The Nuclear Many-Body Problem

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Aim of these lectures:

To give an overview of contemporary nuclear structure theory, i. e. effective interactions, methods that solve the quantum many-body problem, and results of such calculations.

HUGS 2008 Summer School at Jefferson Laboratory

Content of lectures

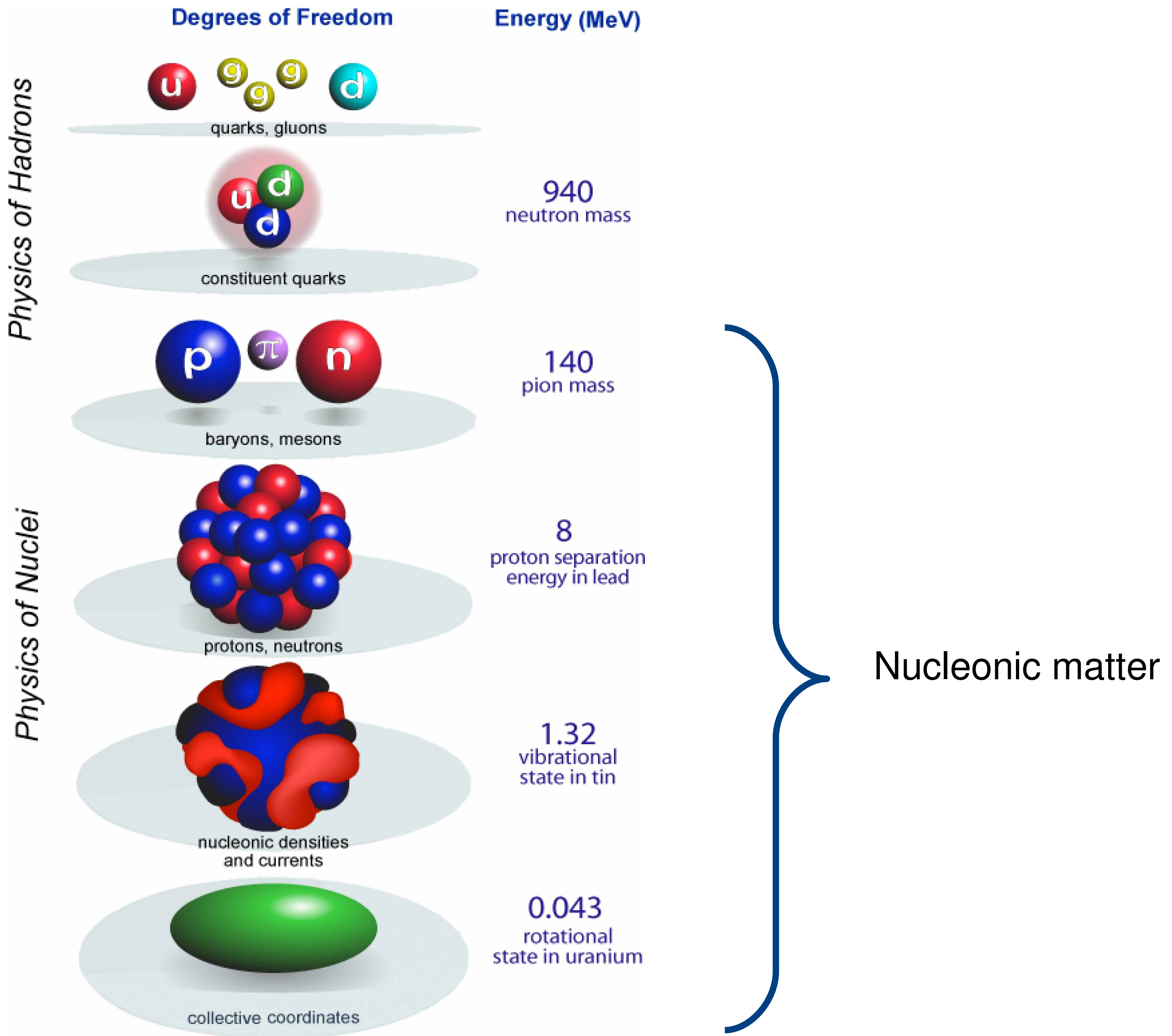
1. The nucleon-nucleon force:
   1. Determination of the Nucleon-Nucleon force from scattering. Chiral EFT and other approaches.
   2. Renormalization and effective interactions for the nuclear many-body problem.

2. Many-body nuclear structure approaches:
   1. Phenomenological models -> shell model with a core
   2. Ab-initio nuclear structure -> building nuclei from the ground up. Overview of many-body theory; No-Core shell model, Green's Function Monte-Carlo, Coupled-Cluster theory.

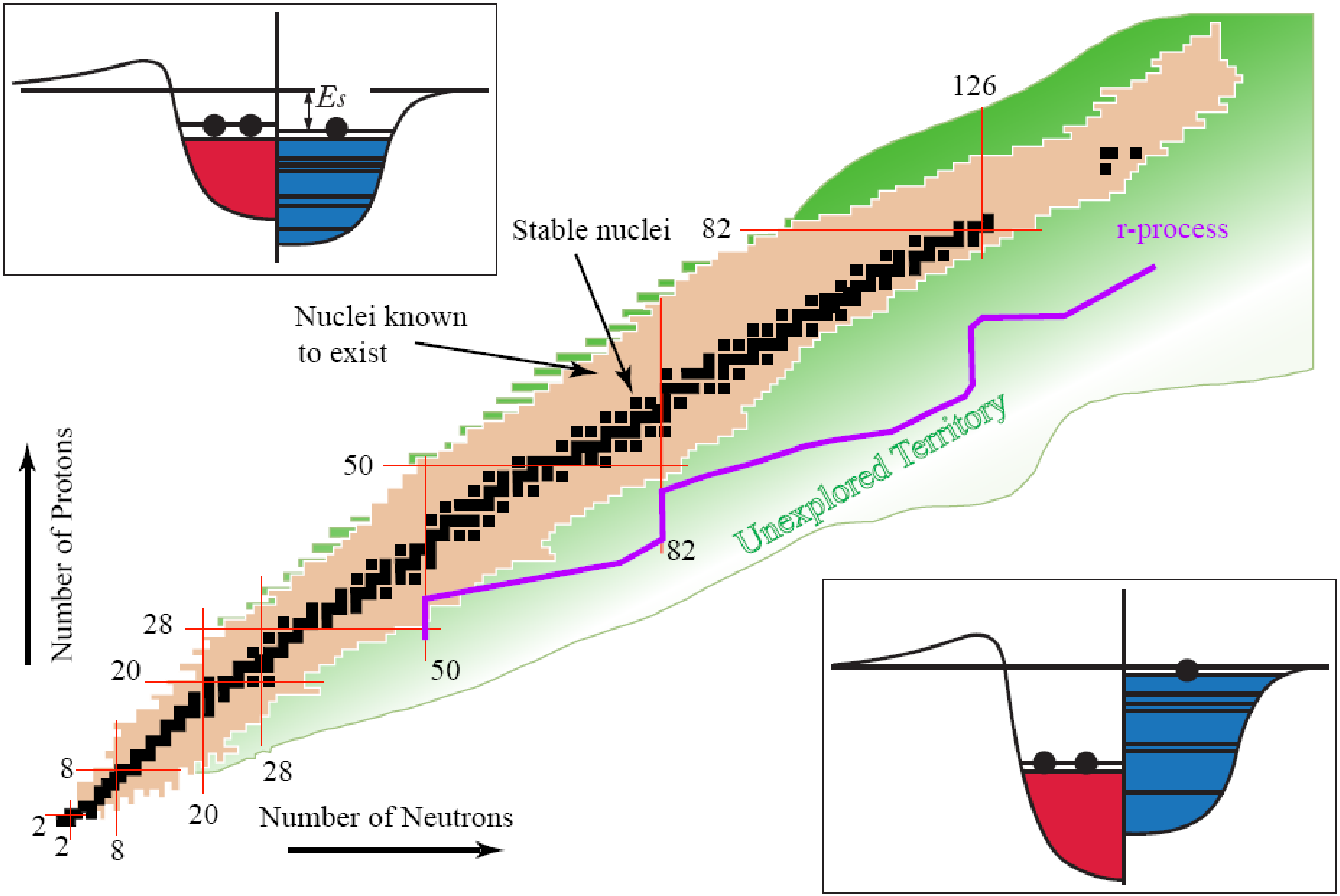
3. Structure of weakly bound and unbound nuclear states.
   1. Berggren basis
   2. Gamow-Shell Model and Coupled Cluster approaches.

