Parton – Hadron Duality and Low–Q^2 Moments

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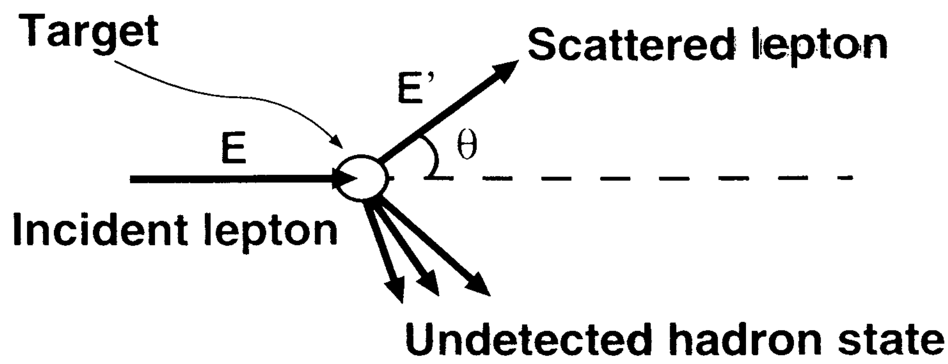
April 17, 2000

OUTLINE:

- INTRODUCTION
- B-G. DUALITY STUDIES
- MOMENTS OF F_2^p at LOW Q^2
- CONCLUSION

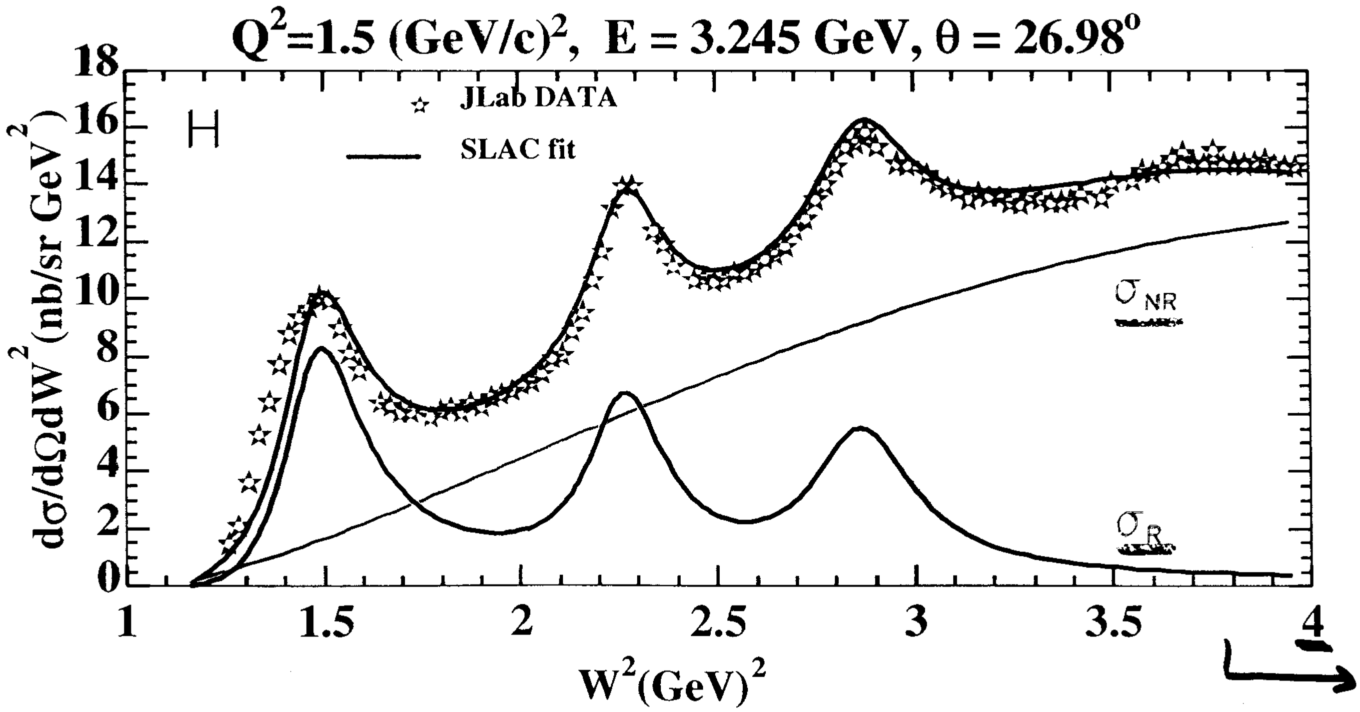
Inclusive electron scattering



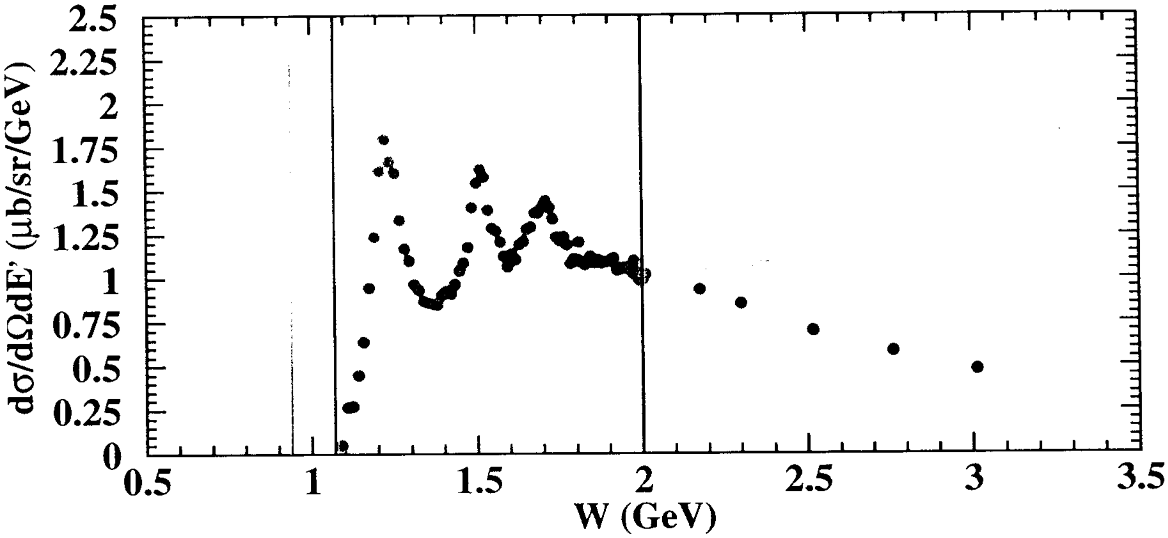
$$x = \frac{Q^2}{2\ M\ \nu}; \quad \xi = \frac{2x}{1+r}\ , \quad r = \sqrt{1 + \frac{4M^2x^2}{Q^2}}$$

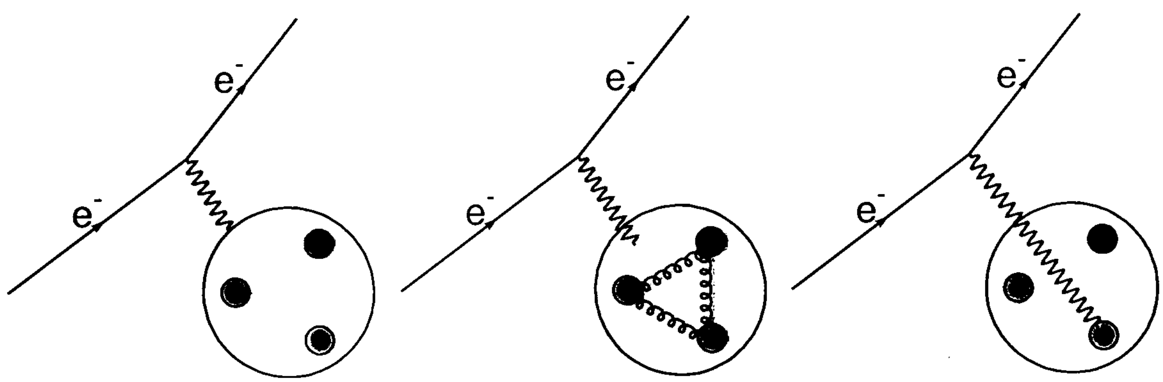
$$(Q^2 \to \infty \quad \xi \to x)$$

$$\frac{d^2\sigma}{d\Omega\ dE'} = \frac{4\alpha^2 E'^{\,2}}{Q^4}[2\,W_1(\nu, Q^2)\ \sin^2(\theta/2) + \underbrace{W_2(\nu, Q^2)}_{F_2 = \nu W_2}\ \cos^2(\theta/2)]$$



