Inclusive and Semi-inclusive Hard Processes (II)

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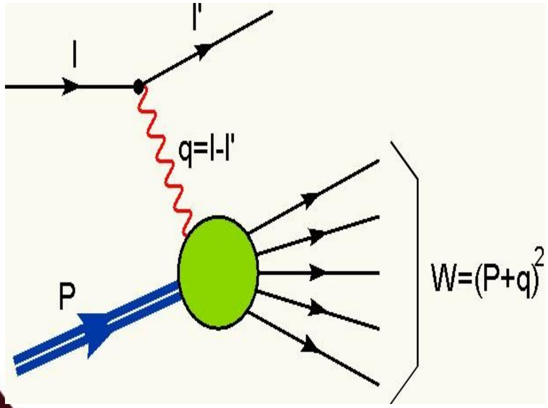
Last Lecture...

- Parton Model:

  Nucleon: No elementary particles ⟶ Constituents: Quarks and Gluons (Partons)

  Strong Interactions: Quantum Chromodynamics (QCD)

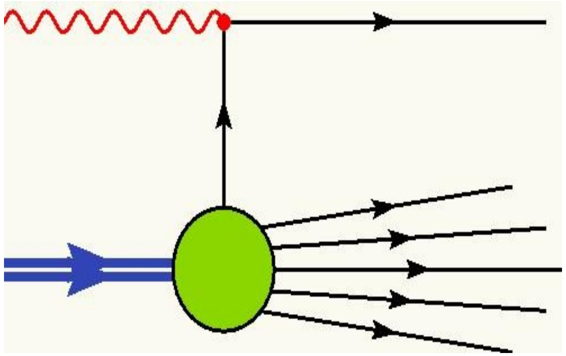
- Inclusive Deep-Inelastic Scattering (DIS):



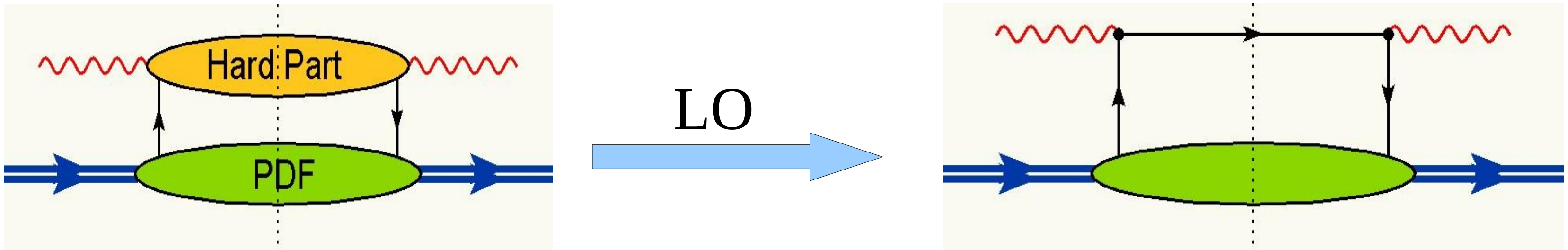
Kinematics:

- large virtuality Q > 1 GeV
- Infinite-Momentum frame, P^+ large

Parton Model:



Factorization of the Cross Section:



$$\boxed{\sigma_{\mathrm{DIS}} \sim (\mathrm{hard}) \otimes (\mathrm{soft})}$$

Hard part:

- Lepton-Parton scattering
- asymptotic freedom $\Longrightarrow \alpha_s(Q)$
- perturbatively calculable

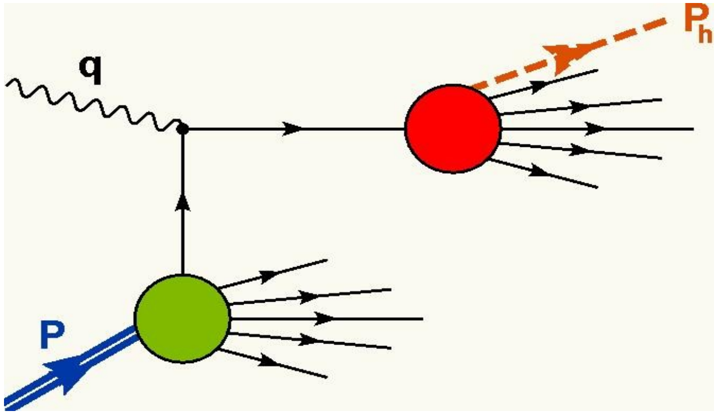
Soft Part:

- non-perturbative $\Longrightarrow$ Experiments, Lattice, Models
- PDFs: $f_1(x)$, $g_1(x)$, $h_1(x)$
- collinear picture
- Universality

Transversity $h_1(x)$ not feasible in inclusive DIS

$\Longrightarrow$ Semi-inclusive DIS

Semi-inclusive DIS



kinematical variables:

$$x_B = \frac{Q^2}{2P\cdot q}\ ;\ Q^2 \quad z_h = \frac{P\cdot P_h}{P\cdot q}\ ;\ \vec{P}_{h\perp}$$

Amplitude:

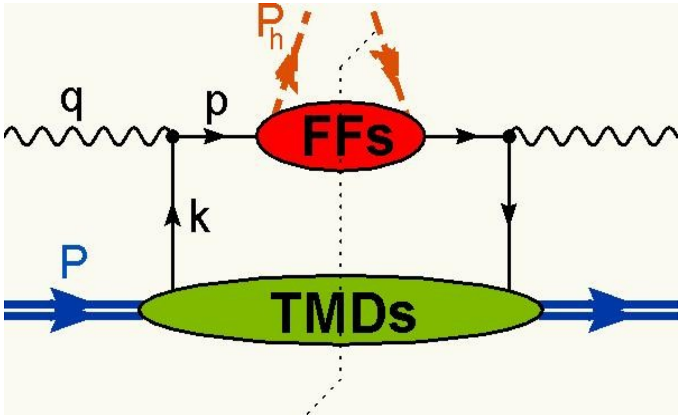
$$iM_{X,X_h} = e_q \langle P_h;\ X_h|\,\bar{\psi}_i(0)\,|0\rangle \gamma^{\mu}_{ij} \langle X|\,\psi_j(0)\,|P,S\rangle$$

Cross section $\Longrightarrow$ squared amplitude

$$d\sigma \propto \sum_{X,X_h} |M_{X,X_h}|^2 (2\pi)^4 \delta^{(4)}(P+q-P_X-P_{X_h}-P_h)\frac{d^3l'}{(2\pi)^3 2E_{l'}}\frac{d^3P_h}{(2\pi)^3 2E_{P_h}}$$

$$d\sigma \propto L_{\mu\nu}W^{\mu\nu}$$

- Hadronic Tensor:



$$2MW^{\mu\nu} = \int d^4k d^4p\, \delta^{(4)}(k+q-p) \mathrm{Tr}[\Phi(k)\gamma^\mu \Delta(p)\gamma^\nu]$$

- Two soft objects: Partonic distribution and fragmentation.

