Nucleon Physics from Lattice QCD (1)

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23rd Annual Hampton University
Graduate Studies Program
2008 June 06

Outline

- **Lecture #1**
  - Lattice QCD overview
    Background, actions, observables
  - Baryon spectroscopy
    Group theory, operator design, spectroscopy results
  - Nucleon Structure Functions
- **Lecture #2**
  - Axial charge couplings and form factors
  - Generalized Parton Distributions (GPDs)
  - Strangeness in the nucleon

Lattice 101

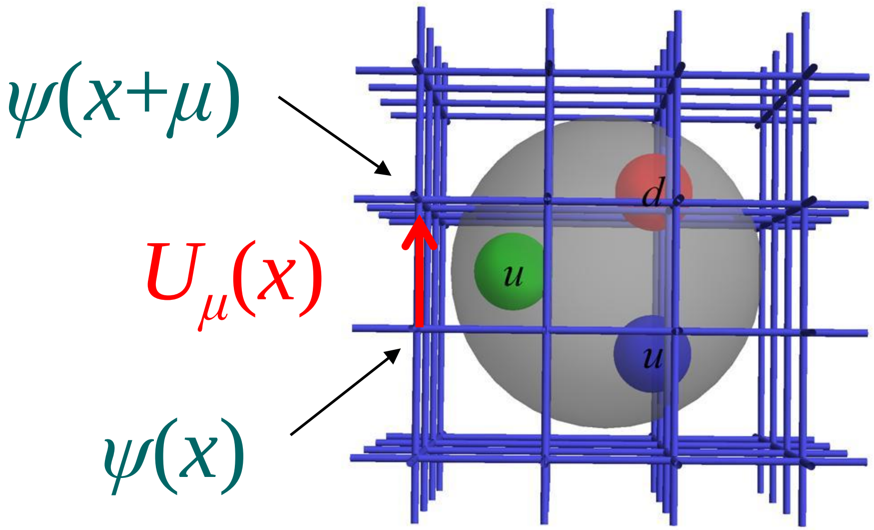
- Book (if you have to pick just one)
  - Degrand and De Tar
    Lattice Methods for Quantum Chromodynamics
    (World Scientific, 2006)

- arXiv article
  - Gupta
    "Introduction to Lattice QCD"
    `arXiv:hep-lat/9807028`

Lattice QCD

- Lattice QCD is a discrete version of continuum QCD theory

Lec. by Mike Peardon



- Physical observables are calculated from the path integral

$$\langle 0|O(\overline{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int [dA][d\overline{\psi}][d\psi]\, O(\overline{\psi},\psi,A)e^{i\int d^4x\,\mathcal{L}^{\mathrm{QCD}}(\overline{\psi},\psi,A)}$$

Lattice QCD

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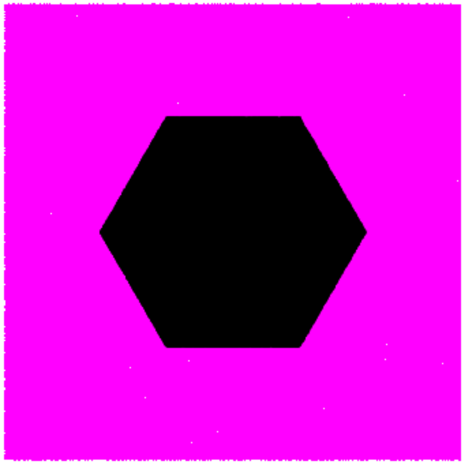
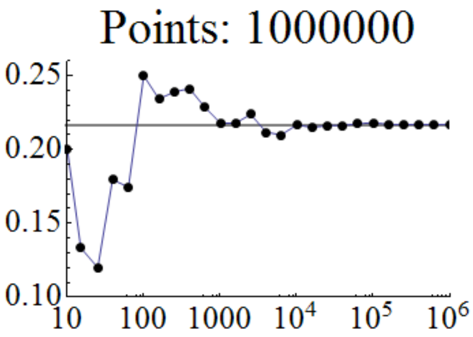
- Use Monte Carlo integration combined with the "importance sampling" technique to calculate the path integral.
- Simple example:





- Take $a \rightarrow 0$ and $V \rightarrow \infty$ in the continuum limit

Lattice Gauge Actions

- General form for improvement up to $O(a^2)$

$$S_g = \frac{\beta}{3}\,\mathrm{ReTr}\Big(c_0\left\langle \mathbb{1} - \square \right\rangle + c_1\left\langle \mathbb{1} - \square\!\square \right\rangle + c_2\left\langle \mathbb{1} - \boxbox \right\rangle\Big)$$

  - Commonly used:
    - Wilson
    - Iwasaki
    - Symanzik-improved
    - doubly blocked Wilson 2 (DBW2)
  - Most gauge actions used today are $O(a^2)$ improved
  - Small discretization effects ($\sim O(\Lambda_{QCD}{}^3 a^3)$) due to gauge choices
  - Most fermion actions are only $O(a)$ improved ($O(\Lambda_{QCD}{}^2 a^2)$)

Lattice Fermion Actions

- (Improved) Staggered fermions (asqtad):
  - Relatively cheap for dynamical fermions (good)
  - Mixing among parities and flavors or "tastes"
  - Baryonic operators a nightmare — not suitable
- Wilson/Clover action:
  - Moderate cost; explicit chiral symmetry breaking
- Twisted Wilson action:
  - Moderate cost; isospin mixing

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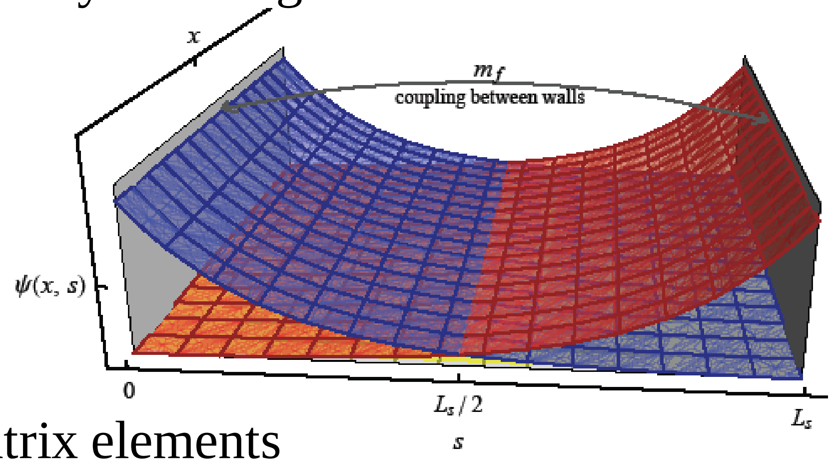
  - Moderate cost; explicit chiral symmetry breaking
- Twisted Wilson action:
  - Moderate cost; isospin mixing
- Chiral fermions
  - **Domain-Wall**/Overlap
  - Automatically $O(a)$ improved,
    good for spin physics and weak matrix elements
  - Expensive $D_{x,s;x',s'} = \delta_{x,x'} D^{\perp}_{s.s'} + \delta_{s,s'} D^{\parallel}_{x.x'}$

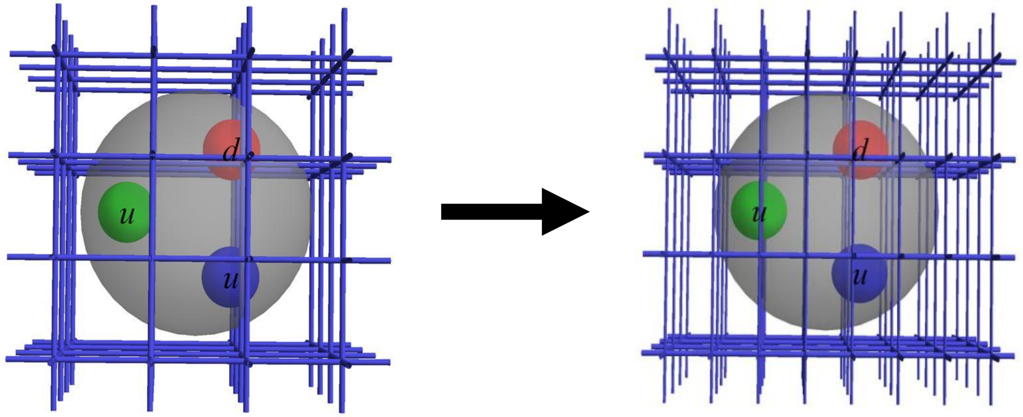


$$
\begin{aligned}
D^{\perp}_{s,s'} &= \frac{1}{2}[(1-\gamma_5)\delta_{s+1,s'} + (1+\gamma_5)\delta_{s-1,s'} - 2\delta_{s,s'}] \\
&- \frac{m_f}{2}[(1-\gamma_5)\delta_{s,L_s-1}\delta_{0,s'} + (1+\gamma_5)\delta_{s,0}\delta_{L_s-1,s'}],
\end{aligned}
$$

Lattice Fermion Actions

- Mixed Action
  - Staggered sea (cheap) with domain-wall valence (chiral)
  - Match the sea Goldstone pion mass to the DWF pion
  - Only mixes with the "scalar" taste of sea pion

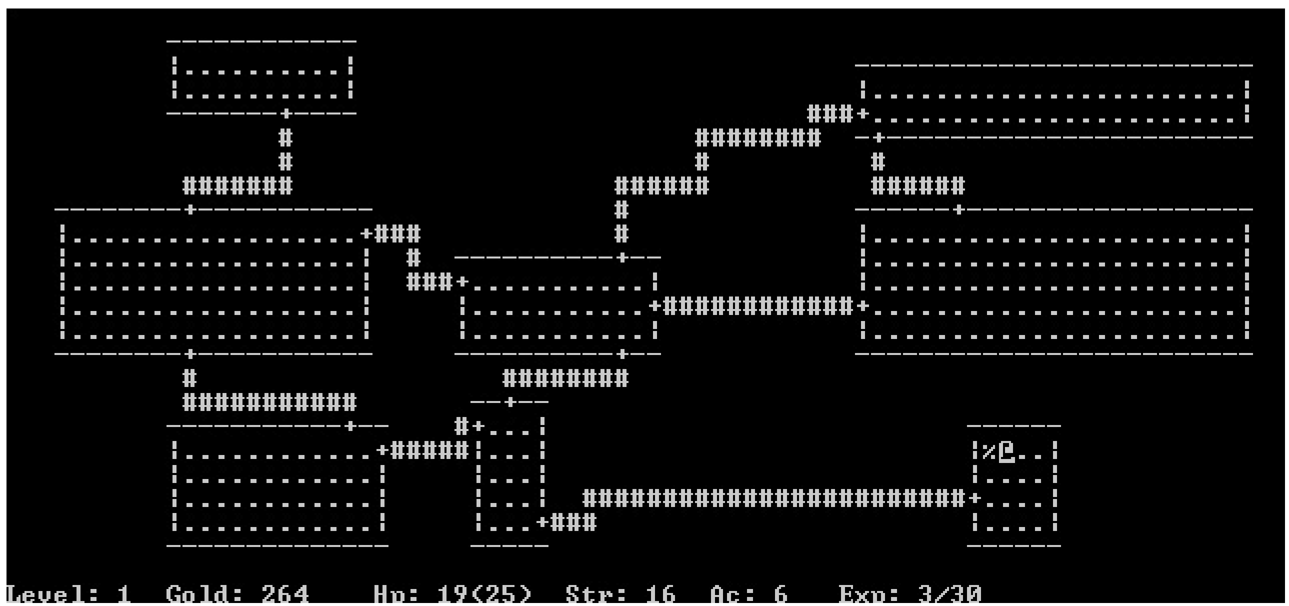
- Anisotropic Wilson/Clover
  - Wilson/Clover fermions with broken space/time symmetry
  - Lattice spacing $a_t < a_{x,y,z}$
  - Complicated but useful for excited-state physics



More details in Mike Peardon's Lecture

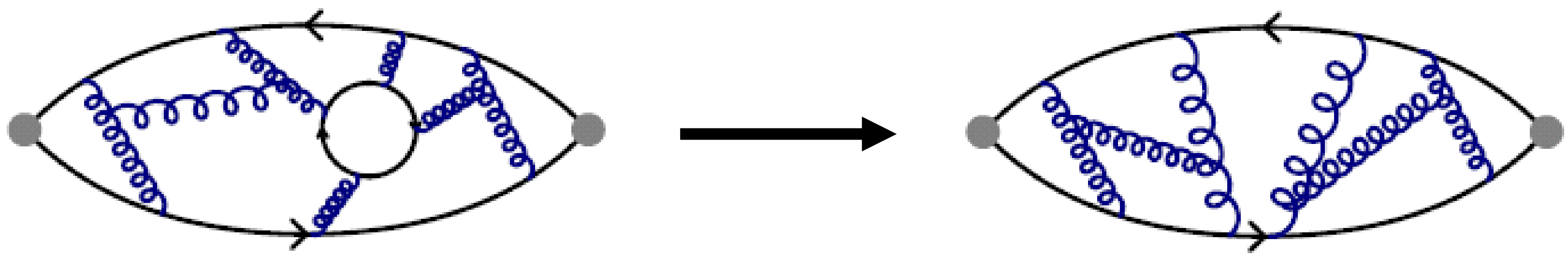
Computational Requirement

- A wide variety of first-principles QCD calculations can be done: In 1970, Wilson started off by writing down the first actions
- Progress is limited by computational resources
  - But assisted by advances in algorithms
- Computer power available for gaming in 1980's:



Poor Man's QCD: Quenched Approximation

- Full QCD: $$\begin{aligned}\langle O\rangle &= \frac{1}{Z}\int [dU][d\psi][d\overline{\psi}]e^{-S_F(U,\psi,\overline{\psi})-S_G(U)}O(U,\psi,\overline{\psi}) \\ &= \frac{1}{Z}\int [dU]\det M e^{-S_G(U)}O(U)\end{aligned}$$
- Quenched: Take det M = constant.