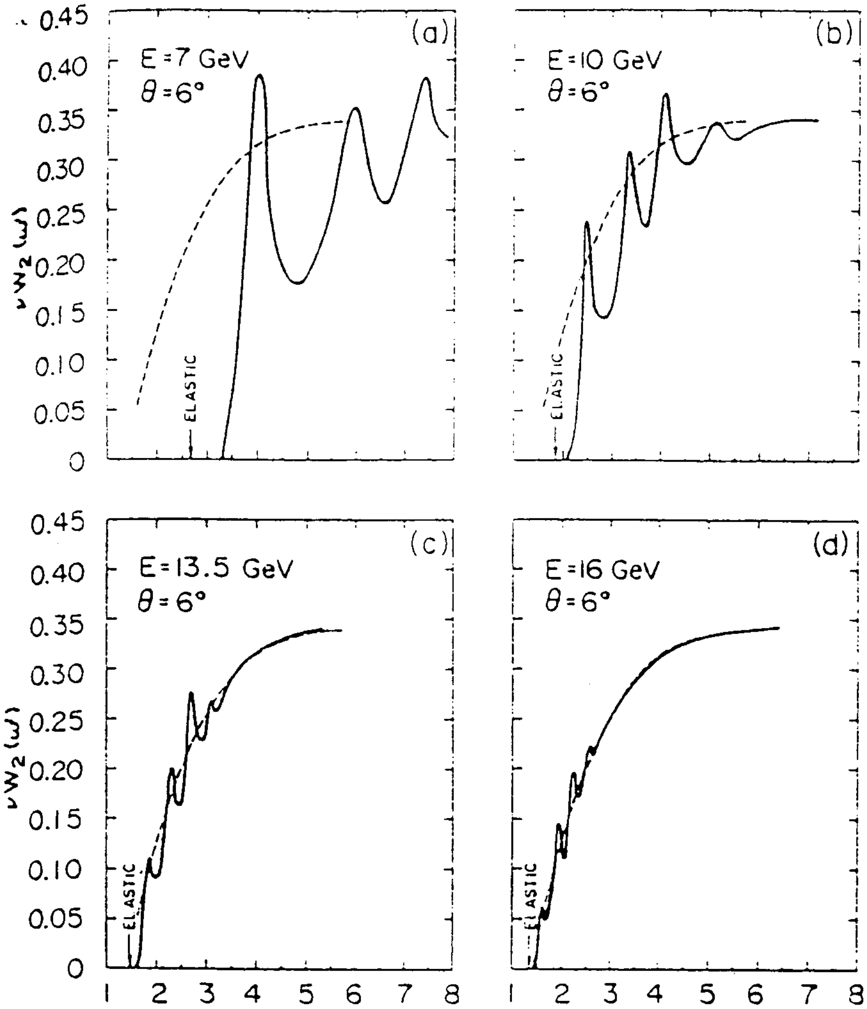
SLAC Data





Elastic Resonance DIS

Bloom - Gilman Duality



Bloom, Gilman
1968

$W^2 = 2$
$Q^2 = 1$ $\quad \omega' = 3$

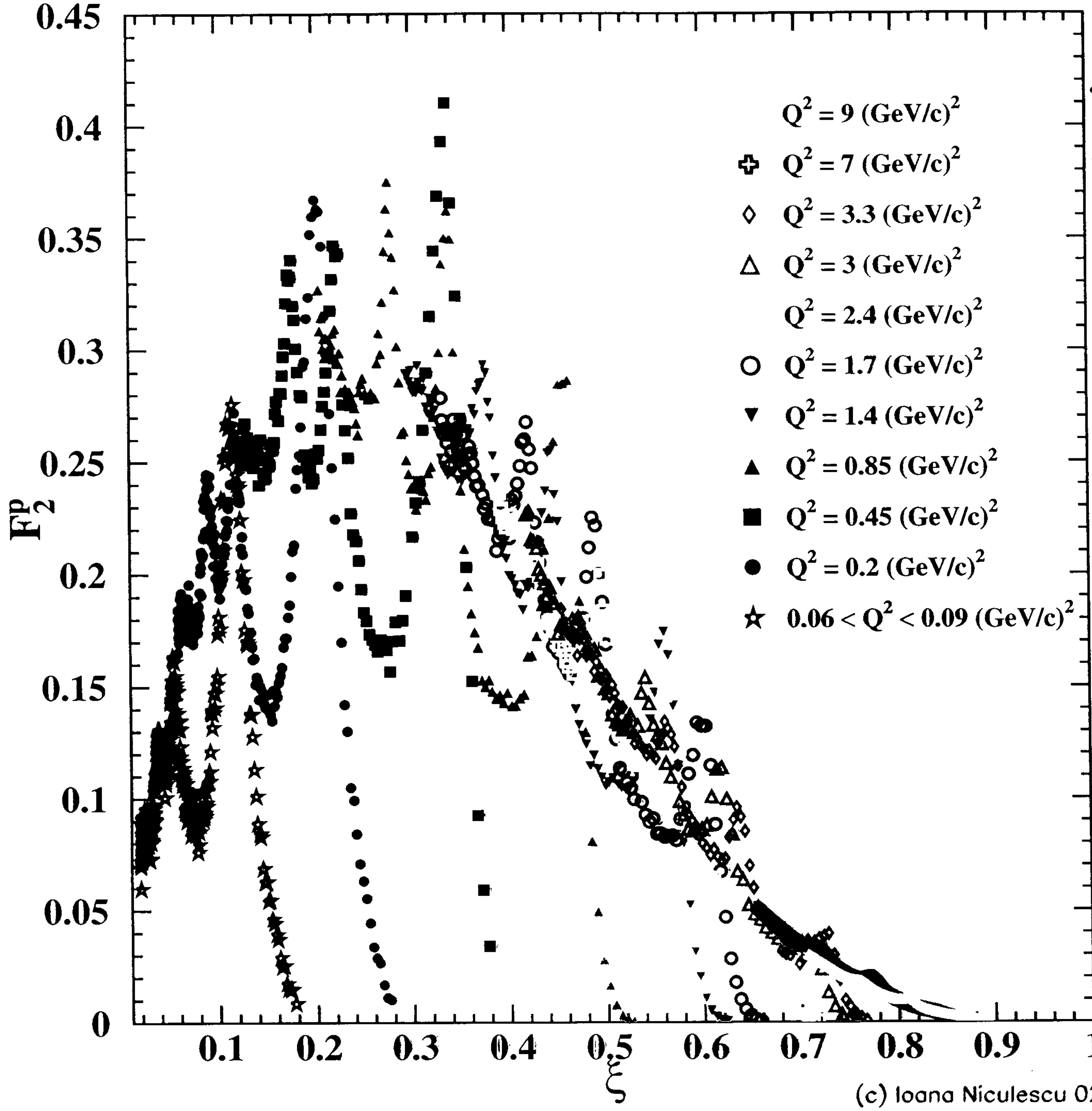
$W^2 = 50$
$Q^2 = 25$ $\quad \omega' = 3$

$$\omega' = 1 + \frac{W^2}{Q^2}$$

"improved" scaling variable.

- "The connection between the behavior of the resonances and the scaling limit hints at a common origin for both."

C. Carlson, N. Mukhopadyay, PR D**41**, 2343, 1990



} SLAC model

JLab 1996

1% sta

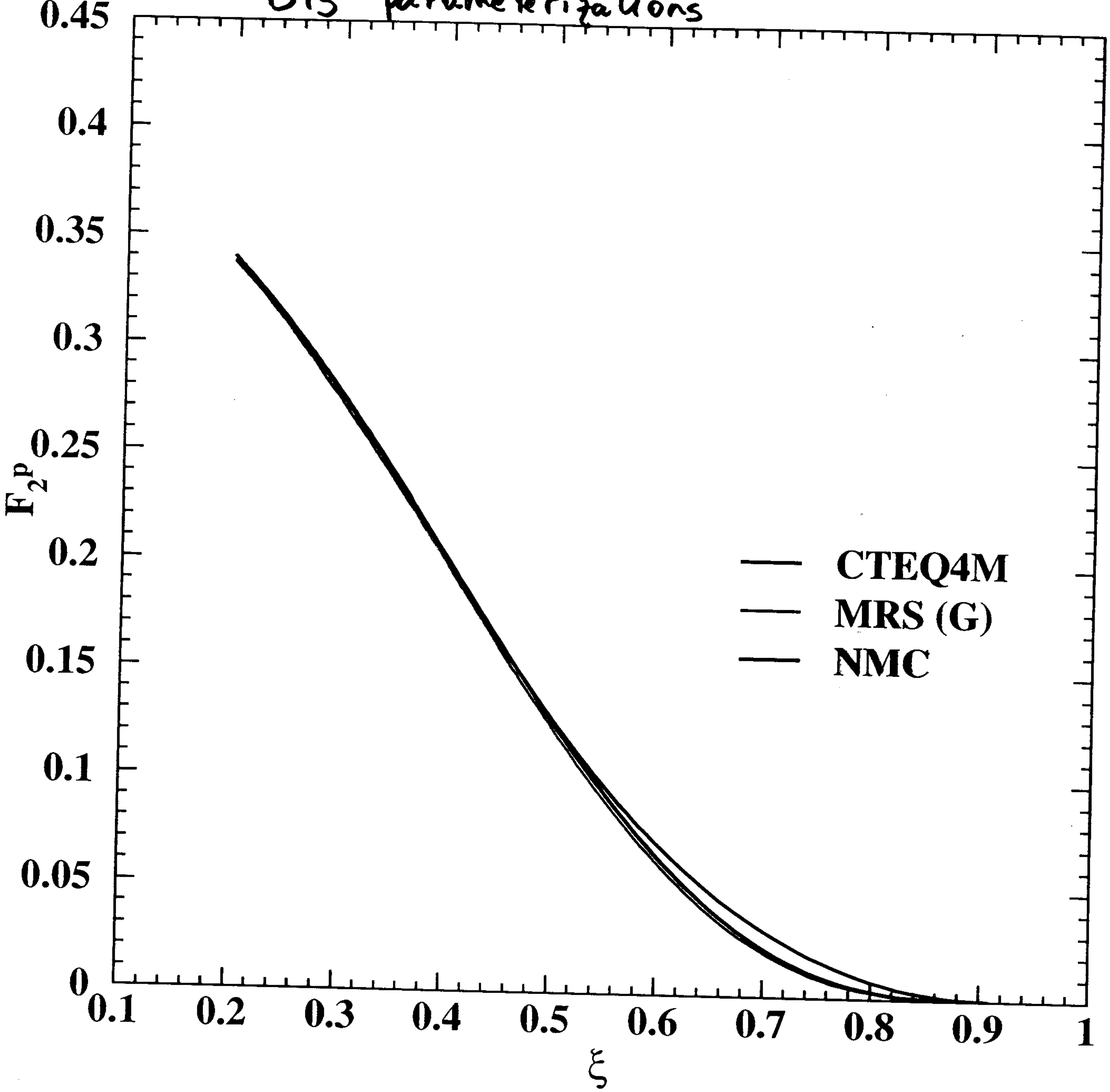
2-3% syst

} SLAC 1975

10% normaliz & rad. corr.

