Inclusive and Semi-inclusive Hard Processes (I)

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Content of Lecture (Part I)

- Elastic Lepton – Nucleon scattering (e + p $\longrightarrow$ e + p)

  $\longrightarrow$ Form Factors

- Inclusive Deep-Inelastic Scattering (DIS) (e + p $\longrightarrow$ e + X)

  $\longrightarrow$ Structure Functions, Parton Model, Parton Distributions, Gauge Links, DGLAP-evolution

- Hard Exclusive Processes (DVCS)

  $\longrightarrow$ Generalized Parton Distributions

Content of the Lecture (Part II)

- Semi-inclusive processes (SIDIS, Drell-Yan, e+e- $\longrightarrow$ hhX)

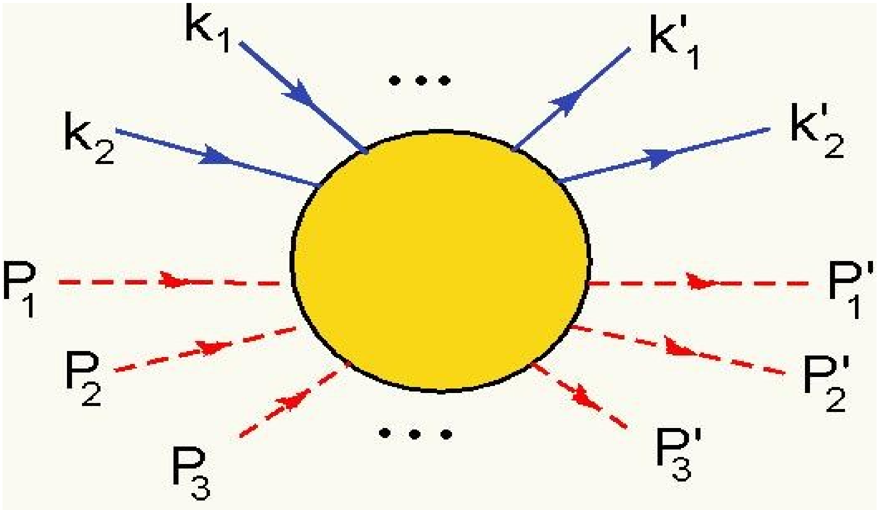
  $\longrightarrow$ Transverse Momentum Dependent Parton Distributions (TMDs), Fragmentation Functions

- Sivers-effect, Collins-effect, etc.

- (Possible) Relations between TMDs and GPDs

The "master formula" for this lecture...

How to treat "soft blobs"... (LSZ-formalism)



on-shell legs: initial and final states

off-shell legs: field operators

$$M = \int \frac{d^4 z_1}{(2\pi)^4}\, \mathrm{e}^{-ik_1\cdot z_1} \ldots \int \frac{d^4 z_n}{(2\pi)^4}\, \mathrm{e}^{-ik_n\cdot z_n} \int \frac{d^4 z'_1}{(2\pi)^4}\, \mathrm{e}^{ik'_1\cdot z'_1} \ldots \int \frac{d^4 z'_m}{(2\pi)^4}\, \mathrm{e}^{-ik'_m\cdot z'_m} \times$$

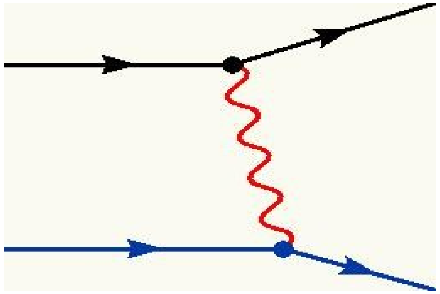
$$\langle P'_1, P'_2, P'_3, \ldots | \mathrm{T}\left(\hat{O}_{f\,1}^{\dagger}(z'_1) \ldots \hat{O}_{f\,m}^{\dagger}(z'_m) \hat{O}_{i\,1}(z_1) \ldots \hat{O}_{i\,n}(z_n) \right) | P_1, P_2, P_3 \ldots \rangle$$

Elastic Lepton-Nucleon scattering

How to get information on the structure of hadrons?
Bombard hadrons with leptons (JLab, HERMES, COMPASS, SLAC, ...)

Simplest process: l + N $\longrightarrow$ l + N

point-like proton: (works for low energies)

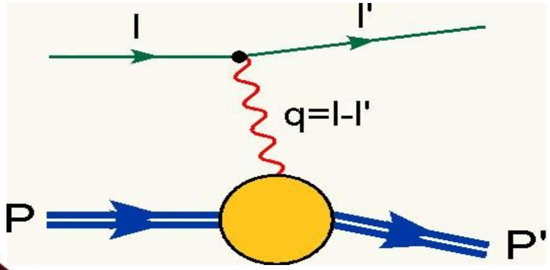


point-like currents: $j^\mu = e\bar{u}(P')\gamma^\mu u(P)$

Cross section:
(in the lab frame)

$$\boxed{\left.\frac{d\sigma}{d\Omega}\right|_{\text{lab}} = \frac{\alpha^2}{4E^2 \sin^4\frac{\Theta}{2}} \frac{E'}{E}\left(\cos^2\frac{\Theta}{2} + \frac{Q^2}{2M^2}\sin^2\frac{\Theta}{2}\right)}$$

Composite proton: (higher momentum transfer Q ~ M)



$$\langle P'|\,\hat{J}^\mu(0)\,|P\rangle = e\bar{u}(P')\left[\gamma^\mu F_1(Q^2) + \frac{\kappa}{2M} i\sigma^{\mu\nu} q_\nu\, F_2(Q^2)\right] u(P)$$

$F_{1,2}(Q^2)$: Dirac / Pauli Form Factors

Cross section for a composite nucleon (Rosenbluth formula):

$$\tau = \frac{Q^2}{4M^2}$$

$$\boxed{\left.\frac{d\sigma}{d\Omega}\right|_{\rm lab} = \frac{\alpha^2}{eE^2\sin^4\frac{\Theta}{2}}\frac{E'}{E}\left[\frac{G_E^2+\tau G_M^2}{1+\tau}\cos^2\frac{\Theta}{2} + 2\tau G_M^2\sin^2\frac{\Theta}{2}\right]}$$

$G_E = F_1 + \kappa\tau F_2$ electric FF $\qquad G_M = F_1 + \kappa F_2$ magnetic FF

- Non-relativistic limit: $Q^2 << M^2$

G_E: FT of charge distribution $\qquad G_M$: FT of magn. moment distribution

$$\boxed{G_E(Q^2) = \int d^3r\,\rho(r)e^{-i\vec{q}\cdot\vec{r}} \simeq e_n - \frac{1}{6}Q^2\langle r^2\rangle + \ldots}$$

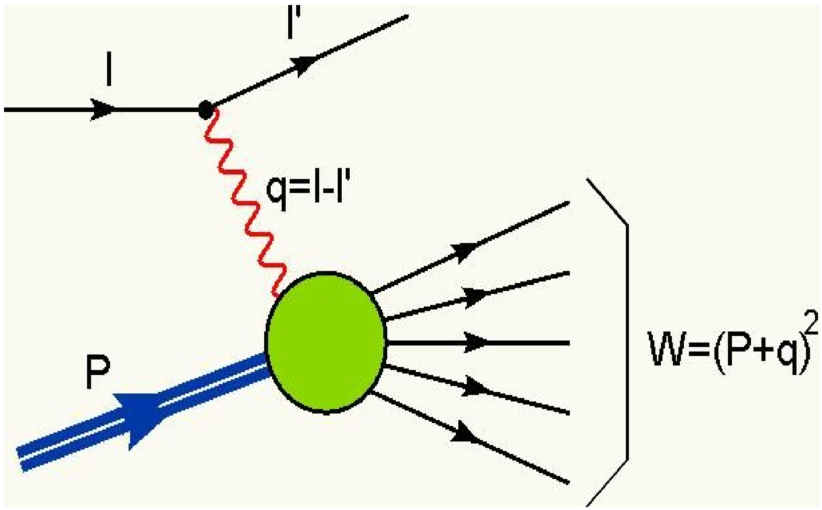
$G_E(0) = e_n$ electric charge $\qquad \langle r^2\rangle = 6\frac{d}{dQ^2}G_E(Q^2)\Big|_{Q^2=0}$ electric charge radius

