Extensions of inclusive-exclusive connection studies

(based on work with Afanasev and Wahlquist)

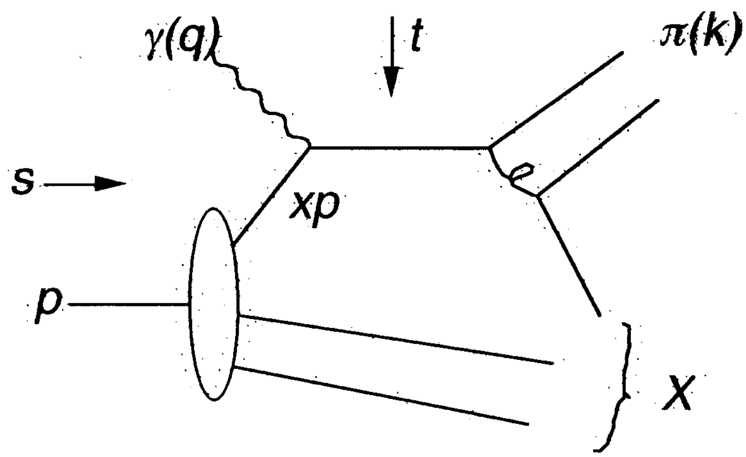
$$\gamma(q) + p \rightarrow \pi(k) + X$$

(or other meson)

Three variables:

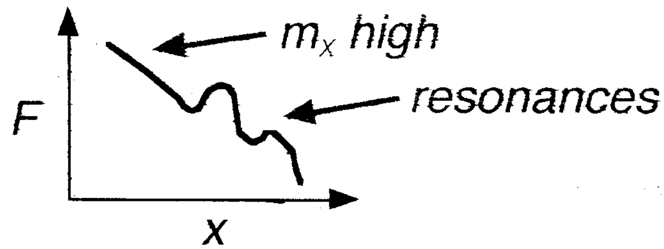
$$\begin{array}{ll} x & \\ Q^2 & \\ t & \equiv (q-k)^2 \end{array}$$

For short distance pion production,



with high Q^2 or t and high m_X, there will be a scaling curve (see below) $F(x)$ [or $F(x, Q^2, t)$, with dependence on Q^2 and t due to radiative corrections]

At lower Q^2 or t, with m_X in the resonance region, resonance bumps appear in $F(x, Q^2, t)$.



Ask:

1. Does the Björkén-like scaling really work at high Q^2 or t?

2. Is the scaling curve from high Q^2 or t a decent average over the resonaance bumps seen at the same x but lower Q^2 or t? (True for all resonances in DIS.)

3. Is the bump to continuum ratio a constant as Q^2 or t changes? (True in DIS for most resonances, but not for the $\Delta(1232)$.)

Elaboration

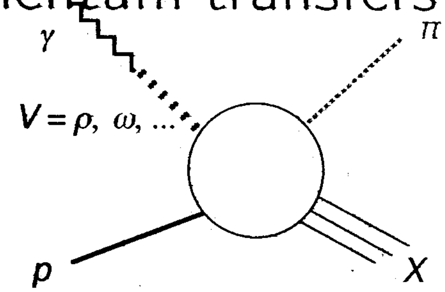
Mainly photoproduction case, $Q^2 = 0$.

- Processes that photoproduce pions, and where to seek duality
- Into the resonance region
- Comments on expectations and feasibility

Processes that photoproduce pions,
and where to seek duality

From low to high momentum transfer (k_T):

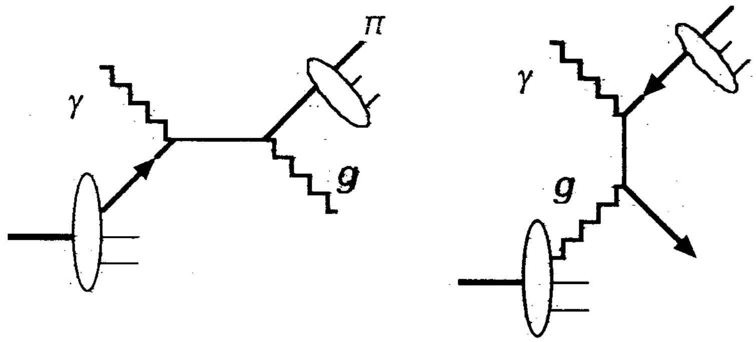
1. Soft processes, at low momentum transfers



(Generally approximated using VMD)

Not part of today's subject.