(c) Ioana Niculescu 02/02/00

Dis parameterizations

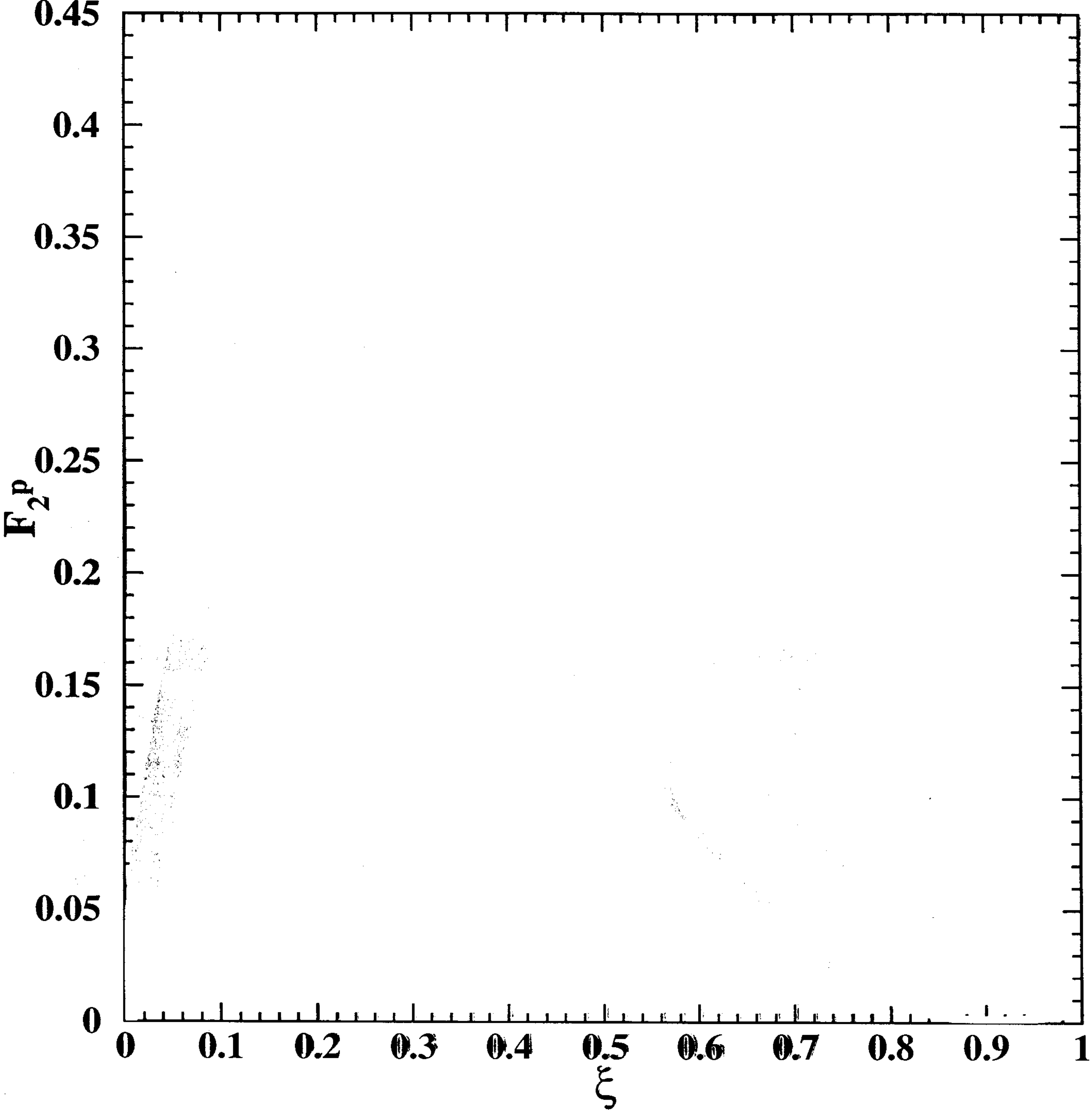


MRS : Martin, Roberts & Stirling

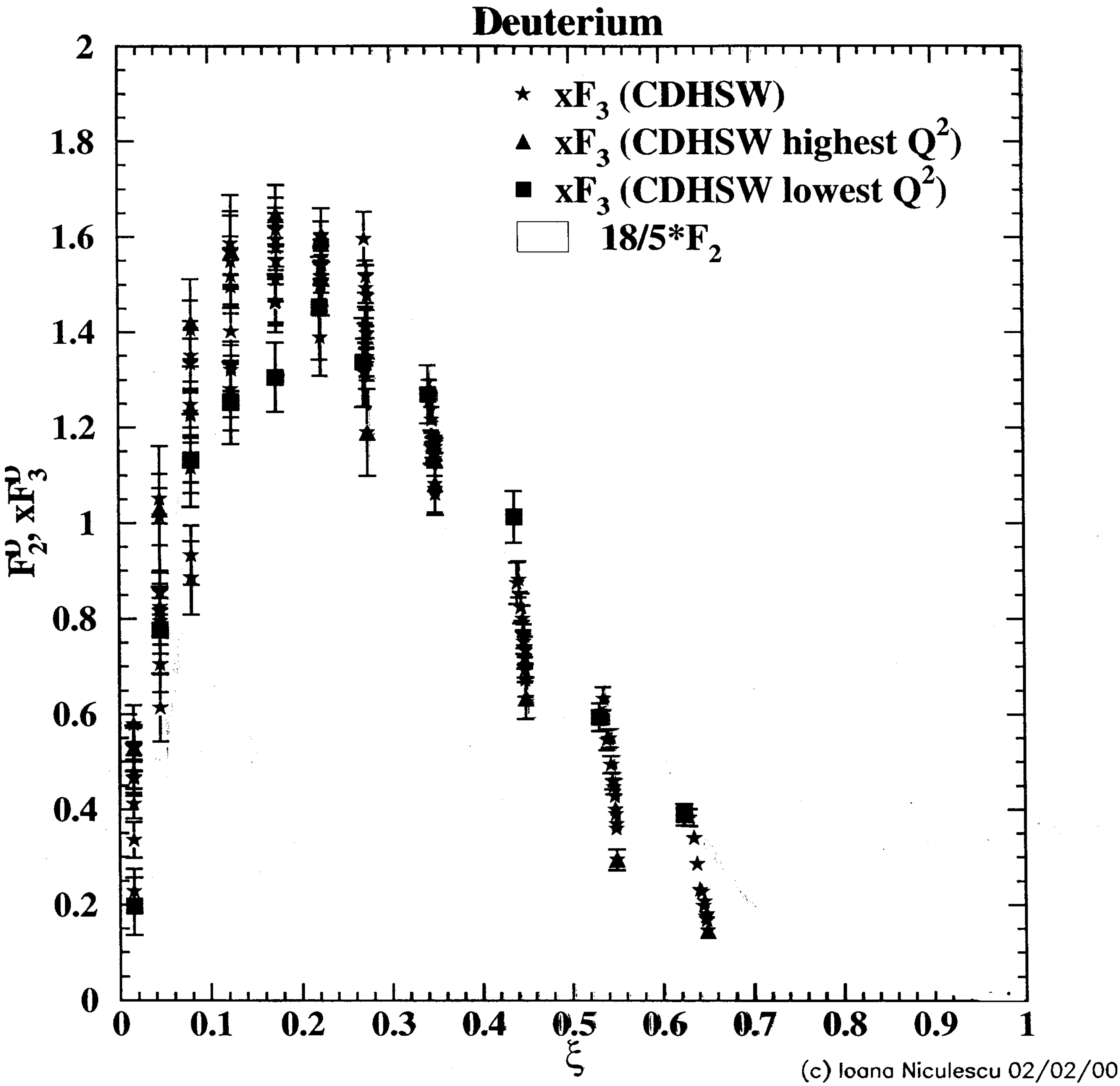
NMC : New Muon Collaboration

CTEQ : Coordinated Theoretical / Exp. Project on QCD Phenomenology ....

Fit to resonance data



Resonances oscillate around one curve even at very low Q^2 (0.2 GeV^2)



Compare F_2^D to xF_3 (valence only?)

Why should Bloom – Gilman duality work?

• De Rújula, Georgi, Politzer – QCD explanation of why the scaling curve would represent the average of the resonance bumps. Ann. Phys. **103**, 315, 1977

$$M_n(Q^2) = A_n(Q^2) + \sum_k \left(\frac{nM_0^2}{Q^2}\right)^k B_{n,k}(Q^2)$$

• Formal separation of **twist–2** term (calculable in pQCD) from **higher twists**

• At **high** $Q^2 \rightarrow$ keep only $A_n(Q^2)$

$$M_n(Q^2) = A_n(Q^2) \equiv A_n(\ln(Q^2))$$

• At **low** $Q^2 \rightarrow$ **higher twists** become important

• Duality is expected to hold if **higher twists** are **small**.