$$\Phi_{ij}(k) = \int \frac{d^4z}{(2\pi)^4}\, e^{ik\cdot z} \langle P,S|\, \bar{\psi}_j(0)\, \psi_i(z)\, |P,S\rangle$$

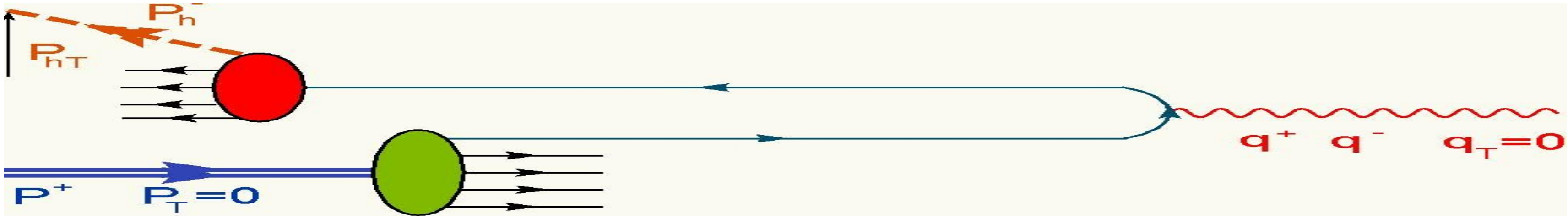
$$\Delta_{ij}(p) = \sum_X \int \frac{d^4z}{(2\pi)^4}\, e^{ip\cdot z} \langle 0|\, \psi_i(z)\, |P_h;\, X\rangle \langle P_h;\, X|\, \bar{\psi}_j(0)\, |0\rangle$$

Simple spin sum in DIS $\Longrightarrow$ Fragmentation correlator in SIDIS

Choose Infinite Momentum Frame (Nucleon and Photon collinear)

$$\boxed{P^+ = \frac{Q}{\sqrt{2}x_B}\,,\ q^+ = -\frac{Q}{\sqrt{2}}\,,\ q^- = \frac{Q}{\sqrt{2}}\,,\ P_T = q_T = 0}$$

$$\boxed{P_h^- = \frac{z_h Q}{\sqrt{2}}\,,\ \vec{P}_{h\perp}}$$



Hadronic Tensor: $2MW^{\mu\nu} \simeq \int dk^+ d^2k_T \int dp^- d^2p_T\, \delta(k^+ + q^+)\delta(q^- - p^-)\delta^{(2)}(\vec{k}_T - \frac{\vec{P}_{h\perp}}{z_h} - \vec{p}_T)\,\mathrm{Tr}[\int dk^- \Phi(k)\gamma^\mu \int dp^+ \Delta(p)\gamma^\nu]$

$$2MW^{\mu\nu} = \int d^2k_T \int d^2p_T\, \delta^{(2)}(\vec{k}_T - \frac{\vec{P}_{h\perp}}{z_h} - \vec{p}_T)\,\mathrm{Tr}[\Phi(x_B, \vec{k}_T)\gamma^\mu \Delta(z_h, \vec{p}_T)\gamma^\nu] + \mathcal{O}(1/Q)$$

TMD Correlators:

$$\Phi_{ij}(x, \vec{k}_T) = \int \frac{dz^- d^2z_T}{(2\pi)^3}\, \mathrm{e}^{ixP^+z^- - i\vec{k}_T\cdot\vec{z}_T} \langle P, S|\, \bar{\psi}_j^q(0)\, \psi_i^q(z^-, 0^+, \vec{z}_T)\, |P, S\rangle$$

$$\Delta_{ij}(z, \vec{p}_T) = \sum_X \int \frac{dz^+ d^2z_T}{(2\pi)^3}\, \mathrm{e}^{i\frac{P_h^-}{z} z^+ - i\vec{p}_T\cdot\vec{z}_T} \langle 0|\, \psi_i(0^-, z^+, \vec{z}_T)\, |P_h;\, X\rangle\langle P_h;\, X|\, \bar{\psi}_j(0)\, |0\rangle$$

TMD parton distributions and FF

- Extract TMDs from Correlators:

$$4A_{ij} = 1\,Tr[1\,A] + \gamma_5\,Tr[\gamma_5\,A] + \gamma^\mu Tr[\gamma_\mu\,A] - \gamma^\mu\gamma_5\,Tr[\gamma_\mu\gamma_5\,A] - \tfrac{1}{2} i\sigma^{\mu\nu}\gamma_5\,Tr[i\sigma_{\mu\nu}\gamma_5\,A]$$

unp. quarks: $\boxed{\frac{1}{2}\mathrm{Tr}\left[\Phi\,\gamma^+\right] = f_1(x,\vec{k}_T^2) + \frac{\vec{k}_T\times\vec{S}_T}{M} f_{1T}^\perp(x,\vec{k}_T^2)}$

long. pol. quarks: $\boxed{\frac{1}{2}\mathrm{Tr}\left[\Phi\,\gamma^+\gamma_5\right] = S_L\,g_{1L} + \frac{\vec{k}_T\cdot\vec{S}_T}{M} g_{1T}}$

