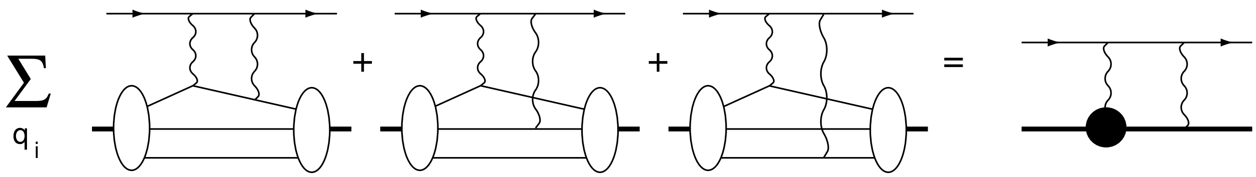
Soft contributions

• ∃ low energy theorem: sum of soft contributions from partonic calculations equals soft contributions from nucleonic calculation.

• Works because there are also soft contributions from cat's ears diagrams.

Pictorial explanation of low energy theorem:

Say that right-hand photon is the soft one.



LHS equivalent to one hard photon, with form factor, and one soft photon on nucleon.

• Repeat: hard parts from cat's ears diagrams are subleading in Q^2 because of momentum mismatches in integrals.

• For imaginary parts, consequence of low energy theorem is that all amplitudes multiplied by same Coulomb phase. $\therefore$ contribution of soft parts to coming calculation of $\mathcal{A}_n$ is zero.

• Real parts and bremsstrahlung: next page.

• The IR divergence in the box is cancelled by an IR divergence from bremsstrahlung, specifically an interference between bremsstrahlung from the electron and bremsstrahlung from the proton. Write

$$\sigma_{1\gamma+2\gamma,soft} = \sigma_{1\gamma}\left(1 + \delta_{2\gamma}^{soft} + \delta_{brems}^{ep}\right)$$

• Because of low energy theorem, take $\delta_{2\gamma}^{soft}$ from nucleonic calculation,

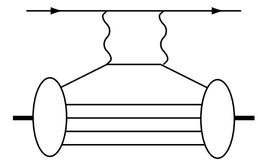
$$\delta_{2\gamma}^{soft} = \frac{e^2}{2\pi^2}\left\{\text{Nucleonic}\left[\ln\left(\frac{\lambda^2}{\sqrt{-\hat{s}\hat{u}}}\right)\ln\frac{\hat{s}}{-\hat{u}}\right] + \frac{\pi^2}{2}\right\}$$

• Take bremsstrahlung from Maximon and Tjon [2000]

• Compare numerically to corresponding Mo and Tsai correction: essentially the same (to 0.1% level) *except* for the $\pi^2/2$ term.

• Thus, since data generally presented with Mo-Tsai correction done, soft corrections give a constant factor $(1+\pi\alpha)$ plus terms that are quite small.

Hard contributions.



Embed partonic calculation in a nucleon.

• Set-up for generalized parton distributions: remove a quark from the proton, and replace it with a quark of different momentum and possibly different helicity.

$$\mathfrak{M}^N_{h,\lambda_2,\lambda_1} = \int_{-1}^{1} \frac{dx}{x} \sum_q \frac{1}{2P^+} \left[\mathfrak{M}^q_{h,+1/2} + \mathfrak{M}^q_{h,-1/2} \right] \bar{u}_{\lambda_2}(p_2) \left[\gamma^+ H^q + \frac{i\sigma^{+\nu} q_\nu}{2M} E^q \right] u_{\lambda_1}(p_1)$$
$$+ \int_{-1}^{1} \frac{dx}{x} \sum_q \frac{1}{2P^+} \left[\mathfrak{M}^q_{h,+1/2} - \mathfrak{M}^q_{h,-1/2} \right] \mathrm{sgm}(x)\ \bar{u}_{\lambda_2}(p_2)\ \gamma^+ \gamma^5 \tilde{H}^q\ u_{\lambda_1}(p_1)$$

• Work in light-front frame, $q^+ \propto q^0 + q^3 = 0$.

• Arguments of GPD's are $H^q(x, \xi = 0, Q^2)$, etc.