2. Fragmentation processes, at moderate k_T:



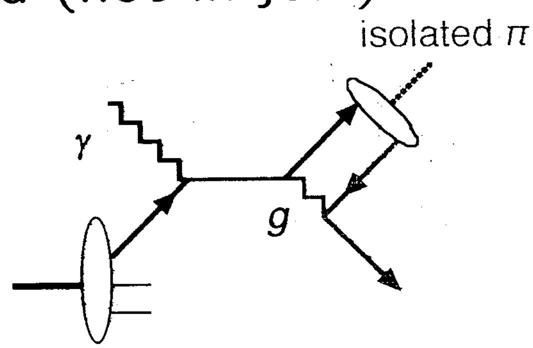
Pions come from fragmentations of quarks or gluons, at relatively long distances from the main interaction region. Partonic subprocess pertubatively calculable; fragmentation is not.

Very useful for studying gluon distributions, but not part of today's subject.

3. Direct pion production, at high k_T

Pions made at short distances,
emerge kinematically isolated (not in jets).
Perturbatively calculable.

Part of today's subject.



With Mandlestam,

$$s = (p+q)^2$$
$$t = (q-k)^2$$
$$u = (p-k)^2$$

$$x \equiv \frac{-t}{s+u-2m_N^2-q^2-m_\pi^2}$$ (general case)

$$m_X^2 = W'^2 = m_N^2 - t\left(\frac{1}{x}-1\right)$$ (exact)

and when Q^2 or t, and m_X, are large,

$x =$ quark momentum fraction

Analogs in DIS

Get the DIS kinematics by $k \to 0$.

Then:

- $t = (q-k)^2 \to q^2 = -Q^2$
- $u = (p-k)^2 \to m_N^2$
- $(m_\pi^2 \to 0)$

Gives:

$$x = \frac{-t}{s - m_N^2 - q^2} = \frac{Q^2}{2p \cdot q}$$

$$m_X^2 = m_N^2 + Q^2 \left(\frac{1}{x} - 1 \right)$$

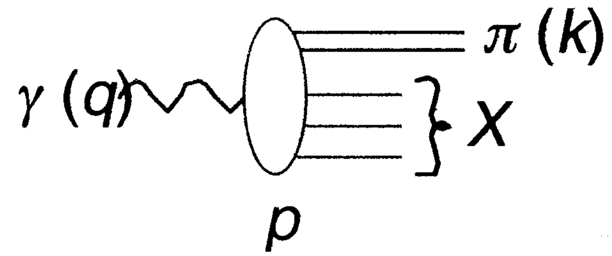
Bloom-Gilman duality in DIS improved by variable x', conventionlly presented as

$$\frac{1}{x'} = \frac{1}{x} + \frac{m_N^2}{Q^2}$$

or

$$x' = \frac{-t}{s - q^2} \ .$$

Forward pion kinematics ($\vec{k} \parallel \vec{q}$ in lab)



$$
\begin{aligned}
z &\equiv \frac{p \cdot k}{p \cdot q} = \frac{k}{\nu} \\
\nu &= |\vec{q}| - m_N x + \mathcal{O}(m_N^2 x^2/\nu)
\end{aligned}
$$

Then

$$
\begin{aligned}
t &= q^2 - 2q \cdot k = -Q^2 - 2\,(\nu - |\vec{q}|)\,k \\
&\approx -Q^2 + 2m_N x \nu z = -Q^2(1-z)
\end{aligned}
$$

and

$$
s + u - 2m_N^2 - q^2 = 2p \cdot q\,(1-z)
$$

Thus

$$
\begin{aligned}
x &= \frac{Q^2}{2p \cdot q} \\
m_X^2 &= m_N^2 + Q^2(1-z)\left(\frac{1}{x} - 1\right)
\end{aligned}
$$