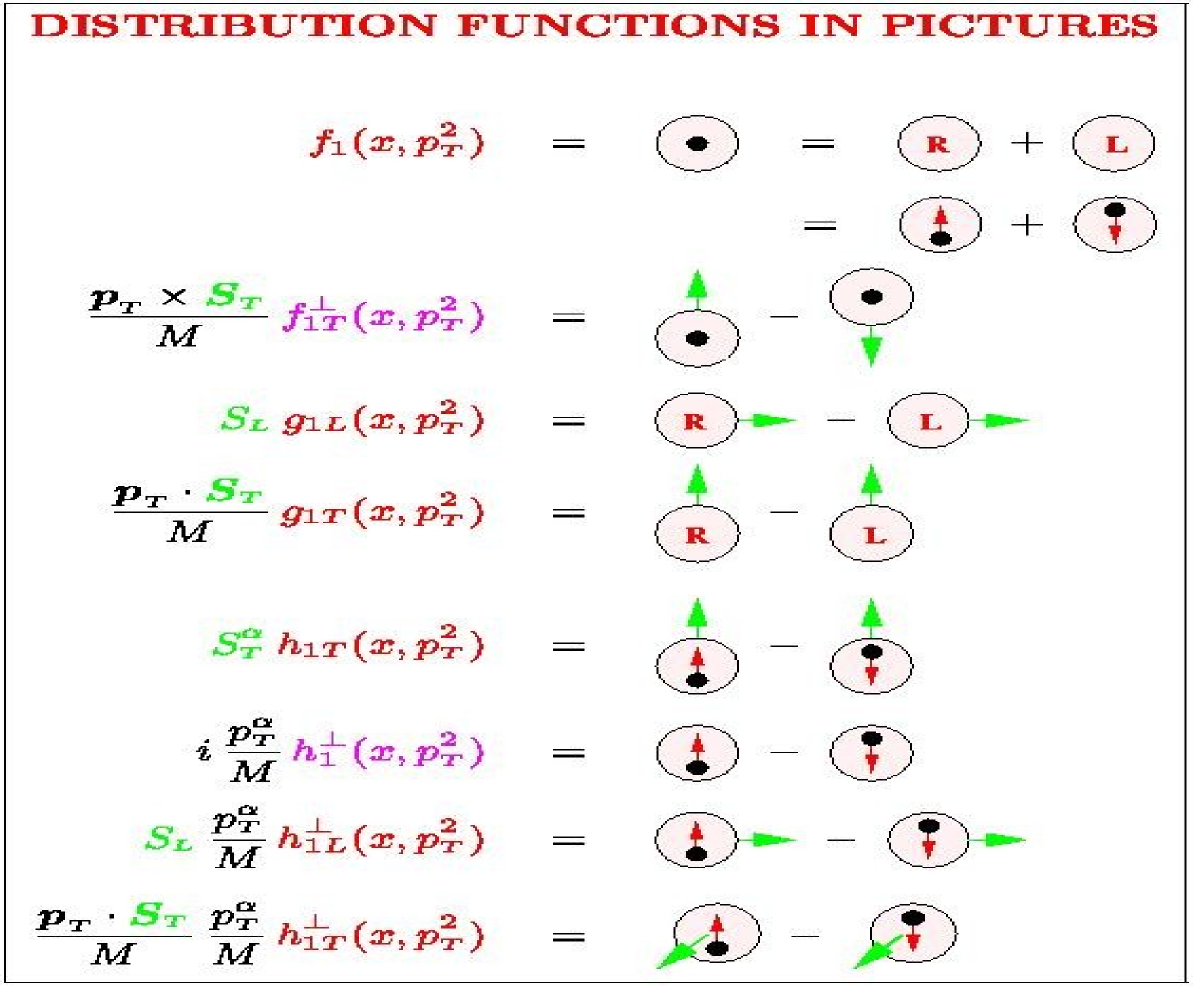
transv. pol. $\boxed{\text{[chiral-odd]: } \mathrm{Tr}\left[\Phi\,\sigma^{\perp+}\right] \longrightarrow (h_1\,,\,h_{1T}^\perp\,,\,h_{1L}^\perp\,,\,h_1^\perp)}$

- Fragmentation functions for a meson (Pion, Kaon,...):

$\boxed{\frac{1}{2}\mathrm{Tr}[\Delta\,\gamma^-] = 2z D_1(z,\vec{p}_T^2)}$ $\boxed{\frac{1}{2}\mathrm{Tr}[\Delta\,i\sigma^{i-}\gamma_5] = -2z\frac{\epsilon_T^{ij}p_T^j}{m_\pi}H_1^\perp(z,\vec{p}_T^2)}$

$H_1^\perp$: Collins-fragmentation function

TMDs in pictures



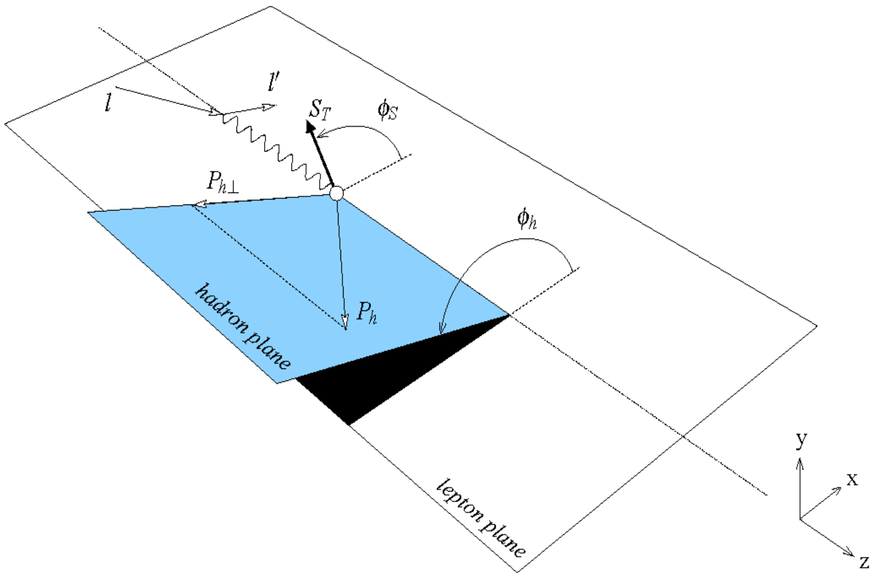
Sivers function

Boer-Mulders function

Sivers- and Boer-Mulders functions vanish under time-reversal!

(forgotten for more than 10 years...)

Single Spin Asymmetries



(Transverse) structure functions in SIDIS:

$$\frac{d\sigma}{dx_B\, dy\, d\phi_s\, d\phi_h\, dP_{h\perp}^2} \propto \sin(\phi_h - \phi_s) F_{UT}^{\sin(\phi_h-\phi_s)}$$

$$+ \sin(\phi_h + \phi_s) F_{UT}^{\sin(\phi_h+\phi_s)} + \ldots$$

18 structure function $\Longrightarrow$ 18 observables

Sivers-Asymmetry:

$$F_{UT}^{\sin(\phi_h-\phi_s)}(x_B, z_h, P_{h\perp}^2, Q^2) = \mathcal{C}\left[-\frac{\vec{P}_{h\perp}\cdot\vec{k}_T}{|\vec{P}_{h\perp}M} f_{1T}^{\perp} D_1\right]$$

Collins-Asymmetry:

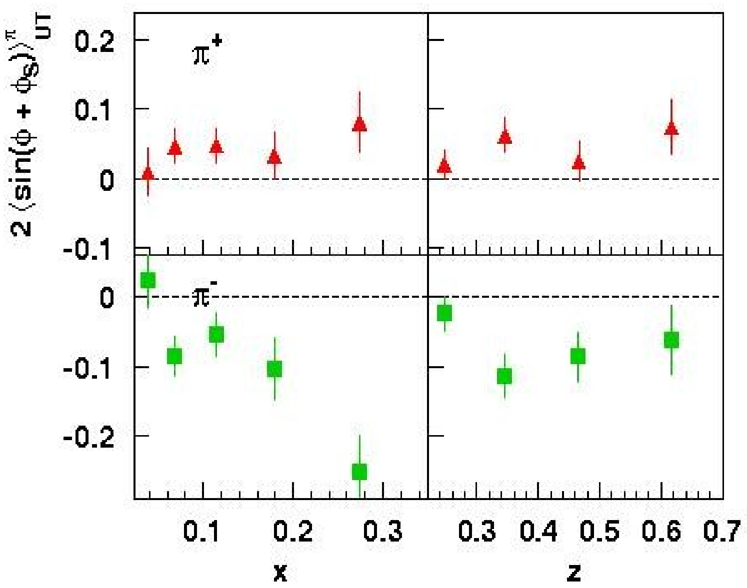
$$F_{UT}^{\sin(\phi_h+\phi_s)}(x_B, z_h, P_{h\perp}^2, Q^2) = \mathcal{C}\left[-\frac{\vec{P}_{h\perp}\cdot\vec{p}_T}{|\vec{P}_{h\perp}m_\pi} h_1 H_1^{\perp}\right]$$