The 2γ corrections to the nucleon form factors become,

$$
\begin{aligned}
\delta G'_M &= \frac{1+\epsilon}{2\epsilon} A - \frac{1-\epsilon}{2\epsilon} C \\
\delta G'_E &= \sqrt{\frac{1+\epsilon}{2\epsilon}}\; B \\
\delta G'_A &= \frac{t}{s-u}\frac{1+\epsilon}{2\epsilon}\,(A-C)\;\;,
\end{aligned}
$$

where the characteristic integrals are,

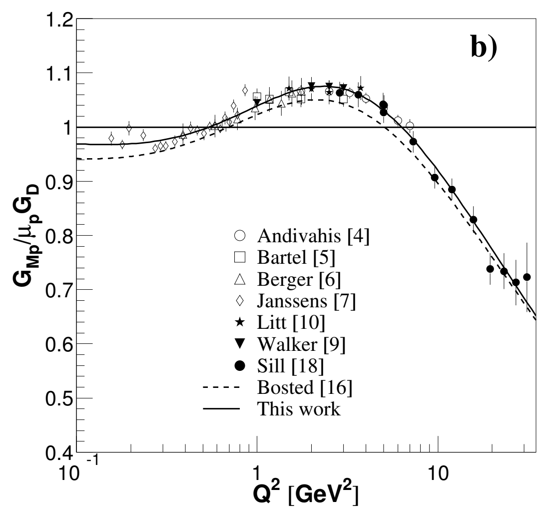
$$
\begin{aligned}
A &= \int_{-1}^{1} \frac{dx}{x}\, \frac{(\hat{s}-\hat{u})\tilde{f}_1^{hard} - \hat{s}\hat{u}\tilde{f}_3}{s-u} \sum_q e_q^2 (H^q + E^q) && \text{"electric GPD"} \\
B &= \int_{-1}^{1} \frac{dx}{x}\, \frac{(\hat{s}-\hat{u})\tilde{f}_1^{hard} - \hat{s}\hat{u}\tilde{f}_3}{s-u} \sum_q e_q^2 (H^q - \tau E^q) && \text{"magnetic GPD"} \\
C &= \int_{-1}^{1} \frac{dx}{x}\, \tilde{f}_1^{hard}\, \mathrm{sgm}(x) \sum_q e_q^2 \tilde{H}^q\;. && \text{"axial GPD"}
\end{aligned}
$$

Final inputs

- G_{Ep}/G_{Mp} from polarization transfer data,

$$G_{Ep} = \left(1 - 0.13(Q^2 - 0.04)\right) \frac{G_{Mp}}{\mu_p}$$

- G_{Mp} from analytic fit of Brash *et al.*, solid line in



• For GPD's use gaussian-valence model like Radyushkin and Diehl *et al.*,

$$
\begin{aligned}
H^q(x,0,Q^2) &= q_v(x)\exp\left(-\frac{(1-x)Q^2}{4x\sigma}\right) \\
\tilde{H}^q(x,0,Q^2) &= \Delta q_v(x)\exp\left(-\frac{(1-x)Q^2}{4x\sigma}\right) \qquad \left[\text{used } \sigma = 0.8 \text{ GeV}^2\right] \\
E^q(x,0,Q^2) &= \frac{\kappa_q}{N^q}(1-x)^2 q_v(x)\exp\left(-\frac{(1-x)Q^2}{4x\sigma}\right)
\end{aligned}
$$

• Valence quark distributions are from Martin, Stirling, Roberts, and Thorne (MRST2002 NNLO fit at baseline $Q_0^2 = 1GeV^2$),

$$
\begin{aligned}
u_v &= 0.262\, x^{-0.69}(1-x)^{3.50}\left(1+3.83\sqrt{x}+37.65x\right) \\
\Delta u_v &= 0.505\, x^{-0.33}(1-x)^{3.428}\left(1+2.179\sqrt{x}+14.57x\right) \\
d_v &= 0.061\, x^{-0.65}(1-x)^{4.03}\left(1+49.05\sqrt{x}+8.65x\right) \\
\Delta d_v &= -0.0185\, x^{-0.73}(1-x)^{3.864}\left(1+35.47\sqrt{x}+28.97x\right)
\end{aligned}
$$

Rosenbluth Plot Results

Plot reduced cross section, normalized to dipole form factor, vs. ϵ.