- "*Almost extinct*" in recent work

- Bad: Uncontrollable systematic error
- Good? Cheap exploratory studies to develop new methods

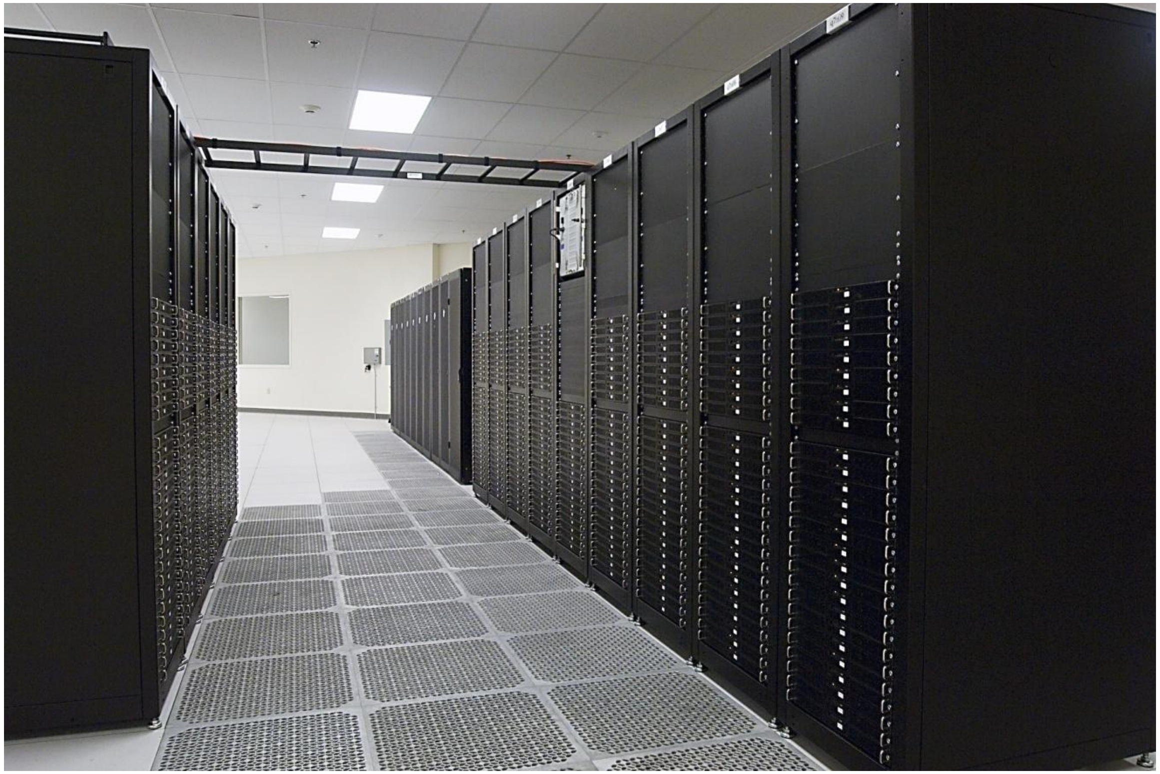
Computational Requirements

- A wide variety of first-principles QCD calculations can be done: In 1970, Wilson started off by writing down the first actions
- Progress is limited by computational resources
  - But assisted by advances in algorithms
- Computer power available today:

- Exciting progress during the last decade

Computational Requirements

2007: The 13 Tflops cluster at Jefferson Lab



Other joint lattice resources within the US: Fermilab, BNL

Non-lattice resources open to USQCD: ORNL, LLNL, ANL

Computational Requirements

- Gauge generation estimate with latest algorithms scales like
  Cost factor: a^{-6}, L^5, M_π^{-3}
- Chiral domain-wall fermions (DWF) at large volume (6 fm) at physical pion mass may be expected in 2011
- But for now….
  need a pion mass extrapolation $M_\pi \rightarrow (M_\pi)_{\mathrm{phys}}$
  (use chiral perturbation theory, if available)

Systematic Errors

- Currently, not at the physical pion-mass point
  XPT uncertainty (parameters used in XPT, etc.)
- Finite lattice spacing
  - Exact: Do multiple lattice-spacing calculations and extrapolate to $a = 0$
  - Otherwise, estimate according to the level of improvement for the gluon and fermion action and operators
- Finite-volume effect
  - Exact: Do multiple volume calculations and extrapolate to $V = \infty$
  - Otherwise, estimate according to previous work
  - Or apply finite-volume XPT to try to correct FVE
- Other Systematics
  - For example: if fitting is involved, what is the dependence on the fit range?

Baryon Resonances

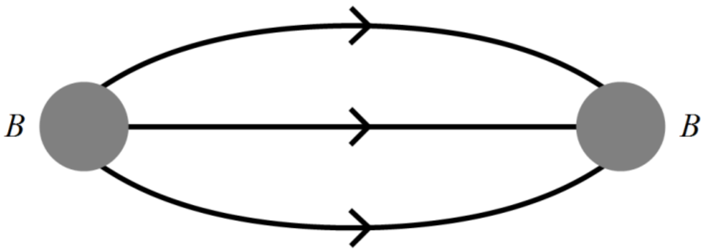
Spectroscopy on Lattice

- Calculate two-point Green function

$$\langle O \rangle = \frac{1}{Z}\int [dU][d\psi][d\overline{\psi}]e^{-S_F(U,\psi,\overline{\psi})-S_G(U)}O(U,\psi,\overline{\psi})$$
$$= \frac{1}{Z}\int [dU]\mathrm{det}M e^{-S_G(U)}O(U)$$

- Spin projection

$$\sum_{\alpha,\beta}\Gamma^{\alpha,\beta}\langle J(X_{\mathrm{snk}})J(X_{\mathrm{src}})\rangle_{\alpha,\beta}$$



- Momentum projection

Two-point correlator

Exp decay

$$\Gamma^{(2),T}_{AB}(t;\vec{p}) = \sum_n \frac{E_n + M_n}{2E_n} Z_{n,A}Z_{n,B}\boxed{e^{-E_n(\vec{P})t}}$$

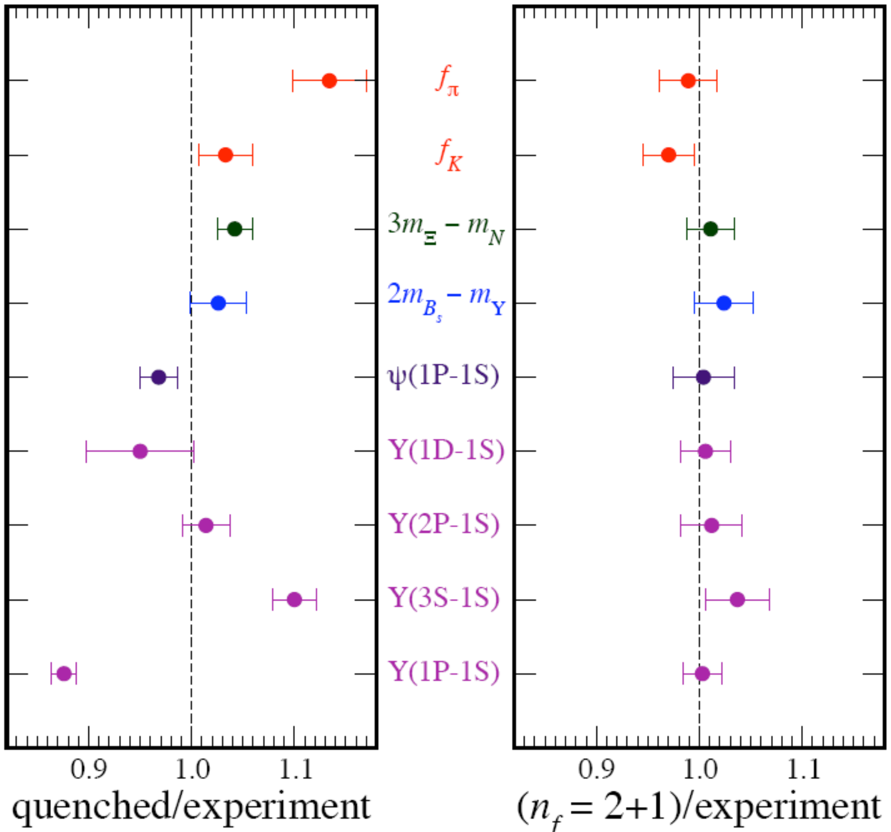
At large enough *t*, the ground-state signal dominates

Why Baryons?

Lattice QCD spectrum

- Successfully calculates many ground states (Nature, ...)

HPQCD

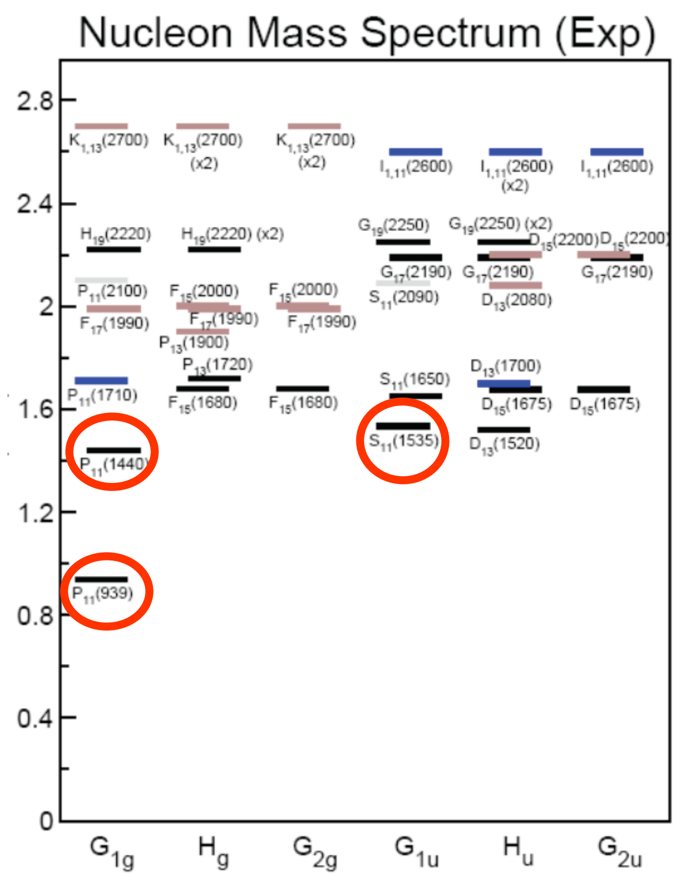


- Predictions: B_c mass, D and D_s decay constants, $D \to Kl\nu$ form factors

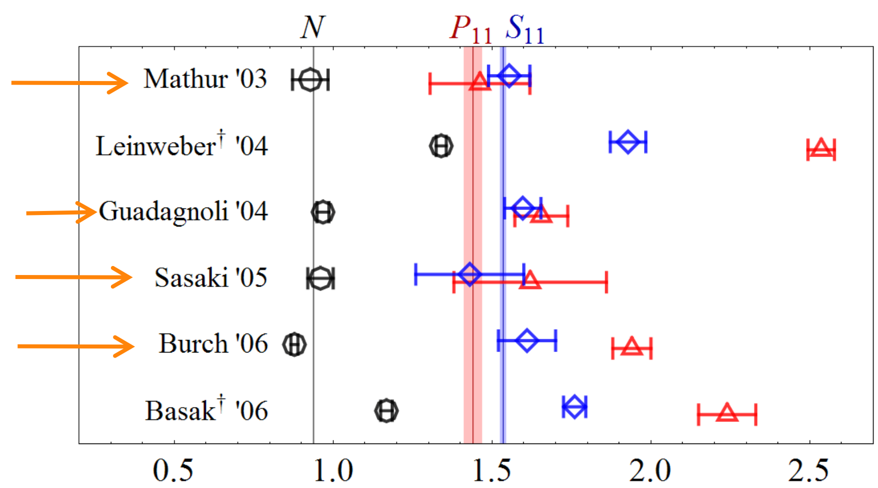
Why Baryons?

Lattice QCD spectrum

- Successfully calculates many ground states (Nature, …)
- Nucleon spectrum, on the other hand… not quite



Example: **Quenched** N, P_{11}, S_{11}



- Systematic errors not included:

Finite volume and lattice spacing;
possible higher excited-state contamination

Strange Baryons

- Strange baryons are of special interest; challenging even to experiment
- Example from PDG Live:

Ξ BARYONS ($S = -2$, $I = 1/2$)

$\Xi^0 = uss$, $\Xi^- = dss$

Ξ^0	$1/2(1/2^+)$	****	$\Xi(1820)$ D_{13}	$1/2(3/2^-)$	***	$\Xi(2370)$	$1/2(?^?)$	•**
Ξ^-	$1/2(1/2^+)$	****	$\Xi(1950)$	$1/2(?^?)$	***	$\Xi(2500)$	$1/2(?^?)$	•*
$\Xi(1530)$ P_{13}	$1/2(3/2^+)$	****	$\Xi(2030)$	$1/2(\geq \frac{5}{2}^?)$	***			
$\Xi(1620)$	$1/2(?^?)$	•*	$\Xi(2120)$	$1/2(?^?)$	•*			
$\Xi(1690)$	$1/2(?^?)$	***	$\Xi(2250)$	$1/2(?^?)$	•**			

• — OMITTED FROM SUMMARY TABLE

Ω BARYONS ($S = -3$, $I = 0$)

$\Omega^- = sss$

Ω^-	$0(3/2^+)$	****
$\Omega(2250)^-$	$0(?^?)$	***
$\Omega(2380)^-$		•**
$\Omega(2470)^-$		•**

Operator Design

- All baryon spin states wanted: $j = 1/2, 3/2, 5/2, \ldots$
- Rotation symmetry is reduced due to discretization

  rotation SO(3) ⇒ octahedral O_h group

	I	J	$6\,C_4$	$8\,C_6$	$8\,C_3$	$6\,C_8$	$6\,C_8'$	$12\,C_4'$
A_1	1	1	1	1	1	1	1	1
A_2	1	3	−2	1	0	−1	1	0
E	2	1	1	1	−1	−1	−1	0
G_1	2	0	1	−1	1	−2	1	0
G_2	2	−4	0	1	0	0	1	−1
T_1	3	2	0	0	1	1	−1	−1
T_2	3	3	0	−1	−1	1	1	0
H	4	−3	−1	0	0	0	−1	1