Power Corrections

$$\frac{d^2\sigma}{d\,\Omega d\,E'} = \frac{\alpha^2}{Q^4}\frac{E'}{E}L^{(e)}_{\mu\nu}(k,q)\,W^{\mu\nu}(p,q)$$

$$W^{\mu\nu} = \int d^4z\,e^{iqz} < p|J_\mu(z)J_\nu(0)|p >$$

- Operator Product Expansion (OPE)

$$iT(J(z)J(0)) = \sum_{\tau,n} C_{\tau,n}(z^2)z_{\mu_1}\ \dots\ z_{\mu_n}O^{\tau}_{\mu_1\dots\mu_n}$$

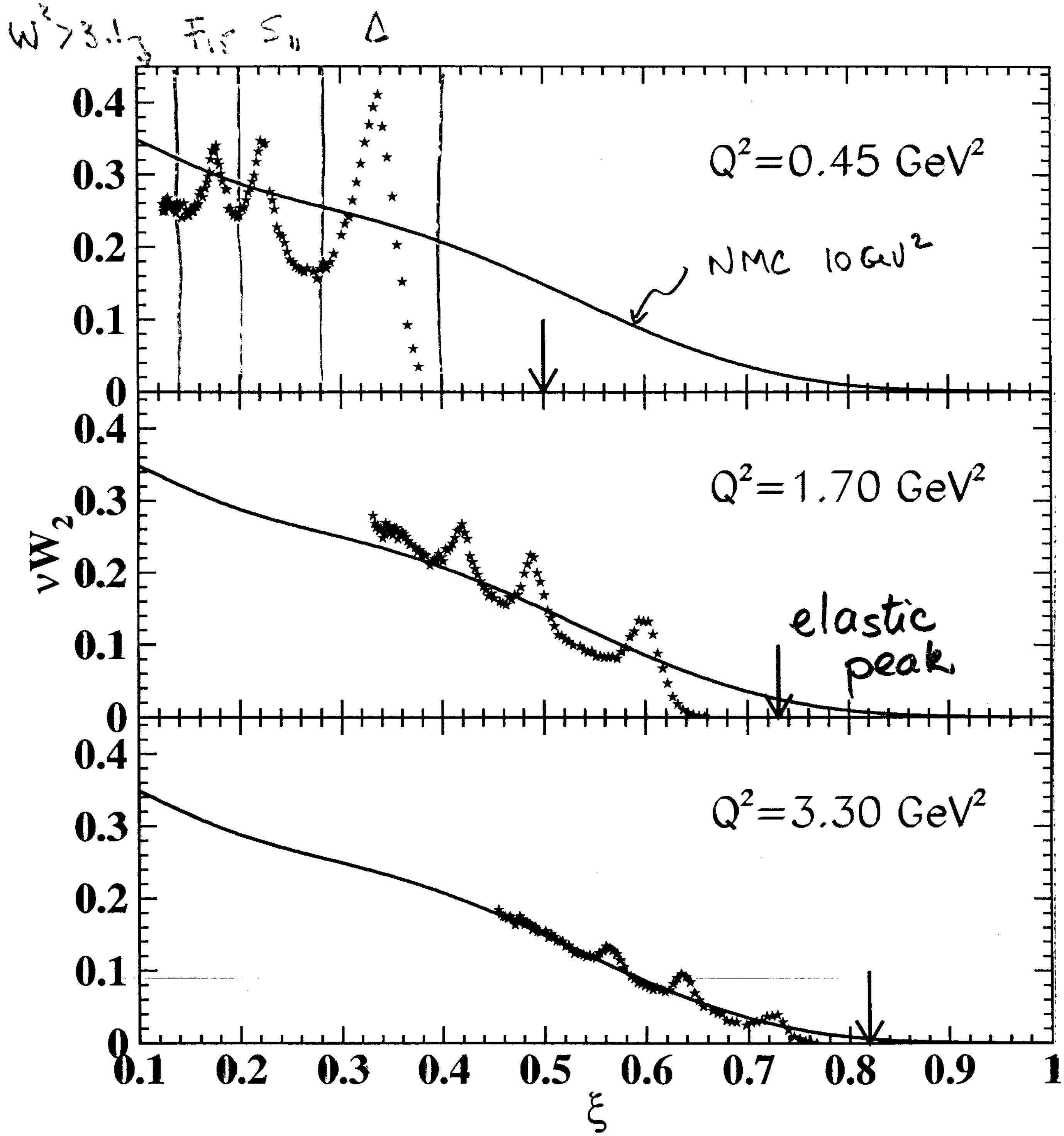
τ labels the twist

- QCD Moments (Cornwall - Norton):

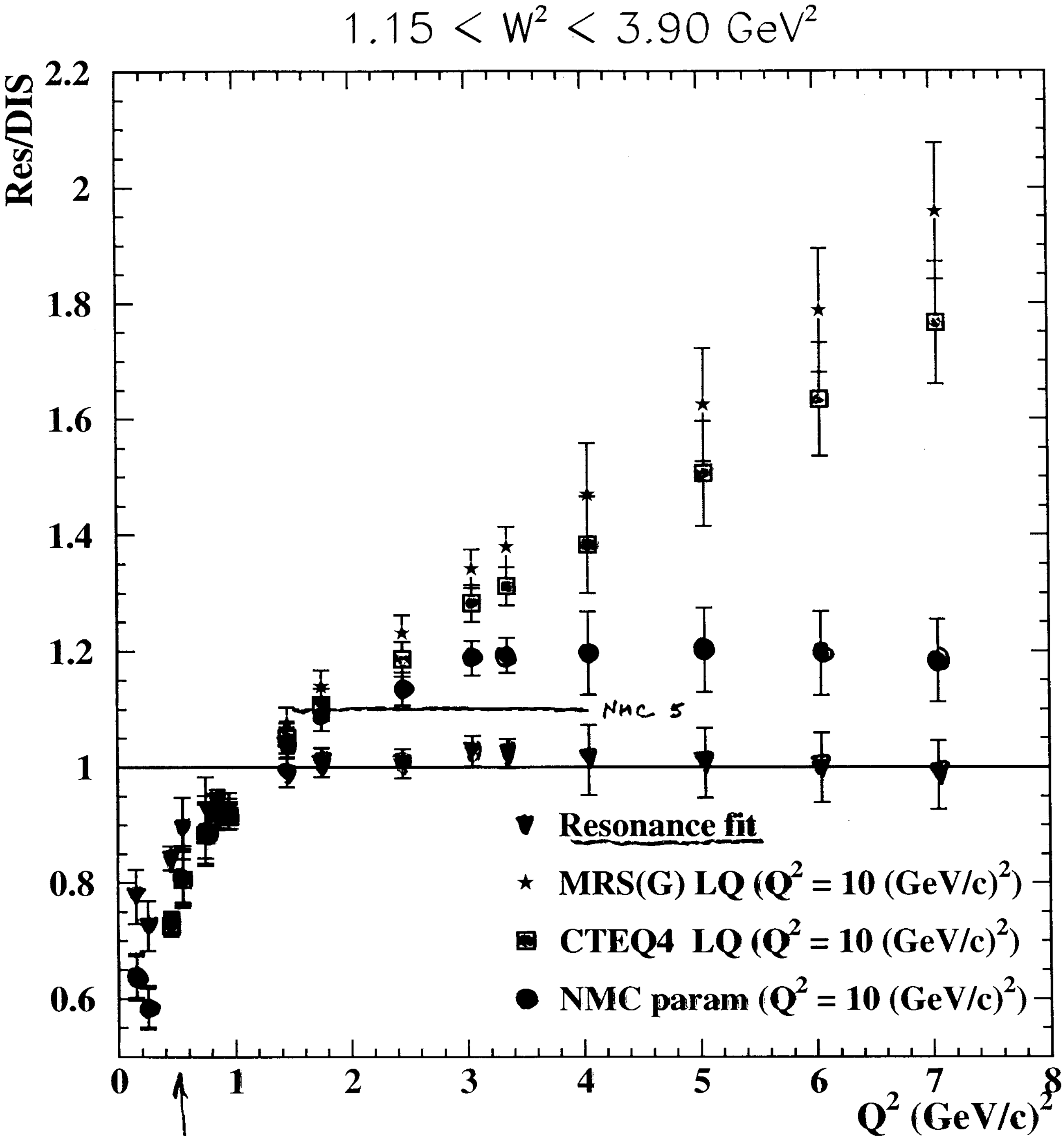
$M_n = \int_0^1 dx\,x^{n-2}F_2(x,Q^2)$

- Using **OPE** $\rightarrow$ twist expansion:

$$M_n(Q^2) = \sum_{i,\tau}\left(\frac{1}{Q^2}\right)^{(\tau-2)/2}C^i_{\tau,n}(Q^2)O^i_{n+2}$$