- experimental fit of G_E (proton): $\qquad G_E(Q^2) = \left(1 - \frac{Q^2}{0.71}\right)^{-2}$

$$\langle r^2\rangle \simeq (0.8\,\mathrm{fm})^2$$

Inclusive Deep-Inelastic Scattering (DIS)

e + p ⟶ e + X (in the one photon approximation):



space-like photon: $q^2 = -Q^2 < 0$

Bjorken-variable: $x_B = \frac{Q^2}{2\, P\cdot q}$

Lepton variable: $y = \frac{P\cdot q}{P\cdot l}$

invariant mass: $W = M^2 + \frac{1-x_B}{x_B} Q^2$

- $x_B \longrightarrow 1$: recovers elastic scattering!
- Lab frame: $y \longrightarrow$ energy transfer, $Q^2 \longrightarrow$ scattering angle.

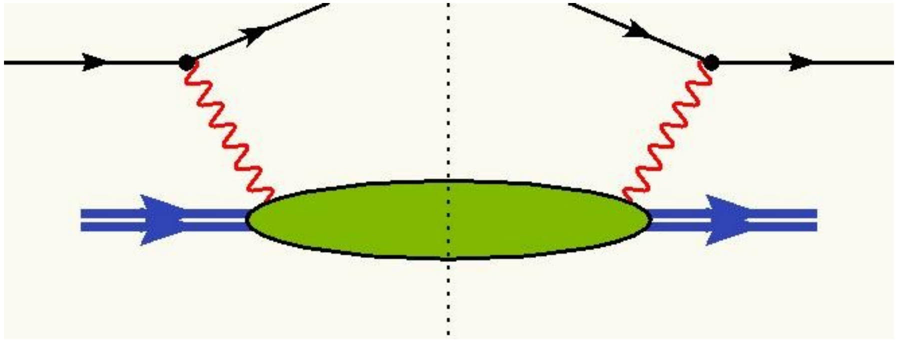
$$\boxed{iM_X = e^2\, \bar{u}(l', \lambda')\gamma^\mu u(l, \lambda)\, \frac{1}{q^2}\, \langle X | \hat{J}_\mu(0) | P, S\rangle}$$

Structure Functions

General formula for the cross section:

$$\boxed{d\sigma = \frac{1}{4J} \times |M_X|^2 \times \left(\frac{d^3\vec{l}'}{(2\pi)^3 2E'} \prod_{n=1}^{n_X} \frac{d^3\vec{p}_n}{(2\pi)^3 2E_n} \right) \times (2\pi)^4 \delta^{(4)}(l + P - l' - \sum_{n=1}^{n_X} p_n)}$$

Squared amplitude:

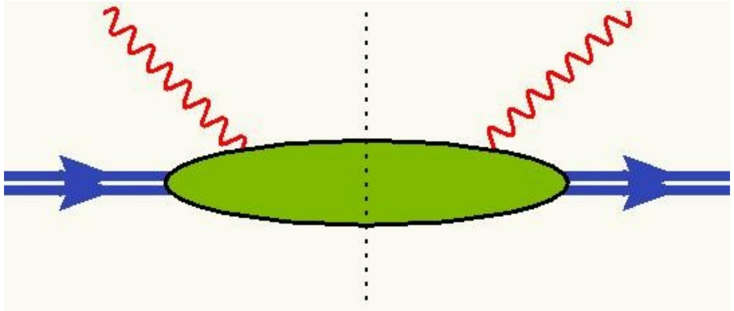


$$\boxed{d\sigma \propto \frac{\alpha^2}{Q^4} L^{\mu\nu} W_{\mu\nu}}$$

Leptonic tensor: $L^{\mu\nu} = \sum_{\lambda'} \left(\bar{u}'\gamma^\nu u\right)\left(\bar{u}'\gamma^\mu u\right)^*$

$$= 2\left(l^\mu l'^\nu + l^\nu l'^\mu - \frac{Q^2}{2} g^{\mu\nu} + \lambda i \epsilon^{\mu\nu\rho\sigma} l'_\rho l_\sigma\right)$$

Hadronic Tensor



$$\langle P,S|J^\mu(0)|X\rangle\langle X|J^\nu(0)|P,S\rangle$$

Sum over *all* hadronic final states:

$$W^{\mu\nu}(P,q)=\sum_X \langle P,S|J^\mu(0)|X\rangle\langle X|J^\nu(0)|P,S\rangle(2\pi)^4\delta^{(4)}(P+q-P_X)$$

$$=\int d^4x e^{iq\cdot x}\langle P,S|J^\mu(x)\sum_X |X\rangle\langle X|J^\nu(0)|P,S\rangle$$

$$=\int d^4x\, e^{iq\cdot x}\langle P,S|[J^\mu(x)\,,\ J^\nu(0)]|P,S\rangle$$

Parameterize hadronic tensor $\longrightarrow$ DIS Structure functions!

Parameterization: Hadronic Tensor

Tensor $W^{\mu\nu}$: Function of P, q, S: $\longrightarrow$

All possible structures $\quad g^{\mu\nu} W_1(q^2, P\cdot q) + q^\mu q^\nu W_2(q^2, P\cdot q) + \ldots$

Restrictions:

- Current conservation $\boxed{\partial_\mu J^\mu = 0 \Longrightarrow q_\mu W^{\mu\nu} = W^{\mu\nu} q_\nu = 0}$
- Parity, Time-reversal, ...

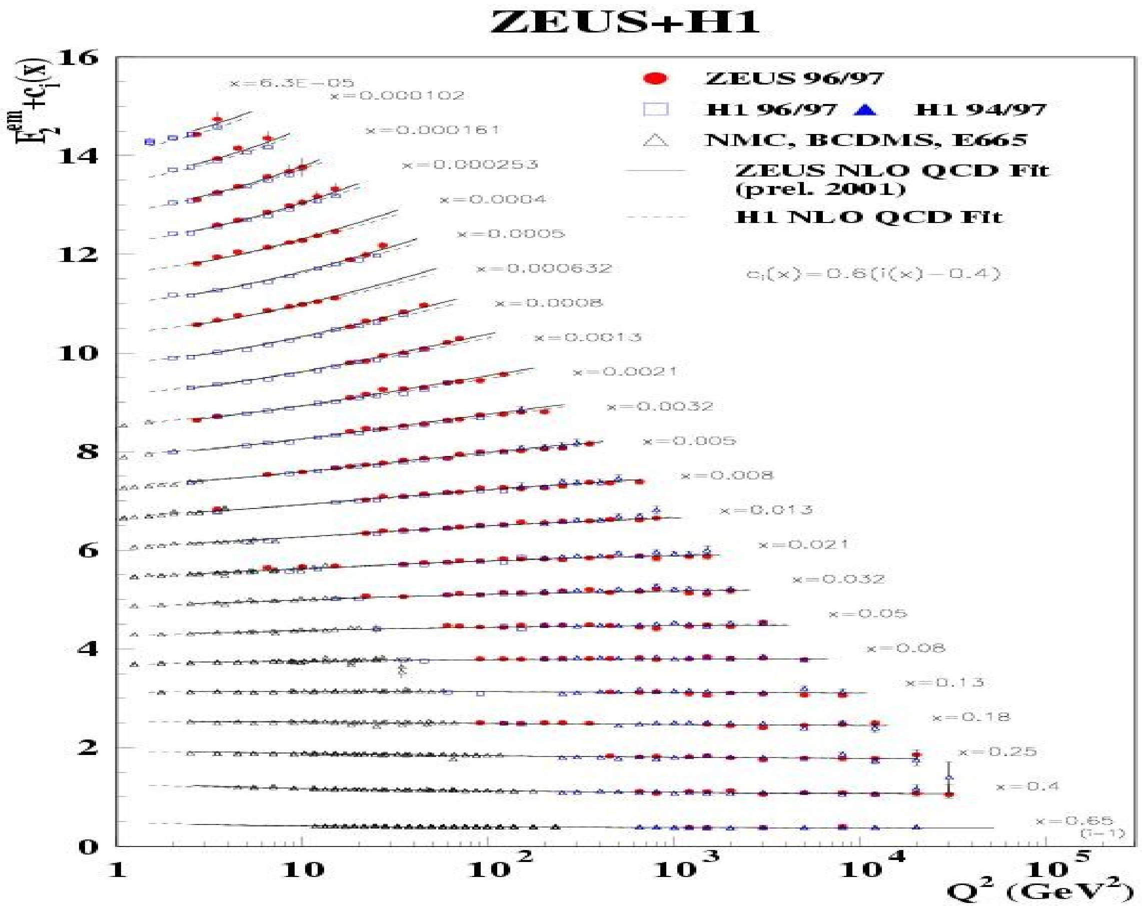
unpolarized hadronic tensor:

$$\boxed{W^{\mu\nu} = -W_1(Q^2, P\cdot q)\left(g^{\mu\nu} + \frac{q^\mu q^\nu}{Q^2}\right) + \frac{1}{M^2} W_2\left(P^\mu + \frac{P\cdot q}{Q^2} q^\mu\right)\left(P^\nu + \frac{P\cdot q}{Q^2} q^\nu\right) + \ldots}$$

Bjorken-Limes: $Q^2 \to \infty,\ P\cdot q \to \infty$, but $x_B = \frac{Q^2}{2P\cdot q}$ finite

$$\lim_{Bj} MW_1(Q^2, P\cdot q) = F_1(x_B) \qquad \lim_{Bj} MW_2(Q^2, P\cdot q) = F_2(x_B)$$

DIS at DESY

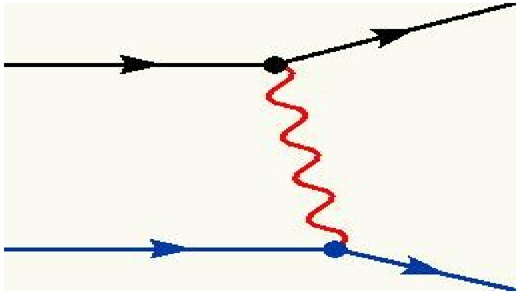


Scaling experimentally observed!

Parton model

What does "scaling" tell us?

Scattering off a point-like fermion:

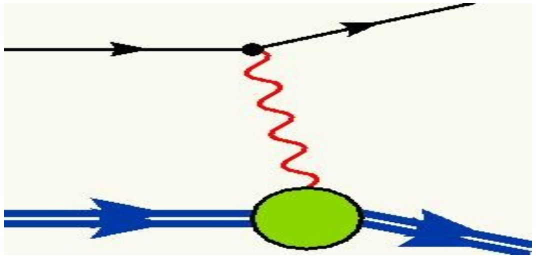


Structure functions:

$$F_1^{\text{point}}(x_B,Q^2) = x_B\delta(1-x_B)$$
$$F_2^{\text{point}}(x_B,Q^2) = \delta(1-x_B)$$

→ scaling

Elastic scattering:



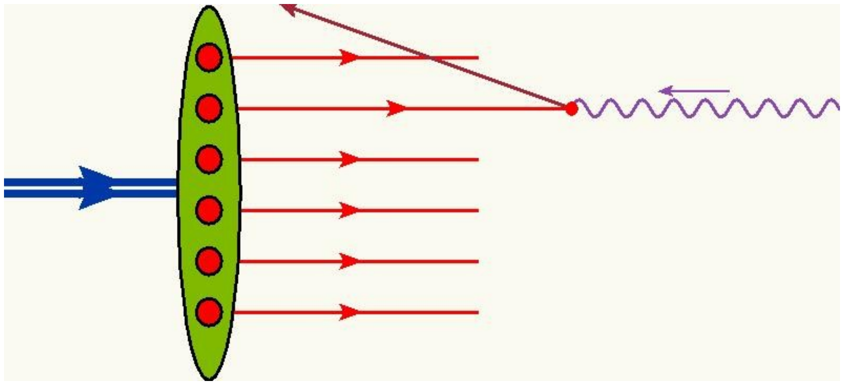
Structure functions:

$$F_1^{\text{elastic}}(x_B,Q^2) = G^2(Q^2)x_B\delta(1-x_B)$$
$$F_2^{\text{elastic}}(x_B,Q^2) = G^2(Q^2)\delta(1-x_B)$$

→ no scaling

Parton Model

- Imagine an ultra-relativistic nucleon moving along the z-axis:



- Lorentz-contraction: Partons are squeezed into transverse plane.
- Time-contraction: Parton are quasi-free.

- After e.m. interaction: Parton $\longrightarrow$ Hadron transition (Hadronization)

- Light cone coordinate:

$$k^{\pm} = \frac{1}{\sqrt{2}}(k^0 \pm k^3) \qquad \vec{k}_T = \left(k^1, k^2\right)$$

$$a \cdot b = a^+ b^- + a^- b^+ - \vec{a}_T \cdot \vec{b}_T$$

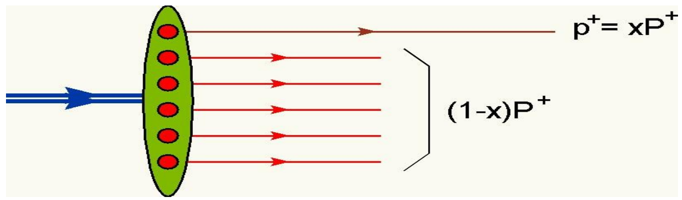
- One-dimensional problem: Hadron, Partons, virt. Photon collinear

$$k^+ = x P^+$$

Parton Distribution Function (PDF)

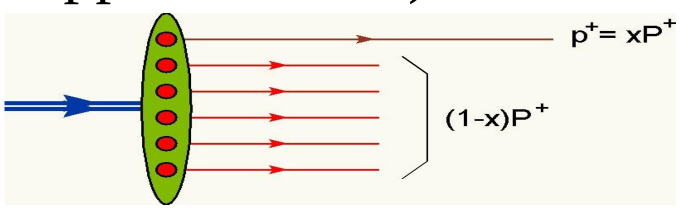
- Introduce probability to find a parton with a certain momentum fraction x $\longrightarrow$ PDF f(x):

$$f^q(x) =$$

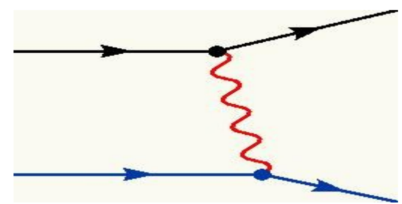


- Separation of DIS cross section into PDF and parton scattering (Impulse approximation)

$$d\sigma \propto$$



$$\otimes$$

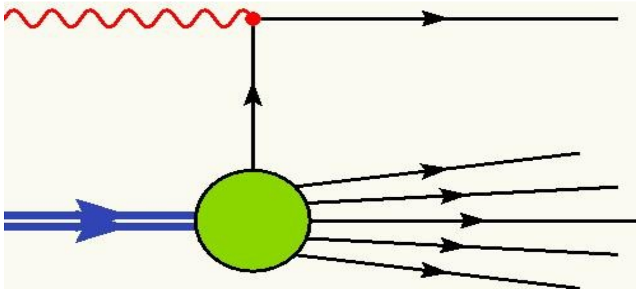


or $F_1(x_B) = \sum_q e_q^2 \int dx\, f^q(x) F_1^{\text{point-like}}(xP^+) = \frac{1}{2}\sum_q e_q^2 \int dx\, f^q(x)\delta(x - x_B) = \frac{1}{2}\sum_q e_q^2\, f^q(x_B)$

and $F_2(x_B) = 2x_B F_1(x_B)$ (Callan-Gross relation)