Into the resonance region

4. Highest momentum transfers:
   "The resonance region"
   FS = Pion $\oplus$ single nucleon or resonance.

Operationally define $F(x,t)$ by

$$E_\pi \frac{d\sigma}{d^3k} = \frac{(s-m_N^2)x^2}{-\pi t} \, \frac{d\hat{\sigma}}{dt}(\gamma q \to \pi q') \; F(x,t)$$

Why? Because, the perturbative formula valid for short distance pion production (proc. 3) is,

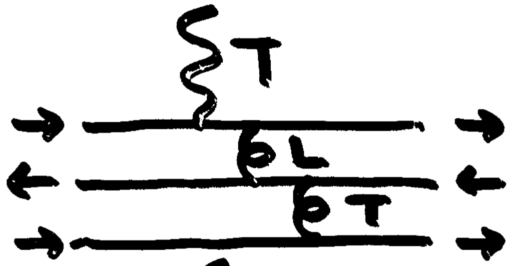
$$E_\pi \frac{d\sigma}{d^3k} = \frac{(s-m_N^2)x^2}{-\pi t} \sum_q \frac{d\hat{\sigma}}{dt}(\gamma q \to \pi q') \, G_{q/T}(x,\mu^2)$$

Thus where perturbation theory works, scaling function $F(x,t)$ is mainly dependent on x . Can relate it to the quark distributions (with weak dependence on the scale μ^2, which we may set to t), as in DIS.

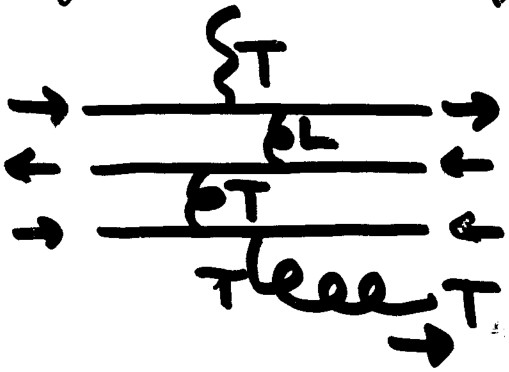
Query: it this scaling function dual, in a Bloom-Gilman like sense, to the bumpier curve one will get in the resonance region?

HIGH Q^2 SCALING OF $N \to qqq$ & $N \to qqqg$.

TRANSVERSE AMPLITUDES.



$G_+ \sim 1/Q^3$



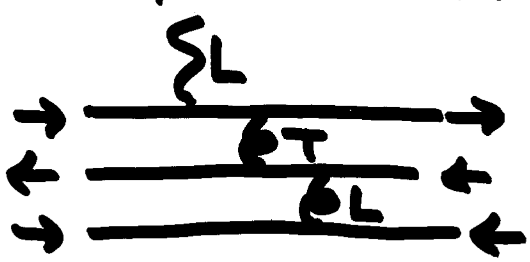
EXTRA QK. PROP.: $1/Q$

VIOLATES TL ALTERNATION: $1/Q$

(ONE HELICITY FLIP)

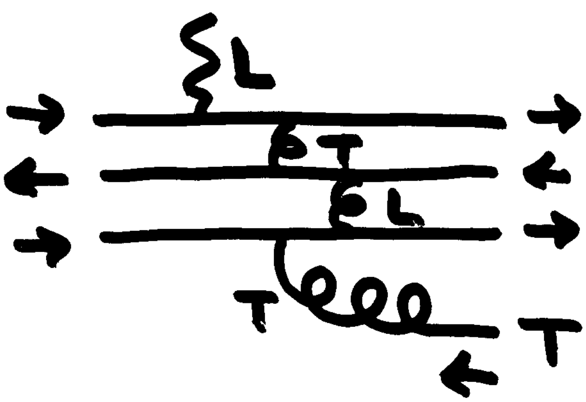
$\therefore \; 1/Q^5$

LONGITUDINAL AMPLITUDES



ONE QUARK HELICITY FLIP: $1/Q$

$\therefore G_0 \sim 1/Q^4$



EXTRA QUARK PROP.: $1/Q$

$\therefore \; 1/Q^4$

RESONANCE BUMPS AND THE SCALING CURVE

RESONANCE PROD ($eN \to e'R$):