Lecture 1

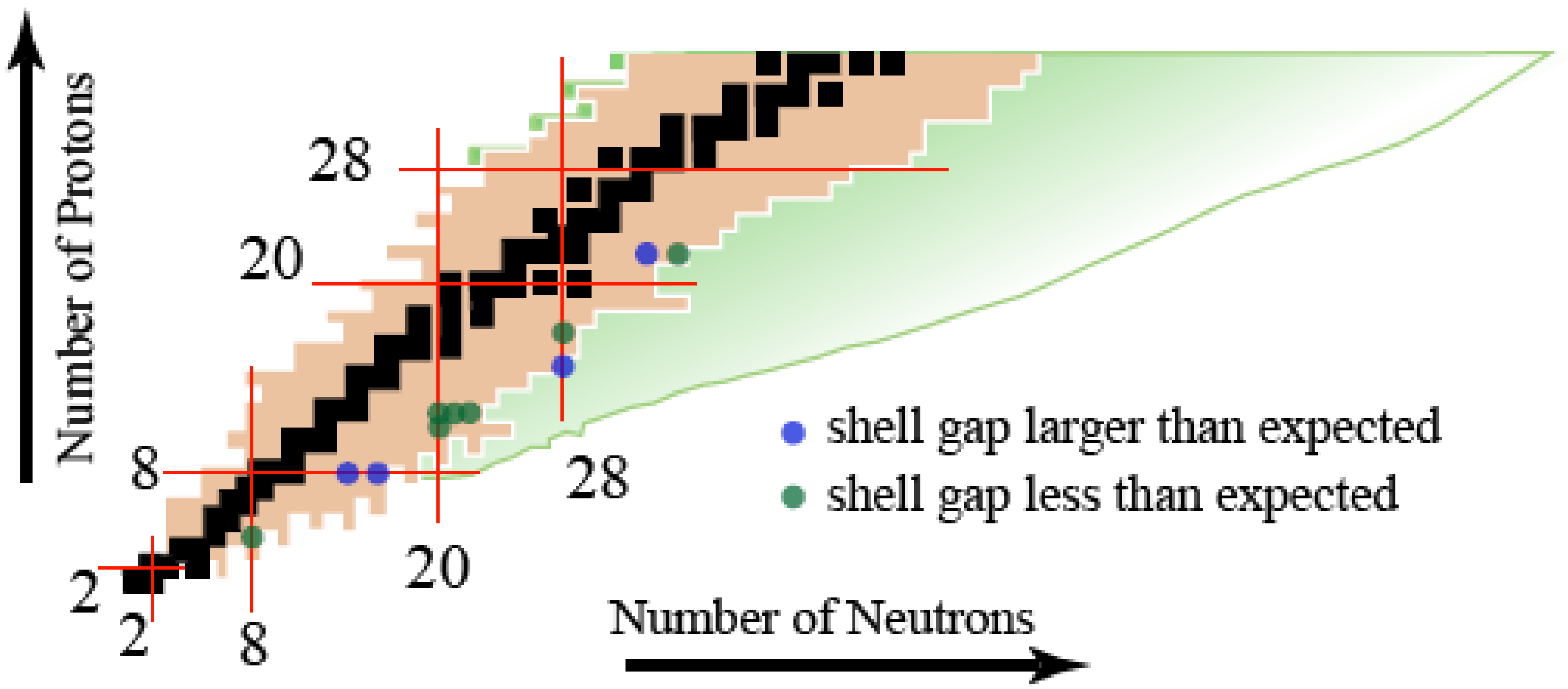
- Overview and challenges of low-energy nuclear structure
- The forces between interacting nucleons
- How can we link low-energy nuclear structure to QCD ?


Degrees of Freedom
Energy (MeV)
Physics of Hadrons
Physics of Nuclei
u
g
g
g
d
quarks, gluons
u
d
d
constituent quarks
940
neutron mass
p
π
n
baryons, mesons
140
pion mass
protons, neutrons
8
proton separation
energy in lead
nucleonic densities
and currents
1.32
vibrational
state in tin
collective coordinates
0.043
rotational
state in uranium
Nucleonic matter

Landscape and consequenses



Changing shell gaps: one of the challenges



The challenge for nuclei

"The first, the basic approach, is to study the elementary particles, their properties and mutual interaction. Thus one hopes to obtain knowledge of the nuclear forces. If the forces are known, one should, in principle, be able to calculate deductively the properties of individual nuclei. Only after this has been accomplished can one say that one completely understands nuclear structure….The other approach is that of the experimentalist and consists in obtaining by direct experimentation as many data as possible for individual nuclei. One hopes in this way to find regularities and correlations which give a clue to the structure of the nucleus….The shell model, although proposed by theoreticians, really corresponds to the experimentalist's approach."

–M. Goeppert-Mayer, Nobel Lecture

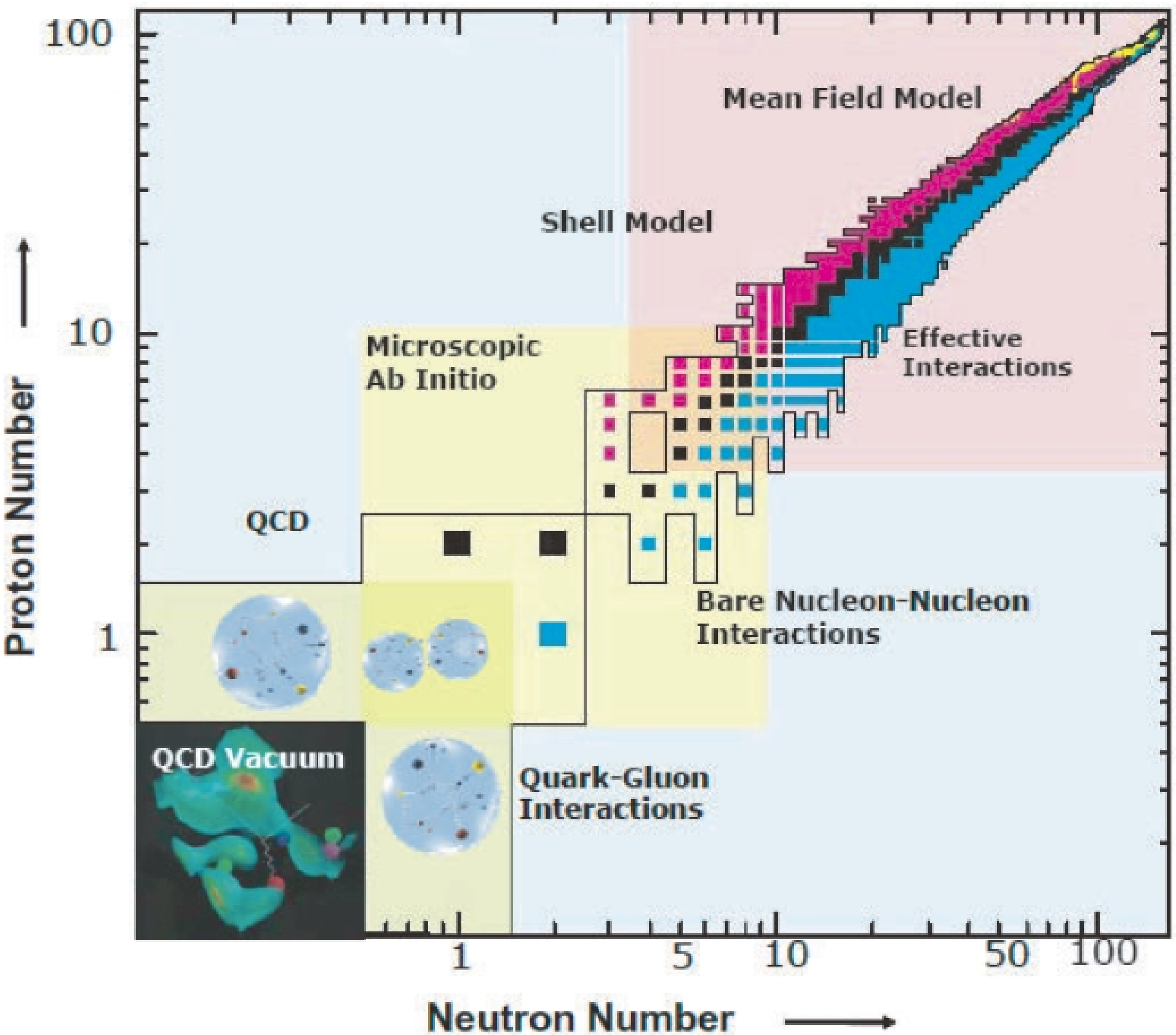
Two ways of doing business (I will focus primarily on the first):

QCD → NN (and NNN) forces → effective H → calculate → predict → experiment

XXX

- **Experiment → effective forces → calculate → predict**
- **Progress involves feedback…**

Overview: theoretical approaches



What are the properties of nuclei?

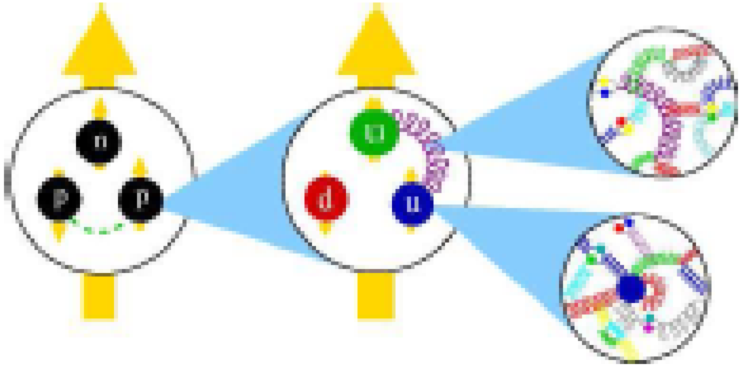
What is the effective interaction?

How does one solve the nuclear many-body problem?

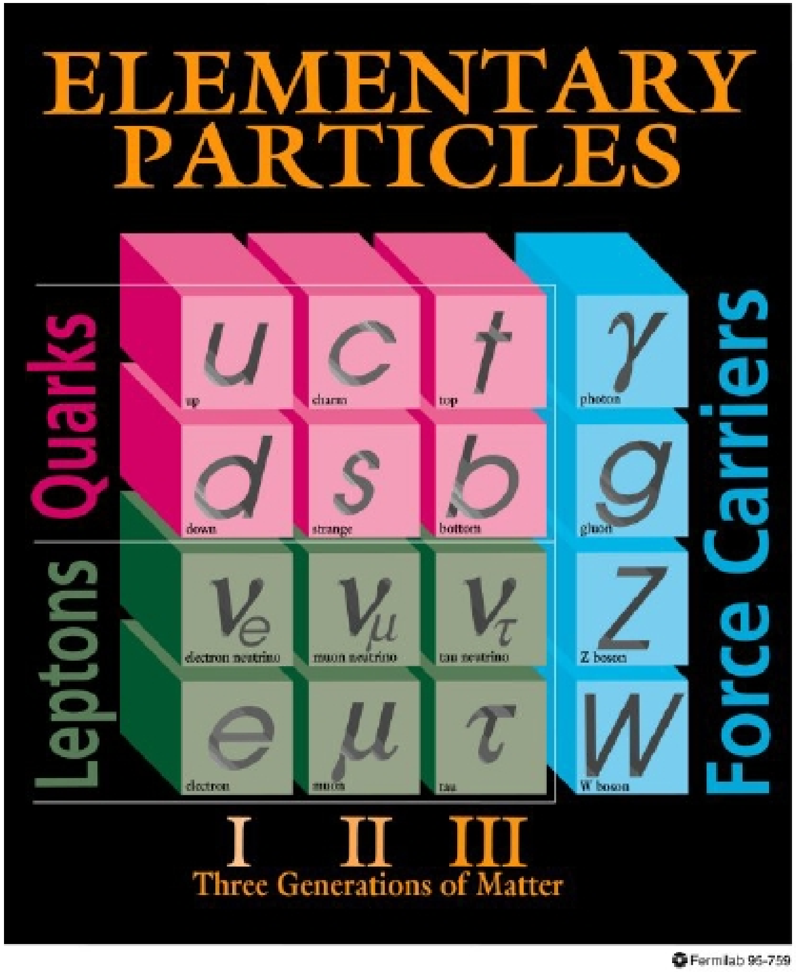
Figure from: A. Richter, INPC 2004

The effective NN interaction

- Nuclei are made of protons and neutrons. These are composite particles



Interplay between nucleonic and sub-nucleonic (quarks and gluons) degrees of freedom in few-body nuclear systems