Recall,

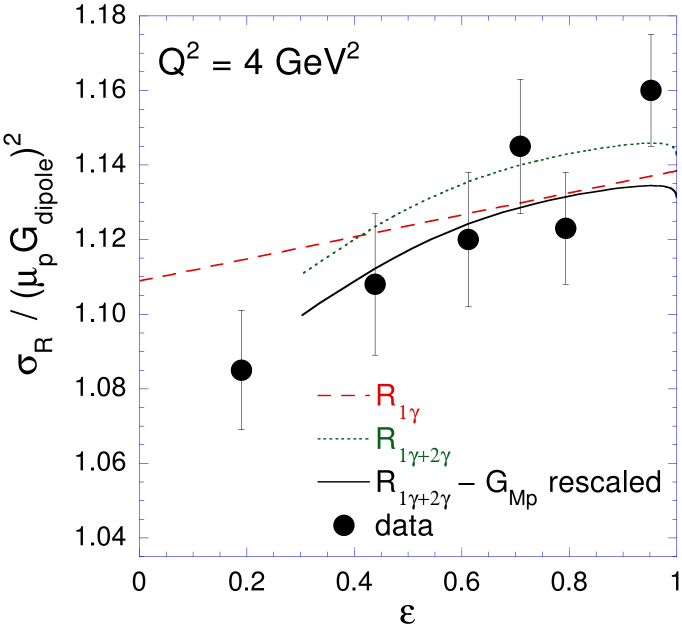
$$\frac{d\sigma}{d\Omega_{Lab}} = \frac{\tau\,\sigma_{NS}}{\epsilon(1+\tau)}\,\sigma_R$$

$$\sigma_R = \left(G_M^2 + \frac{\epsilon}{\tau}G_E^2\right)\left(1 + \delta_{2\gamma}^{soft} + \delta_{brems}^{ep} - \delta_{MT}\right)$$
$$+\ (1+\epsilon)G_M\,\mathrm{Re}\,A + \frac{\sqrt{2\epsilon(1+\epsilon)}}{\tau}G_E\,\mathrm{Re}\,B + (1-\epsilon)G_M\,\mathrm{Re}\,C$$

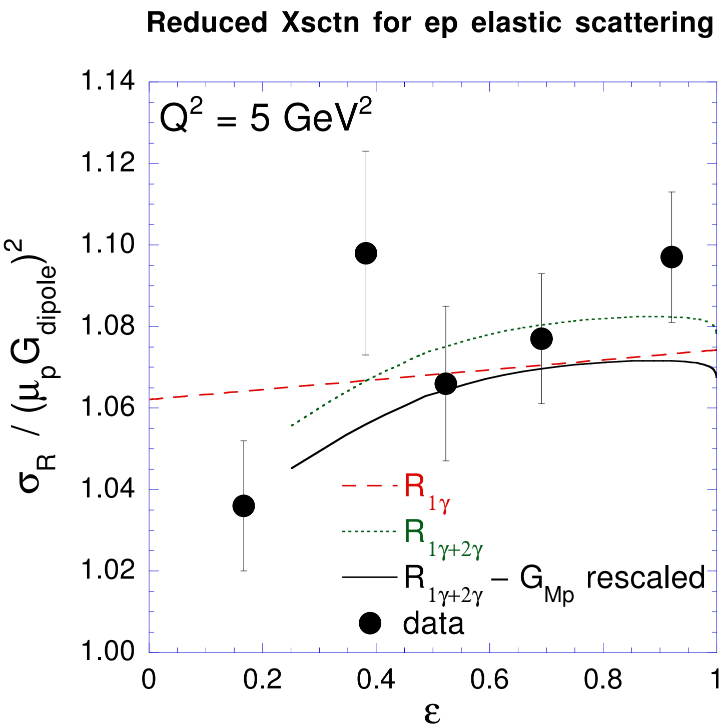
Plots show

$$R \equiv \frac{\sigma_R}{\mu_p^2 G_{dipole}^2} \qquad ; \qquad G_{dipole} \equiv \left(1 + \frac{Q^2}{0.71\ \mathrm{GeV}^2}\right)^{-2}$$