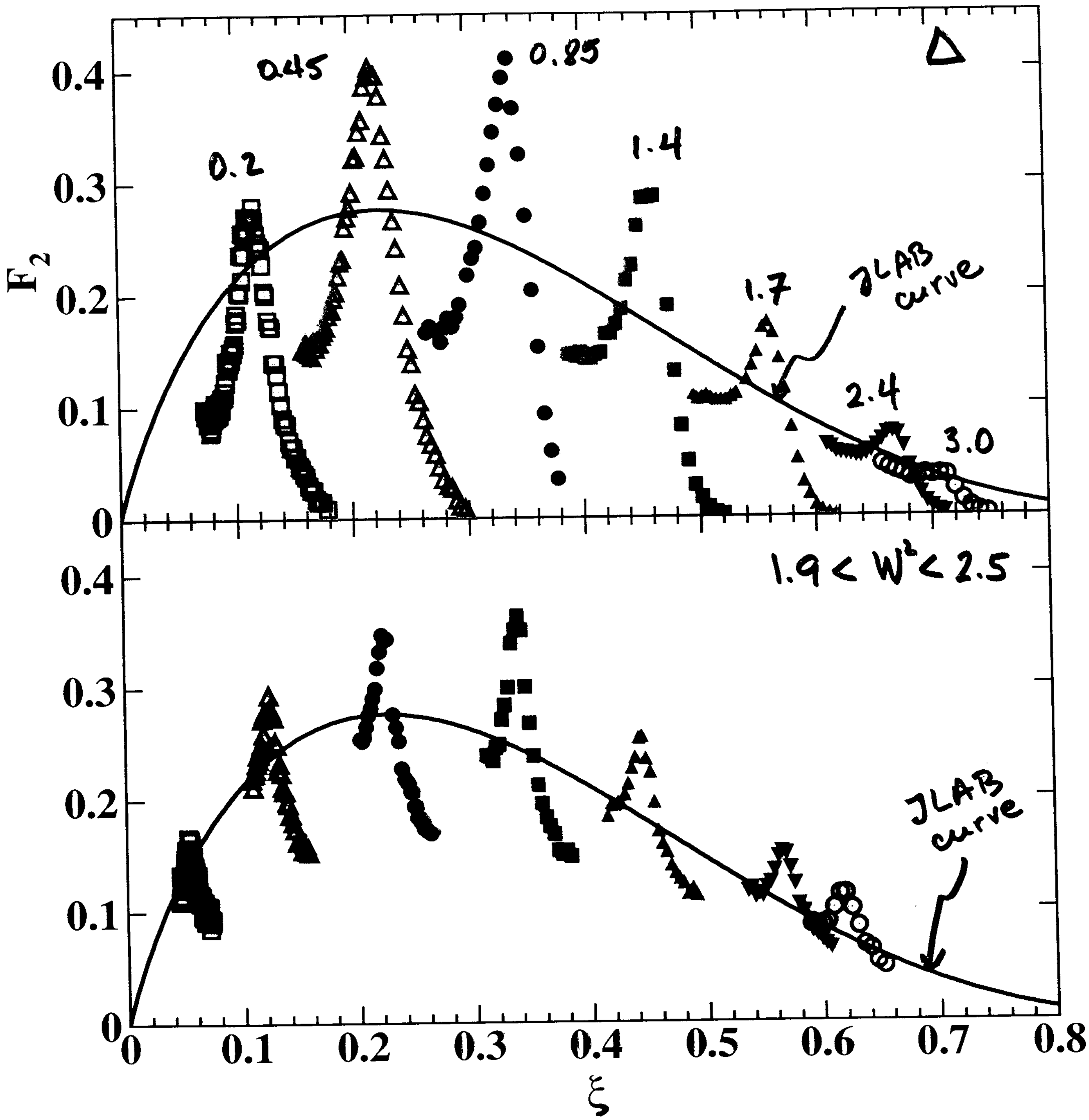


$$I(Res/DIS) = \frac{\int_{\xi_{min}}^{\xi_{max}} \nu W_2(\xi)\, d\,\xi}{\int_{\xi_{min}}^{\xi_{max}} F_2(\xi)\, d\,\xi}$$



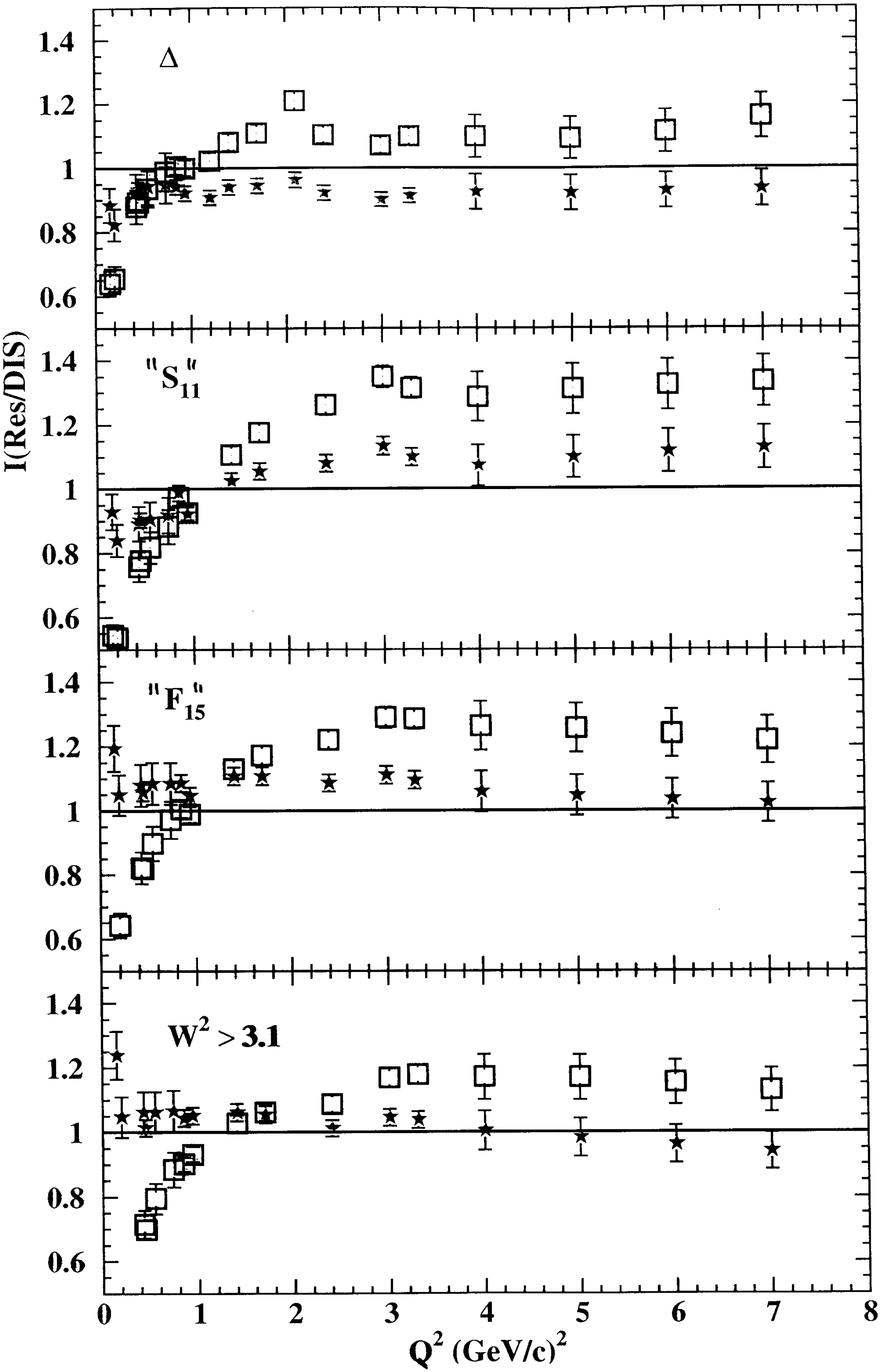
$Q^2 \sim 0.5\ \mathrm{GeV}^2$

BG duality works to ~10%



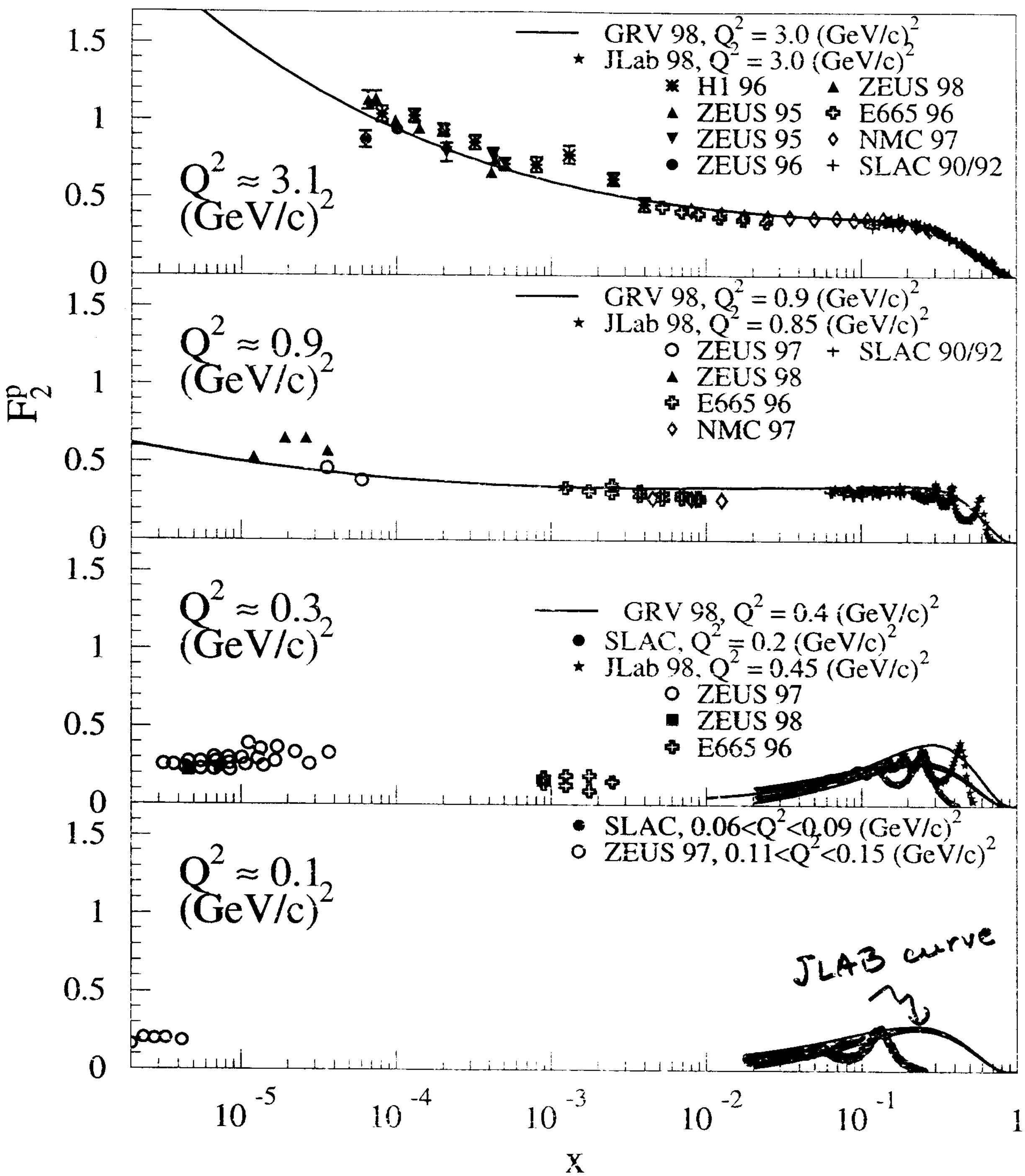
scaling curve → determines the Q^2 dependence of nucleon resonances

Transition F.F. scale as Q^{-4} at ~small Q^2


Δ
"S11"
"F15"
W^2 > 3.1
I(Res/DIS)
Q^2 (GeV/c)^2
1.4
1.2
1
0.8
0.6
0
1
2
3
4
5
6
7
8

F_2 collapse

small Q^2 dependence



GRV - Glück, Reya, Vogt

resonance F_2 - valence-like?

