

ON EXTRACTING POLARIZED PARTON  
DENSITIES FROM SEMI-INCLUSIVE DIS.

ELLIOT LEADER

BIRKBECK COLLEGE, LONDON.

- 1) ARGUE THAT GLOBAL ANALYSIS IN LO OF INCLUSIVE AND SEMI-INCLUSIVE DATA IS DANGEROUS.
- 2) SUGGEST STRATEGY FOR ANALYSIS WITH EMPHASIS ON SAFEGUARDS.



- 1) Danger of fake information in DIS.
- 2) Criticism of LO "PURITY" approach
- 3) Strategic approach to LO analysis.
- 4) Can we determine  $\Delta s(x)$  ?
- 5) Conclusions.

Work done in collaboration with  
E. Christova, Bulgarian Academy  
of Sciences

Sometimes useful to parametrize  
 $\Delta q_v$ ,  $\Delta \bar{q}$  separately.

For simplicity :  $\Delta \bar{u} = \Delta \bar{d} = \lambda \Delta s$

" $\chi^2$ " analysis favours  $\lambda = 1$ "

NONSENSE ! Must be hidden  
biases in minimization procedure.

NOTE : Although measure only  
two observables

$$g_i^p(x, Q^2) \quad g_i^n(x, Q^2)$$

PERFECT DATA  $\Rightarrow$  KNOWLEDGE OF ALL  
 $\Delta u + \Delta \bar{u}$ ,  $\Delta d + \Delta \bar{d}$ ,  $\Delta s$ ,  $\Delta G$

because of evolution eqns.

Another example from study of both inclusive and SMC semi-inclusive :-

"Present semi-inclusive data alone fail to define a  $\Delta d_V$  consistent with those extracted from inclusive data".

MORAL :- Parameter space is complicated. Trust physics,

NOT numerical minimization.

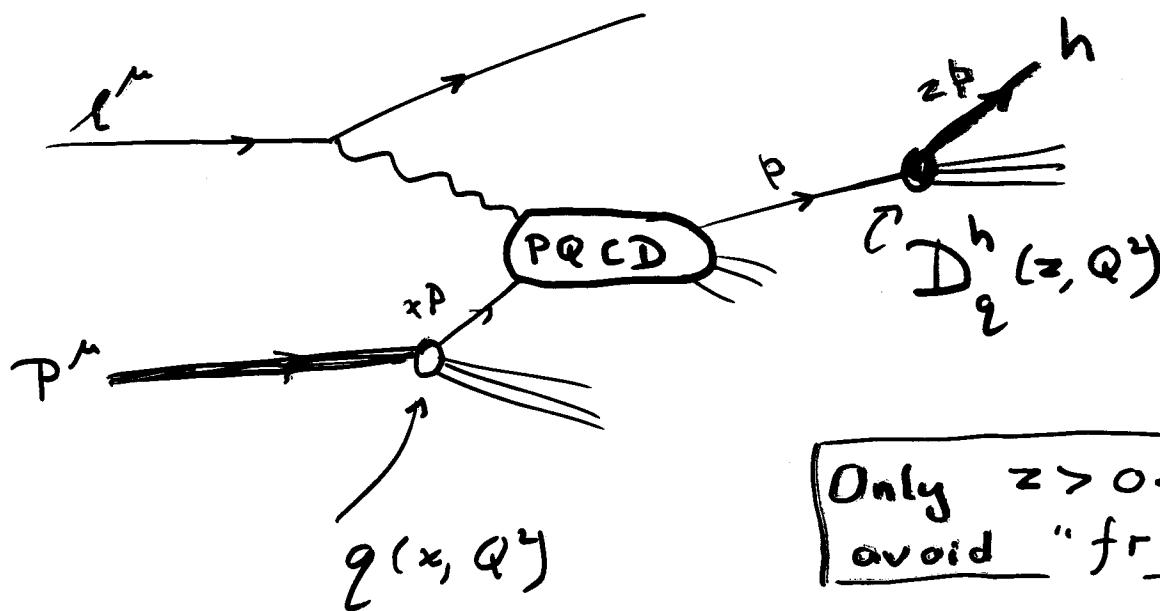
Will discuss LO strategy for semi-inclusive ..... can be generalised to NLO, but much less straightforward.