PDFs for pedestrians

- Take Parton diagram "literally":



Amplitude $A^\mu = \bar{u}_k(p')(-ie_q\gamma^\mu)_{kl}\int d^4x\, \mathrm{e}^{ik\cdot x}\langle X|\psi_l(x)|P,S\rangle$

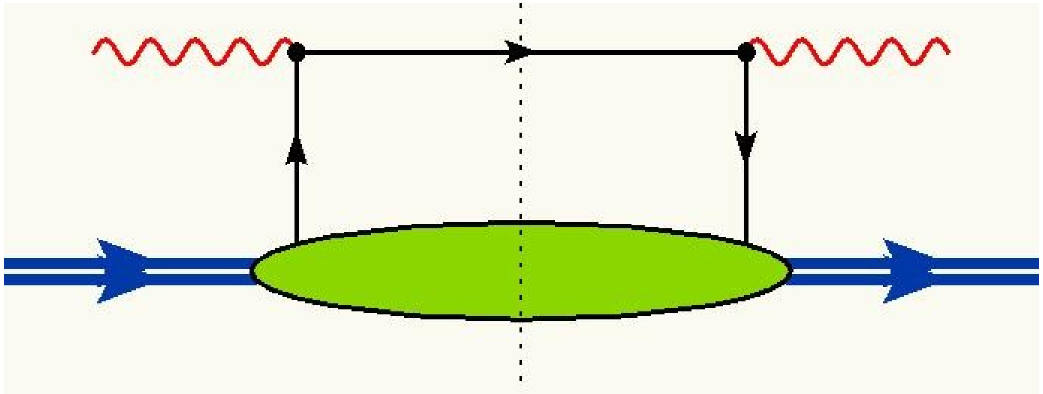
$$iM^\mu = e_q\, \bar{u}_k(p')\gamma^\mu_{kl}\langle X|\,\psi_l(0)|P,S\rangle$$

with Translation $\hat{\mathcal{O}}(x) = \mathrm{e}^{ix\cdot\hat{\mathcal{P}}}\hat{\mathcal{O}}(0)\mathrm{e}^{-ix\cdot\hat{\mathcal{P}}}$

- Hadronic Tensor:

$$W^{\mu\nu} = \sum_q \int \frac{d^3p'}{(2\pi)^3 2E_{p'}} \sum\!\!\!\!\!\!\int_X \delta^{(4)}(p' + P_X - P - q)(iM^\mu)(iM^\nu)^*$$

- "Handbag diagram":



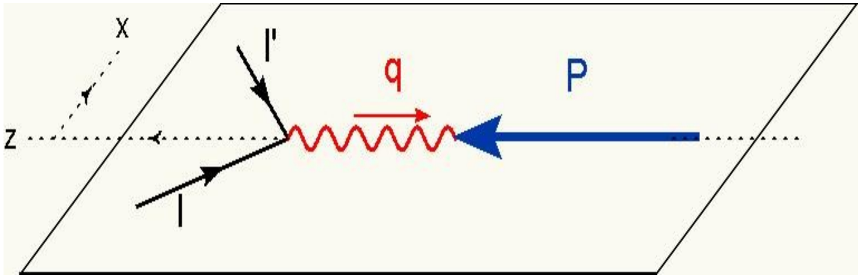
$$W^{\mu\nu} \propto \sum_q e_q^2 \int d^4k\, \delta^+((k+q)^2 - m_q^2)\, \mathrm{Tr}\left[(\not{k} + \not{q} + m_q)\gamma^\mu \Phi(k) \gamma^\nu\right]$$

Quark-quark correlator:

$$\Phi_{ij}(k; P, S) = \int \frac{d^4z}{(2\pi)^4}\, \mathrm{e}^{ik\cdot z} \langle P, S|\, \bar{\psi}_j^q(0)\, \psi_i^q(z)\, |P, S\rangle$$

- Non-perturbative $\longrightarrow$ property of the hadron, universal
- Gives information on the partonic substructure of hadrons
- Encodes all spin information of partons *and* nucleon
- Correlator depends on the full 4-momentum of the quark... (yet!)

- Infinite-Momentum frame:



- assume large photon virtuality Q
- choose nucleon and photon to be collinear
- choose $P^+ \sim O(Q)$ large
  $\longrightarrow P^- = M^2/2P^+$ suppressed

fixed frame: $P^+ = \frac{Q}{\sqrt{2}x_B}$ $\quad P^- = \frac{x_B M^2}{\sqrt{2}Q}$ $\quad q^+ = -\frac{Q}{\sqrt{2}}$ $\quad q^- = \frac{Q}{\sqrt{2}}$

- collinear partons: expect scalar products $k^2 \sim k \bullet P \sim O(Q)$ to be suppressed

$$\boxed{\Longrightarrow \; k^+ = xP^+ \sim \mathcal{O}(Q)\ ,\ \vec{k}_T \sim \mathcal{O}(Q^0)\ ,\ k^- \sim \mathcal{O}(1/Q)}$$

- Insertion into handbag diagram:

$$\delta\Big((k+q)^2 - m_q^2\Big) = \delta\Big(2(q^+ + k^+)(q^- + k^-) - \vec{k}_T^2\Big) \sim \frac{x_B}{Q^2}\delta(x - x_B)$$

$$\not{k} + \not{q} + m_q = (q^- + k^-)\gamma^+ + \cancel{(k^+ + q^+)\gamma^-} - \vec{k}_T \cdot \vec{\gamma}_T + m_q \sim \frac{Q}{\sqrt{2}}\gamma^+ + \ldots$$

- back to the hadronic tensor:

$$W^{\mu\nu} \propto \sum_q e_q^2 \int d^4k\, \delta^+((k+q)^2 - m_q^2)\, \mathrm{Tr}\Big[(\not k + \not q + m_q)\gamma^\mu \Phi(k) \gamma^\nu\Big]$$
$$= \sum_q e_q^2 \int dx \delta(x - x_B) \mathrm{Tr}\Big[\gamma^\nu \gamma^+ \gamma^\mu \int dk^- \int d^2k_T\, \Phi(k)\Big] + \mathcal{O}(1/Q)$$

$$= \sum_q e_q^2 \mathrm{Tr}\Big[\gamma^\nu \gamma^+ \gamma^\mu \Phi(x_B)\Big] + \mathcal{O}(1/Q)$$

- Collinear Correlator:

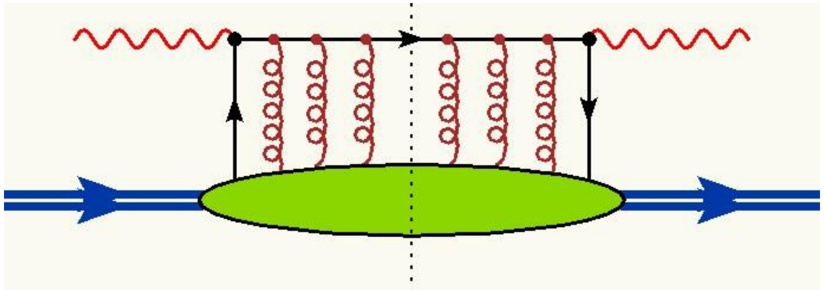
$$\boxed{\Phi_{ij}(x) = \int \frac{dz^-}{2\pi}\, \mathrm{e}^{ixP^+z^-} \langle P, S|\, \bar\psi_j^q(0)\, \psi_i^q(z^-, 0^+, \vec{0}_T)\, |P, S\rangle}$$

- Identify spin-1/2 Partons $\longleftrightarrow$ Quarks. Interaction between Quarks $\longrightarrow$ QCD!

QCD *local* gauge theory $\longrightarrow \Phi$ not color gauge invariant!