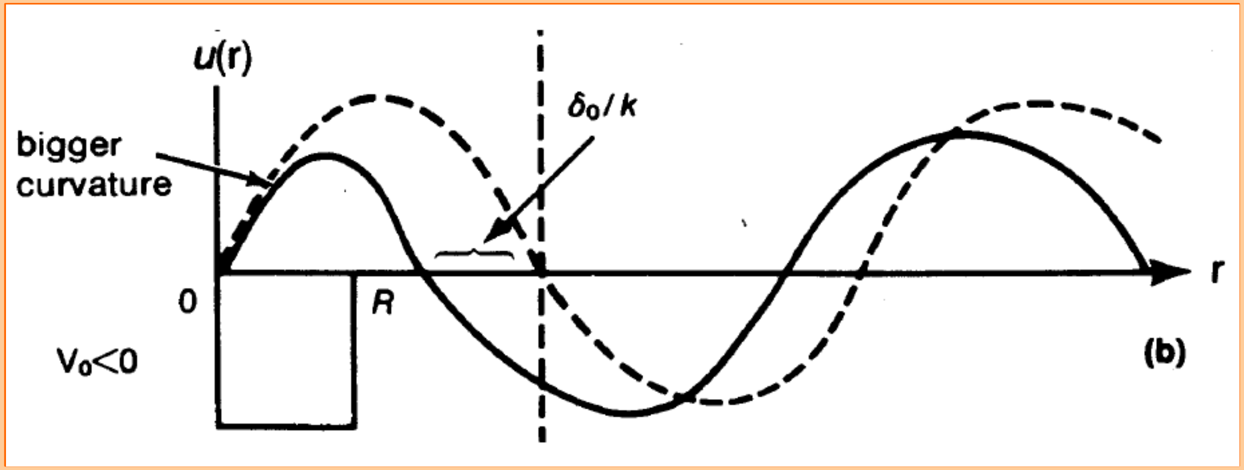


Q: How do we determine the interaction between two nucleons?

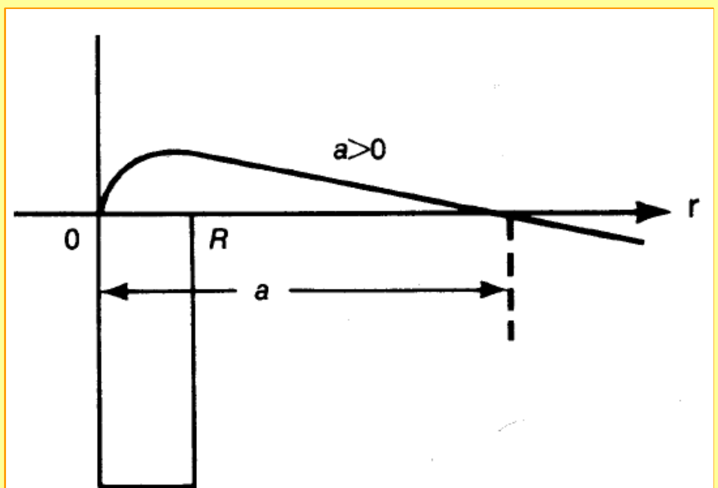
A: Study two-nucleon scattering!

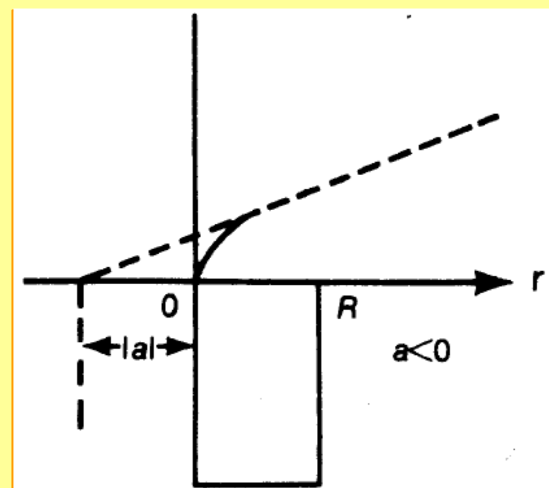
Recapitulation: Scattering theory

Phase shift is a function of relative momentum k; Figure shows s-wave.



Scattering length: $k \cot \delta(k) \approx -\frac{1}{a}; \quad \sigma_{\text{tot}} \approx 4\pi a^2 \quad \text{for} \quad k \to 0$

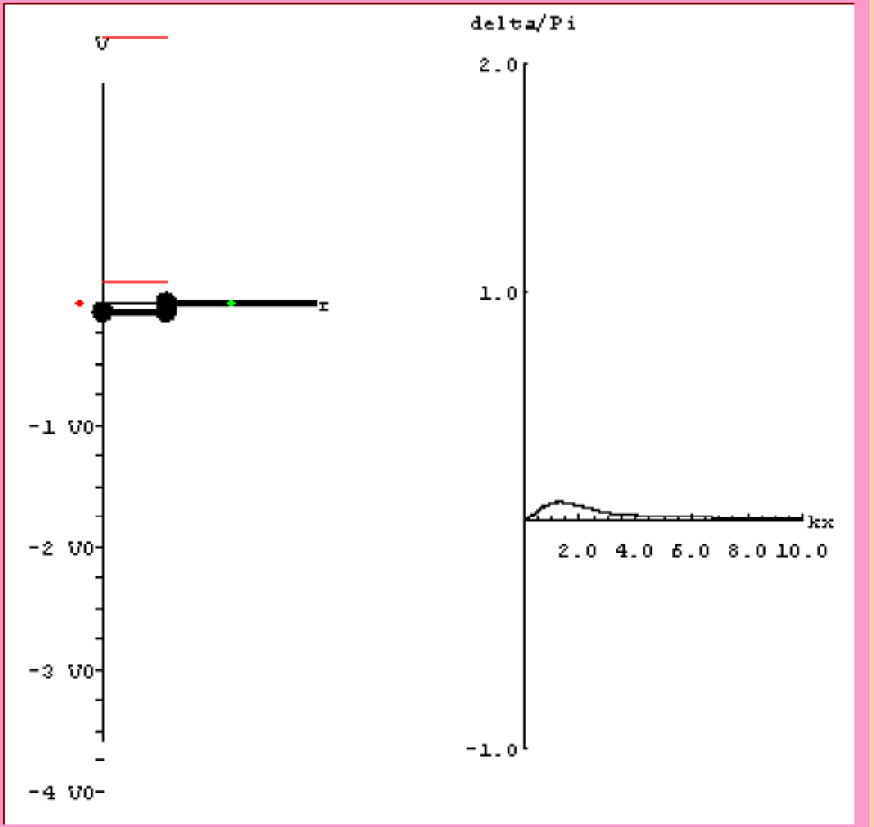




Scattering from a spherical well

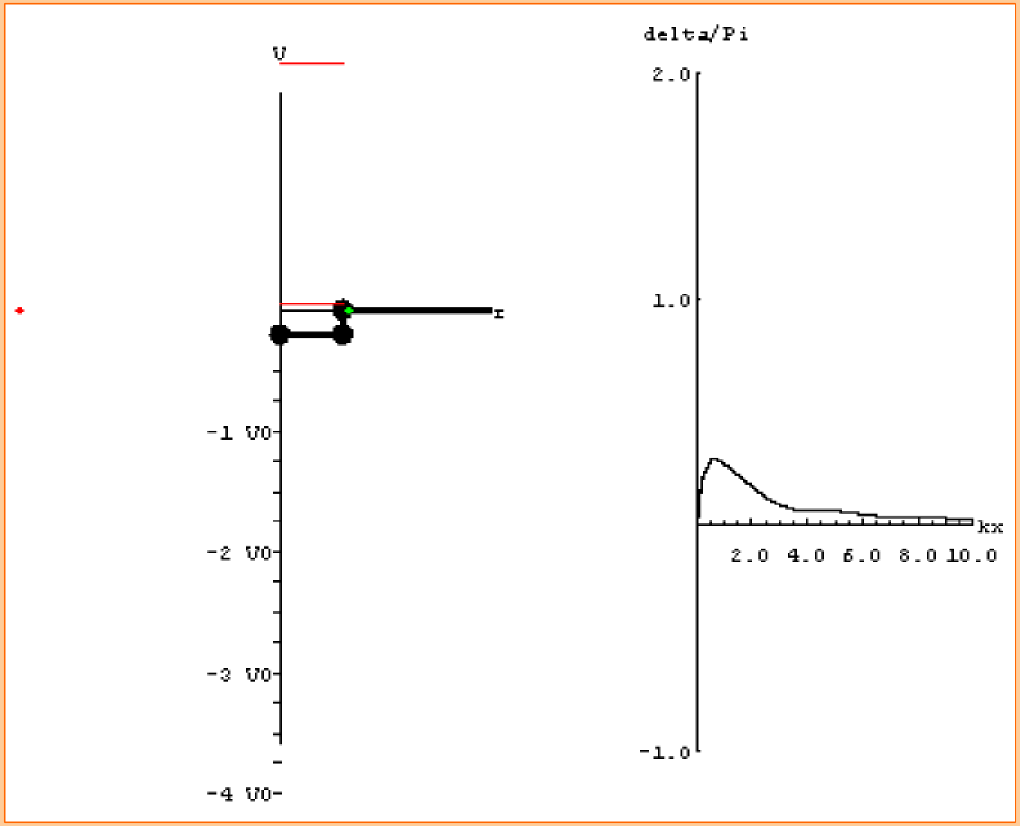
http://people.ccmr.cornell.edu/~emueller/scatter/well.html