## 2) Criticism of LO "PURITY" approach.

Use

$$\bar{\sigma} = \frac{x(P+\lambda)^2}{4\pi\alpha^2} \left[ \frac{2y^2}{1+(1-y)^2} \right] \frac{d^3\sigma}{dx dy dz}$$

$$\Delta \bar{\sigma} = \frac{x(P+\lambda)^2}{4\pi\alpha^2} \left[ \frac{y}{2-y} \right] \left\{ \frac{d^3\sigma_{+-}}{dx dy dz} - \frac{d^3\sigma_{++}}{dx dy dz} \right\}$$



$$\Delta \bar{\sigma}(x, z, Q^2) = \sum_{g, \bar{g}} e_g^2 \Delta q_i(x, Q^2) D_i^h(z, Q^2) \quad \left. \begin{array}{l} \text{Indep.} \\ \text{Fragmentation} \\ \text{and} \end{array} \right\}$$

$$\bar{\sigma}(x, z, Q^2) = \sum_{g, \bar{g}} e_g^2 q_i(x, Q^2) D_i^h(z, Q^2) \quad \left. \begin{array}{l} \text{Same} \\ D_i^h \end{array} \right\}$$

# SMC and HERMES :

$$\frac{\Delta \tilde{\sigma}}{\tilde{\sigma}} = \sum_{q, \bar{q}} P_i^h(x) \cdot \frac{\Delta q_i(x)}{q_i(x)}$$

where

$$\text{PURITY } P_i^h(x) = \frac{e_i^2 q_i(x) \int_{0.2}^1 dz D_i^h(z)}{\sum_{q, \bar{q}} e_i^2 q_i(x) \int_{0.2}^1 dz D_i^h(z)}$$

THEY ASSUME  $P_i^h(x)$  KNOWN.

i) WHY ????

WHY TRUST  $D_i^h(z)$  ?? Errors ??

e.g. Binneweis et al obtain

$D_i^h$  from  $e^+e^- \rightarrow \text{hadrons}$

Fit has 31 parameters.

No errors.

2) How good is LO approximation ???

## Anti Purity summary:

- 1) Gives an absolute status to fragmentation functions that they don't deserve.
- 2) Blocks possibility to check reliability of LO approach.

Discuss a strategic approach which tests LO at each step.

N.B. In principle inclusive + semi-inclusive data could determine also the fragmentation functions

3) A strategic approach to the analysis of semi-inclusive DIS.

Will involve relating certain densities to combinations of data.

May find errors on combinations are big.

This is a fact of life, not a consequence of method of analysis.

Simply, a global analysis hides this fact.

Define

$$\Delta R_{p,n}^{h^+ \pm h^-} = \frac{\Delta \tilde{\sigma}_{p,n}^{h^+} \pm \Delta \tilde{\sigma}_{p,n}^{h^-}}{\tilde{\sigma}_{p,n}^{h^+} \pm \tilde{\sigma}_{p,n}^{h^-}}$$

$h^+$  = any  $\oplus$  charge or sum of them

$h^-$  = Anti particles of  $h^+$

$$\Delta R_{p \pm n}^{h^+ \pm h^-} = \frac{\Delta \tilde{\sigma}_p^{h^+ \pm h^-} \pm \Delta \tilde{\sigma}_n^{h^+ \pm h^-}}{\tilde{\sigma}_p^{h^+ \pm h^-} \pm \tilde{\sigma}_n^{h^+ \pm h^-}}$$

The main test of LO :-

$$\Delta R_{p-n}^{h^+ + h^-}(x, z, Q^2) = \frac{g_1^p(x, Q^2) - g_1^n(x, Q^2)}{F_1^p(x, Q^2) - F_1^n(x, Q^2)}$$

$\underbrace{\qquad\qquad\qquad}_{\text{NB}}$

No  $z$ -dependence

Error on this indicates probable error on parton densities extracted.

9)

"theoretical systematic error"

## Direct evaluation of VALENCE densities:

$$\Delta u_v(x, Q^2) = \frac{1}{2} \left\{ (u_v + d_v) \Delta R_{p+n}^{h^+ h^-}(x, z, Q^2) \right.$$

↗  
No  
 $\rightarrow$

$$+ (u_v - d_v) \Delta R_{p-n}^{h^+ h^-}(x, z, Q^2) \}$$

$z$ -dependence.

$$\Delta d_v(x, Q^2) = \frac{1}{2} \left\{ (u_v + d_v) \Delta R_{p+n}^{h^+ h^-}(x, z, Q^2) \right.$$

$$- (u_v - d_v) \Delta R_{p-n}^{h^+ h^-}(x, z, Q^2) \}$$

Lack of  $z$ -dependence crucial  
to test reliability of LO!