Moments of F_2

Define **Cornwall–Norton moments**:

$$M_n(Q^2) = \int_0^{x_{thr}} dx x^{n-2} \nu W_2(x, Q^2) \qquad (1)$$

Define **Nachtmann moments**:

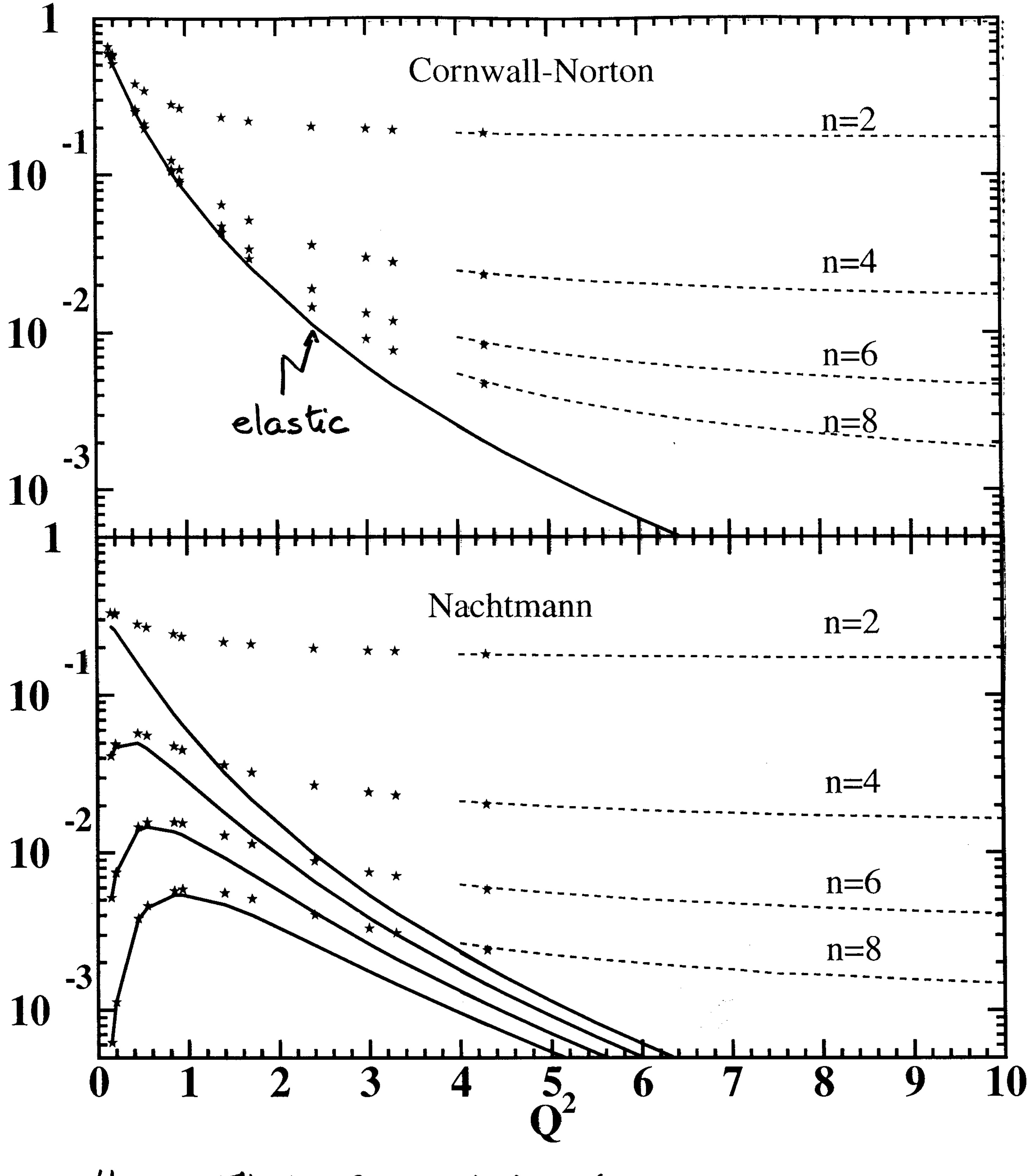
$$M_n(Q^2) = \int_0^{x_{thr}} dx \frac{\xi^{n+1}}{x^3} \left[\frac{3 + 3(n+1)r + n(n+2)r^2}{(n+2)(n+3)}\right] \nu W_2(x, Q^2) \qquad (2)$$

$r = (1 + 4M^2 x^2/Q^2)^{1/2}$,
x_{thr} Bjorken x for pion threshold.

Elastic contribution $(x = 1)$:

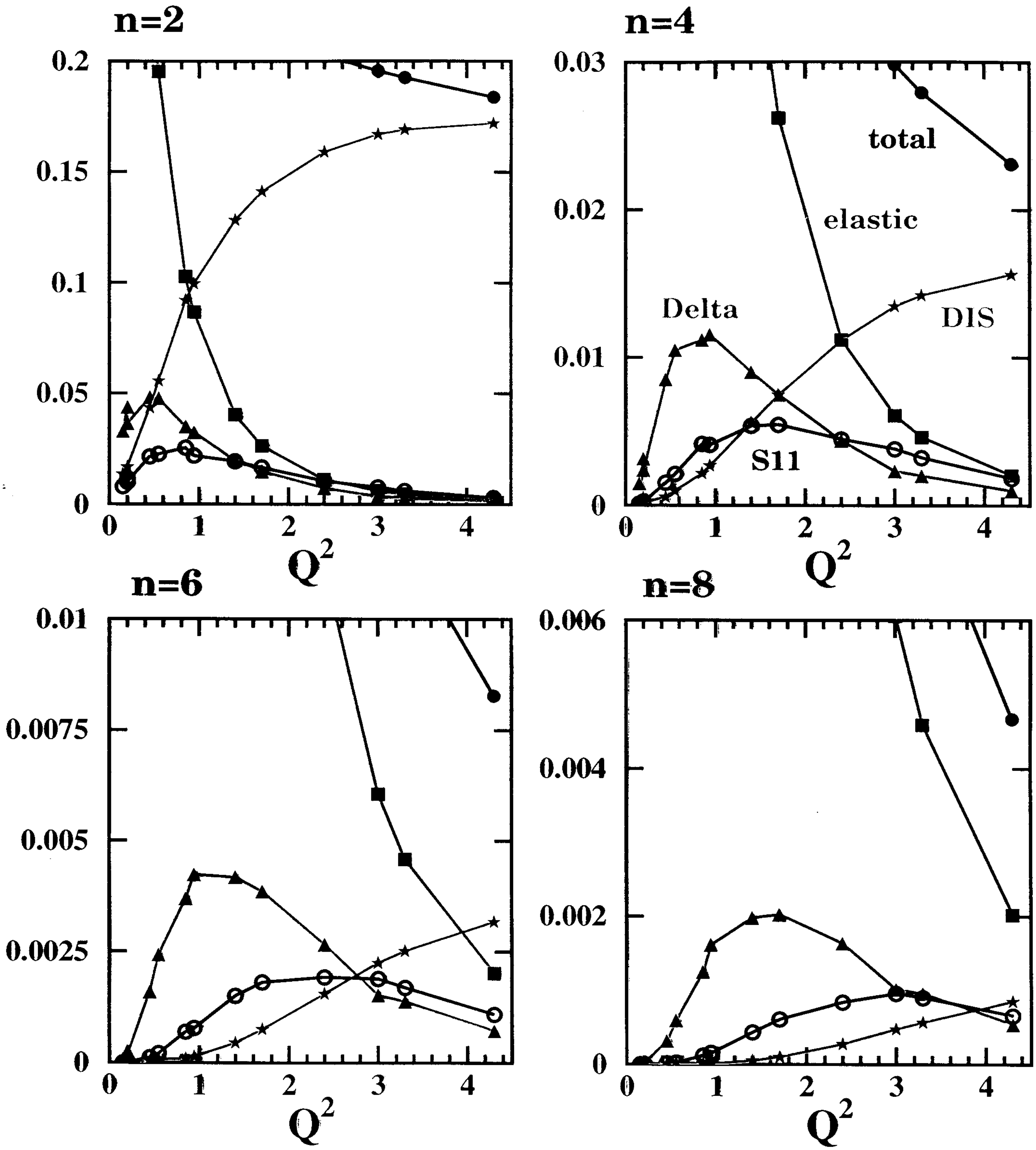
$$\nu W_2(x, Q^2) = \delta(1 - x) \frac{(G_E^2(Q^2) + \frac{Q^2}{4M^2} G_M^2(Q^2))}{(1 + \frac{Q^2}{4M^2})} \qquad (3)$$