System has no bound state



Increase depth of well:

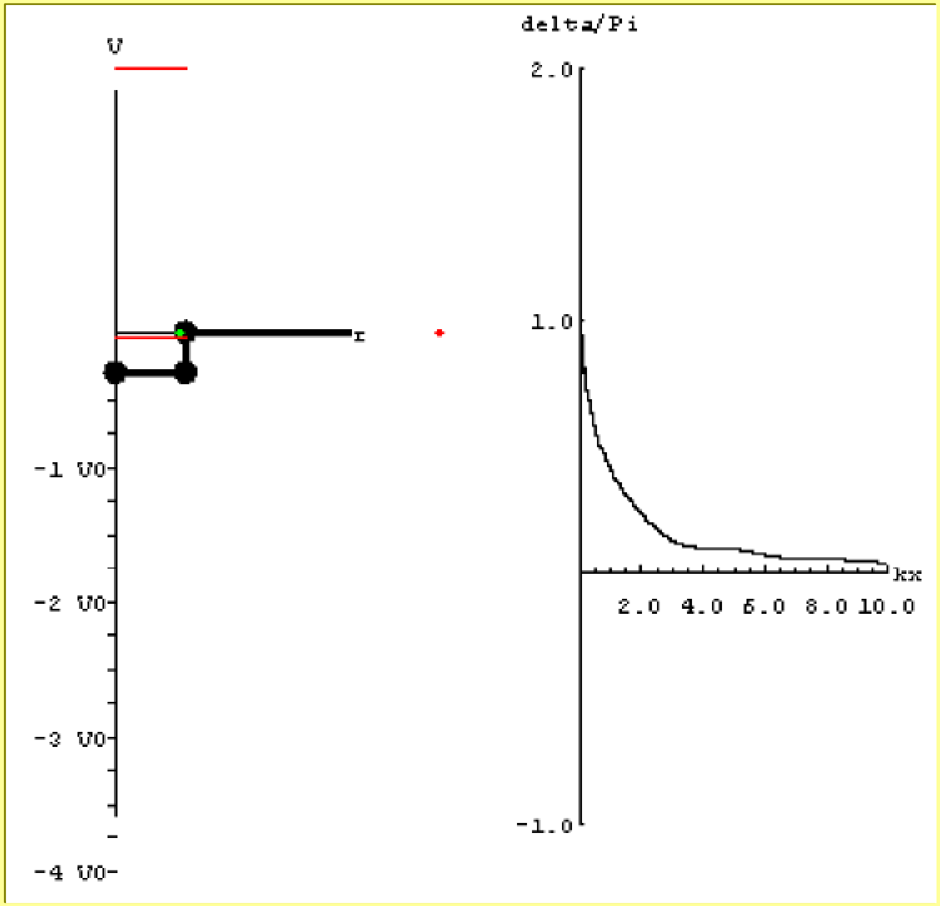
First bound state is about to enter



Scattering from a spherical well

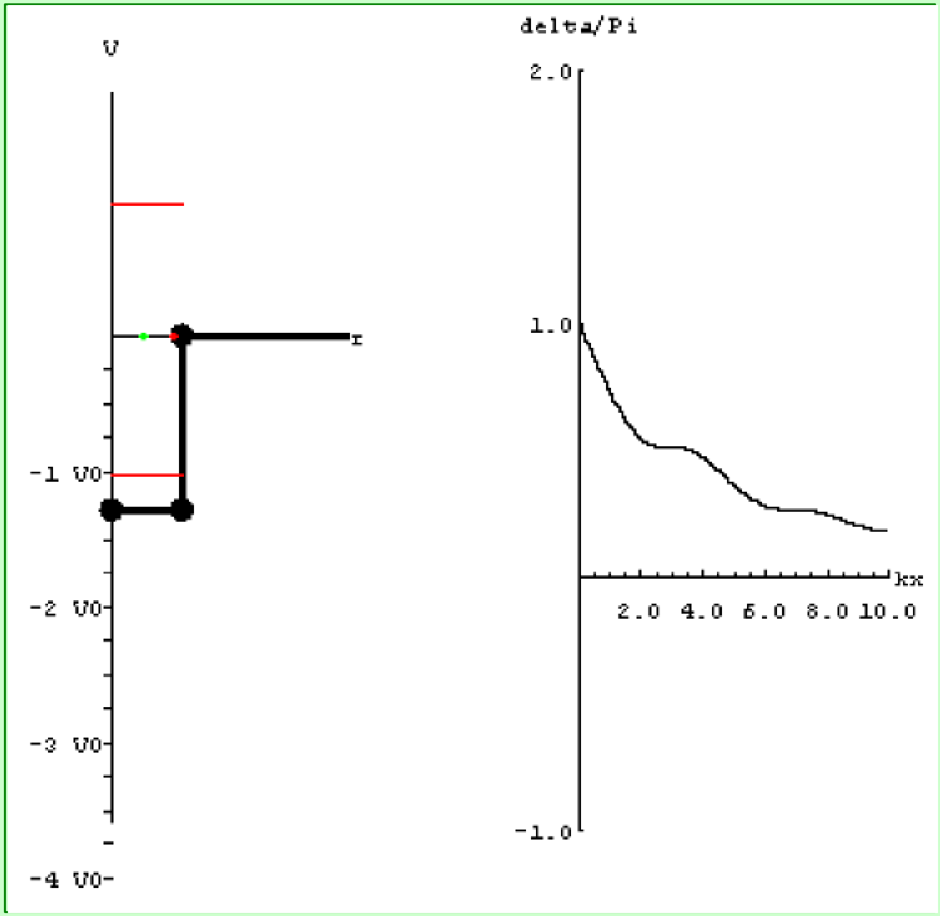
Further increase of depth:

System has one shallow bound state



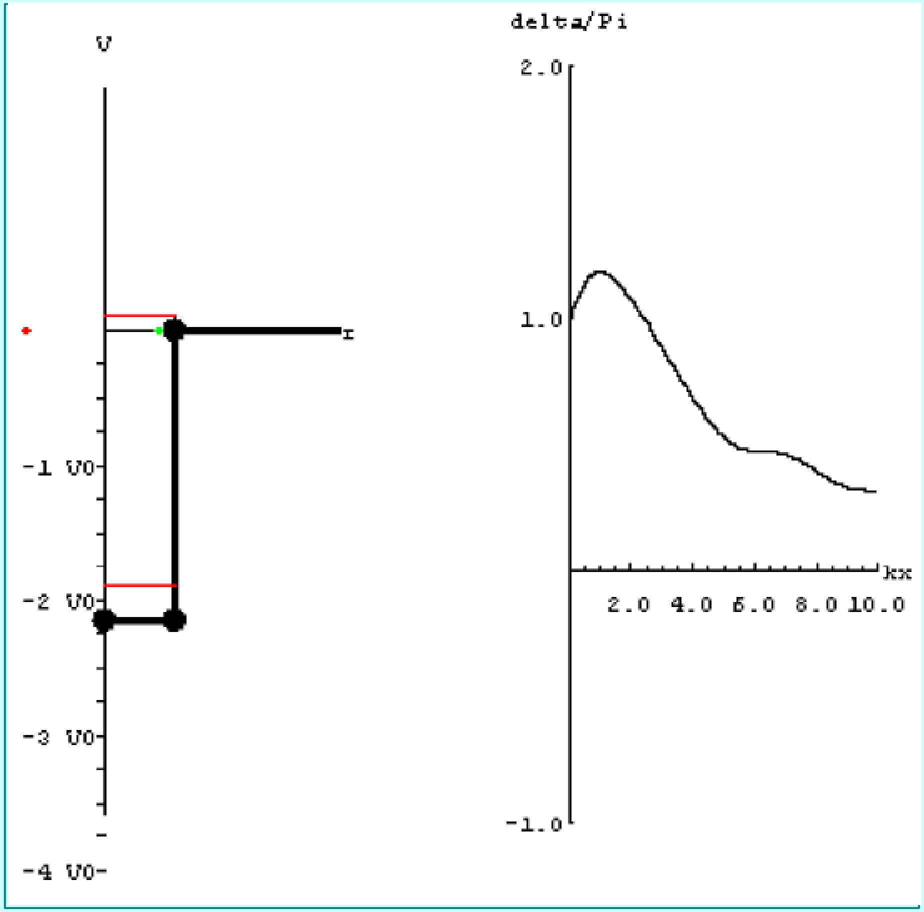
Further increase of depth:

System has one deep bound state

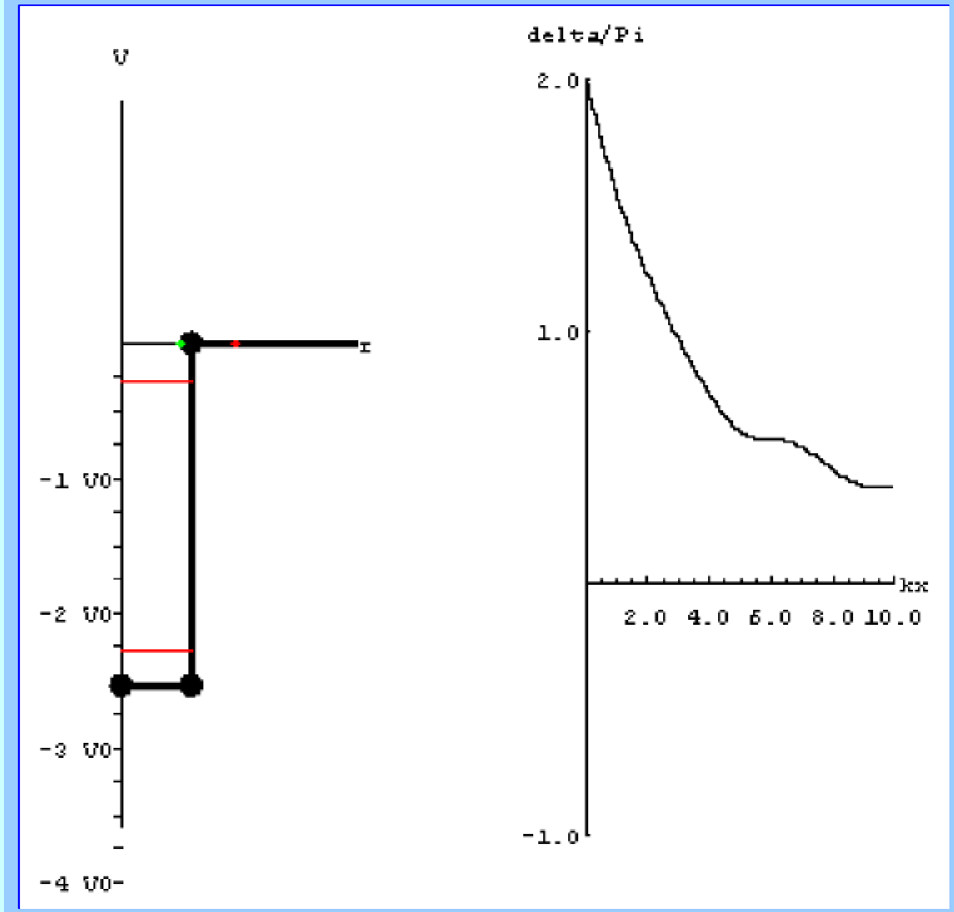


Scattering from a spherical well

Second bound state about to enter

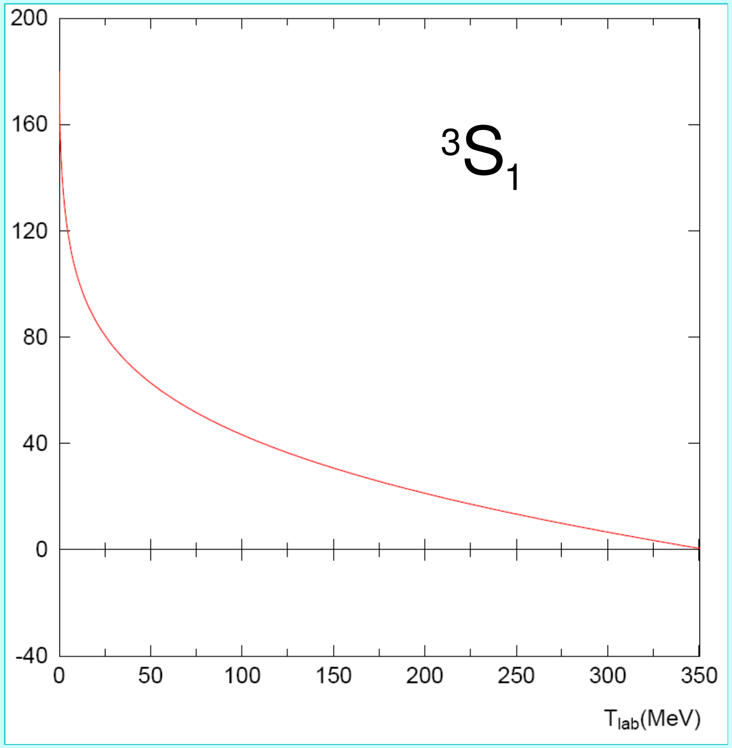


System has two bound state



Nuclear s-wave phase shifts

http://nn-online.org/

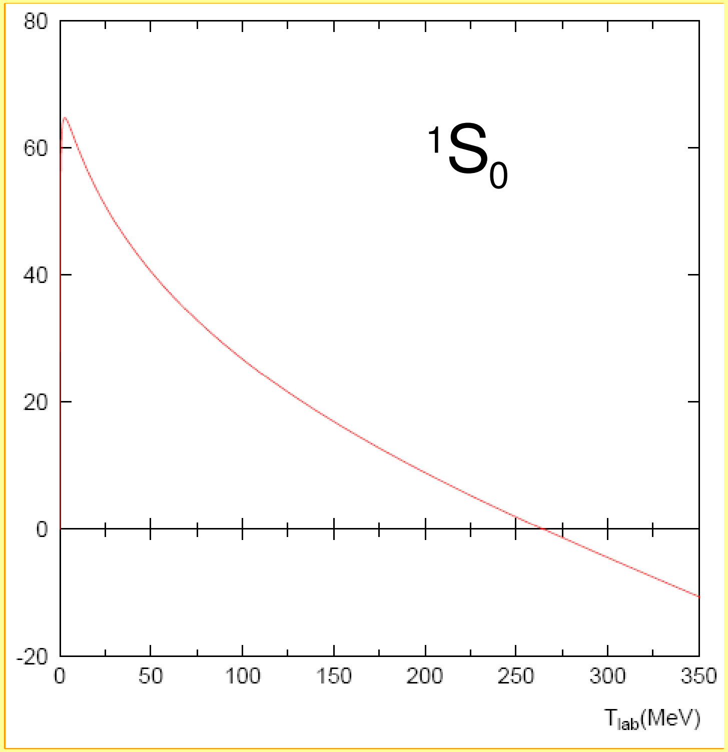


Deuteron is a very weakly bound system!

System has one bound state.

Steep decrease from 180 degrees due to large scattering length a = 5.5 fm.

Acts repulsive due to large (positive) scattering length.



System (barely) fails to exhibit bound state.

Steep rise at 0 due to large scattering length a = -18 fm.

Monotonous decrease due to hard core.

Features of the nucleon-nucleon force

1. The existence of the deuteron with $J^{\pi} = 1^{+}$ indicates that the force between protons and neutrons is attractive at least for the $^3S_1$ partial wave. Interference between Coulomb and nuclear scattering for the proton-proton partial wave $^1S_0$ shows that the NN force is attractive at least for the $^1S_0$ partial wave.
2. It has a short range and strong intermediate attraction.
3. Spin dependent, scattering lengths for triplet and singlet states are different,
4. Spin-orbit force. Observation of large polarizations of scattered nucleons perpendicular to the plane of scattering.

Features of the nucleon-nucleon force

1. Hard core. The s-wave phase shift becomes negative at ≈ 250 MeV implying that the singlet S has a hard core with range $0.4 - 0.5$ fm.
2. Charge independence (almost). Two nucleons in a given two-body state always (almost) experience the same force. Modern interactions break charge and isospin symmetry lightly. That means that the pp, neutron-neutron and pn parts of the interaction will be different for the same quantum numbers.
3. Non-central. There is a tensor force. First indications from the quadrupole moment of the deuteron pointing to an admixture in the ground state of both $l = 2$ ($^3D_1$) and $l = 0$ ($^3S_1$) orbital momenta.

Charge dependence

- After correcting for the electromagnetic interaction, the forces between nucleons (pp, nn, or np) in the same state are almost the same.
- "Almost the same": Charge-independence is slightly broken.
- Equality between the pp and nn forces: Charge symmetry.
- Equality between pp/nn force and np force: Charge independence.

Charge dependence, 1S0 Scattering lengths

Charge-symmetry breaking (CSB), after electromagnetic effects have been removed:

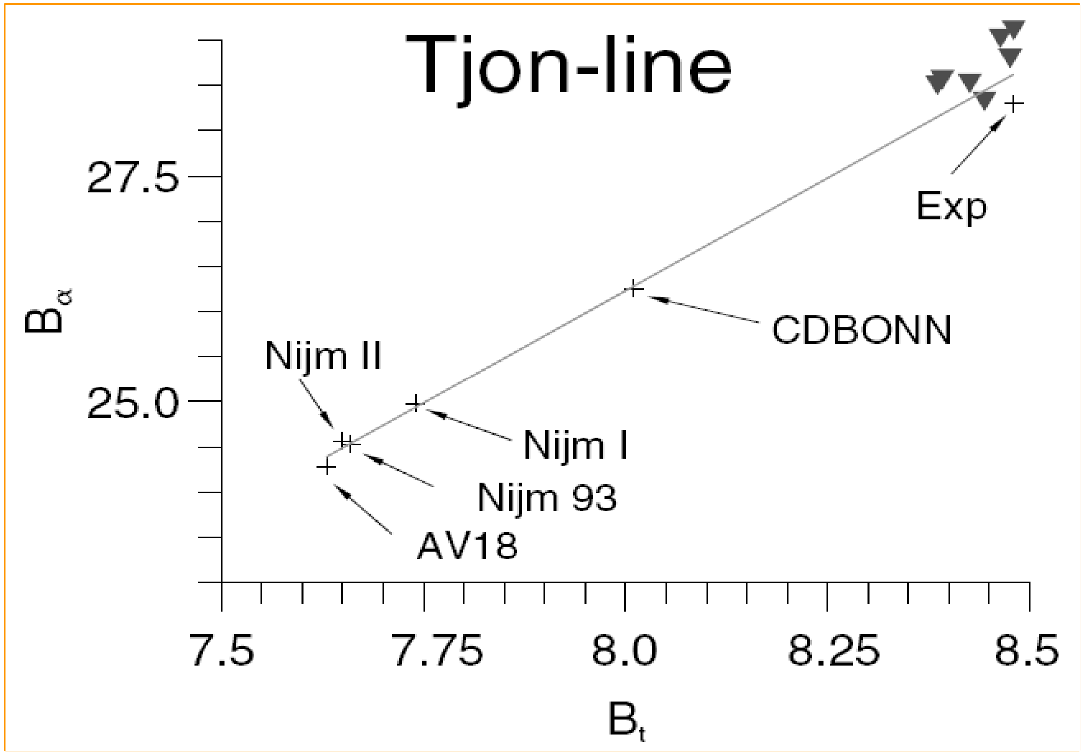
- $a_{pp} = -17.3 \pm 0.4\mathrm{fm}$
- $a_{nn} = -18.8 \pm 0.5\mathrm{fm}$. Note however discrepancy from *nd* breakup reactions resulting in $a_{nn} = -18.72 \pm 0.13 \pm 0.65\mathrm{fm}$ and $\pi^- + d \to \gamma + 2n$ reactions giving $a_{nn} = -18.93 \pm 0.27 \pm 0.3\mathrm{fm}$.