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_m f_\Delta^{-1}}{1+\tau^*}\left\{G_0^2 + (G_+^2+G_-^2)\frac{1}{2} + (1+\tau^*)(G_+^2+G_-^2)\tan^2\frac{\theta}{2}\right\}$$

$$f_\Delta = 1+\frac{2E}{m_N}\sin^2\frac{\theta}{2}, \quad \tau^* = \frac{\nu^2}{Q^2}, \quad \sigma_m = \frac{4\alpha^2 E'^2}{Q^4}\cos^2\frac{\theta}{2}$$

DIS: $$\frac{d\sigma}{d\Omega\, dE'} = \sigma_m\left\{W_2 + 2W_1\tan^2\frac{\theta}{2}\right\}$$

CONNECT: $$\frac{d\sigma}{d\Omega\, dE'} = \frac{d\sigma}{d\Omega\, dm_x}\cdot\frac{m_N}{m_x}f_\Delta = \delta(m_x-m_R)\frac{d\sigma}{d\Omega}\cdot\frac{m_N}{m_R}f_\Delta$$

SHARP RES.

$$\simeq \underbrace{\frac{\Gamma/2\pi}{(m_x-m_R)^2+\Gamma^2/4}}_{2/\pi\Gamma \text{ AT PEAK}}\cdot\frac{m_N}{m_R}\cdot\frac{\sigma_m}{1+\tau^*}\left\{G_0^2+\frac{1}{2}(G_+^2+G_-^2)+\dots\tan^2\frac{\theta}{2}\right\}$$

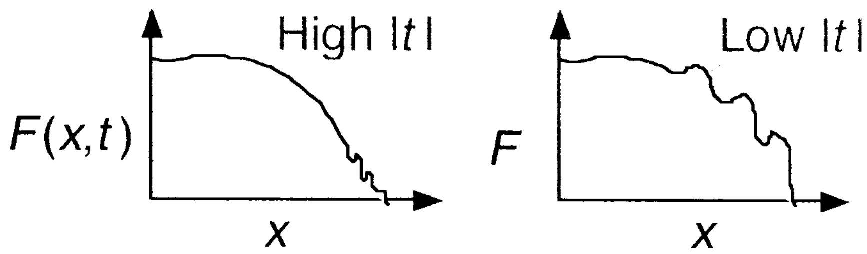
$$\therefore \nu W_2(\text{RES. PEAK}) = \frac{2}{\pi\Gamma}\cdot\frac{m_N}{m_R}\cdot\frac{\nu}{1+\nu^2/Q^2}\left\{G_0^2+\frac{1}{2}(G_+^2+G_-^2)\right\}$$

$$= \underbrace{\frac{\nu}{1+\nu^2/Q^2}}_{\text{CONST., AS } \nu\to\infty}\cdot\frac{m_N}{\pi m_R\Gamma}\cdot\left\{2G_0^2+G_+^2+G_-^2\right\}$$

$$\frac{1}{Q^6} \simeq \left(\frac{1-x}{m_R^2-m_N^2}\right)^3$$

THUS, νW_2 (RES. PEAK) $\propto (1-x)^3$

Comments on expectations and feasibility



$\boxed{1}$ There always is a resonance region. As $|t|$ increases it slides closer to the endpoint $x = 1$. Query: given known behaviors of the scaling curve and the resonance production cross section, do we expect the resonance/scaling curve ratio to be constant?

Answer: Yes (just, by the way, as for DIS).