G_E (G_M) = proton electric (magnetic) form factor



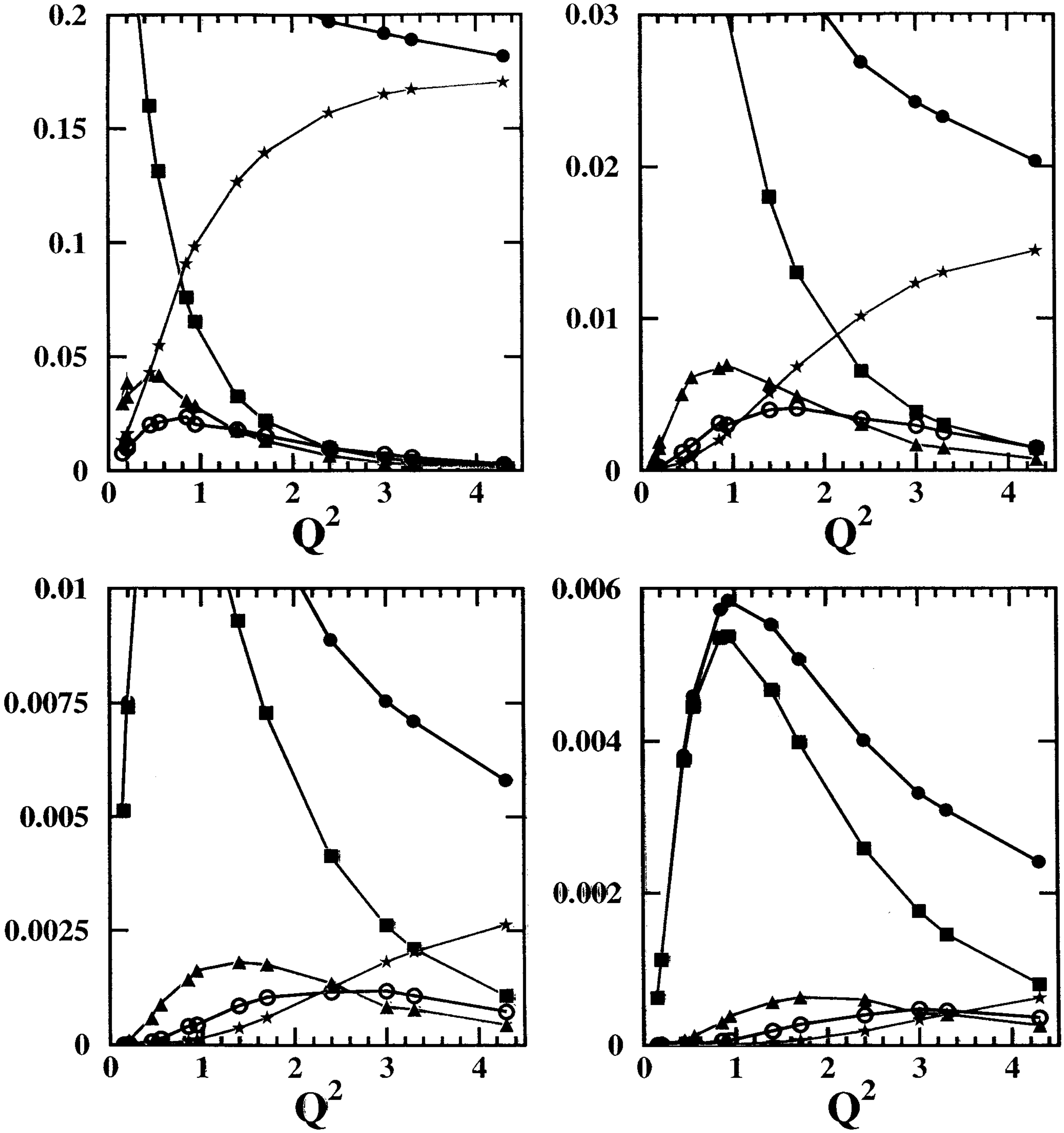
Use JLab + SLAC data (large x)
MRS param (small x)

Cornwall - Norton Moments



Resonance contribution – non-negligible even at $Q^2 \simeq 5$
Moments of F_2 show a smooth transition from DIS to $Q^2 = 0$

Nachtmann Moments



Extraction of higher twist coefficients from QCD moments

see: I.N., S.L., C.K., G.N. in PRD 60, 094001 (1999)

Using higher twist expansion, write the moments:

$$[M_n(Q^2)]^{-1/d_n} = P_1 \ln \frac{Q^2}{\Lambda^2}(1 + \frac{P_2}{Q^2} + \frac{P_3}{Q^4})^{-1/d_n}$$

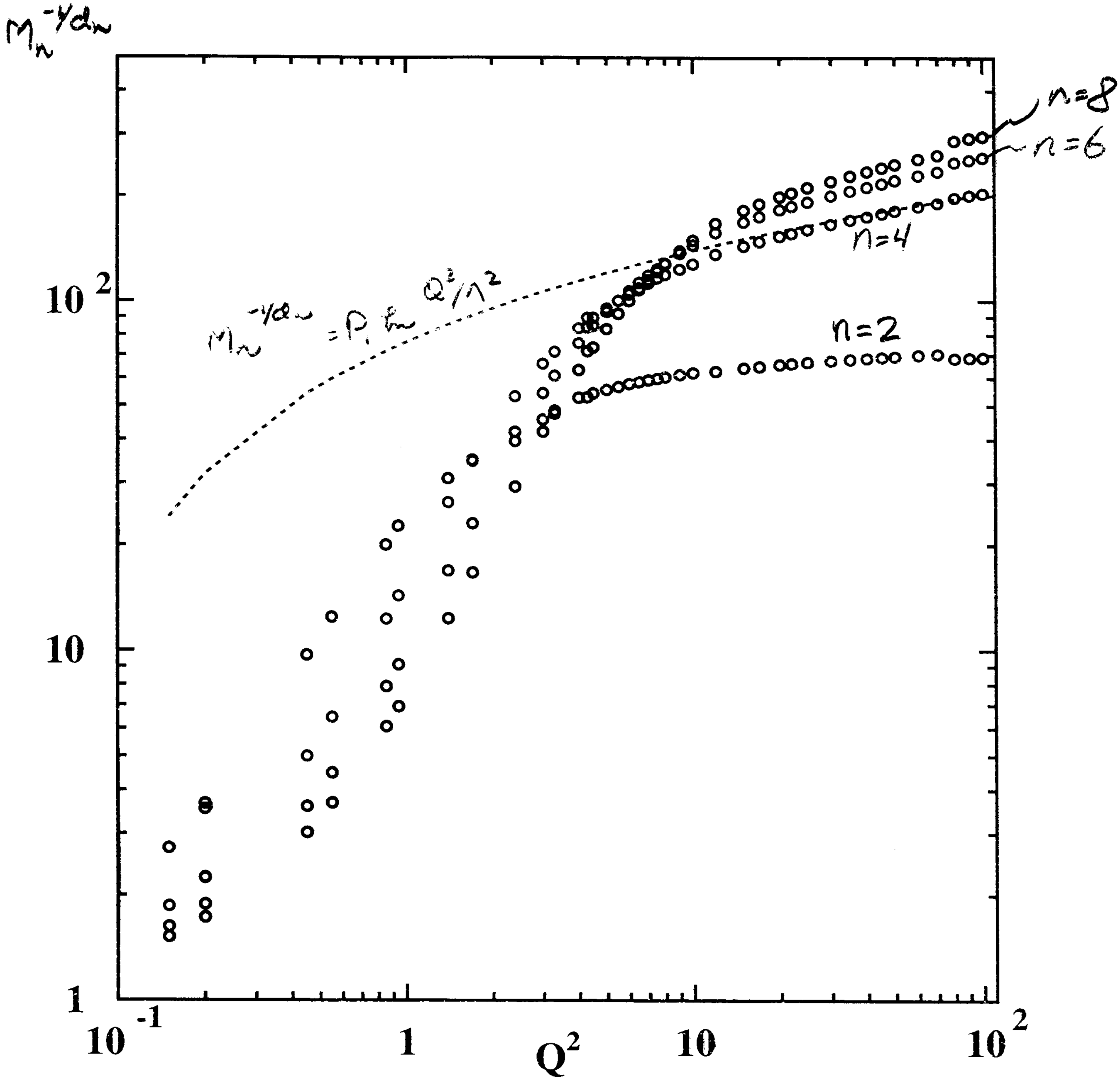
$\Lambda = 0.25$ GeV

Fit $M_n^{-1/d_n} = f(\ln Q^2)$.

Obtain P_1, P_2, P_3.

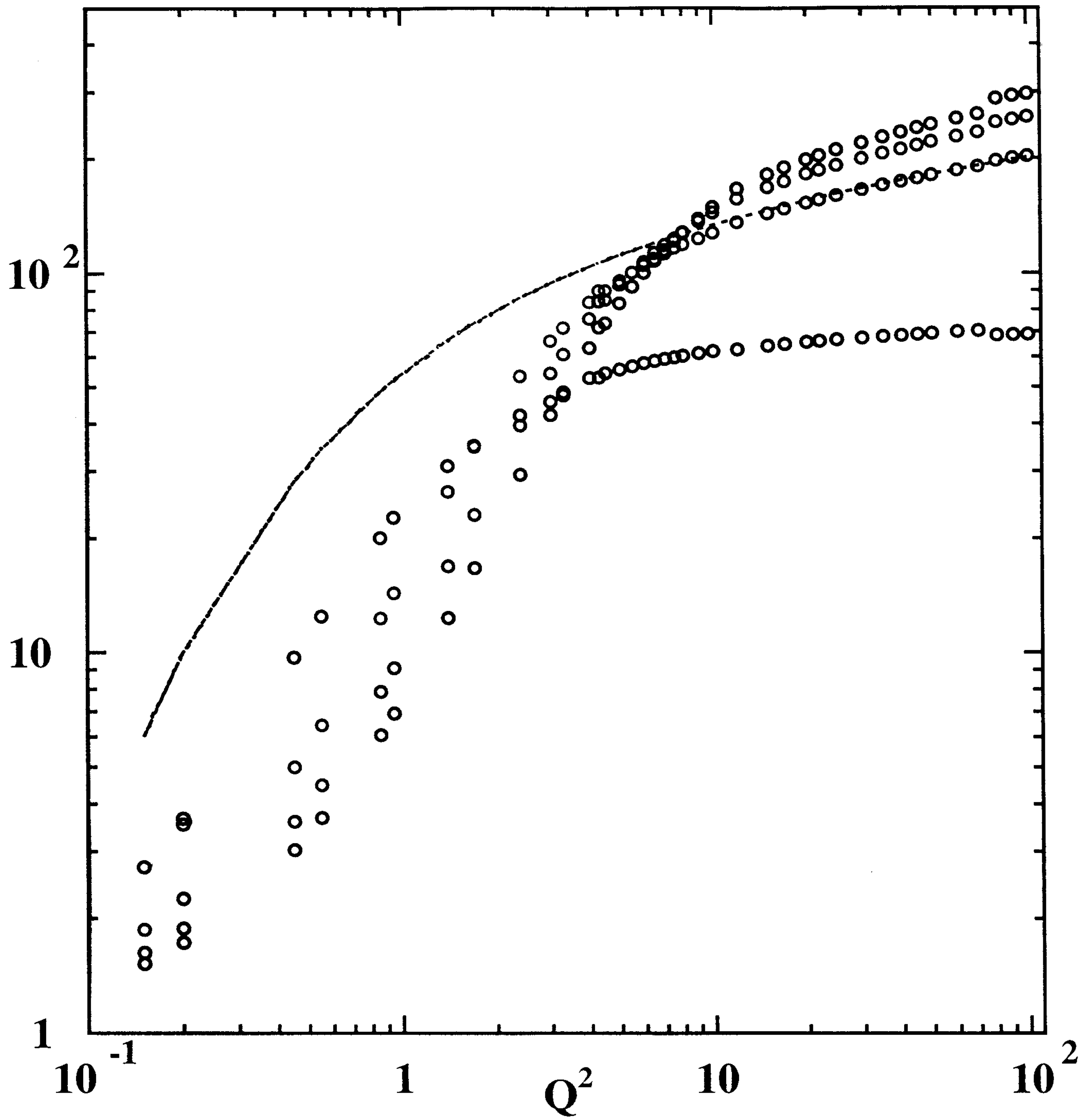
P_1 – determined by high Q^2 region $Q^2 > 20$ GeV^2.