Charge-independence breaking (CIB)

- $a_{pn} = -23.74 \pm 0.02\mathrm{fm}$

1990s: High precision NN potential **models**

- Phenomenological models based on meson exchange.
- Contain about 40 parameters; determined by fit to phase shifts/deuteron.
- Reproduce NN phase shifts with a χ^2/datum very close to 1.0.
- "Nearly perfect" two-body physics.



A. Nogga et al, PRL 85 (2000) 944

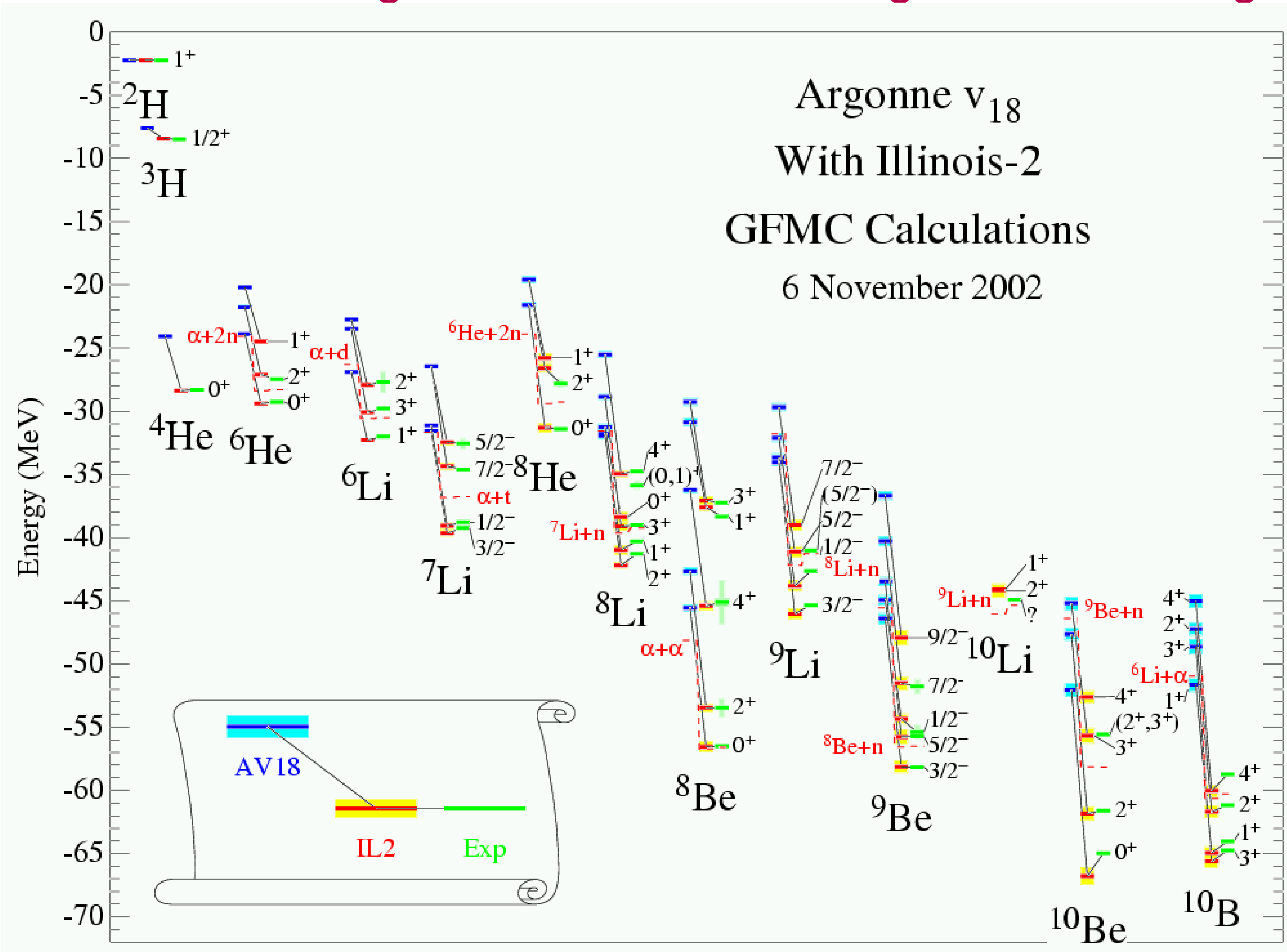
Different two-body potential models disagree on structure of triton and alpha particle.

With additional **three-nucleon forces**, agreement with experiment is possible.

(Three-nucleon force differs for different two-body potentials.)

Four-body forces very small.

Role of TNF in light nuclei: overall binding & level ordering



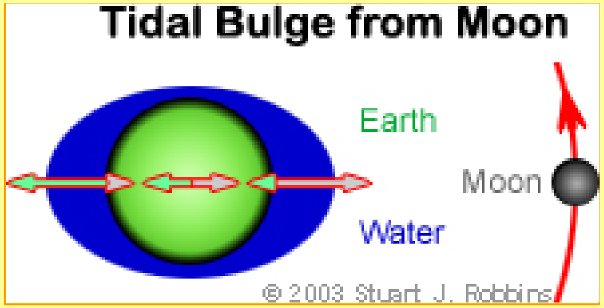
Three-nucleon forces

- Different NN forces must be associated with different three-nucleon forces
- Modern understanding that there are no 'TRUE' NN and 3N forces.
- Question: Is there a consistent and systematic way of relating 3NF's to a given nucleon-nucleon force ?
- Question: Is there a systematic way of linking low-energy nuclear structure with QCD ?
- Answer: YES! Chiral effective field theory.

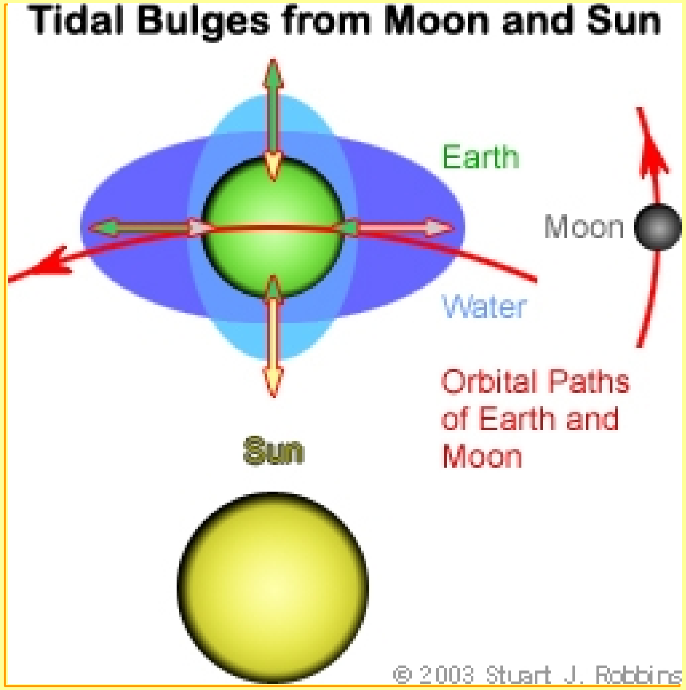
Three nucleon forces: Why?

- Nucleons are not point particles (i.e. elementary).
- We neglected some internal degrees of freedom (e.g. Δ-resonance, “polarization effects”, …), and unconstrained high-momentum modes.

Example from celestial mechanics:
Some tidal effects are included in the two-body interaction

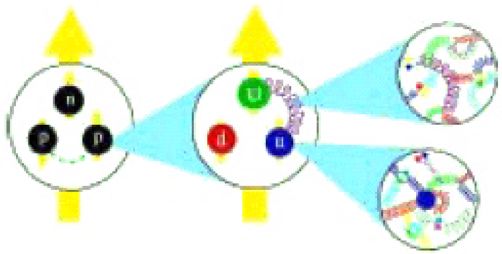


Other tidal effects cannot be included in the two-body interaction! Three-body force unavoidable.



Renormalization group transformation: Removal of “stiff” degrees of freedom at expense of additional forces.

Effective Field Theory



Interplay between nucleonic and sub-nucleonic (quarks and gluons) degrees of freedom in few-body nuclear systems

Its pretty complicated inside a nucleon!!

Starting point is an effective Lagrangian.

$$L_{\pi N} = L_{\pi N}^{(1)} + L_{\pi N}^{(2)} + L_{\pi N}^{(3)} + \cdots$$

- Obeys QCD symmetries (spin, isospin, chiral symmetry)

$$L_{\pi N}^{(1)} = \overline{N}\left(iD_0 - \frac{g_A}{2}\vec{\sigma} \bullet \vec{u} \right)N \approx \overline{N}\left[i\partial_0 - \frac{1}{4 f_\pi^2}\tau \bullet (\pi \times \partial_0 \pi) - \frac{g_A}{2 f_\pi}\tau \bullet (\vec{\sigma} \bullet \nabla)\pi \right]N + \cdots$$

Chiral Perturbation Theory.