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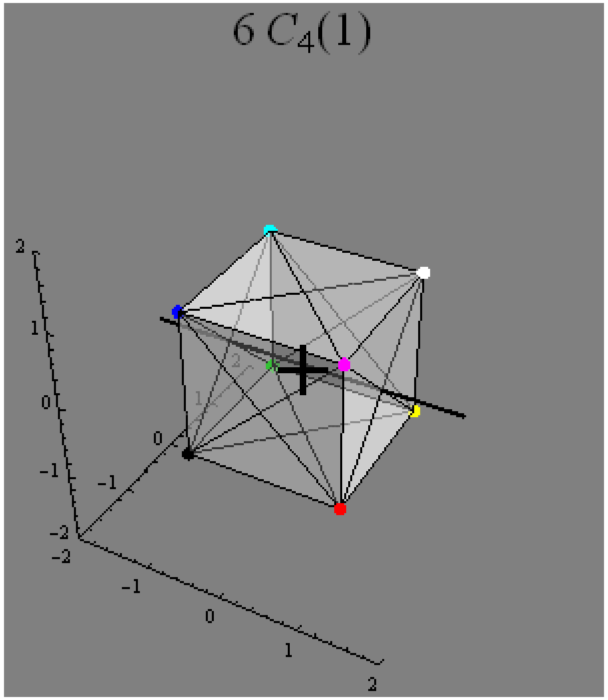
	I	J	6 C_4	8 C_6	8 C_3	6 C_8	6 C'_8	12 C'_4
A_1	1	1	1	1	1	1	1	1
A_2	1	3	-2	1	0	-1	1	0
E	2	1	1	1	-1	-1	-1	0
G_1	2	0	1	-1	1	-2	1	0
G_2	2	-4	0	1	0	0	1	-1
T_1	3	2	0	0	1	1	-1	-1
T_2	3	3	0	-1	-1	1	1	0
H	4	-3	-1	0	0	0	-1	1



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  rotation SO(3) $\Rightarrow$ octahedral O_h group

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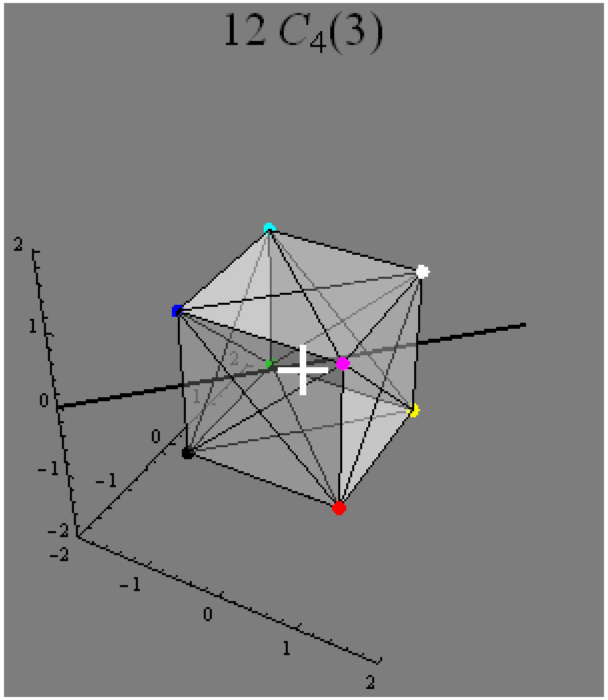
E	2	1	1	1	-1	-1	-1	0
G_1	2	0	1	-1	1	-2	1	0
G_2	2	-4	0	1	0	0	1	-1
T_1	3	2	0	0	1	1	-1	-1
T_2	3	3	0	-1	-1	1	1	0
H	4	-3	-1	0	0	0	-1	1



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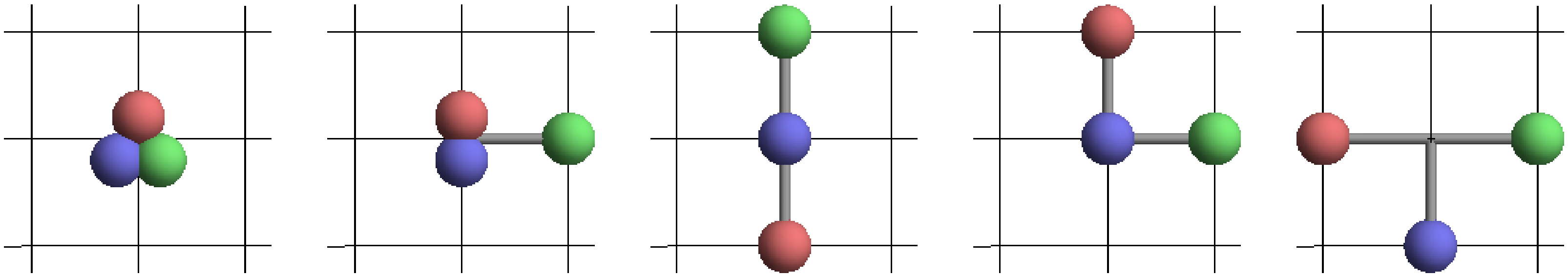
G_2	2	-4	0	1	0	0	1	-1
T_1	3	2	0	0	1	1	-1	-1
T_2	3	3	0	-1	-1	1	1	0
H	4	-3	-1	0	0	0	-1	1

Baryons

j	Irreps
$\frac{1}{2}$	G_1
$\frac{3}{2}$	H
$\frac{5}{2}$	$G_2 \oplus H$
$\frac{7}{2}$	$G_1 \oplus G_2 \oplus H$
$\frac{9}{2}$	$G_1 \oplus 2\,H$
$\frac{11}{2}$	$G_1 \oplus G_2 \oplus 2\,H$
$\frac{13}{2}$	$G_1 \oplus 2\,G_2 \oplus 2\,H$
$\frac{15}{2}$	$G_1 \oplus G_2 \oplus 3\,H$
$\frac{17}{2}$	$2\,G_1 \oplus G_2 \oplus 3\,H$
$\frac{19}{2}$	$2\,G_1 \oplus 2\,G_2 \oplus 3\,H$
$\frac{21}{2}$	$G_1 \oplus 2\,G_2 \oplus 4\,H$
$\frac{23}{2}$	$2\,G_1 \oplus 2\,G_2 \oplus 4\,H$

Operator Design

- Baryon field $\Phi^{ABC}_{\alpha\beta\gamma,ijk}(x) \;=\; \epsilon_{abc}[\tilde{D}^{(3)}_i\tilde{\psi}]_{Aa\alpha}(x)[\tilde{D}^{(3)}_j\tilde{\psi}]_{Bb\beta}(x)[\tilde{D}^{(3)}_k\tilde{\psi}]_{Cc\gamma}(x)$



- Classify states according to symmetry properties
- Projection onto irreducible representations of finite groups
- Number of operator

N^+ Operator type	G_{1g}	H_g	G_{2g}
Single-Site	3	1	0
Singly-Displaced	24	32	8
Doubly-Displaced-I	24	32	8
Doubly-Displaced-L	64	128	64
Triply-Displaced-T	64	128	64
Total	**179**	**321**	**144**

S. Basak et al., Phys. Rev. D72, 094506 (2005)

Variational Method

- Construct the correlator matrix

$$C_\Lambda^{m,n}(t) = \sum_{\vec{x}} \sum_{\lambda} \langle 0 \mid B_\lambda^{\Lambda,m}(\vec{x},t) \bar{B}_\lambda^{\Lambda,n}(0) \mid 0 \rangle$$

- Construct the matrix

$$C_{i\ j}(t) = \langle 0 \mid \mathcal{O}_i(t)^\dagger \mathcal{O}_j(0) \mid 0 \rangle$$

- Solve for the generalized eigensystem of

$$C(t)\psi = \lambda(t,t_0)C(t_0)\psi$$

with eigenvalues

$$\lambda_n(t,t_0) = e^{-(t-t_0)E_n}(1 + \mathcal{O}(e^{-|\delta E|(t-t_0)}))$$

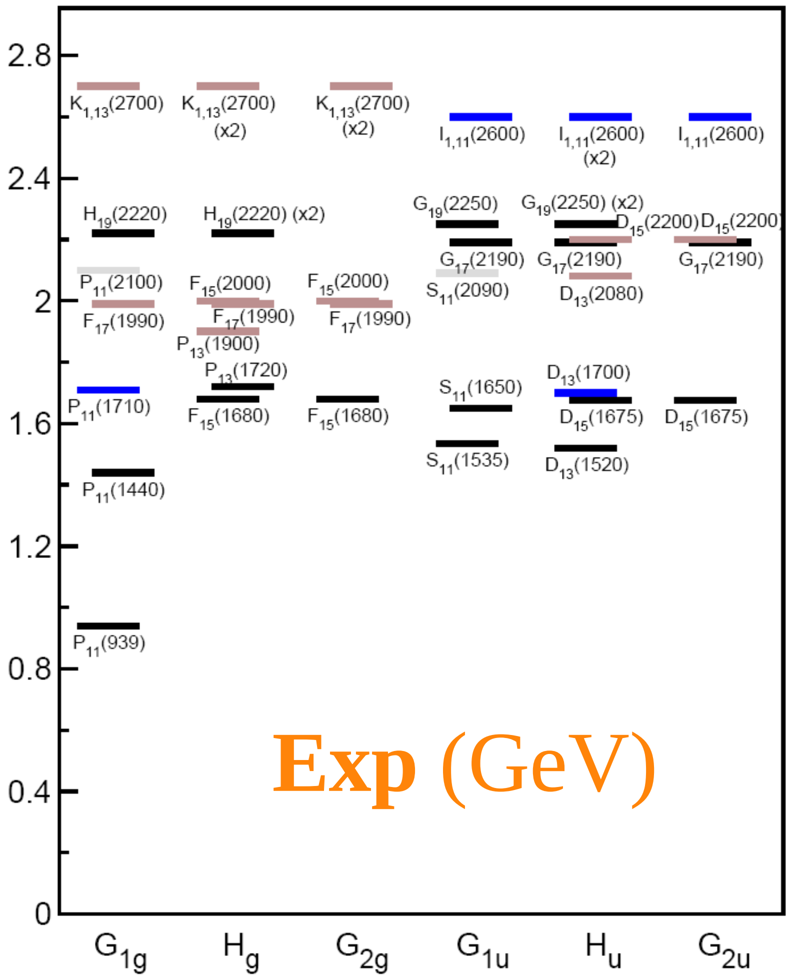
C. Michael, Nucl. Phys. B 259, 58 (1985)

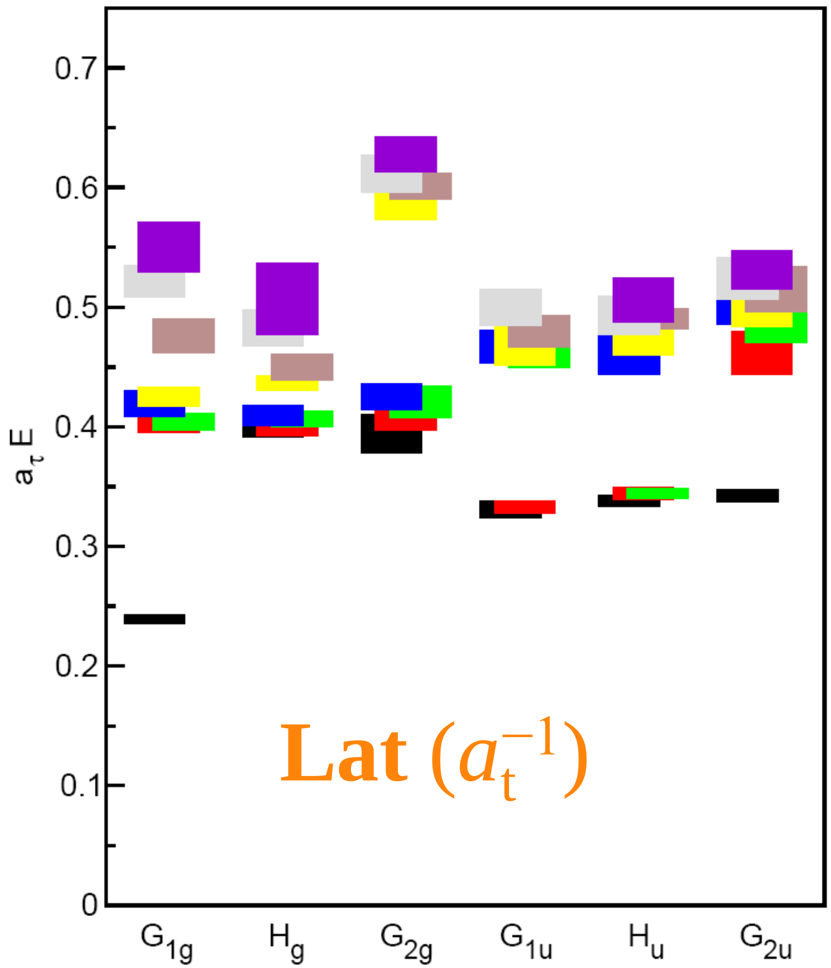
M. Lüscher and U. Wolff, Nucl. Phys. B 339, 222 (1990)

- At large *t*, the signal of the desired state dominates.