Convolution:

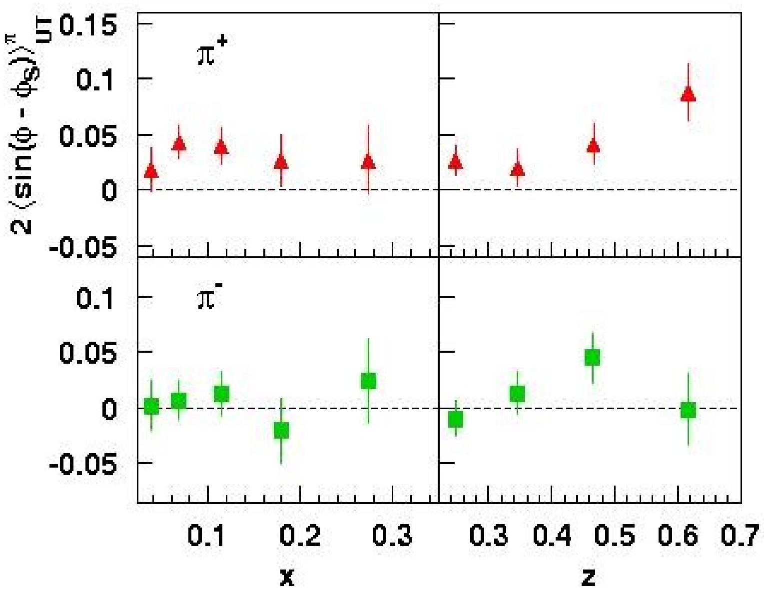
$$\mathcal{C}[w\, f\, D] = x_B \sum_q e_q^2 \int d^2k_T\, d^2p_T\, \delta^{(2)}\left(\vec{k}_T - \frac{\vec{P}_{h\perp}}{z_h} - \vec{p}_T\right) w(\vec{k}_T, \vec{p}_T) f^q(x_B, \vec{k}_T^2) D^q(z_h, \vec{p}_T^2)$$

Experiments (HERMES, also COMPASS, JLab,...)

Collins-Asymmetry



Sivers-Asymmetry

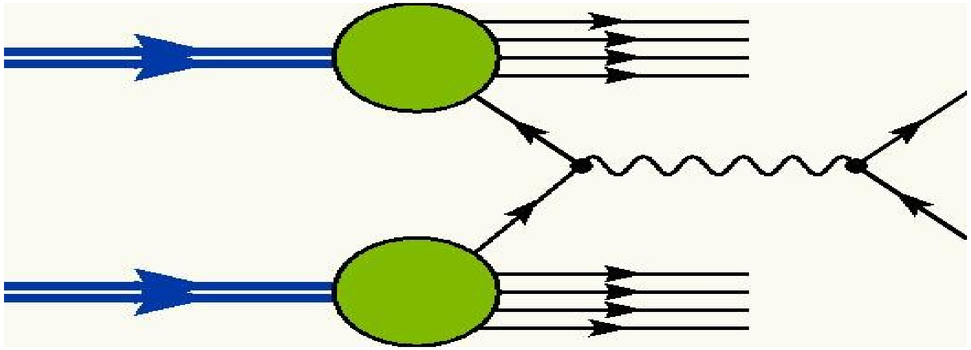


Sivers function must be non-zero!

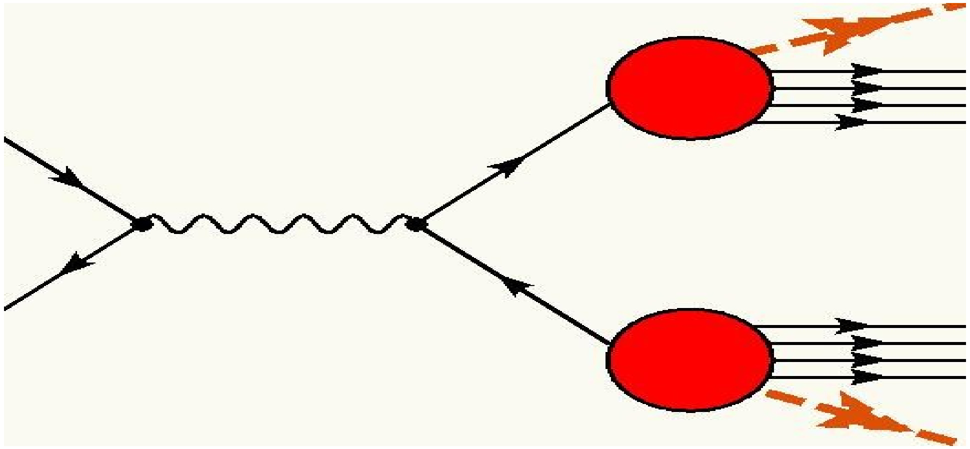
What went wrong?

Other TMD processes...

Drell-Yan process (e.g. at RHIC, COMPASS, GSI, FermiLab,...)



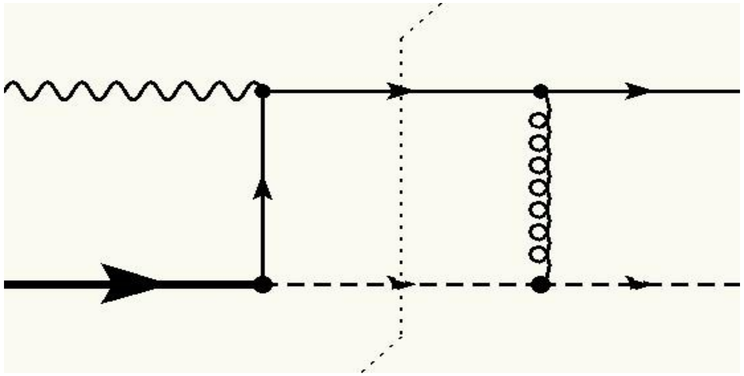
e^+e^- - annihilation (Belle, SLAC, LEP,...)



Final State Interactions

- Till 2002: Sivers-function vanishes due to time-reversal.
- Brodsky, Hwang, Schmidt, 2002: Sivers asymmetry due to final state interactions.

Diquark spectator model: “Rescattering-effect”



- Collins 2002: Hide Final State Interactions in TMD parton distributions

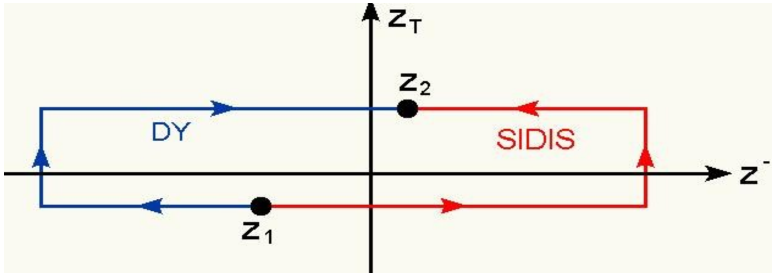
$\Longrightarrow$ Gauge Link! (take it seriously...)