"If you want more accuracy, you have to use more theory (more orders)"

Effective Lagrangian → obeys QCD symmetries (spin, isospin, chiral symmetry breaking)

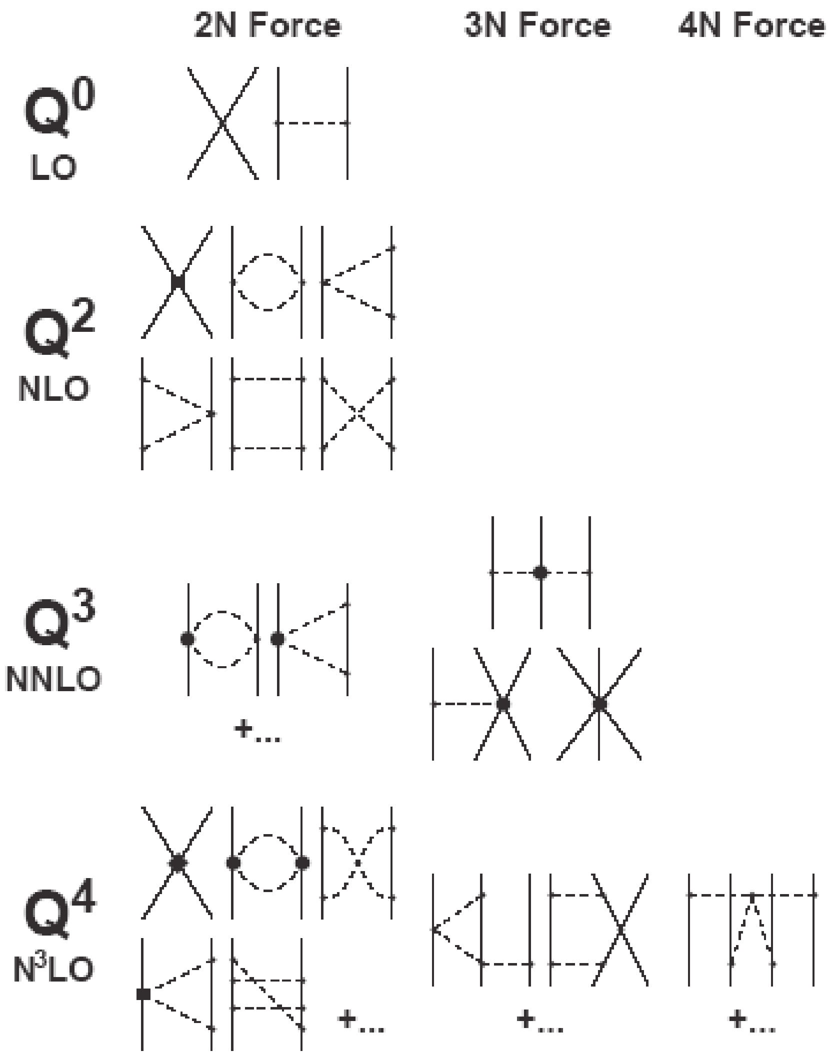
Lagrangian
→ infinite sum of Feynman diagrams.

Expand in $O(Q/\Lambda_{QCD})$

Weinberg, Ordonez, Ray, van Kolck

NN amplitude uniquely determined by two classes of contributions: contact terms and pion exchange diagrams.

24 paramters (rather than 40 from meson theory) to describe 2400 data points with $\chi^2_{dof} \approx 1$



At which order do we get sufficient accuracy ?

Nucleon-Nucleon phase shifts up to

300 MeV for different orders in Chiral Perturbation theory.

Red line from Entem & Machleidt

(Phys. Rev. C 68, 041001 (2003) and

green lines from Epelbaum et al.

(Eur. Phys. J A15, 543 (2002).

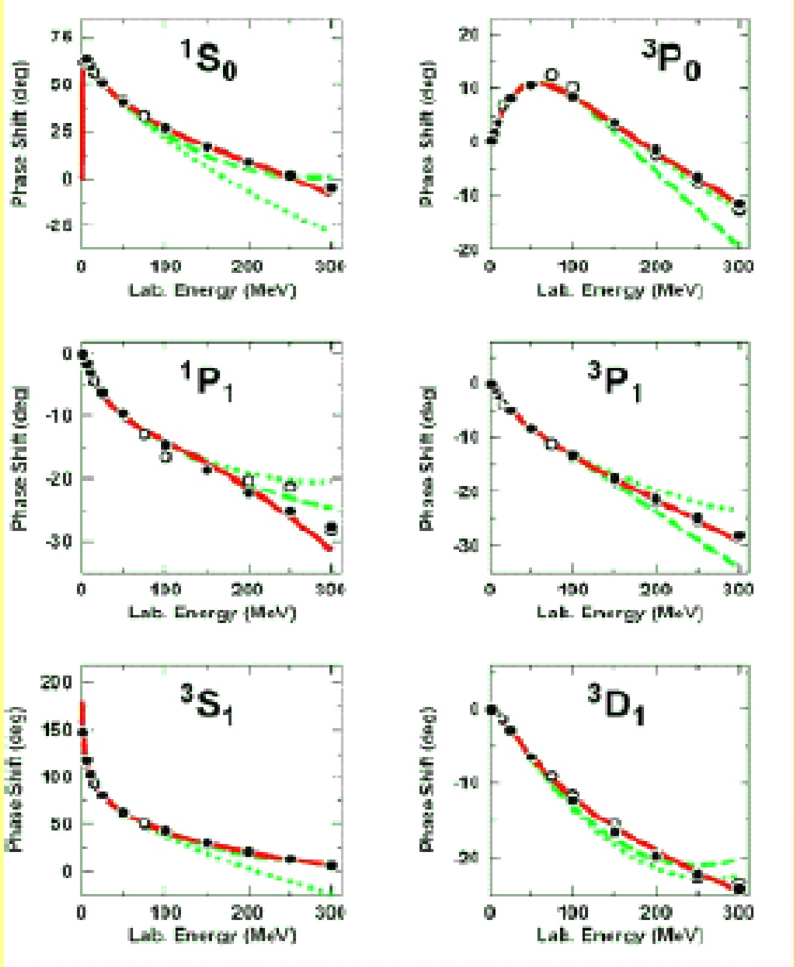
Green dotted line : Next to leading order NLO