Gauge Links for pedestrians

- QCD: Implement gluons into handbag diagram
  $\longrightarrow$ One subclass: Multiparton correlations.



- "Struck" quark scatters off target remnants!
  Field-theoretically:

$$\langle P, S | \bar{\psi} \, A^{\mu_1} \, A^{\mu_2} \, A^{\mu_3} \ldots \psi | P, S \rangle$$

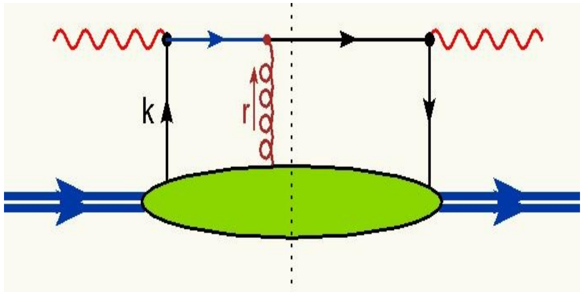
- 1/Q-Expansion: Contribution of A^+-gluons to leading order!

Summing up the gluons $\longrightarrow$ Gauge Link.

$$\boxed{\mathcal{W}[0, z^-] = \mathcal{P} \mathrm{e}^{-ig \int_0^{z^-} d\lambda \, A^+(\lambda n)}}$$

- ensures color gauge invariance!

- Here Example: 1-gluon-exchange



$$W^{\mu\nu} \propto \sum_q e_q^2 \int d^4k\, \delta^+\left((k+q)^2 - m_q^2\right) \times$$

$$\int d^4r \frac{\mathrm{Tr}\left[(\not{k}+\not{q}+m_q)\gamma_\rho(\not{k}+\not{q}-\not{r}+m_q)\gamma^\mu \Phi_A^\rho(k-r,r)\right]}{(k+q-r)^2 - m_q^2 + i0}$$

- Quark-Gluon-Quark Correlator:

$$\boxed{\Phi_A^\rho(k-r,r) = \int \frac{d^4z}{(2\pi)^4} \int \frac{d^4y}{(2\pi)^4}\, \mathrm{e}^{ir\cdot y}\mathrm{e}^{i(k-r)\cdot z} \langle P,S|\, \bar{\psi}_j(0)\, gA^\rho(y)\, \psi_i(z)\, |P,S\rangle}$$

- Infinite Momentum Frame: Gluons are also collinear:

$$r^+ \sim \mathcal{O}(Q)\ ,\ \vec{r}_T \sim \mathcal{O}(Q^0)\ ,\ r^- \sim \mathcal{O}(1/Q)$$

- Investigate longitudinal gluons: $\rho = +$

- 1/Q-Expansion:

$$\delta\big((k+q)^2 - m_q^2\big) \simeq \frac{x_B}{Q^2}\delta(x - x_B) \qquad (k+q-r)^2 - m_q^2 + i0 \simeq 2(k^+ + q^+ - r^+)q^- + i0$$

$$\not{k} + \not{q} + m_q \simeq q^-\gamma^+ \qquad \not{k} + \not{q} - \not{r} + m_q \simeq q^-\gamma^+$$

- Insertion into hadronic tensor:

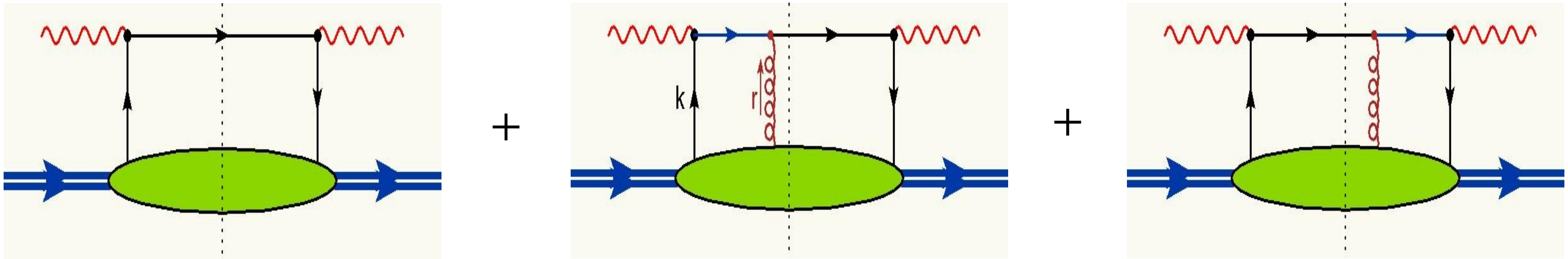
$$W^{\mu\nu} \sim -\frac{1}{2}\sum_q e_q^2 \mathrm{Tr}\left[\gamma^\nu\gamma^+\gamma^\mu \int_{-\infty}^{\infty} dr^+ \, \frac{\Phi_a^+(x_B P^+ - r^+, r^+)}{r^+ - i0}\right]$$

- Add. quark-propagator determines A^+-integration:

with $\int_{-\infty}^{\infty} dr^+ \frac{\mathrm{e}^{ir^+(y^- - z^-)}}{r^+ - i0} = 2\pi i \Theta(y^- - z^-)$

$$\boxed{\int_{-\infty}^{\infty} dr^+ \, \frac{\Phi_a^+(x_B P^+ - r^+, r^+)}{r^+ - i0} = \int \frac{dz^-}{2\pi} \, \mathrm{e}^{ix_B P^+ z^-} \langle P, S|\bar{\psi}_j(0)\Big(-ig\int_{\infty}^{z^-} dy^- A^+(y^-)\Big) q_i(z^-)|P, S\rangle}$$

- We recovered the first order of the gauge link:



Redefine the Quark-Quark Correlator:

$$\Phi_{ij}(x) = \int \frac{dz^-}{2\pi} \, \mathrm{e}^{ixP^+z^-} \langle P,S|\bar{\psi}_j(0)\left(1 - ig\int_0^{z^-} dy^- A^+(y^-) + ...\right) q_i(z^-)|P,S\rangle$$

$$= \int \frac{dz^-}{2\pi} \, \mathrm{e}^{ixP^+z^-} \langle P,S|\bar{\psi}_j(0)\mathcal{W}[0,z^-] q_i(z^-)|P,S\rangle$$

- Gauge Link reduces to unity in light cone gauge: $A^+=0$
  (but important later...)

  - Transverse gluons A_t^i : Contributions to higher twist (1/Q)

Quark Polarization

Projection of different quark spin polarizations:

$$\Phi^{[\gamma^+]} \propto \langle P,S\,|\,\bar{\psi}\,\gamma^+\,\psi\,|\,P,S\,\rangle \Longrightarrow \bar{u}(k,s)\,\gamma^+\,u(k,s) \;=\; 2k^+$$

No Polarization

$$\Phi^{[\gamma^+\gamma_5]} \propto \langle P,S\,|\,\bar{\psi}\,\gamma^+\gamma_5\,\psi\,|\,P,S\,\rangle \Longrightarrow \bar{u}(k,s)\,\gamma^+\gamma_5\,u(k,s) \;=\; 2ms^+$$

Longitudinal Polarization

$$\Phi^{[i\sigma^{i+}\gamma_5]} \propto \langle P,S\,|\,\bar{\psi}\,i\sigma^{i+}\gamma_5\,|\,P,S\,\rangle \Longrightarrow \bar{u}(k,s)\,i\sigma^{i+}\gamma_5\,u(k,s) \;=\; 2k^+s_T^i$$

Transverse Polarization

Collinear PDFs

unpolarized distribution:

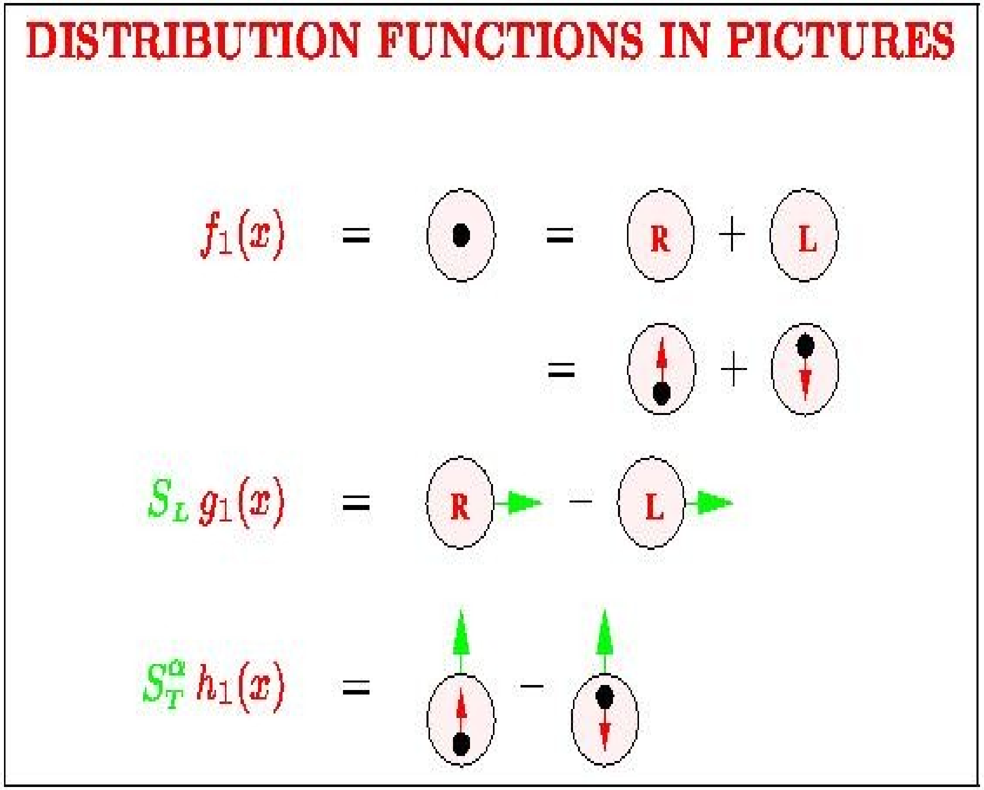
$$\Phi^{[\gamma^+]}(x) = f_1(x)$$

helicity distribution:

$$\Phi^{[\gamma^+\gamma_5]}(x) = S_L\, g_1(x)$$

transversity distribution:

$$\Phi^{[i\sigma^{i+}\gamma_5]}(x) = S_T^i\, h_1(x)$$



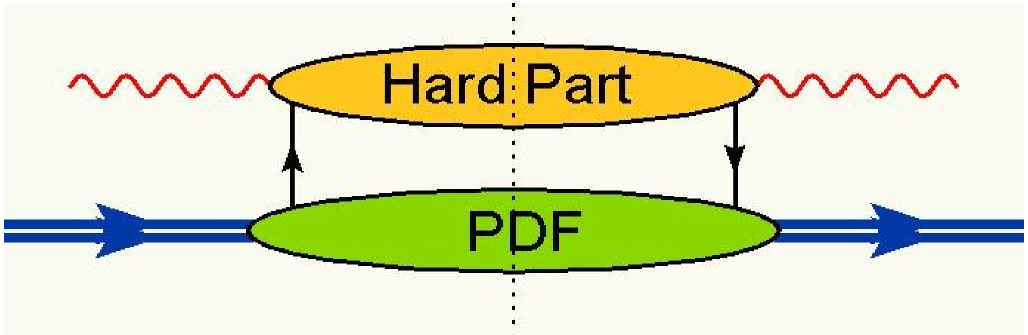
Transversity *chirally-odd* → ~~incl. DIS~~

Beyond leading order...

- Corrections to the "naive" parton model: Power corrections... (not here)
  Up till now: only leading order in α_s

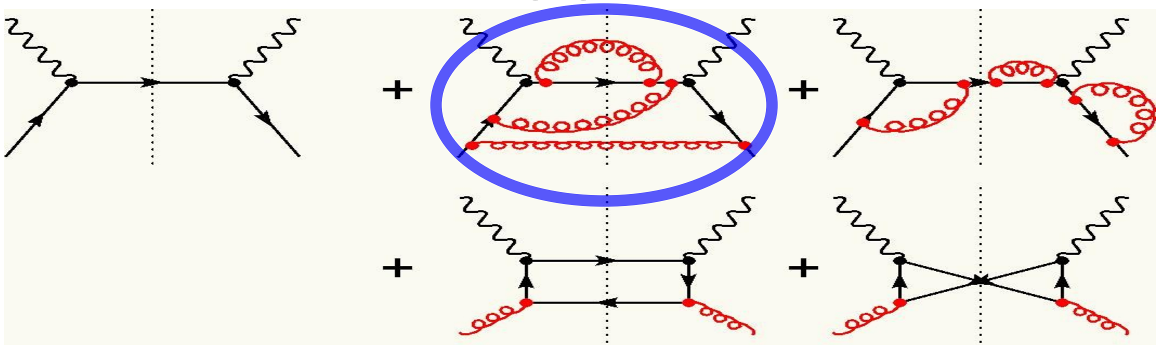
- Formal factorization theorem to all orders [Collins, Soper, Sterman, 1988]

$$F_1(x_B, Q^2) = \sum_q \int_{x_B}^1 \frac{dx}{x} f_q(x, \mu) \, H_{1,q}\Big(\frac{x_B}{x}, \frac{Q}{\mu}, \alpha_s(\mu)\Big) + \mathcal{O}(1/Q)$$



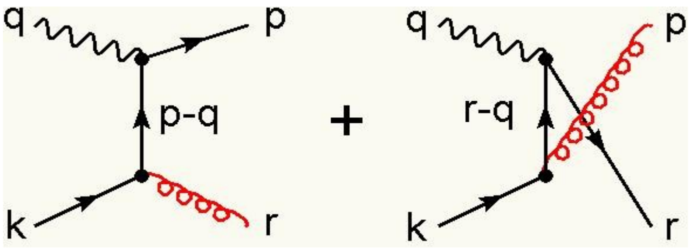
- Hard part pertubatively calculable:
  Scaling violated for higher orders
  High-p_T jets: Outgoing partons no more collinear

- Hard part (including gluon corrections of the partonic CS)



- Example: Only gluon radiation: Let's calculate:

$|M|^2 = \Big|$



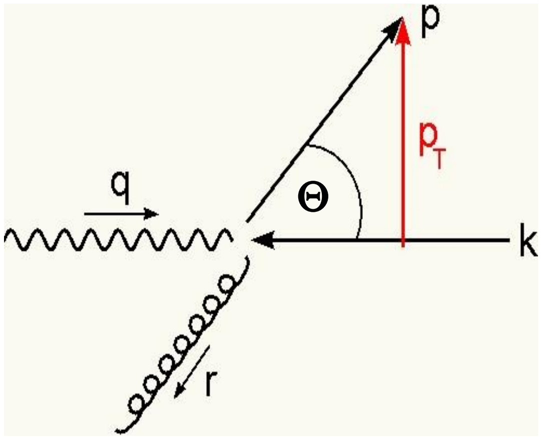
$\Big|^2 = 32\pi^2 e_q^2 \alpha\alpha_s C_F\left[-\frac{t}{s} - \frac{s}{t} + \frac{2uQ^2}{st}\right]$

Mandelstam variables: $s=(k+p)^2\ ,\ t=(k-r)^2\ ,\ u=(r-q)^2$

$$s+t+u=-Q^2$$

- Let's restrict ourselves to small angle scattering: -t << s , gluon radiation dominates over photon-gluon fusion

- <u>photon-parton c. m. - frame:</u> $\vec{q} + \vec{k} = 0$



- Differential partonic cross section (cm-frame for -t << s):

$$|M|^2 \implies d\hat{\sigma} \simeq \frac{1}{16\pi s^2}|M|^2 dp_T^2$$

$$\boxed{\frac{d\hat{\sigma}}{dp_T^2} = e_q^2 \frac{2\pi\alpha\alpha_s}{s}\frac{P_{qq}(z)}{p_T^2}}$$

$z = \frac{Q^2}{2k\cdot q} \simeq \frac{x_B}{x}$ "partonic" Bjorken-variable P_{qq} : probability of a quark emitting a gluon

- <u>Integrated partonic cross section:</u>

$$\hat{\sigma}(z) = \int_{\mu^2}^{p_{T,\max}=\mathcal{O}(Q^2)} dp_T^2 \frac{d\hat{\sigma}}{dp_T^2} \simeq e_q^2 \frac{2\pi\alpha\alpha_s}{s} P_{qq}(z) \ln\left(\frac{Q^2}{\mu^2}\right)$$

Divergence for $p_T \longrightarrow 0$, use μ^2 as a cut-off.