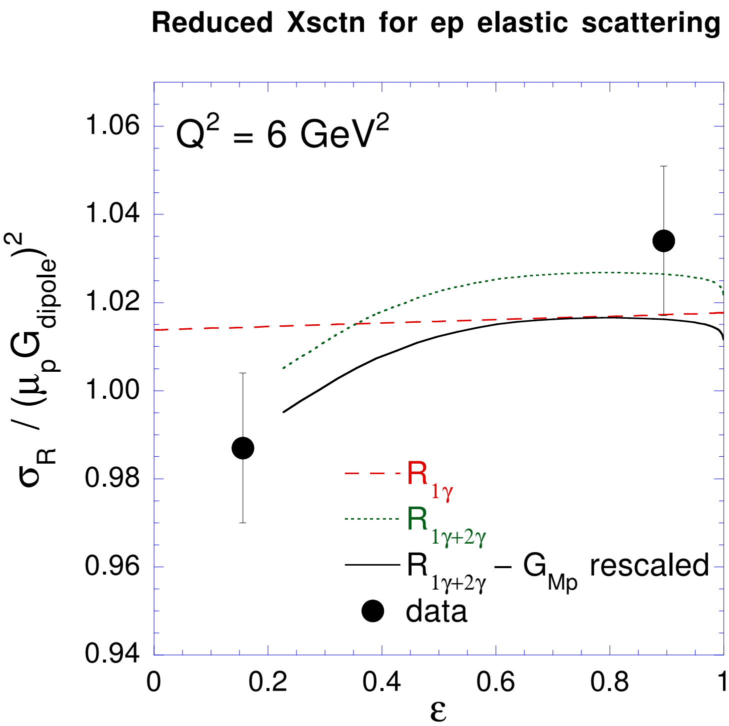
Reduced Xsctn for ep elastic scattering



dashed (red): LO result, w/ Brash *et al.* G_{Mp}
dotted (green): our full result, w/ Brash *et al.* G_{Mp}
full curve (black): our full result, w/ Brash *et al.* $G_{Mp} \times 0.995$.



Reduced Xsctn for ep elastic scattering
$Q^2 = 5\ GeV^2$
$\sigma_R\ /\ (\mu_p G_{dipole})^2$
1.14
1.12
1.10
1.08
1.06
1.04
1.02
1.00
0
0.2
0.4
0.6
0.8
1
ε
$R_{1\gamma}$
$R_{1\gamma+2\gamma}$
$R_{1\gamma+2\gamma} - G_{Mp}$ rescaled
data


Reduced Xsctn for ep elastic scattering
$Q^2 = 6\ GeV^2$
$\sigma_R / (\mu_p G_{dipole})^2$
1.06
1.04
1.02
1.00
0.98
0.96
0.94
0
0.2
0.4
0.6
0.8
1
ε
$R_{1\gamma}$
$R_{1\gamma+2\gamma}$
$R_{1\gamma+2\gamma}$ – G_{Mp} rescaled
data

Re: Rosenbluth plots

• Polarization transfer determined form factors do not fit data, if just Mo-Tsai (e.g) are only radiative corrections applied.

• Including hard two-photon exchange corrections changes the slope in ϵ and reconciles the Rosenbluth and polarization transfer data. Dependence in ϵ not linear.

• Should do reanalysis of extraction of G_{Mp} and G_{Ep} from data using full HO corrections. Beyond today's scope. Did show that reducing present good G_{Mp} fit by (1/2)% could improve fit to data.