$N_f = 0$ Study: Nucleon

- Anisotropic Wilson action, hep-lat/0609019
  $V = 12^3 \times 48$, $a_s \sim 0.1$ fm, $a_s/a_t \sim 3$, $M_\pi \sim 700$ MeV





Pion-Mass Dependences

- Examples of a $N_f = 2+1$ study
  - Isotropic mixed action: DWF on staggered sea,
  - $M_\pi \sim 300\text{–}750$ MeV, $L \sim 2.5$ fm
  - Number of operator:

Flavor	$G_{1g/u}(2)$	$H_{g/u}(4)$
N	3	1
Δ	1	2
Λ	4	1
Σ	4	3
Ξ	4	3
Ω	1	2

  - Naïve chiral extrapolation

j	Irreps
$\frac{1}{2}$	G_1
$\frac{3}{2}$	H
$\frac{5}{2}$	$G_2 \oplus H$
[illegible]	$2\,G_1 \oplus 2\,G_2 \oplus 3\,H$
$\frac{21}{2}$	$G_1 \oplus 2\,G_2 \oplus 4\,H$
$\frac{23}{2}$	$2\,G_1 \oplus 2\,G_2 \oplus 4\,H$

This calculation:

Three quarks in a baryon located at a single site

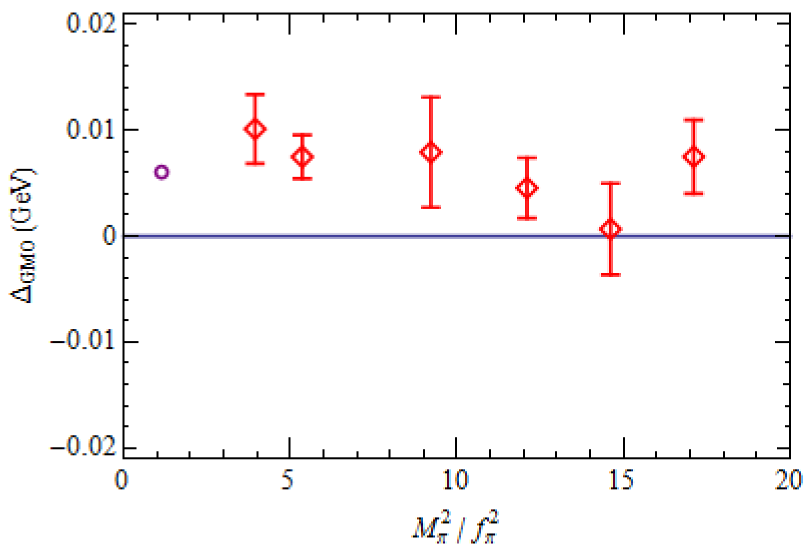
Pion-Mass Dependences

2+1-flavor mixed action

- SU(3) flavor symmetry breaking
  - Gell-Mann-Okubo relation

$$\Delta_{GMO} = \frac{3}{4}M_\Lambda + \frac{1}{4}M_\Sigma - \frac{1}{2}M_N - \frac{1}{2}M_\Xi$$

- Mass differences are close to experimental numbers



Pion-Mass Dependences

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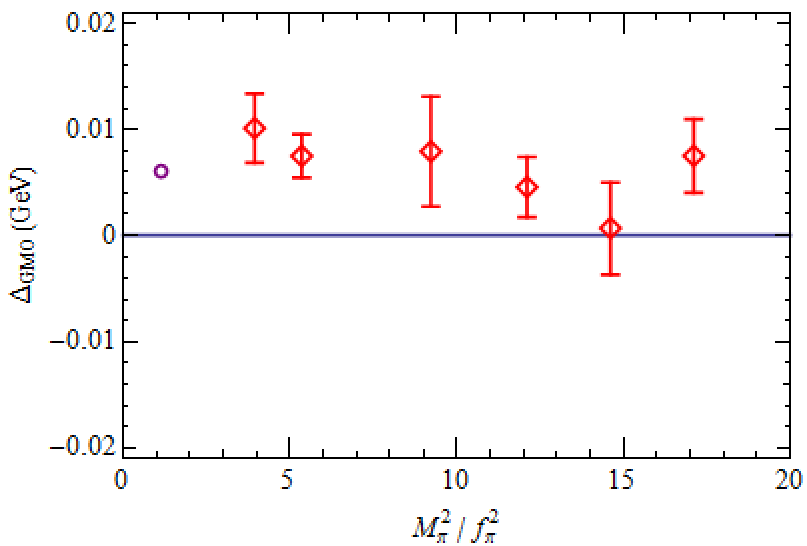
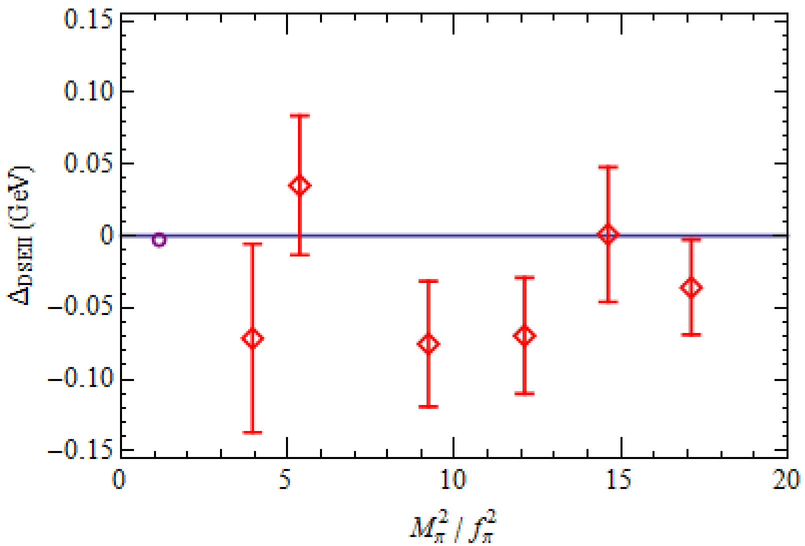
$$\Delta_{GMO} = \frac{3}{4}M_\Lambda + \frac{1}{4}M_\Sigma - \frac{1}{2}M_N - \frac{1}{2}M_\Xi$$

  - Decuplet Equal-Spacing Relation

$$\Delta_{DESII} = \frac{1}{2}(M_{\Sigma^*} - M_\Delta) + \frac{1}{2}(M_\Omega - M_{\Xi^*}) - M_{\Xi^*} + M_{\Sigma^*}$$

- Mass differences are close to experimental numbers

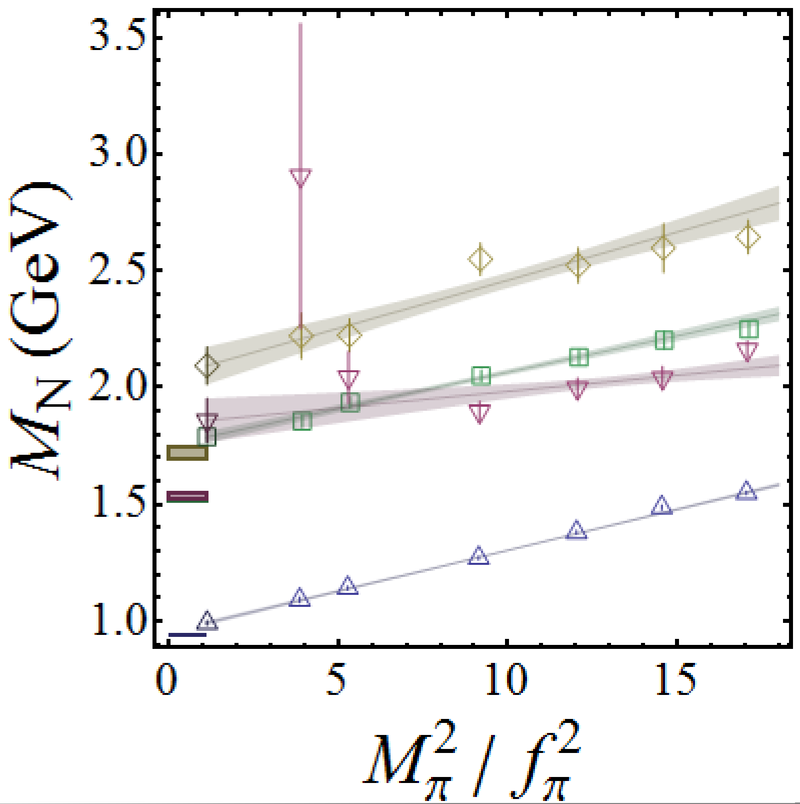




Pion-Mass Dependences

2+1-flavor mixed action

- The non-strange baryons (N)
- Symbols: J^P = 1/2+ △, 1/2− ▽ , 3/2+ ◇, 3/2− □
  N N(1535) N(1720) N(1520)

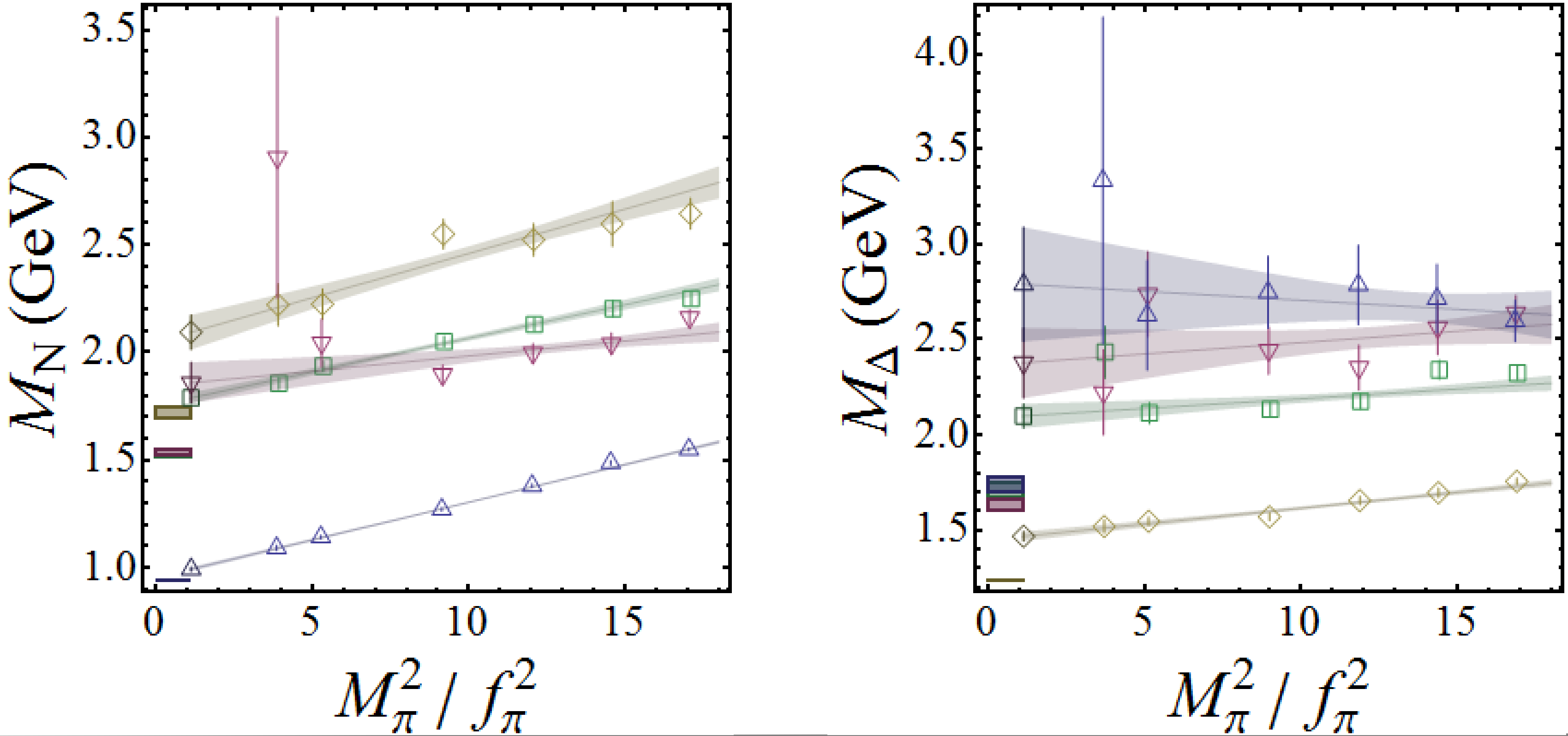


Pion-Mass Dependences

2+1-flavor mixed action

- The non-strange baryons (N and Δ)
- Symbols: $J^P = 1/2^+$ △ , $1/2^-$ ▽ , $3/2^+$ ◇ , $3/2^-$ □

$1/2^+$	$1/2^-$	$3/2^+$	$3/2^-$
N	$N(1535)$	$N(1720)$	$N(1520)$
	$\Delta(1620)$	Δ	$\Delta(1700)$

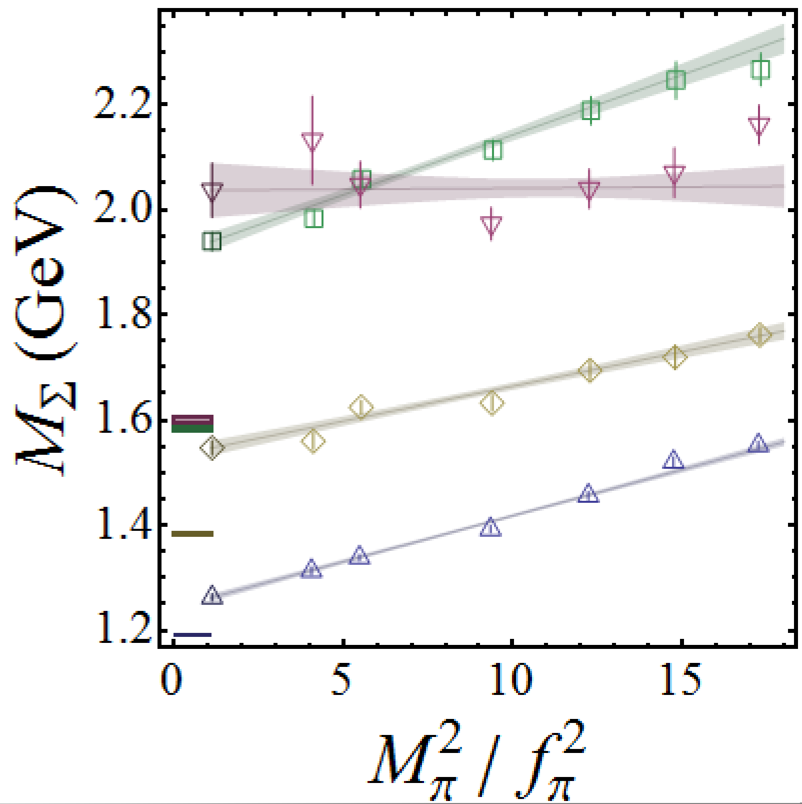


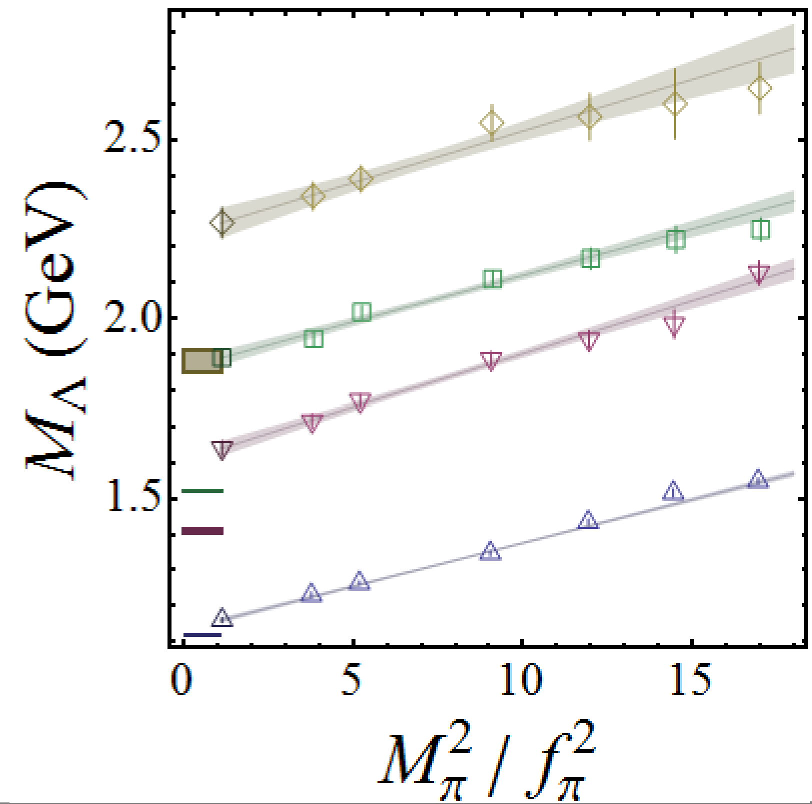
Pion-Mass Dependences

2+1-flavor mixed action

- The singly strange baryons: (Σ and Λ)
- Symbols: J^P = 1/2$^+$ △, 1/2$^-$ ▽, 3/2$^+$ ◇, 3/2$^-$ □