Omit details of argument. The starting points are the expected $(1-x)^3$ scaling curve behavior for $x \to 1$ and the the counting rules for $A + B \to C + D$ at fixed CM angle at high s or t,

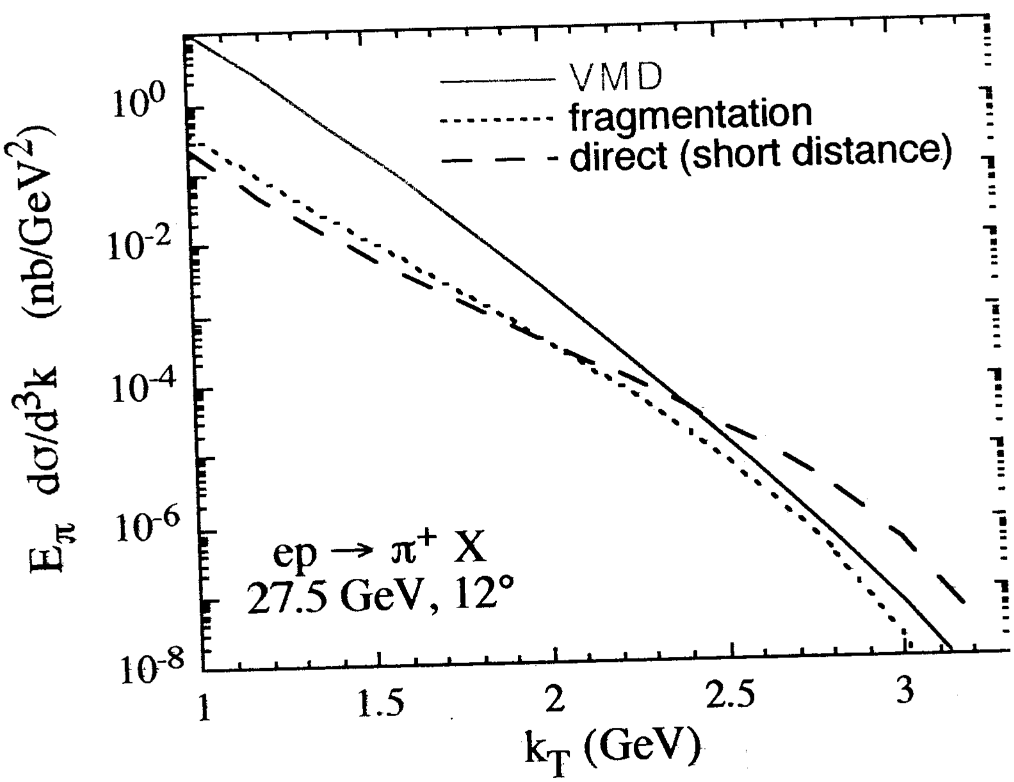
$$\frac{d\sigma}{dt} \propto s^{2-n_A-n_B-n_C-n_D} \ ;$$