- Transition Parton $\longrightarrow$ Hadron: Convolution with PDF

$$\frac{1}{x_B}F_2(x_B,Q^2)=\sum_q e_q^2 \underbrace{\int_{x_B}^1 \frac{dx}{x} f_1^q(x)\left(\delta\left(1-\frac{x_B}{x}\right)+\frac{\alpha_s}{2\pi}P_{qq}\left(\frac{x_B}{x}\right)\ln\left(\frac{Q^2}{\mu^2}\right)\right)}$$

Recover the "old" expression by redefinition of the PDF: $\equiv f_1^q(x_B,Q^2)$

- Scaling violation is a property of PDFs: DGLAP-evolution

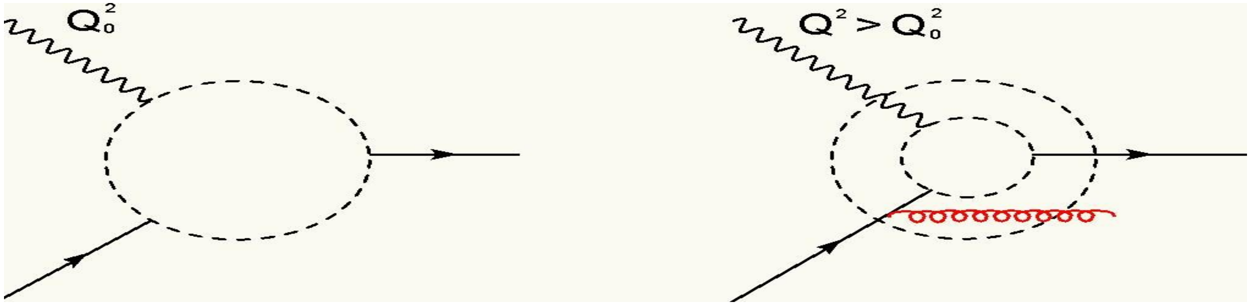
$$\boxed{\frac{\partial}{\partial \ln Q^2} f_1^q(x,Q^2)=\frac{\alpha_s}{2\pi}\int_{x_B}^1 \frac{dx}{x}\left(f_1^q(x,Q^2)P_{qq}\left(\frac{x_B}{x}\right)+g(x,Q^2)P_{qg}\left(\frac{x_B}{x}\right)\right)}$$

$$\boxed{\frac{\partial}{\partial \ln Q^2} g(x,Q^2)=\frac{\alpha_s}{2\pi}\int_{x_B}^1 \frac{dx}{x}\left(f_1^q(x,Q^2)P_{gq}\left(\frac{x_B}{x}\right)+g(x,Q^2)P_{gg}\left(\frac{x_B}{x}\right)\right)}$$

"Splitting functions": $P_{qq}(z)=C_F\frac{1+z^2}{(1-z)_+}+2\delta(1-z)$, ...

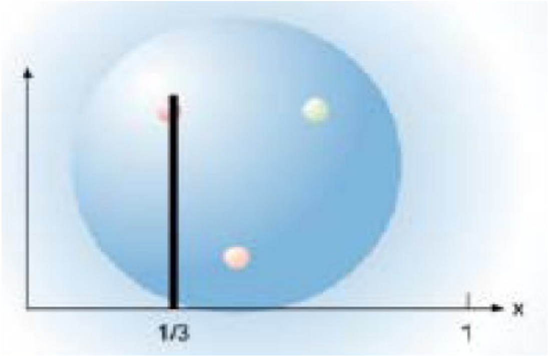
interpreted as: Probability to find "a gluon inside a quark" etc.

- Physical interpretation of DGLAP evolution equations:

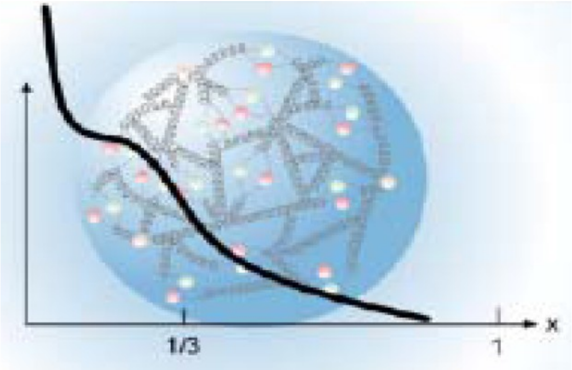


higher resolution $\longrightarrow$ gluon radiation $\longrightarrow$ lower x

- Evolution: makes comparison of different experiments possible.
- Universality: PDFs property of hadrons, not of processes.



free quarks



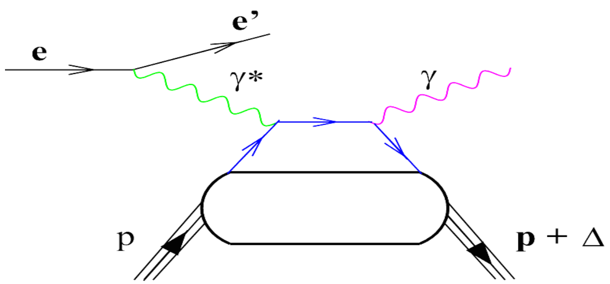
interacting quarks

Exclusive processes, GPDs in a nutshell...

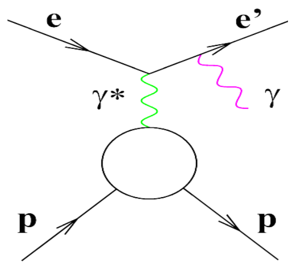
- Complete specification of the final state, most simple final state:

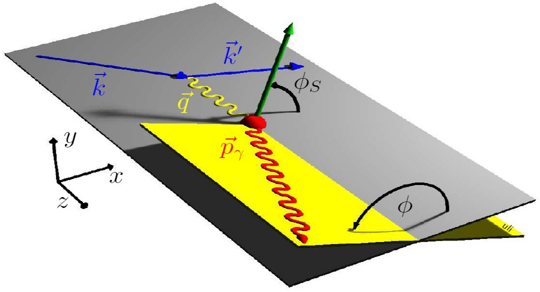
$l + N \rightarrow l + N + \gamma$ Deep-Virtual Compton Scattering (DVCS)

DVCS



Bethe-Heitler



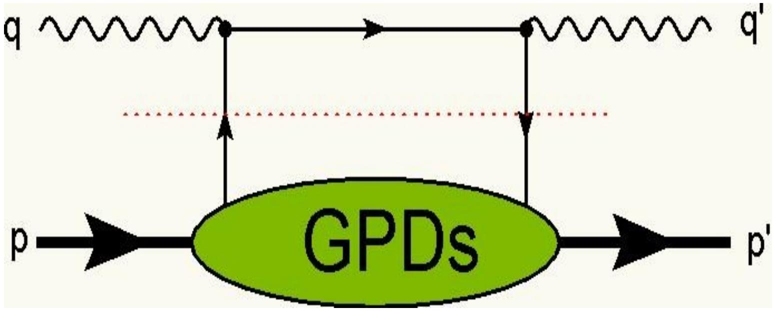


$$\frac{d^4\sigma}{dx_B\, dQ^2\, d|t|\, d\phi} \propto |T_{DVCS} + T_{BH}|^2 = |T_{DVCS}|^2 + |T_{BH}|^2 + \underbrace{T_{DVCS} T_{BH}^* + T_{DVCS}^* T_{BH}}_{I}$$