Green dashed line : Next-to-next-to leading order NNLO

Red Line : N3LO

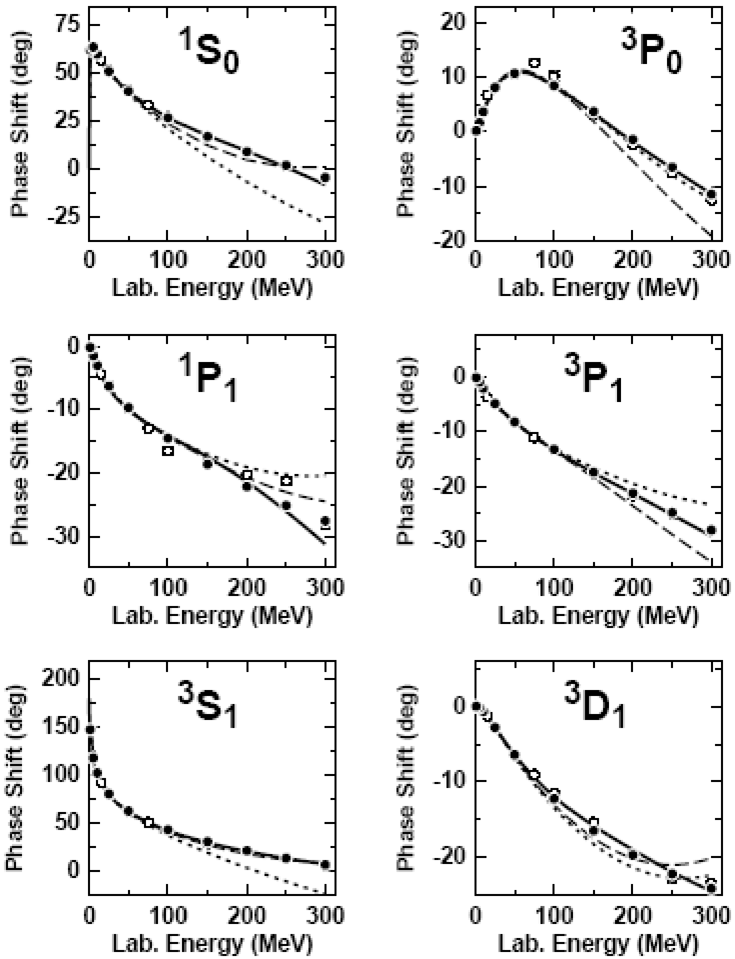
Phase shifts reproduced to χ^2/datum=1

About 40+ parameters

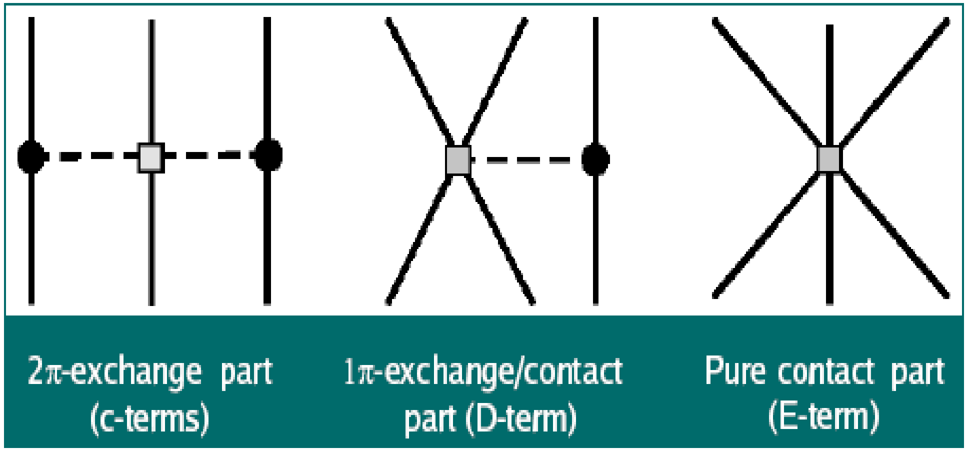


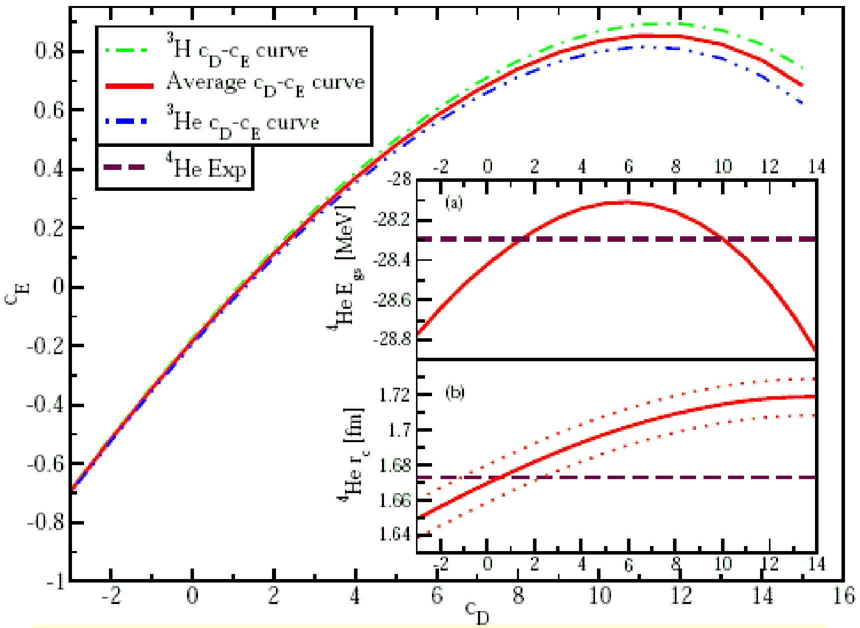
D. R. Entem and R. Machleidt, Phys.Rev. C68 (2003) 041001

Three-nucleon force at NNLO



dashed -> NLO
dot -> NNLO
Solid -> N3LO





Challenge: Deliver the best NN and NNN interactions with their roots in QCD.

Chiral interactions and role 3NF in light nuclei.

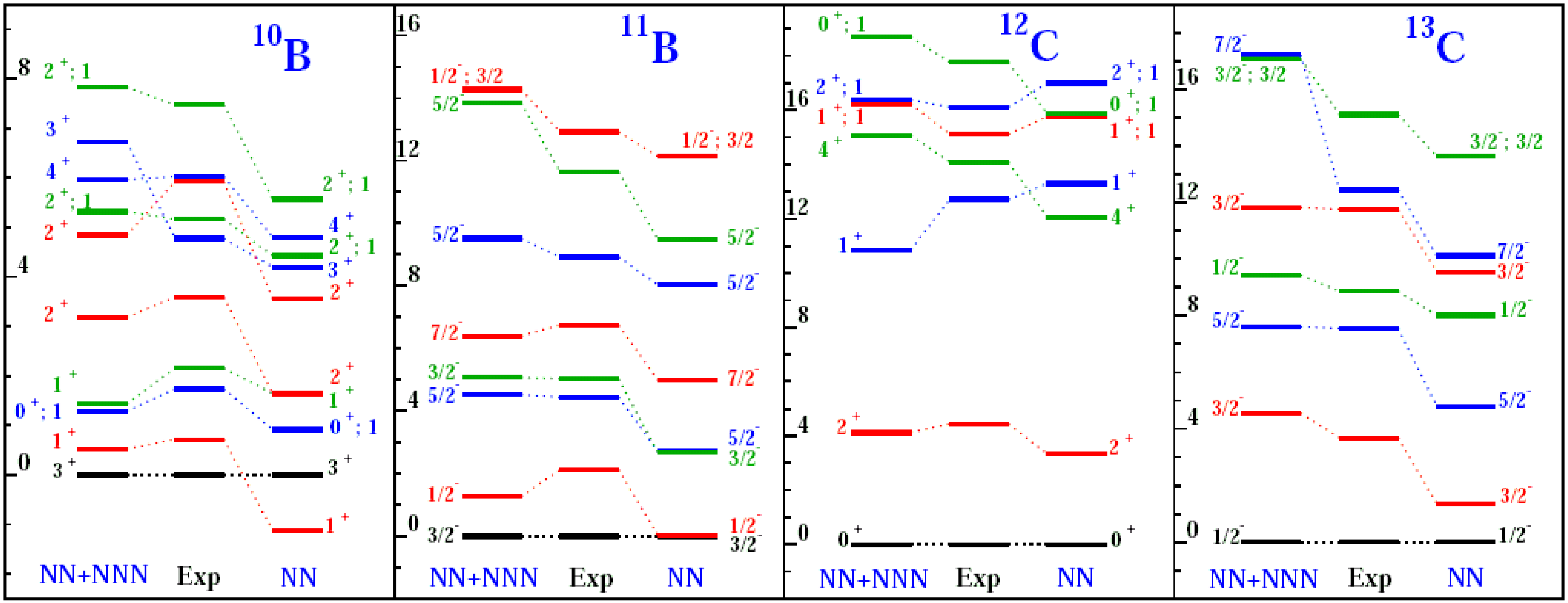
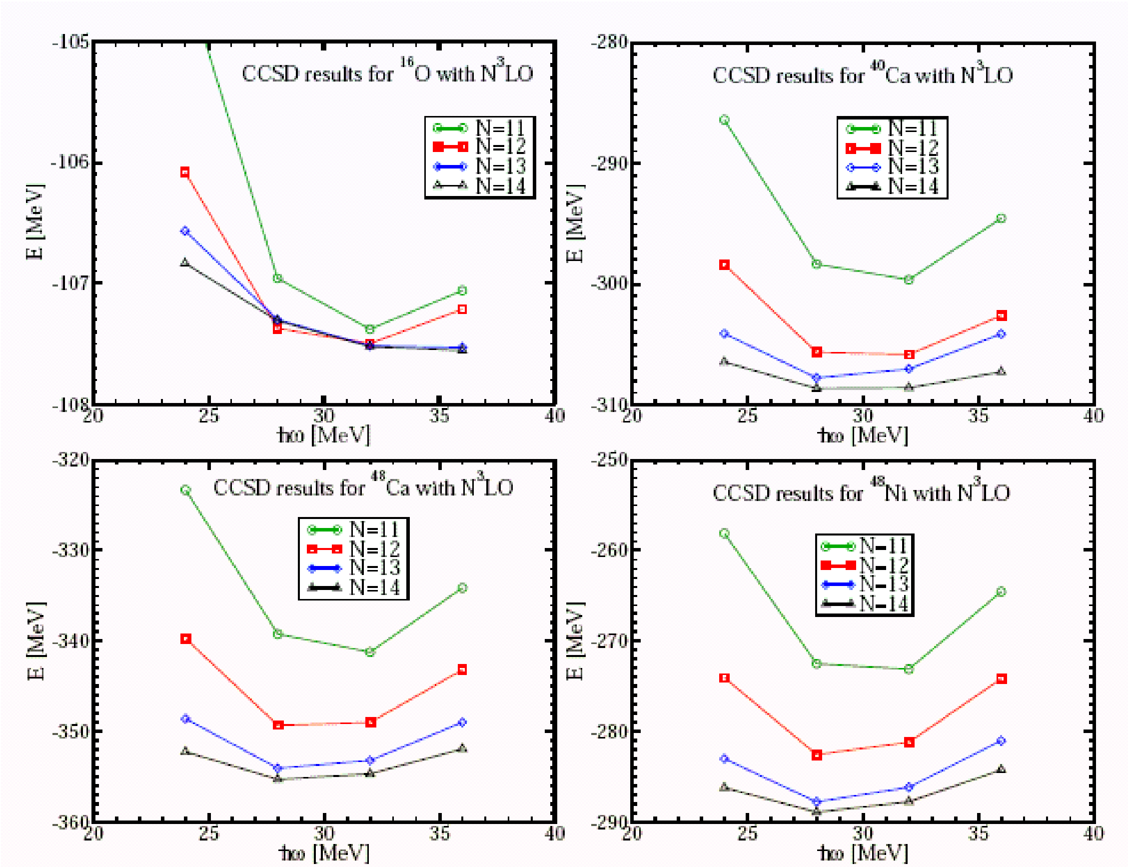


FIG. 4: States dominated by p-shell configurations for ^{10}B, ^{11}B, ^{12}C, and ^{13}C calculated at $N_{\max} = 6$ using $\hbar\Omega = 15$ MeV (14 MeV for ^{10}B). Most of the eigenstates are isospin T=0 or 1/2, the isospin label is explicitly shown only for states with T=1 or 3/2. The excitation energy scales are in MeV.

P. Navratil et al.,
Arxiv:nucl-th/0701038

Chiral interactions in medium sized nuclei



Nucleus	E/A	V/A	Q	$\Delta E/A$	$< r^2 >_{ch}^{1/2}$	$< r^2 >_{ch}^{1/2}$ (Exp)
^{4}He	-5.99	-22.75	0.90	1.08		1.673(1)
^{16}O	-6.72	-30.69	1.08	1.25	2.72(5)	2.737(8)
^{40}Ca	-7.72	-36.40	1.18	0.84	3.25(9)	3.4764
^{48}Ca	-7.40	-37.97	1.21	1.27	3.24(9)	3.4738
^{48}Ni	-6.02	-36.04	1.20	1.21	3.52(15)	?

Coupled Cluster approach to medium sized nuclei with bare Chiral Interactions.

G. Hagen et al., to be submitted to PRL.

Summary on microscopic interactions

- High precision two-body potential models available
  - Fit two-nucleon scattering data with χ^2/datum close to 1.
  - Yield different results for triton and alpha particle.
- Calculations in light systems require three-nucleon forces
  - No "best" potential.
  - Different TNF for each two-nucleon potential.
- Chiral effective field theory
  - Based on QCD symmetries.
  - permits systematic construction of nuclear forces (parametrization of our ignorance).
  - TNF arise naturally.