1/2$^+$ △	1/2$^-$ ▽	3/2$^+$ ◇	3/2$^-$ □
Σ	Σ(1620)	Σ*	Σ(1580)
Λ	Λ(1405)	Λ(1890)	Λ(1520)



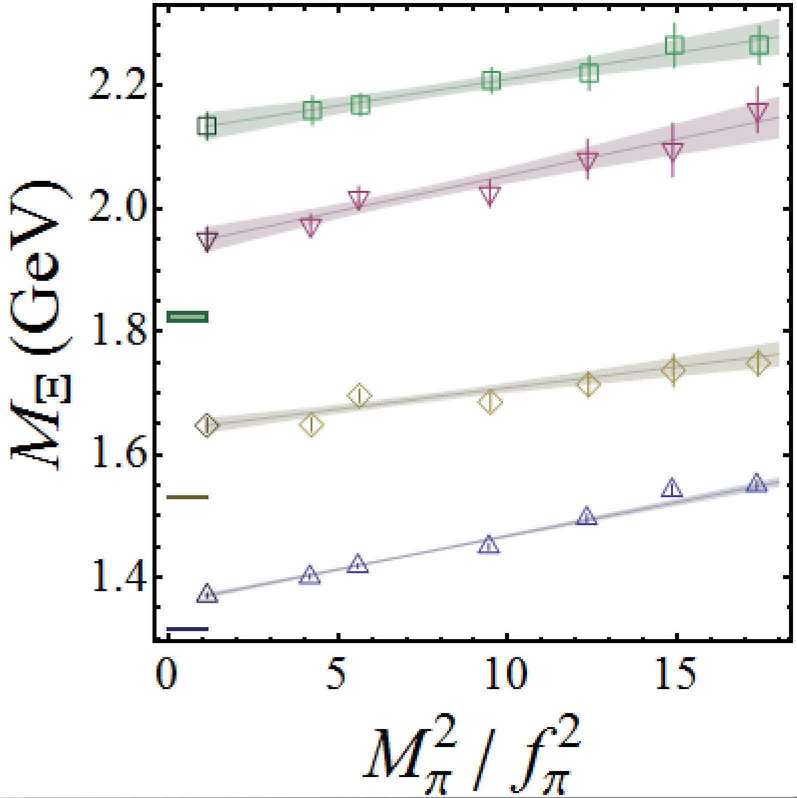


Pion-Mass Dependences

2+1-flavor mixed action

- The less known baryons (Ξ)
- Symbols: J^P = 1/2⁺ △, 1/2⁻ ▽ , 3/2⁺ ◇, 3/2⁻ □

Ξ Ξ(1690)? Ξ(1530) Ξ(1820)



Pion-Mass Dependences

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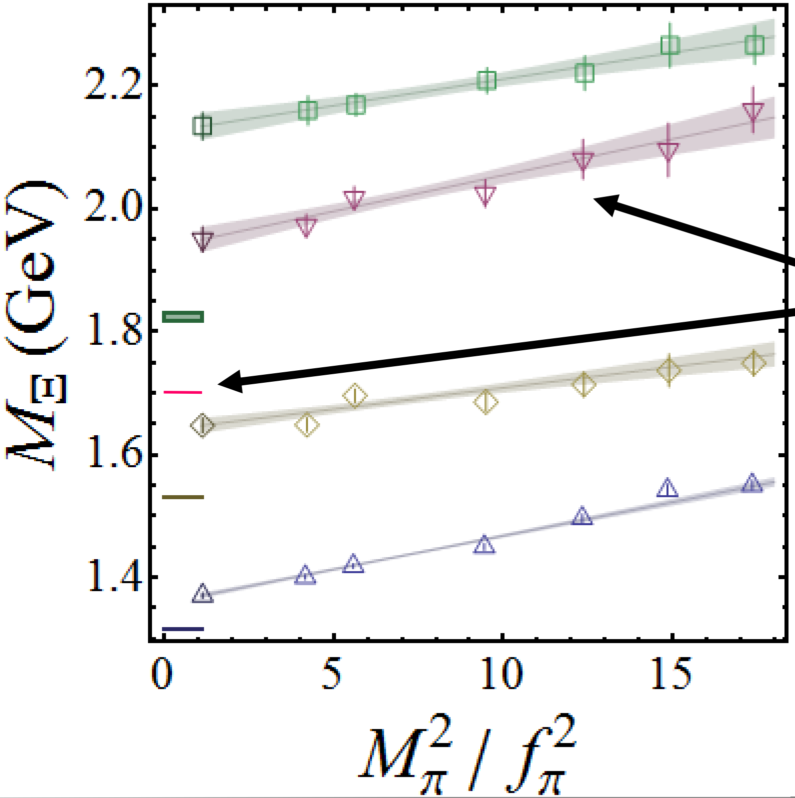
Ξ Ξ(1690)? Ξ(1530) Ξ(1820)



- Babar at MENU 2007: Ξ(1690)^0 negative parity −1/2

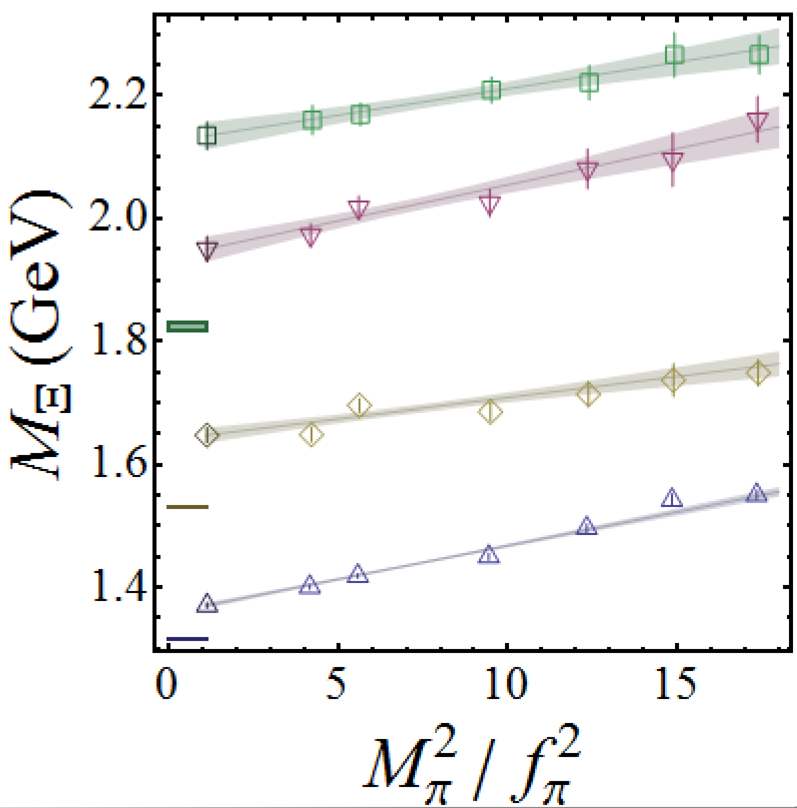
Pion-Mass Dependences

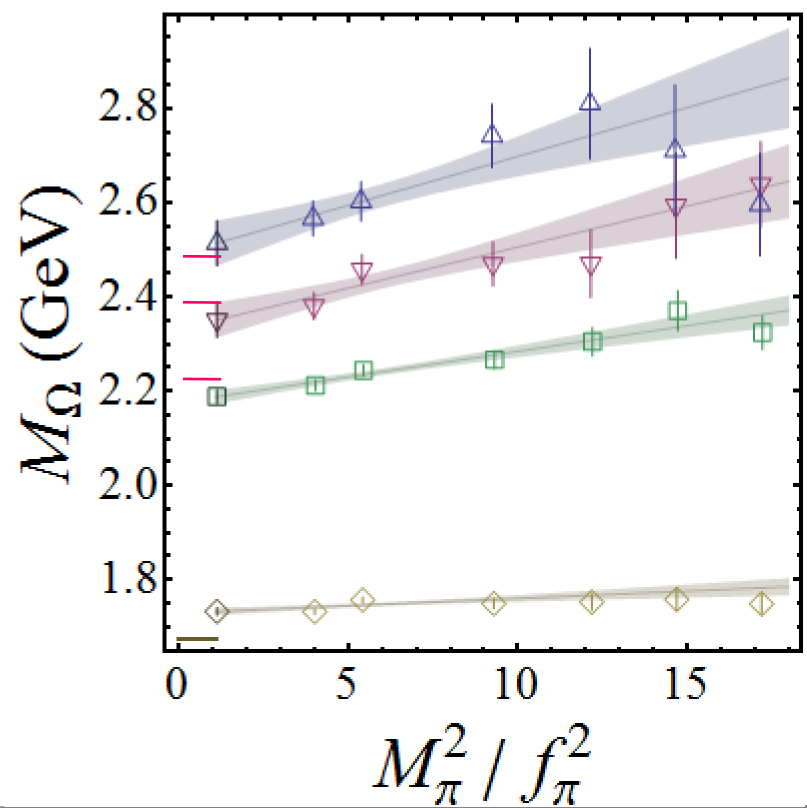
2+1-flavor mixed action

- The less known baryons (Ξ and Ω)
- Symbols: J^P = 1/2$^+$ △, 1/2$^-$ ▽ , 3/2$^+$ ◇, 3/2$^-$ □

Ξ Ξ(1690)? Ξ(1530) Ξ(1820)

Could they be Ω(2250), Ω(2380), Ω(2470)?

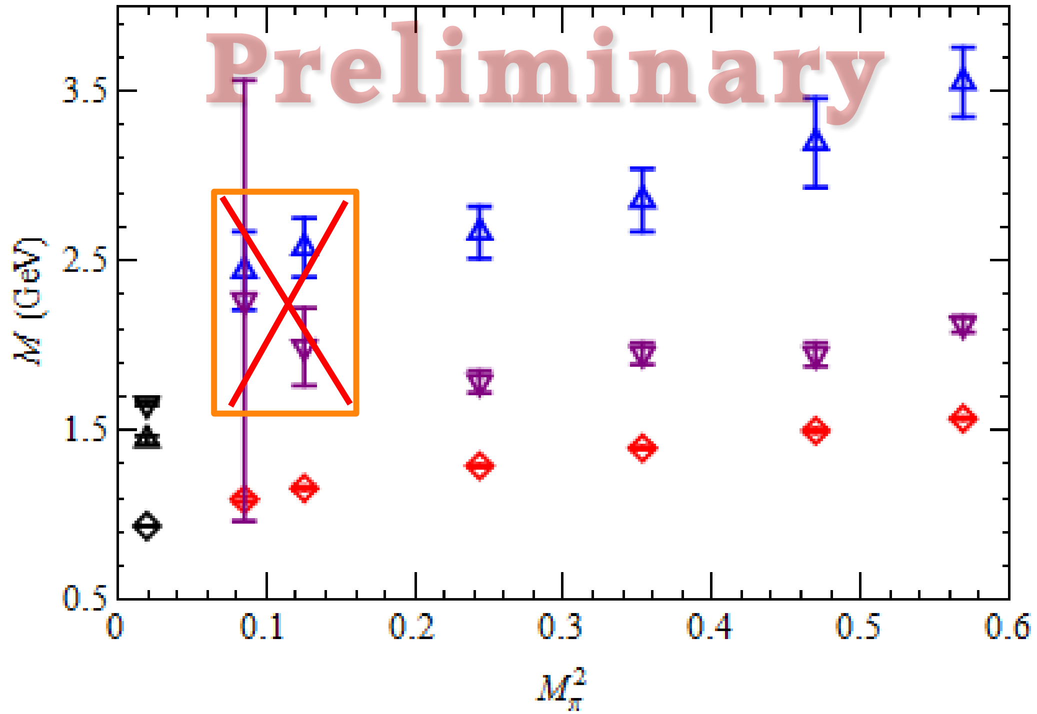




Roper in Full QCD

- $N_f = 2+1$ mixed action (DWF+asqtad) calculation ($L \sim 2.5$ fm)
- Symbols: J^P

  ◇ $1/2^+$ ***N***
  ▽ $1/2^-$ $\boldsymbol{S_{11}}$
  △ $1/2^+$ $\boldsymbol{P_{11}}$



- Finite-volume effects starting at 350 MeV pion
- Prove or disprove Roper as the first radial excited state of nucleon?