Interference term I: measured in azimuthal asymmetries

$$I \propto \underbrace{\cos\phi[\ldots]}_{\text{charge asym.}} + \underbrace{\sin\phi[\ldots] + \sin(\phi - \phi_s)[\ldots]}_{\text{spin asymmetries}} + \ldots$$

- DVCS-handbag contribution:



$$P = \tfrac{1}{2}(p+p') \qquad \Delta = p' - p$$

Skewness-parameter:

$$\Delta^+ = -2\xi P^+ \qquad t = -\frac{\vec{\Delta}_T^2 + 4\xi^2 M^2}{2(1-\xi^2)}$$

- GPDs $\longrightarrow$ "off-diagonal" matrix elements of quark-quark operator:

$$\boxed{F_{ij}(x,\xi,\vec{\Delta}_T) = \int \frac{dz^-}{2(2\pi)}\, e^{ixP^+z^-} \langle p' | \bar{\psi}_j(-\tfrac{z^-}{2}) \left[-\tfrac{z^-}{2} ; \tfrac{z^-}{2} \right] \psi_i(\tfrac{z^-}{2}) | p \rangle}$$

- Project out quark polarizations:

unpolarized: $(H\,,\,E)(x,\xi,t)$ longitudinally: $(\tilde{H}\,,\,\tilde{E})$

transverse [chirally-odd]: $(H_T\,,\,E_T\,,\,\tilde{H}_T\,,\,\tilde{E}_T)$

- Limiting cases of GPDs:

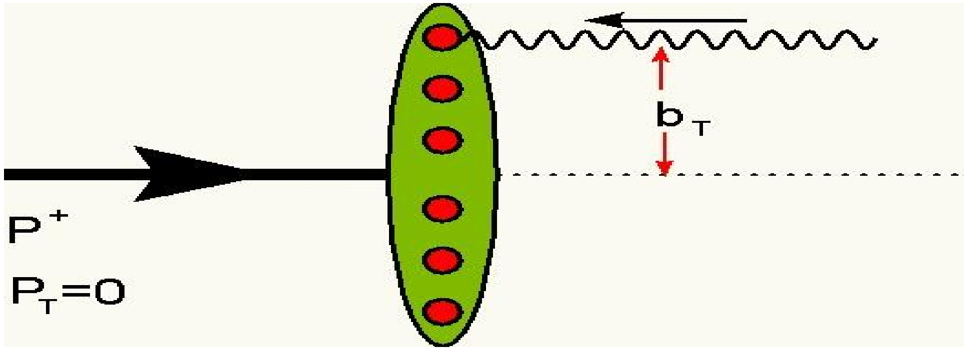
$$H^{q,g}(x,0,0) = q(x)$$

$$\tilde{H}^{q,g}(x,0,0) = \Delta q(x)$$

$$\sum_q e_q \int dx\, H^q(x,\xi,t) = F_1(t)$$

Impact parameter space

- Impact Parameter Space: (ξ=0, P_T=0) [M. Burkardt, PRD62, 071503]



- Impact parameter b_T and transv. momentum transfer Δ_T $\longrightarrow$ FT

$$\mathcal{F}_{ij}(x,\vec{b}_T) = \int \frac{d^2\Delta_T}{(2\pi)^2}\, \mathrm{e}^{-i\vec{\Delta}_T\cdot\vec{b}_T}\, F_{ij}(x,0,\vec{\Delta}_T)$$

- Impact parameter space $\longrightarrow$ "diagonal" matrix element $z_{1/2} = \mp\frac{z^-}{2}n_- + b_T$

$$\boxed{\mathcal{F}_{ij}(x,\vec{b}_T) = \int \frac{dz^-}{2(2\pi)}\, \mathrm{e}^{ixP^+z^-}\langle P^+;\vec{0}_T|\bar{\psi}_j(z_1)\,[z_1;z_2]\,\psi_i(z_2)|P^+;\vec{0}_T\rangle}$$

- (ξ=0, P_T=0): density interpretation.

Summary

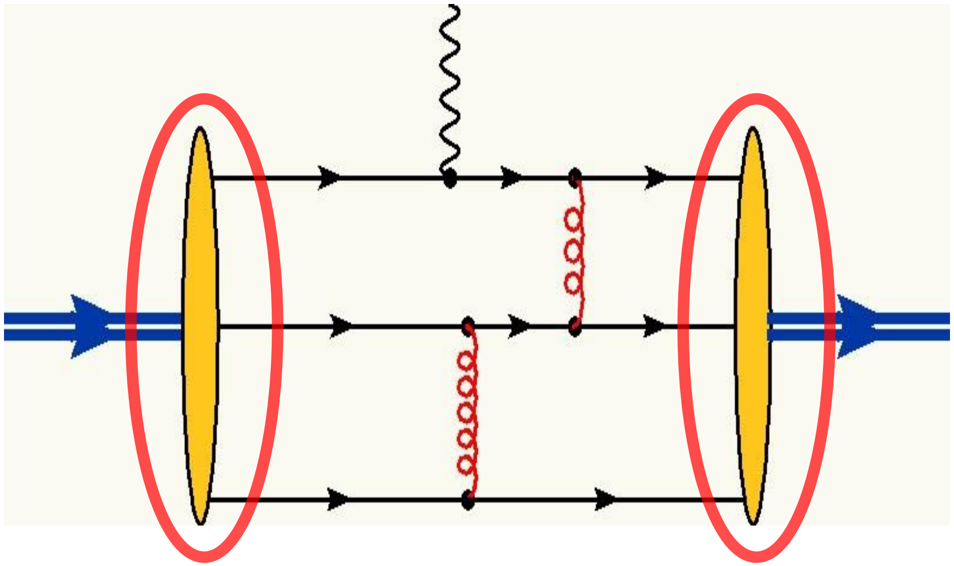
- Form Factors: give overall information about nucleons
  $\longrightarrow$ charge distribution, radius,...

- Deep-Inelastic Scattering: Scaling of structure function
  $\longrightarrow$ Parton-Model, PDFs, one-dimensional information
  about the partonic substructure
  diagonal matrix elements, Gauge link,
  scaling violations, DGLAP evolution

- Hard Exclusive processes: Generalized parton distributions, DVCS
  Impact Parameter space
  $\longrightarrow$ three-dimensional picture of the nucleon

What about large Q^2 ?
We should be sensitive to the substructure of hadrons.

Picture a la Brodsky-LePage:



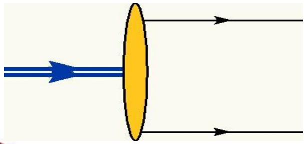
- "Light cone" wave function: non-perturbative object
- Transition hadron$\longrightarrow$ parton
- Fock-space decomposition:

$$|N;P\rangle = \sum_{qqq} \psi_{qqq}|qqq\rangle + \sum_{qqqg} \psi_{qqqg}\psi_{qqqg} + \ldots$$

- Form Factor: Only valence wave function $\longrightarrow$ counting rules

$$F_N(Q) \sim (Q)^{-}4\ ,\ \ F_\pi(Q) \sim Q^{-}2$$

- Pion Distribution Amplitude:



$$\phi_\pi(x) \propto \int d^2k_T \psi_{q\bar{q}/\pi}(x,\vec{k}_T) \propto \int dz^- \, e^{ixP^+z^-} \langle 0|\bar{\psi}(0)\gamma^+\gamma_5\psi(z^-)|\pi\rangle$$