n_A counts constituents of particle A. Hence,

$$\frac{d\sigma}{dt}(\gamma + p \to \pi + R) \propto s^{-7}$$

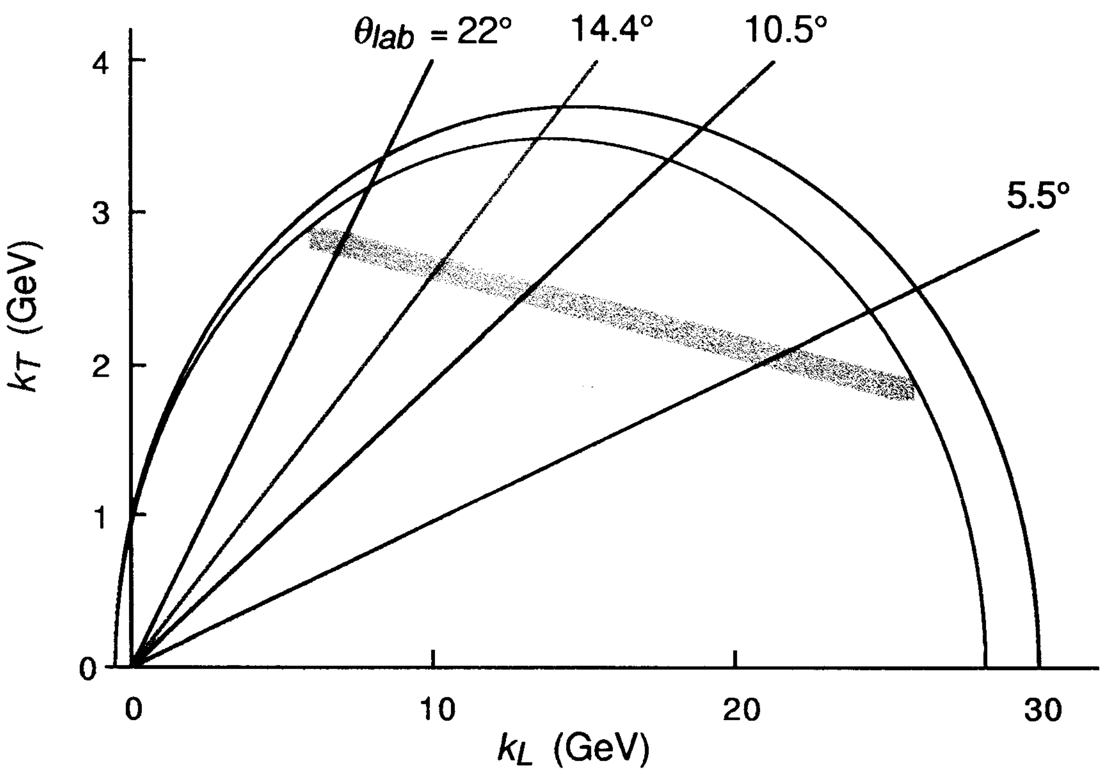
This falloff is just right for the resonance bump to slide down the scaling curve, always above it by the same ratio.

$\boxed{2}$ Is there a region where direct (or isolated or short distance) pion production dominates?



$\boxed{3}$ Where are the points of comparison between the scaling region and the resonance region?

Show picture.

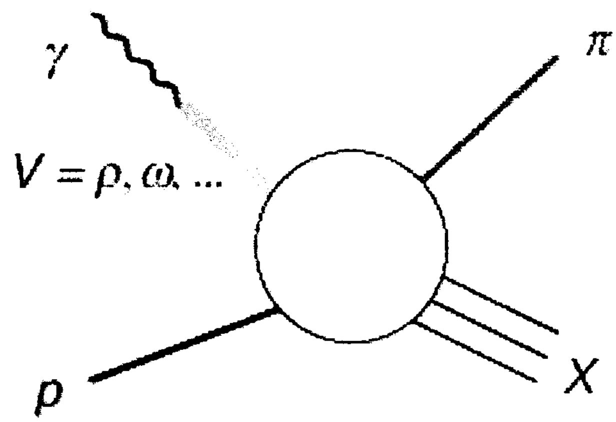


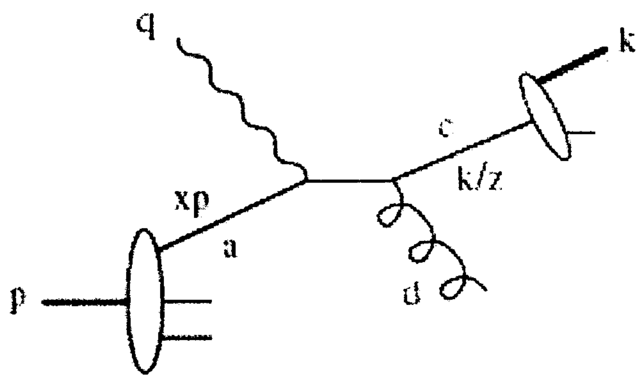
SOME POSSIBILITIES FOR $E_\gamma = 30$ GeV

- RESONANCE REGION $[m_N^2 \leq m_X^2 \leq (2\,\text{GeV})^2]$ BETWEEN TWO ELLIPTICAL CURVES
- DIRECT PION PRODUCTION DOMINANT ABOVE GREY LINE.
- (GREY LINE CAN BE MOVED DOWN BY USING $Q^2 > 0$ PHOTONS, DIMINISHING VMD CONTRIB.)