Roper in Full QCD

- $N_f = 2+1$ mixed action (DWF+asqtad) calculation ($L \sim 2.5$ fm)
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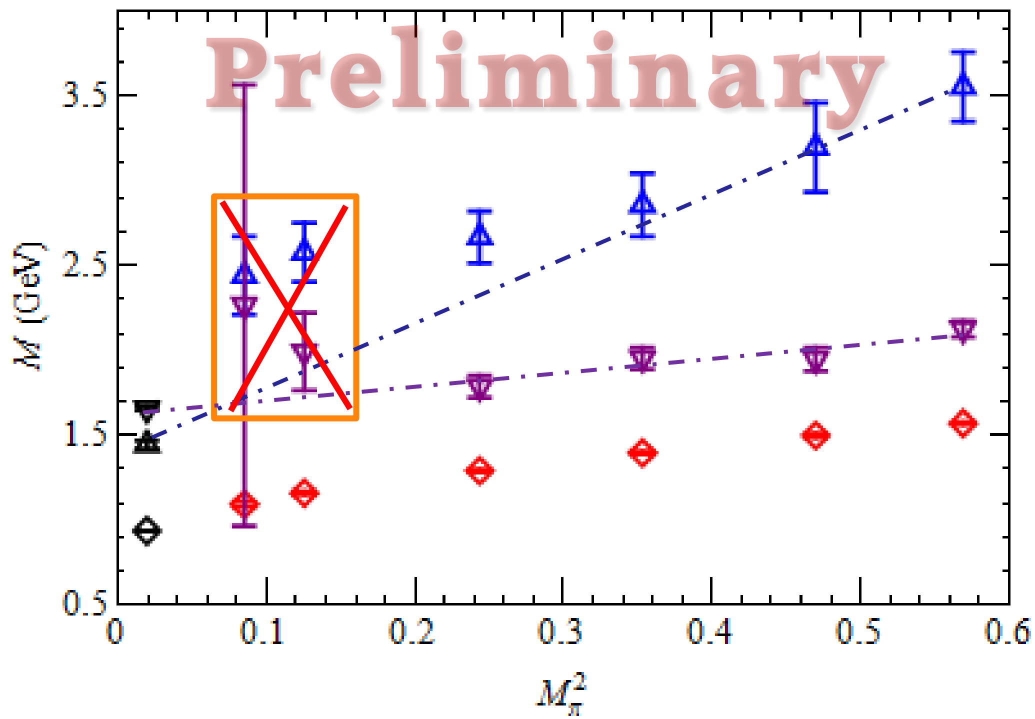
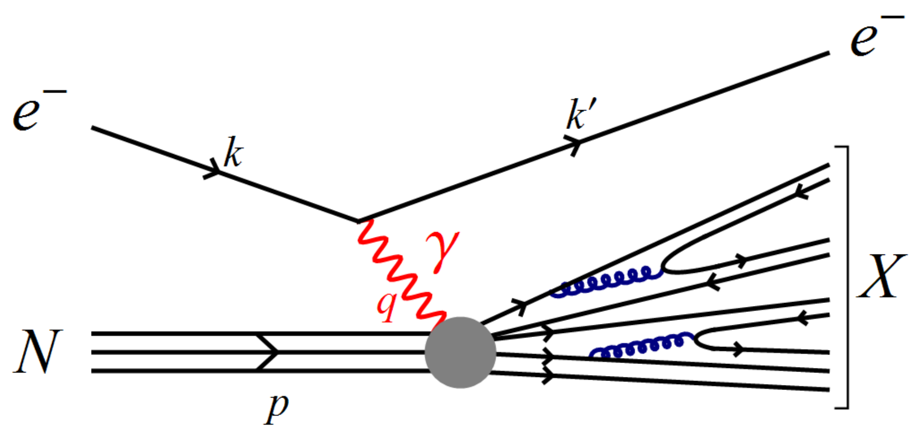


- Finite-volume effects starting at 350 MeV pion.?
- Prove or disprove Roper as the first radial excited state of nucleon?
- Not a crazy possibility (see the hand-drawn extrapolation lines)
- Stay tuned on future $N_f = 2+1$ lattice calculations

Nucleon Structure

Deep Inelastic Scattering

- Probing nucleon structure



$$\sigma \sim L^{\mu\nu} W_{\mu\nu},$$
$$W_{\mu\nu} = i \int d^4x e^{iqx} \langle N | T\{J^\mu(x), J^\nu(0)\} | N \rangle$$

- The symmetric, unpolarized, spin-averaged

$$W^{\{\mu\nu\}}(x,Q^2) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x,Q^2) + \left(p^\mu - \frac{\nu}{q^2} q^\mu\right)\left(p^\nu - \frac{\nu}{q^2} q^\nu\right) \frac{F_2(x,Q^2)}{\nu}$$

- The anti-symmetric, polarized

$$W^{[\mu\nu]}(x,Q^2) = i\epsilon^{\mu\nu\rho\sigma} q_\rho \left(\frac{s_\sigma}{\nu}(g_1(x,Q^2) + g_2(x,Q^2)) - \frac{q \cdot s p_\sigma}{\nu^2} g_2(x,Q^2)\right)$$

Moments of Structure Functions

- No light-cone operator directly calculated on the lattice
- Operator product expansion
  - Polarized

$$
\begin{aligned}
2\int dx\, x^n g_1(x,Q^2) &= \sum_{q=u,d} e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\langle x^n\rangle_{\Delta q} \\
2\int dx\, x^n g_2(x,Q^2) &= \frac{n}{(n+1)}\sum_{q=u,d}\Big[2e_{2,n}^{(q)}(\mu^2/Q^2, g(\mu))d_n^q(\mu) \\
&+ e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\langle x^n\rangle_{\Delta q}\Big]
\end{aligned}
$$

  - Unpolarized

$$
\begin{aligned}
2\int dx\, x^{n-1} F_1(x,Q^2) &= \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\langle x^n\rangle_q \\
\int dx\, x^{n-2} F_2(x,Q^2) &= \sum_{q=u,d} c_{2,n}^{(q)}(\mu^2/Q^2, g(\mu))\langle x^n\rangle_q
\end{aligned}
$$

- e_1, e_2, c_1, c_2 are Wilson coefficients
- $\langle x^n\rangle_q,\ \langle x^n\rangle_{\Delta q},\ d_n$ are the forward nucleon matrix elements

Nucleon Structure Functions

- Matrix element $\langle P,S \,|\, O \,|\, P,S \rangle$

  - Unpolarized

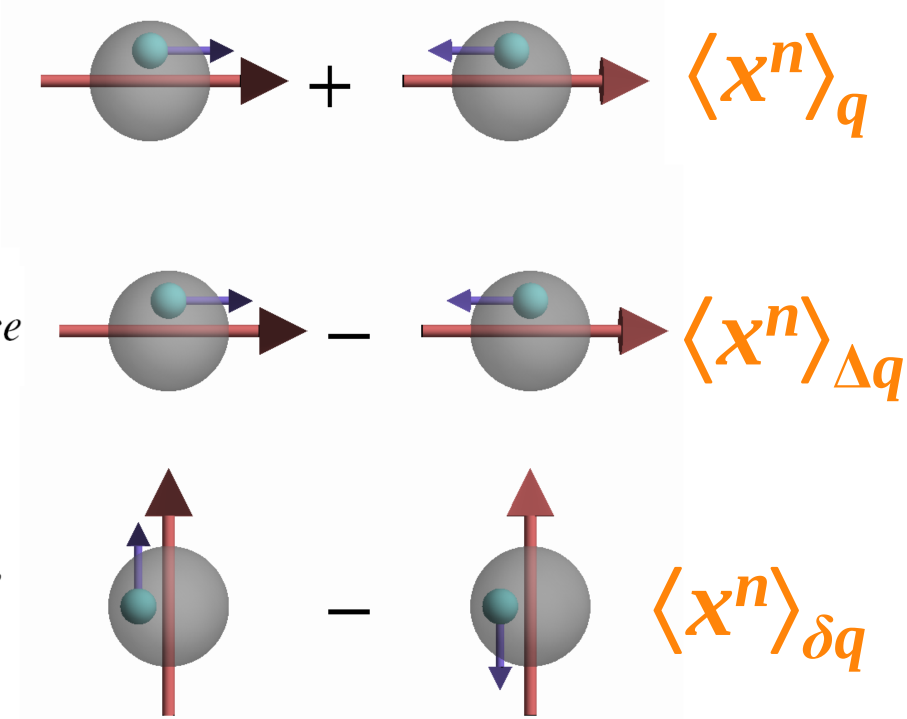
$$\mathcal{O}^{q}_{\mu_1\mu_2\cdots\mu_n} = \left(\frac{i}{2}\right)^{n-1} \bar{q}\gamma_{\mu_1} \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_n}\, q - trace$$

  - Polarized

$$\mathcal{O}^{5q}_{\sigma\mu_1\mu_2\cdots\mu_n} = \left(\frac{i}{2}\right)^{n-1} \bar{q}\gamma_{\sigma}\gamma_5 \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_n}\, q - trace$$

  - Transversity

$$\mathcal{O}^{\sigma q}_{\rho\nu\mu_1\mu_2\cdots\mu_n} = \left(\frac{i}{2}\right)^{n} \bar{q}\sigma_{\rho\nu} \overleftrightarrow{D}_{\mu_1} \cdots \overleftrightarrow{D}_{\mu_n}\, q - trace$$



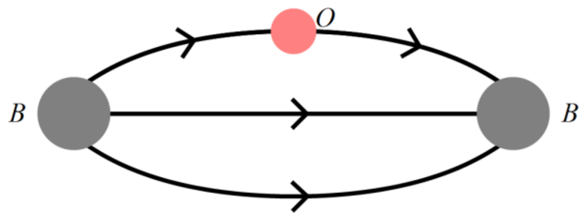
Implementation on the Lattice

- Interpolating field

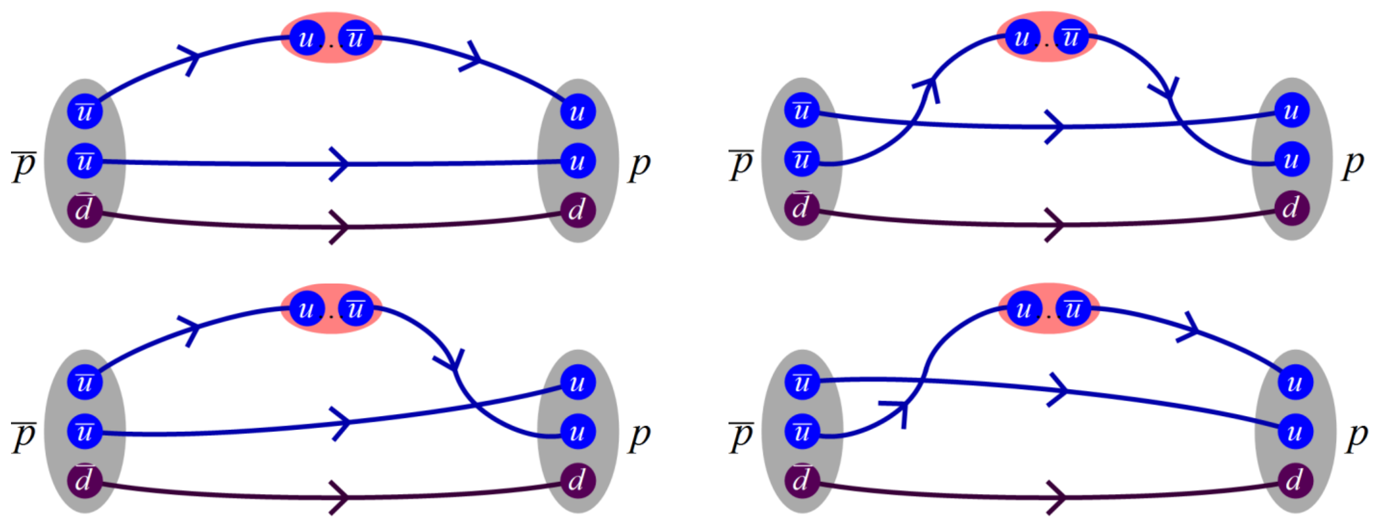
$$J_\alpha(\vec{p},t) = \sum_{\vec{x},a,b,c} e^{i\vec{p}\cdot\vec{x}}\epsilon^{abc}\left[u_a^T(y_1,t)C\gamma_5 d_b(y_2,t)\right]u_{c,\alpha}(y_3,t)\phi(y_1-x)\phi(y_2-x)\phi(y_3-x)$$

- Three-point Green function

$$\sum_{\alpha,\beta}\Gamma^{\alpha,\beta}\langle J(X_{\mathrm{snk}})O(X_{\mathrm{int}})J(X_{\mathrm{src}})\rangle_{\alpha,\beta}$$



- Contractions: *u* insertion, connected



Implementation on the Lattice

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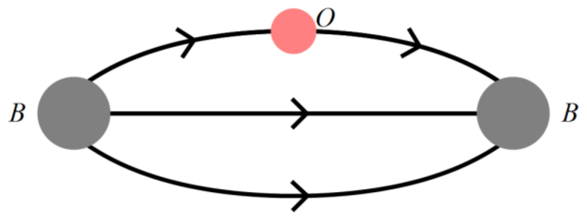
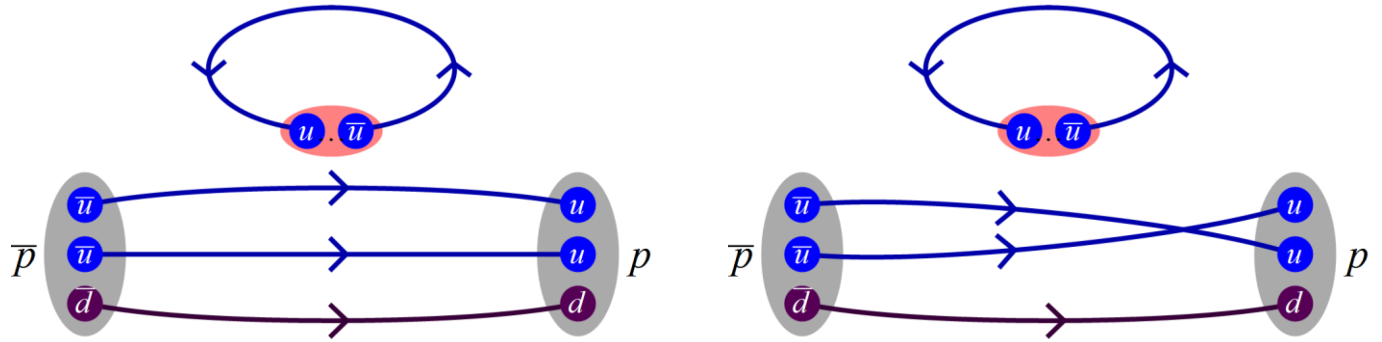
$$J_\alpha(\vec{p},t) = \sum_{\vec{x},a,b,c} e^{i\vec{p}\cdot\vec{x}} \epsilon^{abc} \left[u_a^T(y_1,t) C\gamma_5 d_b(y_2,t)\right] u_{c,\alpha}(y_3,t)\phi(y_1-x)\phi(y_2-x)\phi(y_3-x)$$

- Three-point Green function

$$\sum_{\alpha,\beta} \Gamma^{\alpha,\beta} \langle J(X_{\rm snk}) O(X_{\rm int}) J(X_{\rm src})\rangle_{\alpha,\beta}$$