Polarization Results

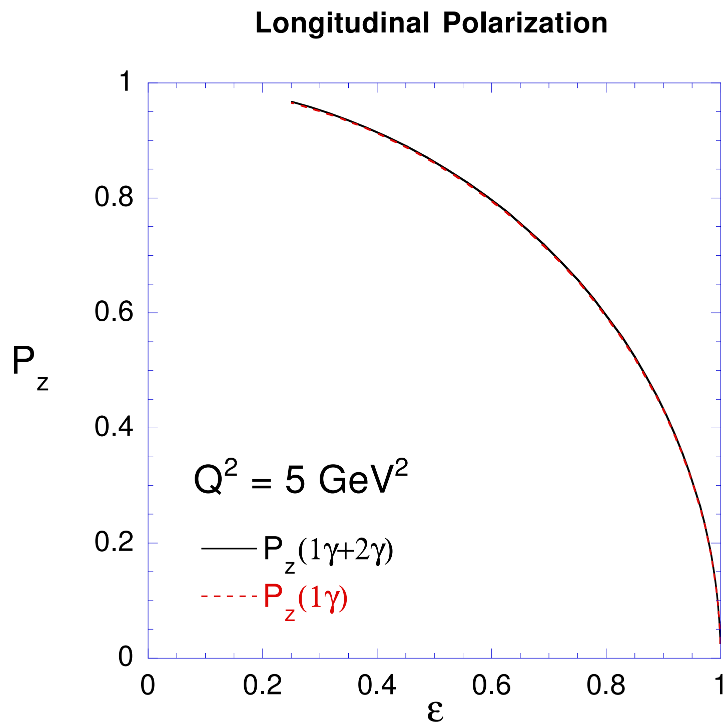
Analyzing powers and polarizations:

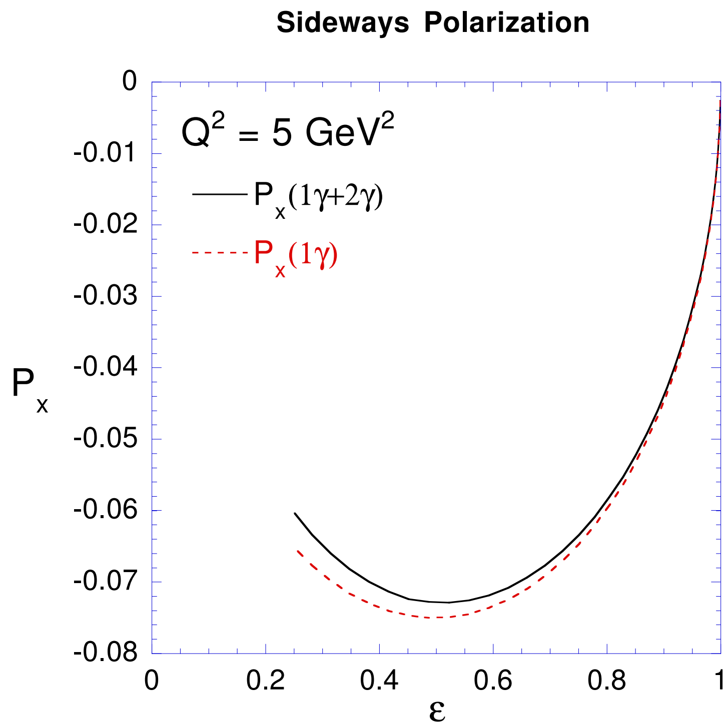
$$\sigma_R \mathcal{A}_x = -(2h_e)\sqrt{\frac{2\epsilon(1-\epsilon)}{\tau}}\left\{G_E G_M + \sqrt{\frac{1+\epsilon}{2\epsilon}} G_M \operatorname{Re} B + G_E \operatorname{Re} C\right\} = \sigma_R \mathcal{P}_x$$

$$\sigma_R \mathcal{A}_y = \sqrt{\frac{2\epsilon(1+\epsilon)}{\tau}}\left\{G_E \operatorname{Im} A - \sqrt{\frac{1+\epsilon}{2\epsilon}} G_M \operatorname{Im} B\right\} = \sigma_R \mathcal{P}_y$$