P_2, P_3 – determined by low Q^2 (2 - 10 GeV^2) region.



P_1 fit to moments for $Q^2 \geqslant 20\ GeV^2$

$\Lambda = 0.250$ GeV

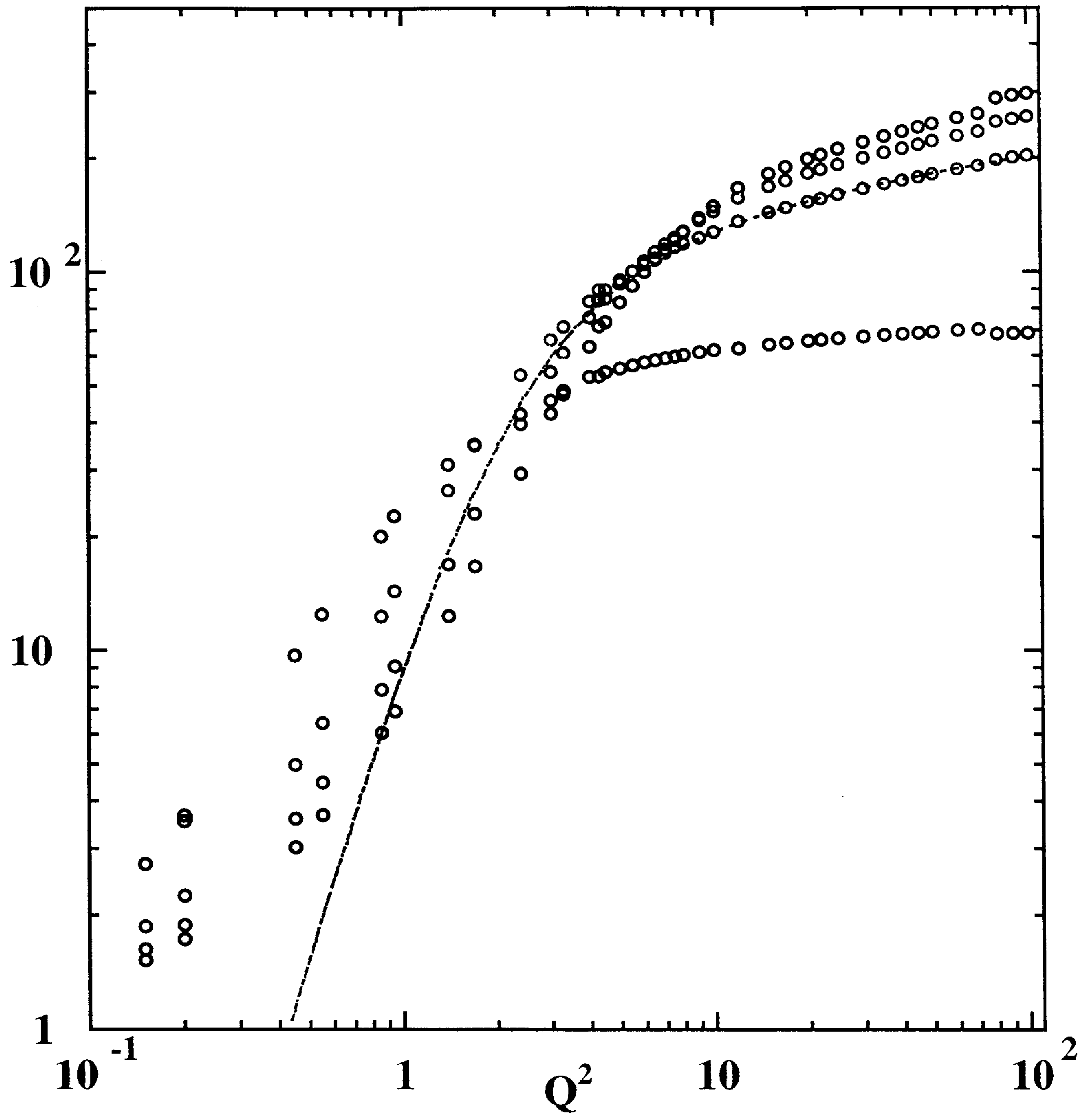


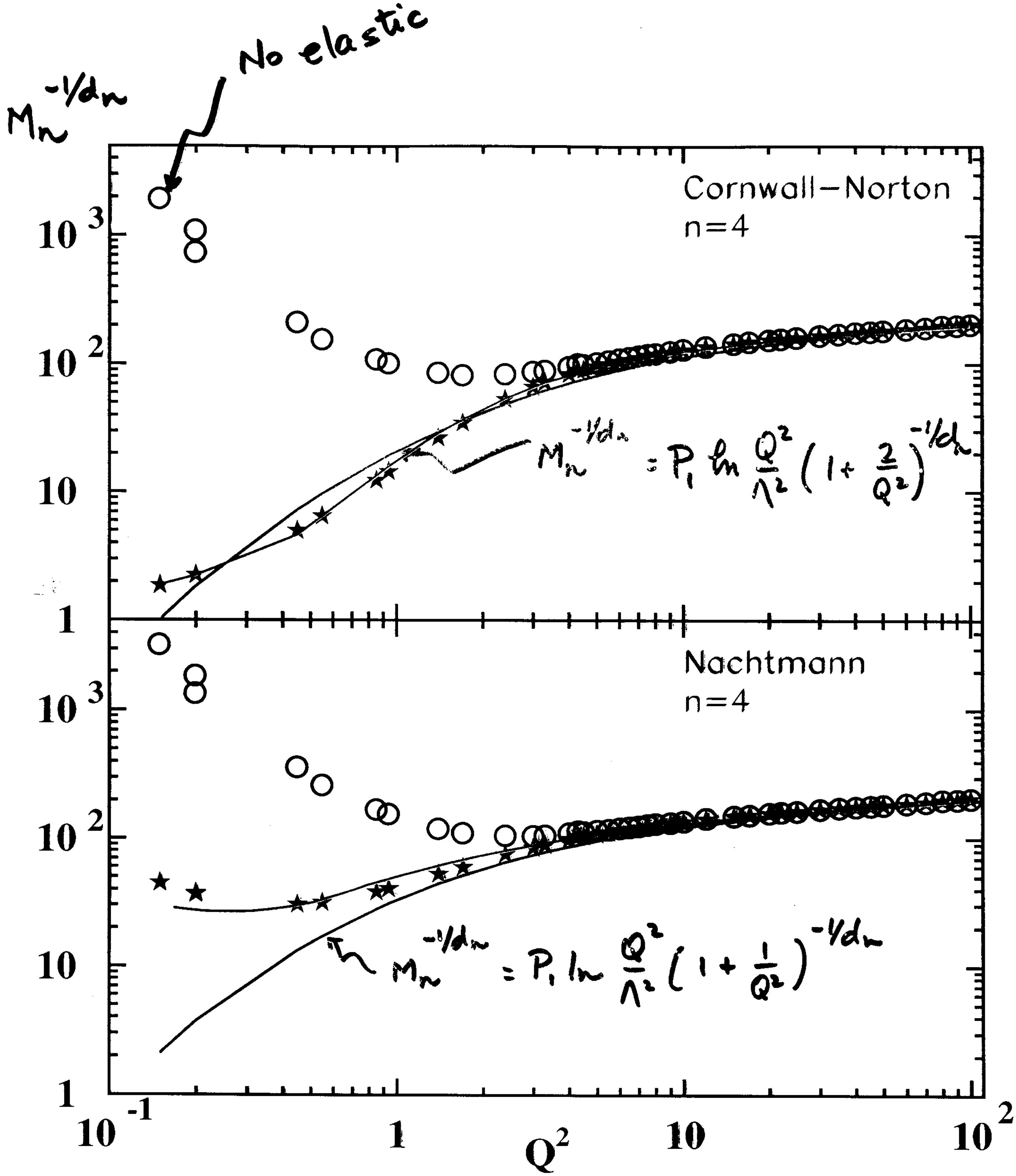
$$M_n^{-1/d_n} = P_1 \ln \frac{Q^2}{\Lambda^2} \left(1 + \frac{P_2}{Q^2}\right)^{-1/d_n}$$

Fit to M_n down to $Q^2 = 2\ \mathrm{GeV}^2$

$$M_n^{-1/d_n} = P_1 \ln \frac{Q^2}{\Lambda^2}\left(1 + \frac{P_2}{Q^2} + \frac{P_3}{Q^4}\right)^{-1/d_n}$$

Fit M_n down to $Q^2 = 2\ \mathrm{GeV}^2$





P_1 from PRD 60 094001

Fit Mn down to $Q^2 = 0.15\ \mathrm{GeV}^2$

OPE does not work in this regime

CONCLUSIONS:

- Global B-G duality holds to better than 10% down to $Q^2 \sim 0.5$.
- Local B-G duality holds for all four resonance regions.
- Resonances oscillate around one single curve even at very low Q^2 (0.1). This curve is not DIS.
- F_2 at low Q^2 seems to be sensitive only to valence quark distribution.
- Moments of F_2 below $Q^2 \sim 0.5$ are governed by elastic contribution.
- OPE is no longer adequate for this low Q^2 region.