Gauge link for TMDs

- k_T-dependence $\longrightarrow$ more complicated gauge link

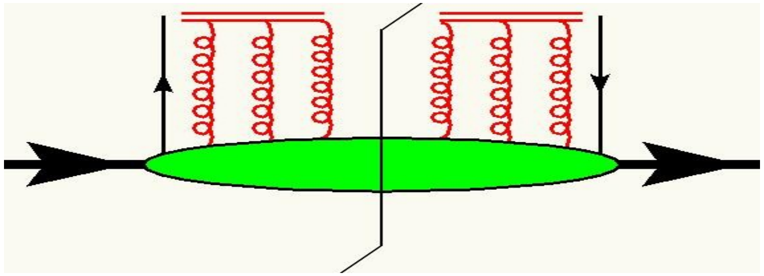
$$\boxed{\mathcal{FT}\langle P|\,\bar{\psi}(z_1)\mathcal{W}[z_1;z_2]\psi(z_2)\,|P\rangle\Big|_{z_1^+=z_2^+=0}}$$

$$\boxed{\mathcal{W}[z_1;z_2]=\mathcal{P}\mathrm{e}^{-ig\int_{z_1}^{z_2}ds\cdot A(s)}}$$



- Light-cone gauge $A^+=0$ doesn't help anymore!

- Describes *Initial (DY)* and *Final (SIDIS)* State Interactions



- Time-reversal: switches Wilson-lines ISI $\longleftrightarrow$ FSI

$$f_{1T}^{\perp}\Big|_{DIS}=-f_{1T}^{\perp}\Big|_{DY}\qquad h_1^{\perp}\Big|_{DIS}=-h_1^{\perp}\Big|_{DY}$$

How to extract TMDs from data?

Example: Sivers function [procedure of Efremov, Goeke, Schweitzer...]

Sivers-asymmetry: k_T - convolution of $f_{1T}^{\perp} \otimes D_1$

Deconvolution:
Gaussian ansatz (model): $$\boxed{f_{1T}^{\perp}(x,\vec{k}_T^2) = f_{1T}^{\perp}(x)\frac{\exp(-\vec{k}_T^2/\langle\vec{k}_T^2\rangle)}{\pi\langle\vec{k}_T^2\rangle}}$$ D_1 accordingly

Asymmetry much simpler: $$\boxed{A_{UT}^{\mathrm{Sivers}} = a_G \frac{\sum_q e_q^2 f_{1T}^{\perp,(1),q}(x_B) D_1^q(z_h)}{\sum_q e_q^2 f_1^q(x_B) D_1^q(z_h)}}$$

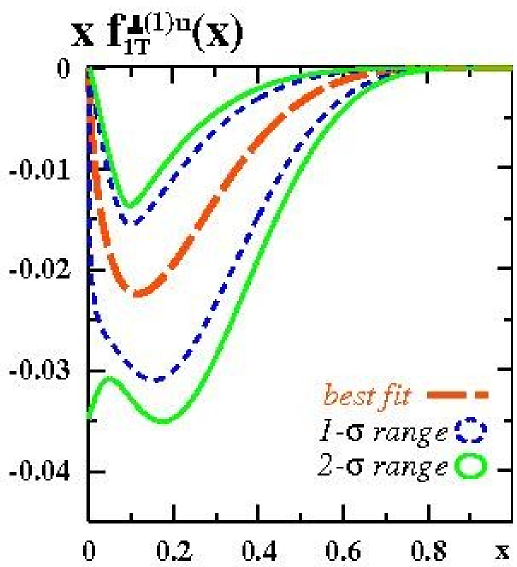
a_G: model-dep., involves Gaussian width etc.

Further assumption: $f_{1T}^{\perp,u} = -f_{1T}^{\perp,d} + \mathcal{O}(1/N_c)$

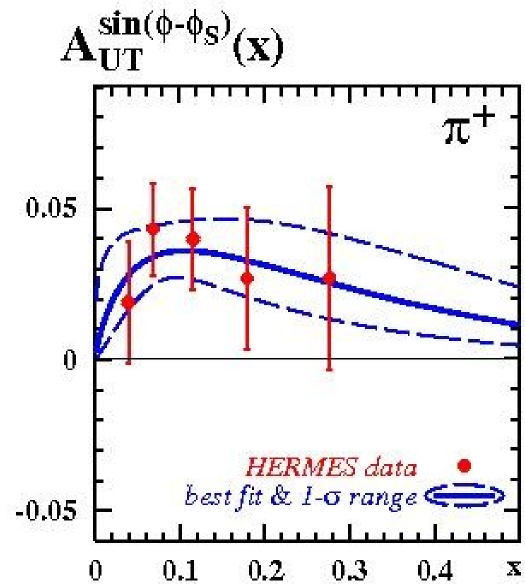
Ansatz for Sivers-function: $$\boxed{x f_{1T}^{\perp,(1)u}(x_B) = A x^b (1-x_B)^5}$$

Inserting back into asymmetry and fitting to HERMES π^+ - data:

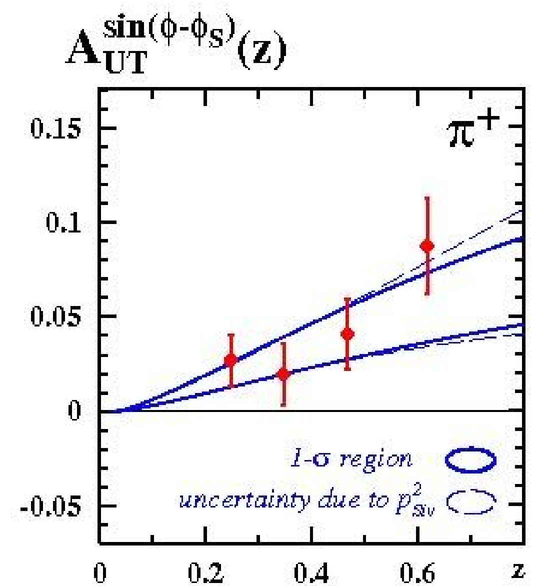
$$A = -0.17\,,\ b = 0.66$$



1. Ansatz



2. Fit to HERMES data



3. Cross check

- Fit describes pion data reasonably well.
  Works also for COMPASS data (deuterium target)

- Sivers-effect was also measured for kaons. Fit not satisfying around x=0.1...
  Sea-quark effect? Sea-quark Sivers-function relevant?

- Sivers function can be extracted also from Drell-Yan. Test sign change.