- Contractions: *u* insertion, *dis*connected



Implementation on the Lattice

- Interpolating field

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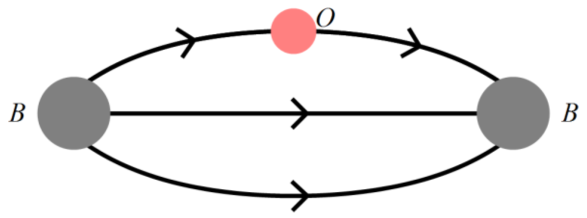
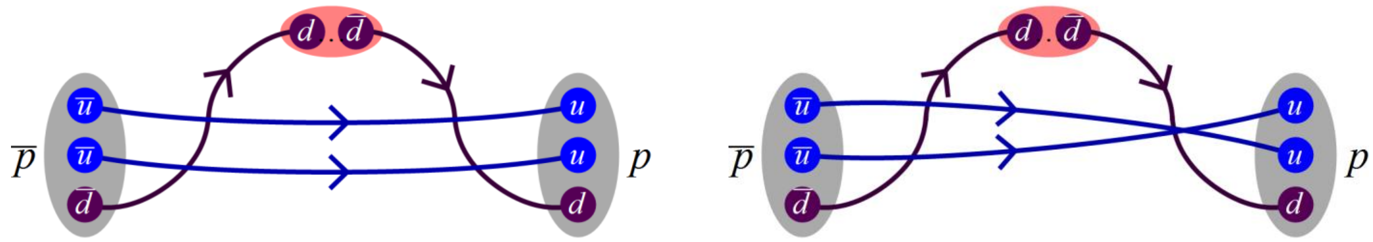


- Contractions: *d* insertion, connected



Implementation on the Lattice

- Interpolating field

$$J_\alpha(\vec{p},t) = \sum_{\vec{x},a,b,c} e^{i\vec{p}\cdot\vec{x}}\epsilon^{abc}\left[u_a^T(y_1,t)C\gamma_5 d_b(y_2,t)\right]u_{c,\alpha}(y_3,t)\phi(y_1-x)\phi(y_2-x)\phi(y_3-x)$$

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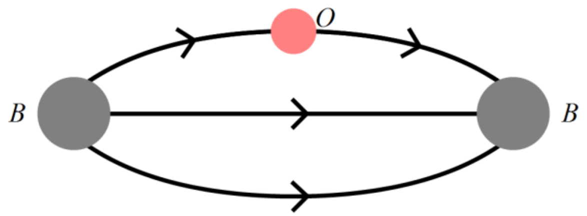
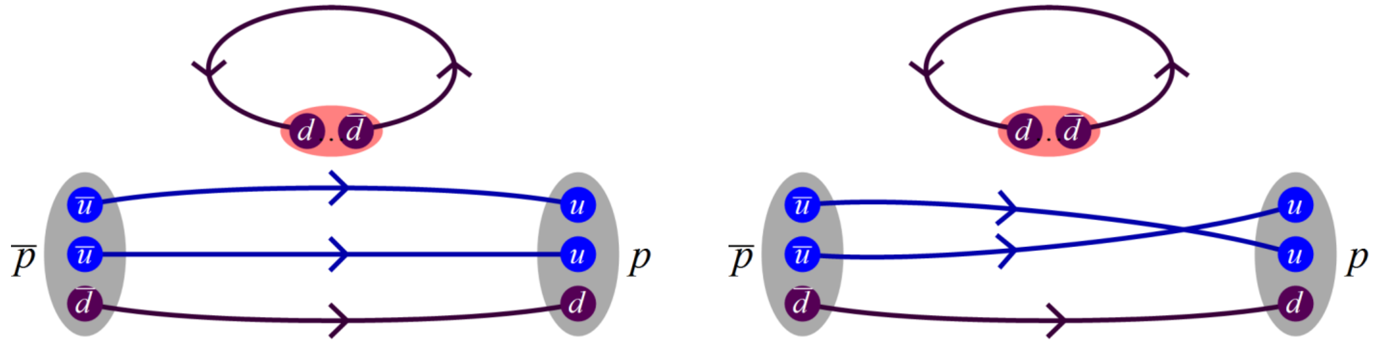
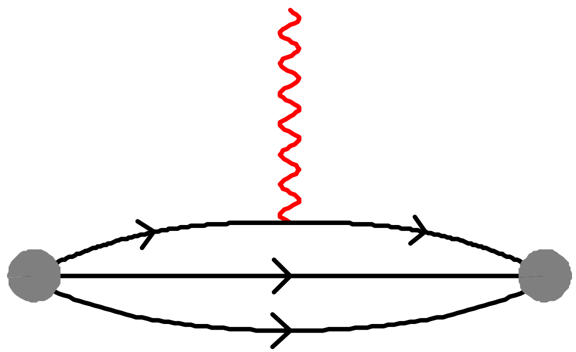


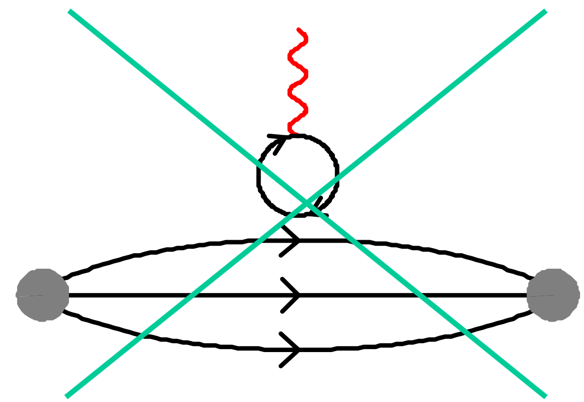
- Contractions: *d* insertion, *dis*connected



Isospin Quantities

- Disconnected contractions are noisy; mostly ignored





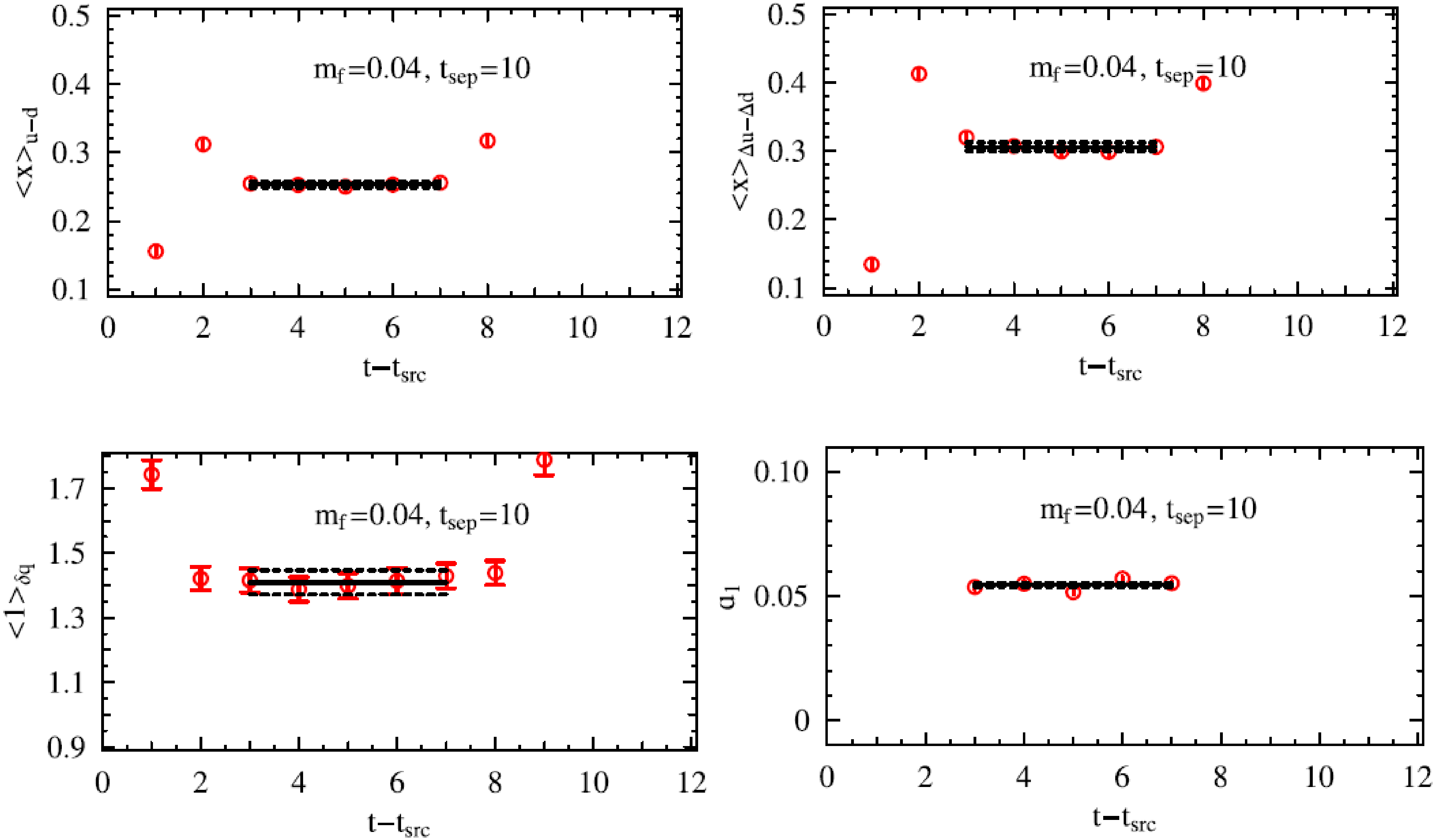
- Calculate isospin quantity where disconnected contribution cancelled

- Use ratios to cancel out the unwanted factors

$$\frac{\Gamma^{BB}_{\mu,GG}(t_i,t,t_f,\vec{p}_i,\vec{p}_f\,;\,T)}{\Gamma^{BB}_{GG}(t_i,t_f,\vec{p}_f\,;\,T)}\sqrt{\frac{\Gamma^{BB}_{PG}(t,t_f,\vec{p}}{\Gamma^{BB}_{PG}(t,t_f,\vec{p}}}\sqrt{\frac{\Gamma^{BB}_{GG}(t_i,t,\vec{p}_f\,;\,T)}{\Gamma^{BB}_{GG}(t_i,t,\vec{p}_i\,;\,T)}}\sqrt{\frac{\Gamma^{BB}_{PG}(t_i,t_f,\vec{p}_f\,;\,T)}{\Gamma^{BB}_{PG}(t_i,t_f,\vec{p}_i\,;\,T)}}$$

Plateaux

- Example: 2f DWF, $M_\pi \sim 700$ MeV, $a \sim 0.12$ fm, $L \sim 2$fm



Nucleon Structure Functions

- List of operators: lowest moments only

$\langle x \rangle_q$	$\langle x \rangle_{\Delta q}$
momentum fraction	helicity distribution
$\mathcal{O}^q_{44} = \overline{q}\left[\gamma_4 \overset{\leftrightarrow}{D}_4 - \frac{1}{3}\sum_k \gamma_k \overset{\leftrightarrow}{D}_k\right] q$	$\mathcal{O}^{5q}_{\{34\}} = i\overline{q}\gamma_5\left[\gamma_3 \overset{\leftrightarrow}{D}_4 + \gamma_4 \overset{\leftrightarrow}{D}_3\right] q$
$\mathbf{3}^+_1$	$\mathbf{6}^-_3$
$R_{\langle x \rangle_q} = \frac{C_{\text{3pt}}^{\Gamma,\mathcal{O}^q_{44}}}{C_{\text{2pt}}} = m_N \langle x \rangle_q$	$R_{\langle x \rangle_{\Delta q}} = \frac{C_{\text{3pt}}^{\Gamma,\mathcal{O}^{5q}_{\{34\}}}}{C_{\text{2pt}}} = m_N \langle x \rangle_{\Delta q}$
${\mathcal{P}^q_{44}}^{-1} = \gamma_4 p_4 - \frac{1}{3}\sum_{i=1,3}\gamma_i p_i$	${\mathcal{P}^{5q}_{34}}^{-1} = i\gamma_5\left(\gamma_3 p_4 + \gamma_4 p_3\right)$

$\langle 1 \rangle_{\delta q}$	d_1
transversity	twist-3 matrix element
$\mathcal{O}^{\sigma q}_{34} = \overline{q}\gamma_5\sigma_{34} q$	$\mathcal{O}^5 q_{[34]} = i\overline{q}\gamma_5\left[\gamma_3 \overset{\leftrightarrow}{D}_4 - \gamma_4 \overset{\leftrightarrow}{D}_3\right] q$
$\mathbf{6}^+_1$	$\mathbf{6}^+_1$
$R_{\langle 1 \rangle_{\delta q}} = \frac{C_{\text{3pt}}^{\Gamma,\mathcal{O}^{\sigma q}_{\{34\}}}}{C_{\text{2pt}}} = \langle 1 \rangle_{\delta q}$	$R_{d_1} = \frac{C_{\text{3pt}}^{\Gamma,\mathcal{O}^{5q}_{[34]}}}{C_{\text{2pt}}} = d_1$
${\mathcal{P}^{\sigma q}_{34}}^{-1} = \gamma_5\sigma_{34}$	${\mathcal{P}^{5q}_{[34]}}^{-1} = i\gamma_5\left(\gamma_3 p_4 - \gamma_4 p_3\right)$

Nucleon Structure Functions

- Chiral extrapolation formulae for each quantity

Chen et al., Nucl.Phys. A707, 452 (2002); Phys. Lett. B523, 107 (2001)

W. Detmold et al., Phys. Rev. D66, 054501 (2002); Phys. Lett. 87, 172001 (2001)

$$\langle x\rangle_{u-d} = C\left[1-\frac{3g_A^2+1}{(4\pi f_\pi)^2}m_\pi^2\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + e(\mu^2)\frac{m_\pi^2}{(4\pi f_\pi)^2}$$

$$\langle x\rangle_{\Delta u-\Delta d} = \tilde{C}\left[1-\frac{2g_A^2+1}{(4\pi f_\pi)^2}m_\pi^2\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \tilde{e}(\mu^2)\frac{m_\pi^2}{(4\pi f_\pi)^2}.$$

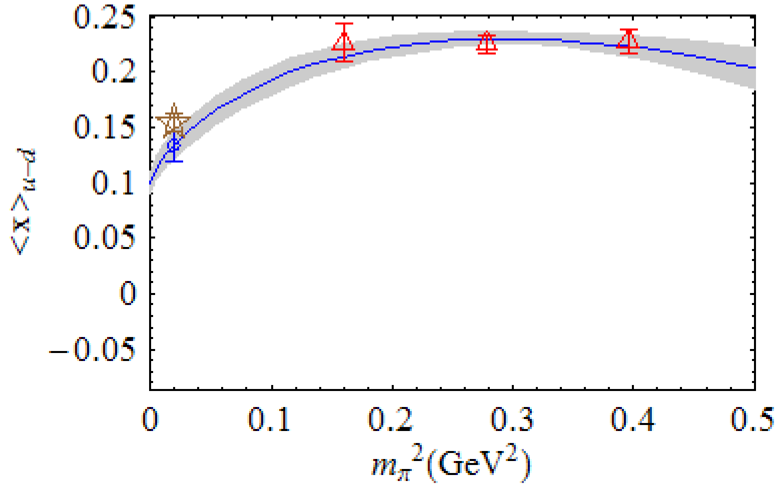
$$\langle x\rangle_{\delta u-\delta d} = \tilde{C}'\left[1-\frac{4g_A^2+1}{2(4\pi f_\pi)^2}m_\pi^2\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \tilde{e}'(\mu^2)\frac{m_\pi^2}{(4\pi f_\pi)^2}$$