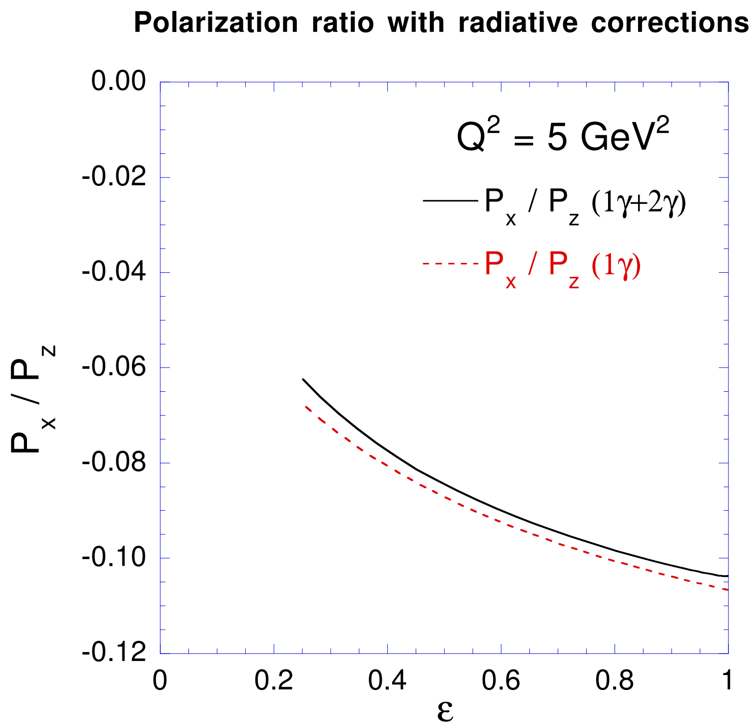
$$\sigma_R \mathcal{A}_z = -(2h_e)\sqrt{1-\epsilon^2}\left\{G_M^2 + G_M \left(\operatorname{Re} A + \operatorname{Re} C\right)\right\} = -\sigma_R \mathcal{P}_z$$

h_e = electron helicity $= \pm 1/2$.

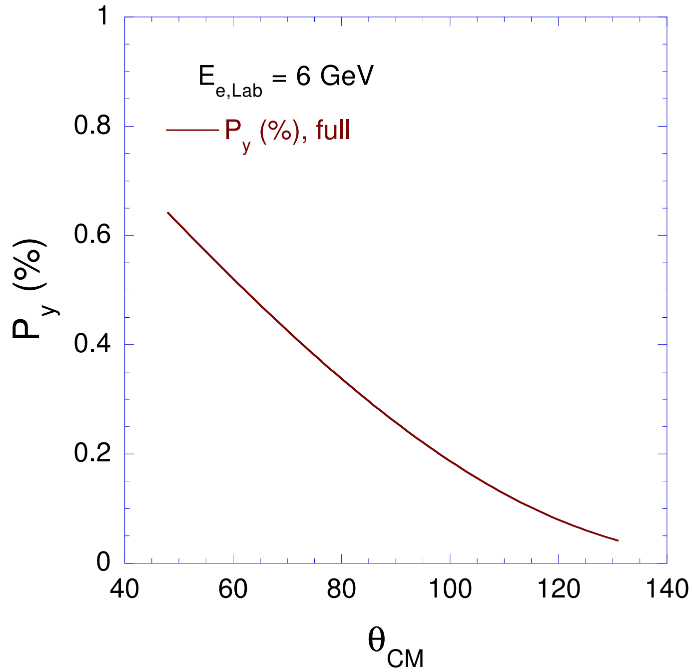

Longitudinal Polarization
1
0.8
0.6
0.4
0.2
0
P_z
Q^2 = 5 GeV^2
$P_z(1\gamma+2\gamma)$
$P_z(1\gamma)$
0
0.2
0.4
0.6
0.8
1
ε