Extraction of Transversity

Collins effect: $h_1 \otimes H_1^{\perp}$

[Anselmino et al.]: Again, use Gaussian ansatz for deconvolution

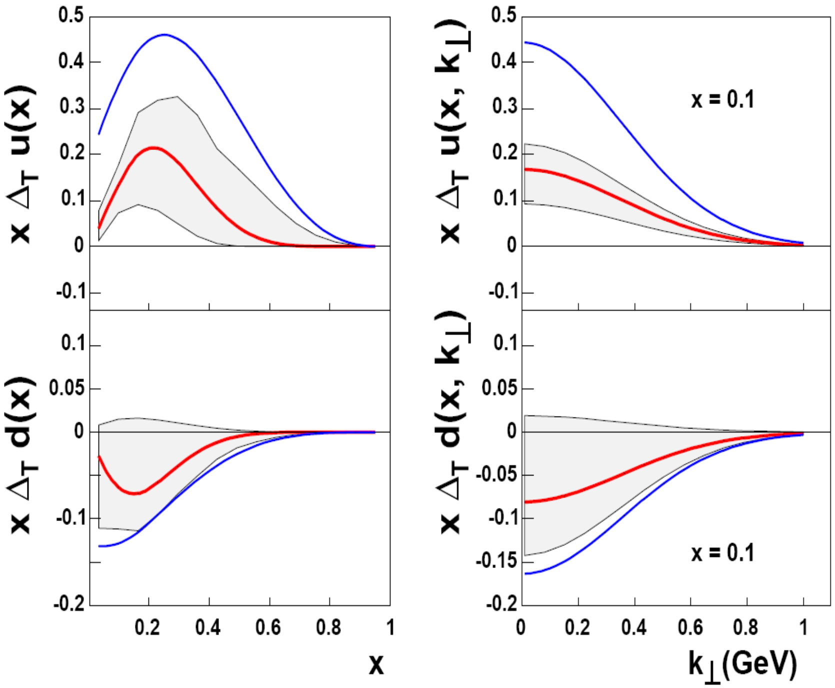
However, Collins FF is needed. Extraction from BELLE-data (e^+e^- - annihilation).

First extraction of u- and d-quarks transversity:

Recent improvements of fits: reduced error bars.

Transversity doesn't seem to be small.

More improvements needed. (Evolution etc.)



(Possible) relations between TMDs and GPDs

Trivial Relations are well-known:

$$\boxed{f_1(x) = H(x,0,0) = \int d^2k_T\, f_1(x,\vec{k}_T^2) = \int d^2b_T \mathcal{H}(x,\vec{b}_T^2)}$$

$$\boxed{g_1(x) = \tilde{H}(x,0,0) = \int d^2k_T\, g_{1L}(x,\vec{k}_T^2)}$$

$$\boxed{h_1(x) = H_T(x,0,0) = \int d^2k_T\, h_1(x,\vec{k}_T^2)}$$

→ model-independent, integrated relations

also for twist-3 PDFs e(x), $g_T(x)$, ...

Non-trivial Relations

Non-trivial relations for "T-odd" parton distributions:

M. Burkardt [Nucl.Phys. A735, 185], [PRD66, 114005]

Step 1: Average transverse of unpolarized partons in a transversely polarized nucleon:

$$\langle k_T^i \rangle_T(x) = \int d^2k_T\, k_T^i\, \tfrac{1}{2}\left[\Phi^{[\gamma^+]}(\vec{S}_T) - \Phi^{[\gamma^+]}(-\vec{S}_T)\right] \propto f_{1T}^{\perp,(1)}(x)$$

Step 2: Impose parity and time reversal:

$$\Phi(x,\vec{k}_T;-\vec{S}_T) = \mathcal{FT}\left[\langle P,-S_T|\,\bar{\psi}\gamma^+\mathcal{W}_{\text{SIDIS}}\psi\,|P,-S_T\rangle\right]$$

$$\longrightarrow \quad \mathcal{FT}\left[\langle P,+S_T|\,\bar{\psi}\gamma^+\mathcal{W}_{\text{DY}}\psi\,|P,+S_T\rangle\right]$$

Non-trivial Relations

Step 3: Derivatives of gauge links:

$$\langle k_T^i \rangle_T(x) \propto \int d^2k_T \int d^2z_T\, k_T^i e^{ik\cdot z} \langle \bar\psi\, \gamma^+ \left(\mathcal{W}_{\text{SIDIS}} - \mathcal{W}_{\text{DY}} \right) \psi \rangle$$

$$i\partial_T^i$$

$$i\partial_T^i \left(\mathcal{W}_{\text{SIDIS}} - \mathcal{W}_{\text{DY}} \right)\Big|_{z_T=0} \propto \int dy^- [-\tfrac{z^-}{2}; y^-]\, gF^{+i}(y^-)\, [y^-; \tfrac{z^-}{2}]$$

$$\equiv 2[-\tfrac{z^-}{2}; \tfrac{z^-}{2}]\, I^i(\tfrac{z^-}{2})$$

$$\boxed{\langle k_T^i \rangle(x) = \int \frac{dz^-}{2(2\pi)} e^{ixP^+z^-} \langle P, S_T | \bar\psi(-\tfrac{z^-}{2}) \gamma^+ [-\tfrac{z^-}{2}; \tfrac{z^-}{2}]\, I^i(\tfrac{z^-}{2})\, \psi(\tfrac{z^-}{2}) | P, S_T \rangle}$$

collinear “soft gluon pole” matrix element

Non-trivial Relations

Step 4: Impact parameter space: $z_{1/2} = \mp \frac{z^-}{2} n_- + b_T$

$$\langle k_T^i \rangle(x) = \int d^2 b_T \int \frac{dz^-}{2(2\pi)} e^{ixP^+z^-} \langle P^+; \vec{0}_T; S_T | \bar{\psi}(z_1) \gamma^+ [z_1; z_2] I^i(z_2) \psi(z_2) | P^+; \vec{0}_T; S_T \rangle$$

Impact parameter representation for GPD E

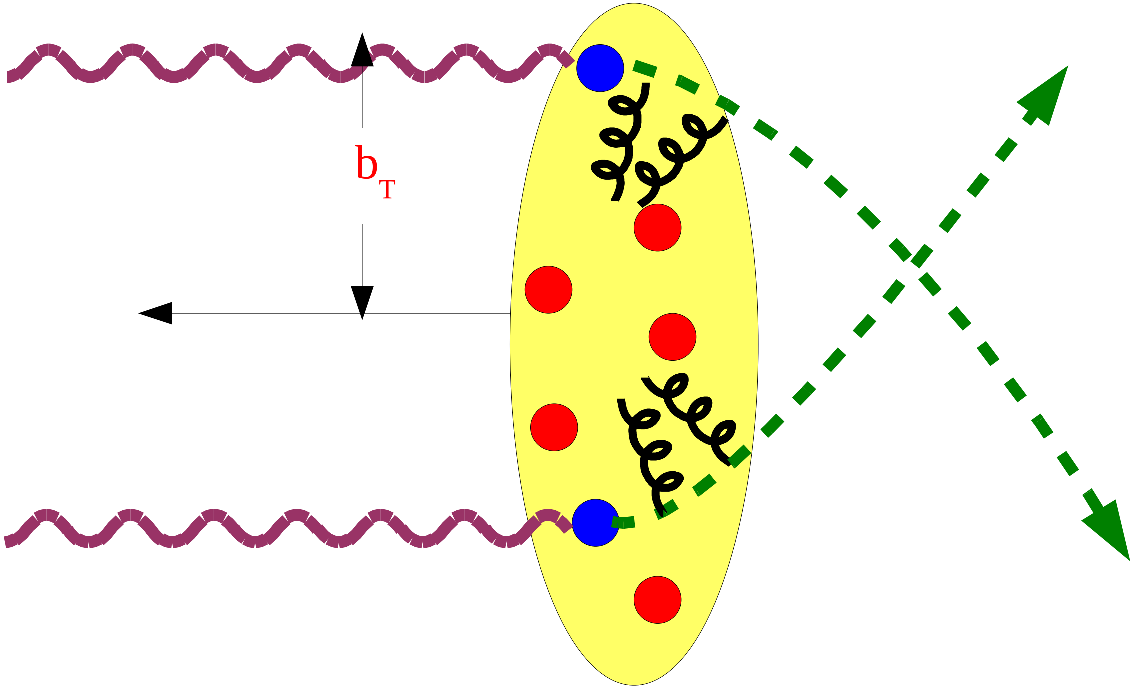
Assume factorization of final state interactions and spatial distortion:

$$\boxed{\langle k_T^i \rangle = -M \epsilon_T^{ij} S_T^j f_{1T}^{\perp,(1)}(x) \simeq \int d^2 b_T \, \mathcal{I}^i(x, \vec{b}_T) \, \frac{\vec{b}_T \times \vec{S}_T}{M} \, \frac{\partial}{\partial b_T^2} \mathcal{E}(x, \vec{b}_T^2)}$$

$\mathcal{I}^i(x, \vec{b}_T^2)$: Lensing Function = net transverse momentum

Physical picture of the Relation

Intuitive picture of the Final State Interactions:



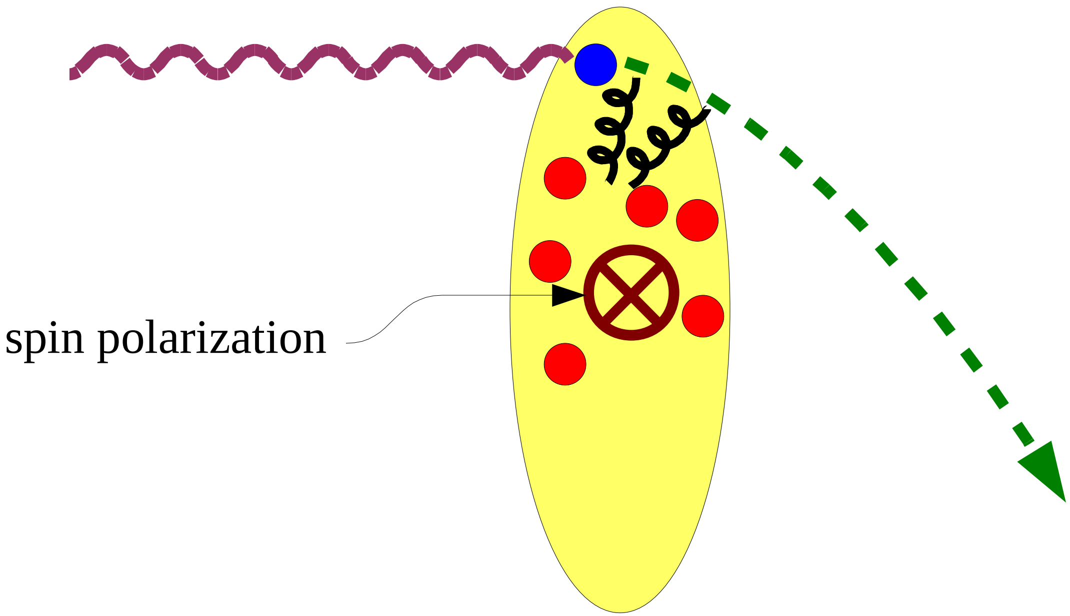
Final State interactions are assumed to be attractive

→ Lensing!

Physical picture of the Relation

Intuitive picture of the Sivers asymmetry:

Spatial distortion in the transverse plane due to polarization!



Mechanism leads to non-zero Sivers asymmetry!

Predictions

Intuitive picture seems to work "numerically":

Distortion effect given by flavor dipole moment:

$$\boxed{d^{q,i} = \int dx \int d^2 b_T\, b_T^i \frac{\vec{b}_T \times \vec{S}_T}{M} \mathcal{E}'(x, \vec{b}_T^2) = -\frac{\epsilon_T^{ij} S_T^j}{2M} \int dx E^q(x,0,0) = -\frac{\epsilon_T^{ij} S_T^j}{2M} \kappa^q}$$

with flavor dipole moment $\kappa^{u/p} \simeq 1.7 \quad \kappa^{d/p} \simeq -2.0$

$$\boxed{f_{1T}^{\perp,(1)}(x) \propto \int d^2 b_T\, \mathcal{I}(x, \vec{b}_T) \frac{\vec{b}_T \times \vec{S}_T}{M} \mathcal{E}'(x, \vec{b}_T^2)}$$

Predicts opposite signs of u- and d- Sivers functions.

- in agreement with large-N_c prediction [Pobylitsa, 2003]

  model calculations in spectator models, MIT-bag model, etc.

Predictions

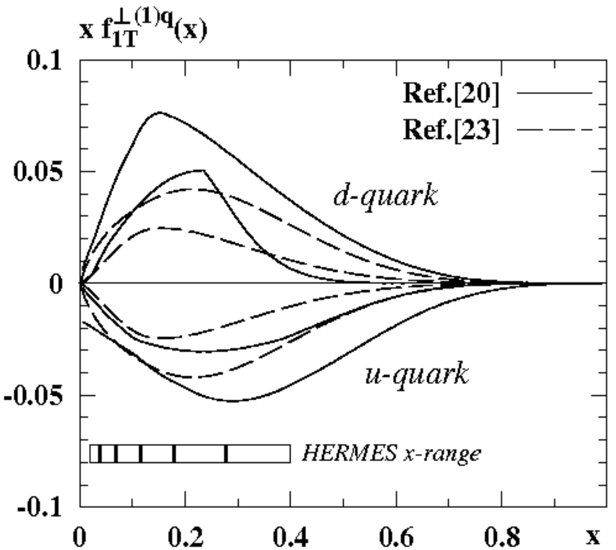
Intuitive picture also predicts the absolute sign