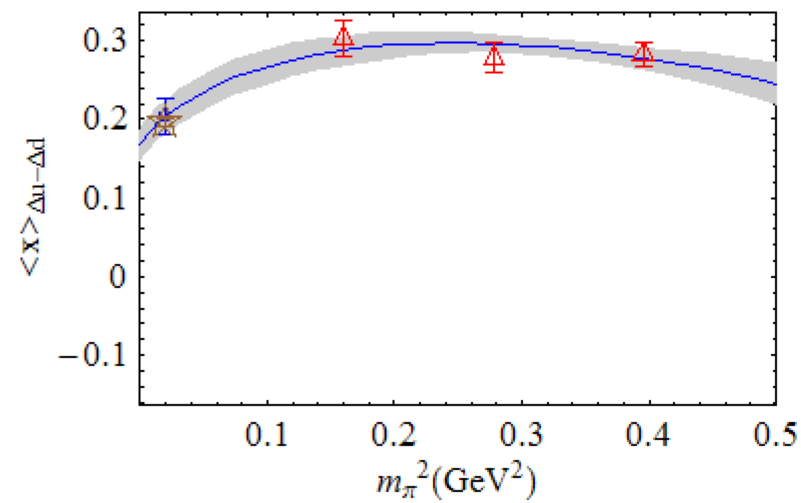
Linear ansatz

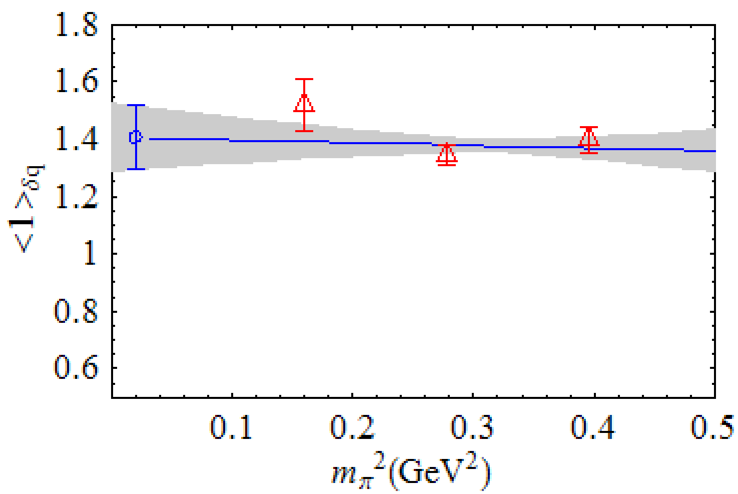
- Renormalization
  - Analytically: Lattice perturbation theory
  - Numerically: RI/MOM-scheme nonperturbative renormalization

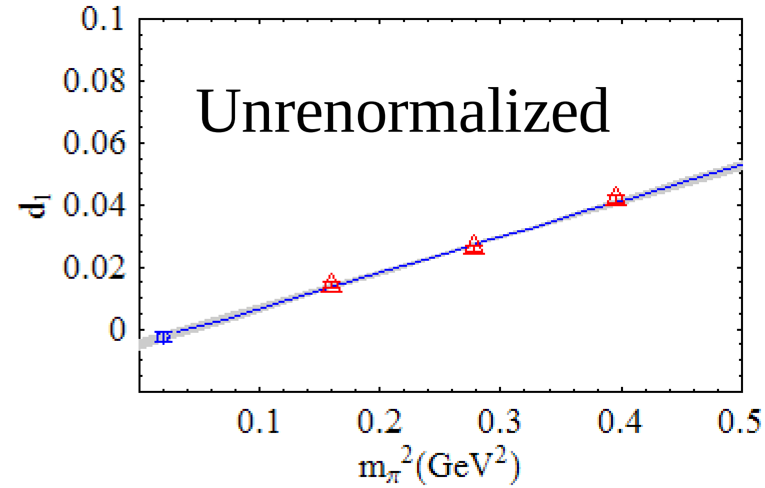
Nucleon Structure Functions

- Example: 2+1 DWF, $M_\pi \sim 320$–620 MeV, $a \sim 0.12$ fm, $L \sim 3$ fm
- Chiral extrapolations: lowest moments only



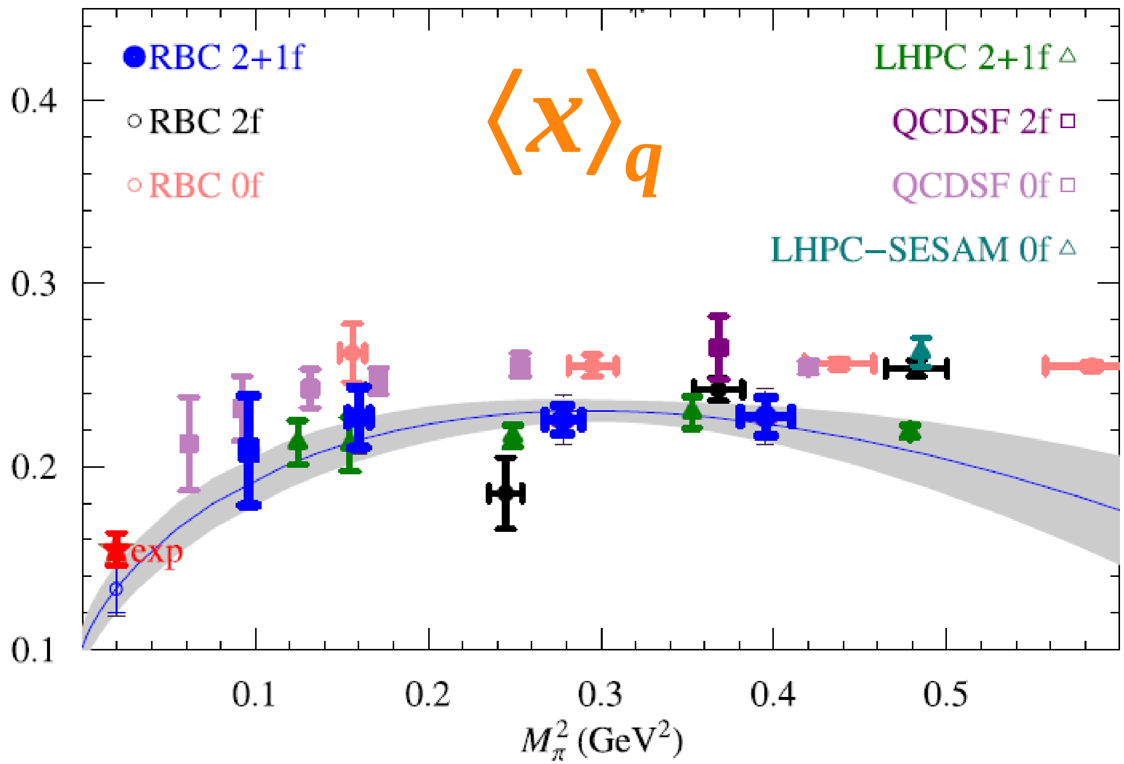






Nucleon Structure Functions

- World data: the first moment of the momentum fraction



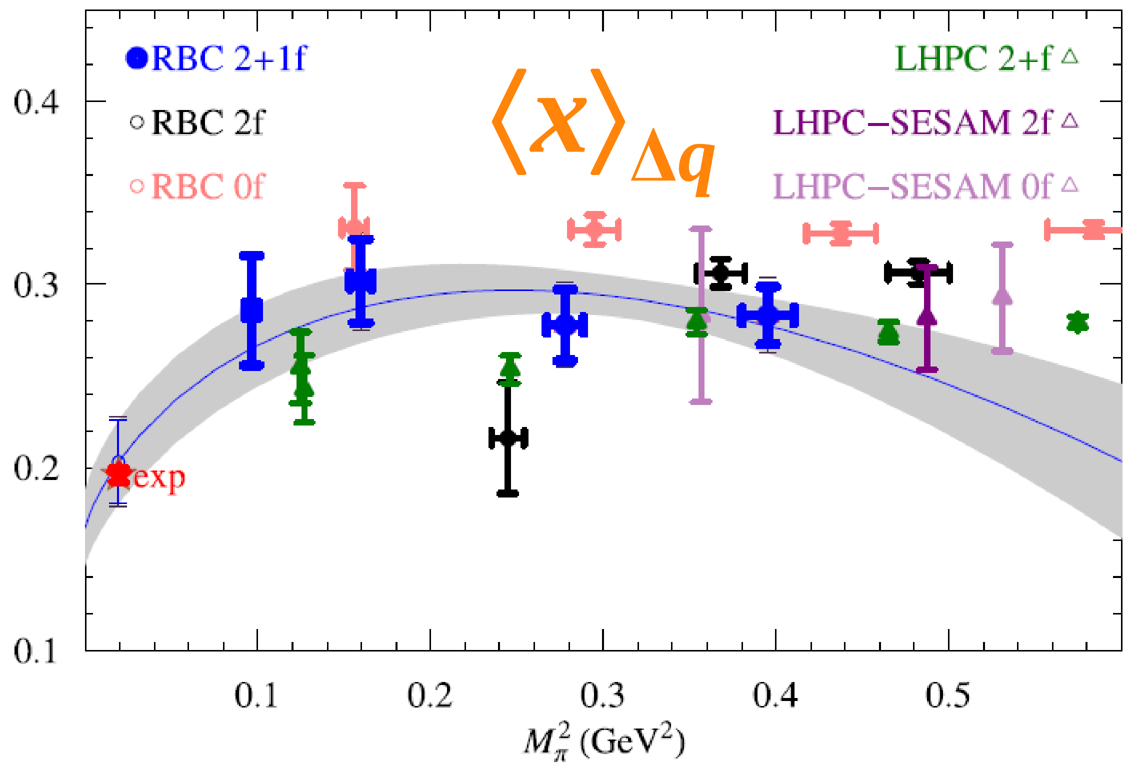
HWL et al., 0802.0863[hep-lat]; M. Guertler et al., PoS(LAT2006)107;

D. Pleiter et al., PoS(LAT2006)120 ; K. Orginos et al., Phys.Rev.D73:094507, 2005;

D. Renner et al., PoS(LAT2006)121; D. Dolgov et al., Phys. Rev. D66, 034506 (2002)

Nucleon Structure Functions

- World data: the first moment of the helicity distribution



HWL et al., 0802.0863[hep-lat]; M. Guertler et al., PoS(LAT2006)107;
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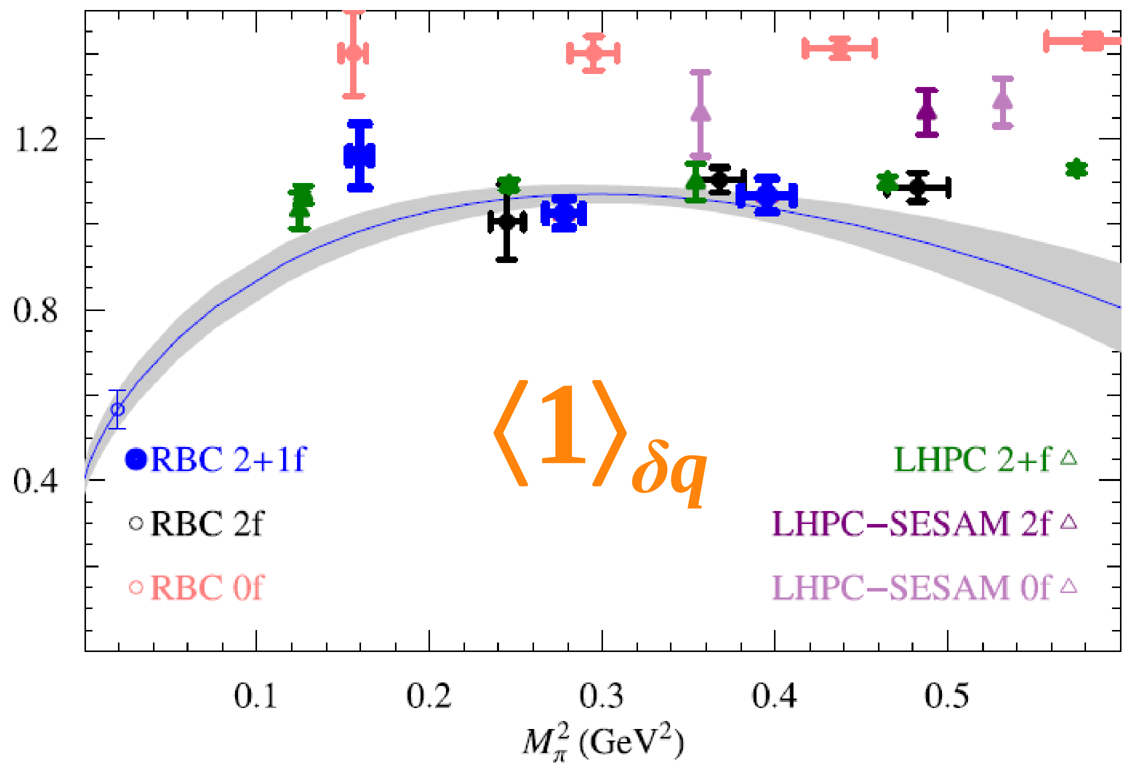
D. Renner et al., PoS(LAT2006)121; D. Dolgov et al., Phys. Rev. D66, 034506 (2002)

Nucleon Structure Functions

- World data: zeroth moment of the transversity



HWL et al., 0802.0863[hep-lat]; M. Guertler et al., PoS(LAT2006)107;
D. Pleiter et al., PoS(LAT2006)120 ; K. Orginos et al., Phys.Rev.D73:094507, 2005;
D. Renner et al., PoS(LAT2006)121; D. Dolgov et al., Phys. Rev. D66, 034506 (2002)

Nucleon Structure Functions: Higher moments

- Example:
  unpolarized moments

  D. Dolgov et al., Phys. Rev. D66, 034506 (2002)

- Symbols:
  - Diamonds: 0f LHPC-SESAM
  - Triangles: 0f QCDSF
  - Squares: 2f LHPC-SESAM

- $n \geq 4$: mixings with lower-dimension operators