For  $h^\pm = \pi^\pm$ ,  $SU(2) \Rightarrow$  simplification:

$$\Delta u_v = \frac{1}{15} \left\{ 4 (4u_v - d_v) \Delta R_p^{\pi^+ \pi^-} + (4d_v - u_v) \Delta R_n^{\pi^+ \pi^-} \right\}$$

$$\Delta d_v = \frac{1}{15} \left\{ 4 (4d_v - u_v) \Delta R_n^{\pi^+ \pi^-} + (4u_v - d_v) \Delta R_p^{\pi^+ \pi^-} \right\}$$

## Sea quarks and fragmentation functions.

Best to import from INCLUSIVE

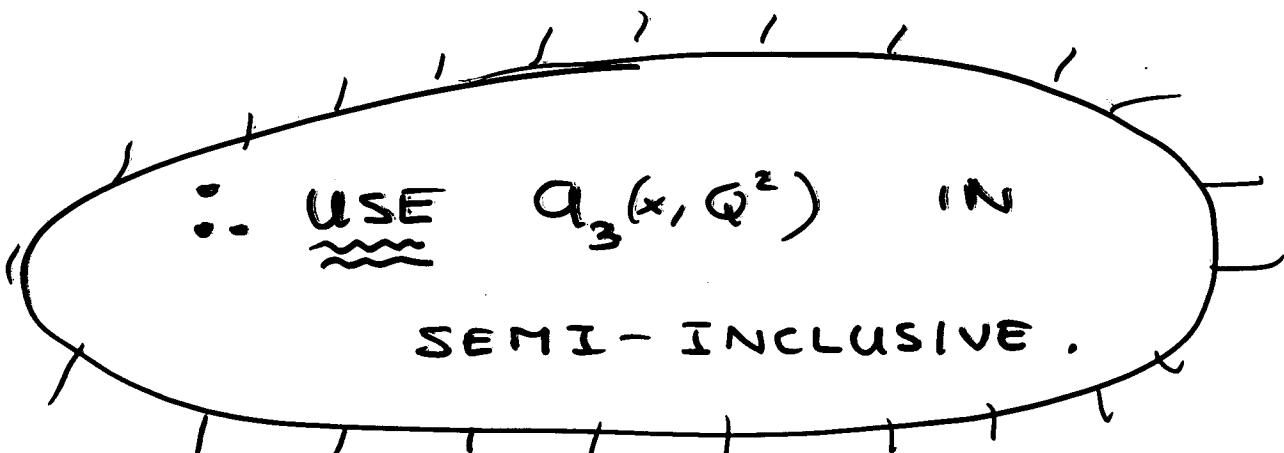
$$Q_3(x, Q^2) = (\Delta u + \Delta \bar{u}) - (\Delta d + \Delta \bar{d})$$

Very well determined, without any influence of sea or gluons.

$$g_1^p(x, Q^2) - g_1^n(x, Q^2) = \frac{1}{6} \alpha_3 \otimes \left[ 1 + \frac{\alpha_s(Q^2)}{2\pi} \delta C_2 \right]$$

↑                      ↑  
positive              negative  
 $\underbrace{\hspace{10em}}$

∴ well determined.



## Examples of use of $a_3(x, Q^2)$

1)

$$\Delta \bar{u} - \Delta \bar{d} = \frac{1}{2} \left\{ a_3 + \Delta d_v - \Delta u_v \right\}$$

Some theories expect this large.

We feel error on RHS make it unreliable.

$$2) 4 \underbrace{D_u^{h^+ h^-}(z, Q^2) - D_d^{h^+ h^-}(z, Q^2)}_{a_3(z, Q^2)} = 9 \left\{ \frac{\Delta \tilde{\sigma}_p^{h^+ h^-}(x, z, Q^2) - \Delta \tilde{\sigma}_n^{h^+ h^-}(x, z, Q^2)}{a_3(x, Q^2)} \right\}$$

independent  
of  $x$

For  $h^\pm = \pi^\pm$ ,  $SU(2) \Rightarrow$  simplification:

$$D_u^{\pi^+ \pi^-}(z, Q^2) = D_d^{\pi^+ \pi^-}(z, Q^2) = 3 \left\{ \frac{\Delta \tilde{\sigma}_p^{\pi^+ \pi^-}(x, z, Q^2) - \Delta \tilde{\sigma}_n^{\pi^+ \pi^-}(x, z, Q^2)}{a_3(x, Q^2)} \right\}$$

Interesting to compare with fragmentation functions from  $e^+ e^- \rightarrow$  hadrons !

Can we measure  $\Delta \bar{q}$  in LO?

Need  $(\Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d}) \equiv a_+(x, Q^2)$

Then

$$\Delta \bar{u} + \Delta \bar{d} = \frac{1}{2} \left\{ a_+ - \Delta u_v - \Delta d_v \right\}$$

Combine with

$$\Delta \bar{u} - \Delta \bar{d} = \frac{1}{2} \left\{ a_3 - \Delta u_v + \Delta d_v \right\}$$

BUT :- Even if  $a_+$  determined accurately, LO errors may make LHS unreliable.