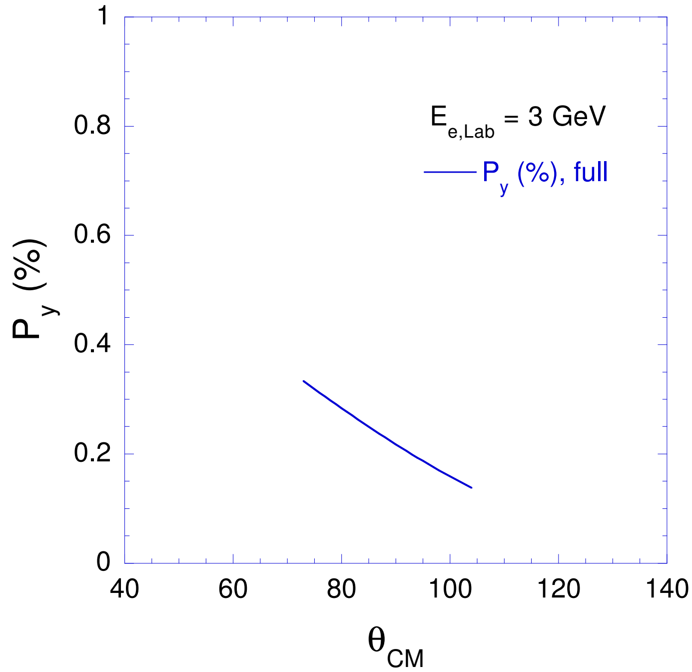
Sideways Polarization
$Q^2 = 5\ GeV^2$
$P_x(1\gamma+2\gamma)$
$P_x(1\gamma)$
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
-0.08
P_x
0
0.2
0.4
0.6
0.8
1
ε


Polarization ratio with radiative corrections
Q^2 = 5 GeV^2
P_x / P_z (1γ+2γ)
P_x / P_z (1γ)
P_x / P_z
ε
0.00
-0.02
-0.04
-0.06
-0.08
-0.10
-0.12
0
0.2
0.4
0.6
0.8
1

Normal Polarization or Analyzing Power

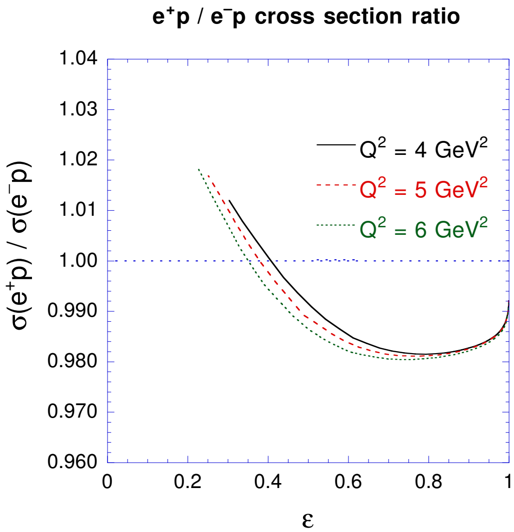


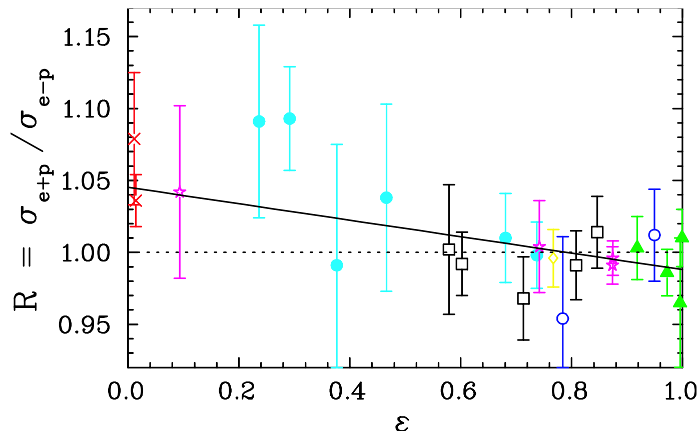
Normal Polarization or Analyzing Power



(Cut off at ends when $-t = M^2$ or $-u = M^2$.)

Electron-Positron Ratio Results





(data figure from Arrington)

Closing Comments

• Presented a partonic calculation of the two-photon exchange corrections to elastic electron-proton scattering.

• Valid for high Q^2, say $Q^2 >> M^2$

• In comparing to data, used G_{EP}/G_{Mp} from polarization measurements.

• Find that in Rosenbluth plot two-photon exchange corrections give additional slope, sufficient to reconcile Rosenbluth and polarization data.

• Detail: Soft photon corrections shifted the data but did not introduce a slope (compared to existing Mo-Tsai corrections). Change in slope came from hard (both photons energetic) corrections.

more closing comments

- For sideways and longitudinal polarization, corrections small.
- For normal direction, predict $\mathcal{O}(1/2\%)$ polarization
- Predict $\mathcal{O}(\mathrm{few}\%)$ effects in positron-proton/electron-proton cross section ratio.

The End