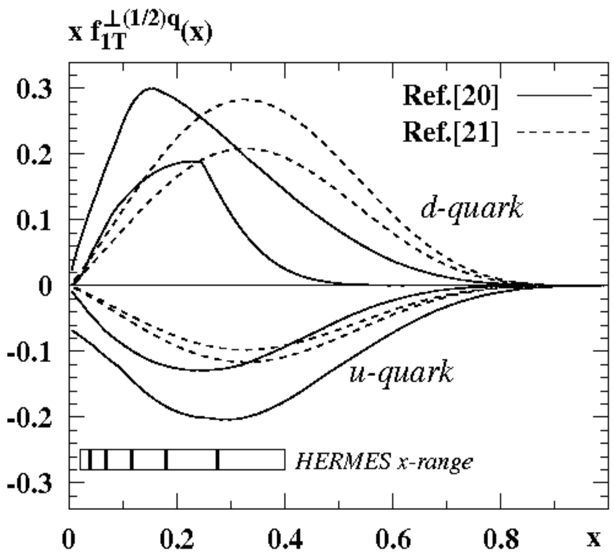
if:

Final state interactions are attractive, $\mathcal{I}(x, \vec{b}_T) < 0$

$$\boxed{f_{1T}^{\perp,u} < 0} \qquad \boxed{f_{1T}^{\perp,d} > 0}$$

Confirmed by HERMES, COMPASS data:





Fits taken from:

[20] Anselmino et al., PRD72 (05)

[21] Vogelsang, Yuan, PRD72 (05)

[23] Collins et al., hep-ph/0510342

Chiral-odd Relation

- Av. transv. momentum of transv. pol. partons in an unpol. hadron:

$$\langle k_T^i \rangle^j(x) = \int d^2k_T\, k_T^i\, \frac{1}{2}\left(\Phi^{[i\sigma^{i+}\gamma^5]}(S) + \Phi^{[i\sigma^{i+}\gamma^5]}(-S)\right)$$

$$\Rightarrow \quad \boxed{-2M^2\, h_1^{\perp,(1)}(x) \simeq \int d^2b_T\, \vec{b_T}\cdot\vec{\mathcal{I}}(x,\vec{b}_T)\,\frac{\partial}{\partial b_T^2}\left(\mathcal{E}_T + 2\tilde{\mathcal{H}}_T\right)(x,\vec{b}_T^2)}$$

- Spatial distortion in transv. plane of transv. pol. quarks quantified by

$$\boxed{\kappa_T = \int dx \left(E_T + 2\tilde{H}_T\right)(x,0,0)}$$

- <u>Lattice QCD, const. quark model:</u> $\kappa_T^u > 0$ and $\kappa_T^d > 0$

$\Rightarrow$ Boer-Mulders function negative for u- and d-quarks!

[in agreement with large-N_c models.]

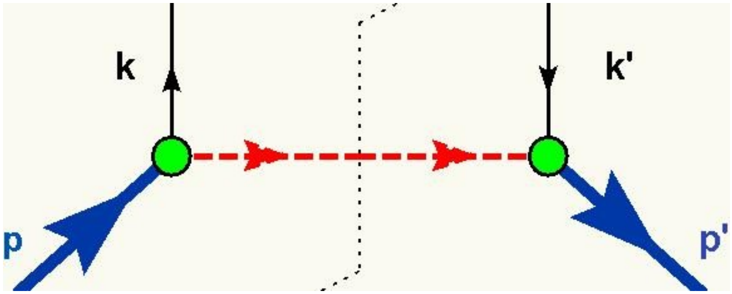
Relations in Spectator Models

Explicit checks of relations in a diquark spectator model:

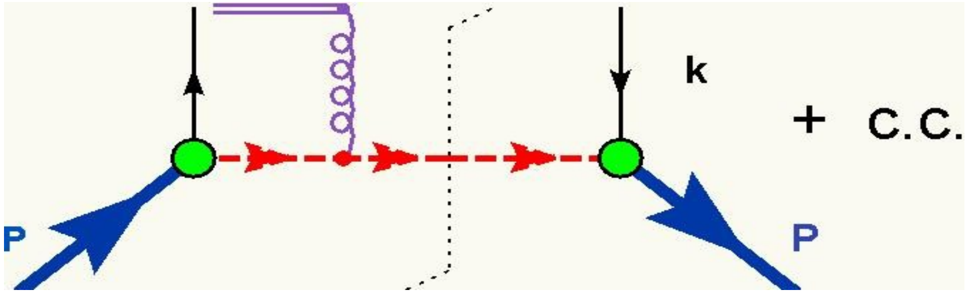
[Burkardt, Hwang, PRD69, 074032], [Meissner, Metz, Goeke, PRD76, 034002]

Lowest order calculations:

GPDs:



(T-odd) TMDs:



Non-trivial relations are *exactly* fulfilled!

$$-M\epsilon^{ij} S_T^j f_{1T}^{\perp,(1)} = \int d^2 b_T\, \mathcal{I}^i \frac{\vec{b}_T \times \vec{S}_T}{M} \mathcal{E}'$$

$$-2M\, h_1^{\perp,(1)} = \int d^2 b_T\, \frac{\vec{b}_T \cdot \vec{\mathcal{I}}}{M} \left(\mathcal{E}_T + 2\tilde{\mathcal{H}}_T\right)'$$

Relations in Spectator Models

In the diquark-spectator model:

•Relations between *arbitrary* moments:

$$\boxed{f_{1T}^{\perp,(n)}(x) \propto E^{(n)}(x)\,,\; 0 \leq \mathrm{n} \leq 1}$$

•TMD: $f^{(n)}(x) \sim \int d^2k_T\,(\vec{k}_T^2)^n f(x,\vec{k}_T^2)$

•GPD: $E^{(n)}(x) \sim \int d^2\Delta_T\,(\vec{\Delta}_T^2)^{n-1} E(x,0,-\frac{\vec{\Delta}_T^2}{(1-x)^2})$

•Relation between GPDs and *T-even* TMDs:

$$\boxed{h_{1T}^{\perp,(n)}(x) \sim \tilde{H}_T^{(n)}(x)}$$

→ No FSI / Lensing function needed!

•Relations also for gluon-GPDs and gluon-TMDs.

•Relations are likely to be broken for higher order diagrams.

Mother functions

Relations between functions:

$$
\begin{tikzcd}
 & \text{GPCFs} \arrow[r, "\Delta = 0"] \arrow[d, "\int dk^-"] & \text{PCFs} \arrow[d, "\int dk^-"] \\
\text{Wigner functions} \arrow[d, "\int d^2 k_T"] & \text{GTMDs} \arrow[l, "\text{Impact parameter}"'] \arrow[r, "\Delta = 0"] \arrow[d, "\int d^2 k_T"] & \text{TMDs} \arrow[d, "\int d^2 k_T"] \\
\text{I.P. GPDs} & \text{GPDs} \arrow[l, "\text{Impact parameter}"'] \arrow[r, "\Delta = 0"] & \text{PDFss}
\end{tikzcd}
$$

(G)PCFs: (Generalized) Parton Correlation Functions
(G)TMDs: (Generalized) Transv. Momentum Depend.

Which GPDs and TMDs have the same mother functions?

Summary

- Semi-inclusive DIS: structure functions kT – convolutions of TMDs + fragmentation functions
- TMDs: provides a deeper insight into the (spin) substructure of nucleons.
- Gauge link more complicated, physically relevant.

- SIDIS yields access to chirally-odd functions such as transversity

- Possible, non-trivial relations between TMDs and GPDs.