How to get  $a_+(x, Q^2)$  ?

$$\Delta \tilde{\sigma}_{p+n}^{\pi^+ + \pi^-} = \frac{1}{9} \left\{ 5 a_+ D_u^{\pi^+ + \pi^-} + 4 \cancel{\Delta S D_s^{\pi^+ + \pi^-}} \right\}$$

neglect

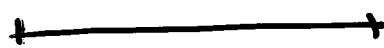
Get

$$a_+(x, Q^2) \approx \frac{3}{5} \left\{ \frac{\Delta \tilde{\sigma}_p^{\pi^+ + \pi^-}(x, z, Q^2) + \Delta \tilde{\sigma}_n^{\pi^+ + \pi^-}(x, z, Q^2)}{\Delta \tilde{\sigma}_p^{\pi^+ + \pi^-} - \Delta \tilde{\sigma}_n^{\pi^+ + \pi^-}} \right\} a_3(x, Q^2)$$

$\nearrow$   
independent  
of  $z$ .

If  $K^0$  can be detected, without  
neglect of  $\Delta s$ , have

$$a_+(x, Q^2) = [u + \bar{u} + d + \bar{d}] \left\{ \frac{\Delta \tilde{\sigma}_{p+n}^{K^+ + K^-} - \Delta \tilde{\sigma}_{p+n}^{K^0 + \bar{K}^0}}{\tilde{\sigma}_{p+n}^{K^+ + K^-} - \tilde{\sigma}_{p+n}^{K^0 + \bar{K}^0}} \right\}$$



Other things can measure :-

$$D_u^{h^+} - D_u^{h^-}; \quad D_d^{h^+} - D_d^{h^-}; \quad D_u^{\pi^\pm} - - - - -$$

$\uparrow$  Test usual assumption

$$D_d^{K^+} = D_d^{K^-}$$

Can we determine  $\Delta s$  and compare it with  $\Delta \bar{u}$  and  $\Delta \bar{d}$  in LO?

Can give explicit expression for  $\Delta s$  (E.Christova + E.L) BUT PESSIMISTIC in LO:

For pions compare

$$\Delta u D_u^\pi \text{ with } \Delta s D_s^\pi$$

$$|\Delta u| \gg |\Delta s| \quad \text{and} \quad |D_u^\pi| \gg |D_s^\pi|$$

$\Rightarrow \Delta s$  term  $\approx$  error of LO approx.

For kaons somewhat better:

$$|\Delta u| \gg |\Delta s| \quad \text{but} \quad |D_u^K| \approx |D_s^K|$$

BUT: similar to inclusive case.

There NLO causes 100% change  
in  $\Delta s$ .  $\therefore$  still pessimistic.

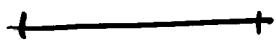
Only hope:

$$h = \phi !$$

Now expect

$$|D_u^{\phi}| \ll |D_s^{\phi}|$$

so LO determination may be  
reliable.



Preliminary work in

Phys. Lett. B 468 (1999) 299.

NLO treatment in preparation.

$$\Delta \tilde{\sigma}^h = \left\{ e_i^i \right\} \Delta g_i \otimes D_i^h + \frac{\alpha_s}{2\pi} \left[ \Delta g_i \otimes \Delta C_{g2} \otimes D_2^h + \Delta g_i \otimes \Delta C_{gG} \otimes D_G^h + \Delta G \otimes \Delta C_{GG} \otimes D_i^h \right] \}$$

## Summary.

- 1) The complex parameter space  $\Rightarrow$  danger of fake results in global analysis.
- 2) The LO approach using PURITY is unreliable. It does not take account of errors on fragmentation functions and fails to test crucial aspects of LO.
- 3) A systematic step-by-step analysis in LO is possible, with tests of the reliability of LO at each step.
- 4) It does not seem possible to determine  $\Delta s$  reliably in LO, except if  $\phi$  production can be measured. Also  $\Delta \bar{u}$ ,  $\Delta \bar{d}$  not reliably determined in LO.
- 5) Many results can be extended to NLO. Should be possible to fix  $\Delta \bar{u}$ ,  $\Delta \bar{d}$ ,  $\Delta s$ , but care with Scheme dependence.

- 6) To really optimize the analysis  
it is important to take all  
data in same  $x, z, Q^2$  bins.
- 7) In principle, with good enough data, can  
determine also the fragmentation functions. ---  
provided we take  $a_3(x, Q^2)$  from  
inclusive DIS. This is a valid  
strategy.