Go into the resonance region with fixed q^2 and diminishing t. Will we see an inclusive-exclusive conection as in the DIS case?

To see scaling, in additon to large s, t, u, and m_X; also competing processes, such as the soft or VMD process, and fragmentation, must be small.





VMD serious at 12 GeV. Use isolation cut to emphasize direct process. Decrease size of VMD process by using spacelike off-shell photon, rather than real photon,

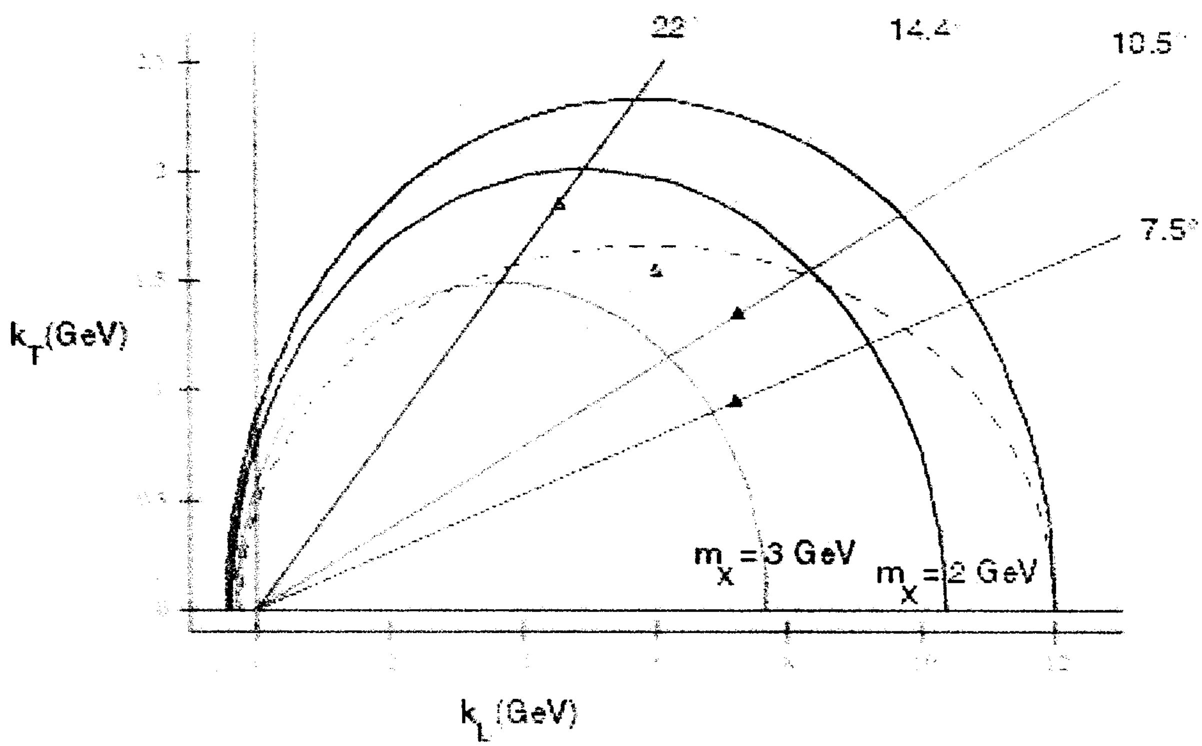
$$\frac{1}{m_\rho^2} \rightarrow \frac{1}{Q^2 + m_\rho^2}$$

Consider latter.

Have means to calculate direct process and estimate VMD process.

$$\gamma(q) + p \to \pi^+(k) + X$$

$$E_\gamma = 12 \text{ GeV}, \; Q^2 = 1 \text{ GeV}^2$$



Solid ellipses show $m_X = m_N$, 2 GeV, 3 GeV.

Dashed ellipse is for fixed $x = 0.5$.

Direct processes dominate above a to right of small triangles.