The Nuclear Many-Body problem
Lecture 5

- Peculiarities at the nuclear driplines
- Many-body open quantum systems.
- Unification of Reaction and Structure

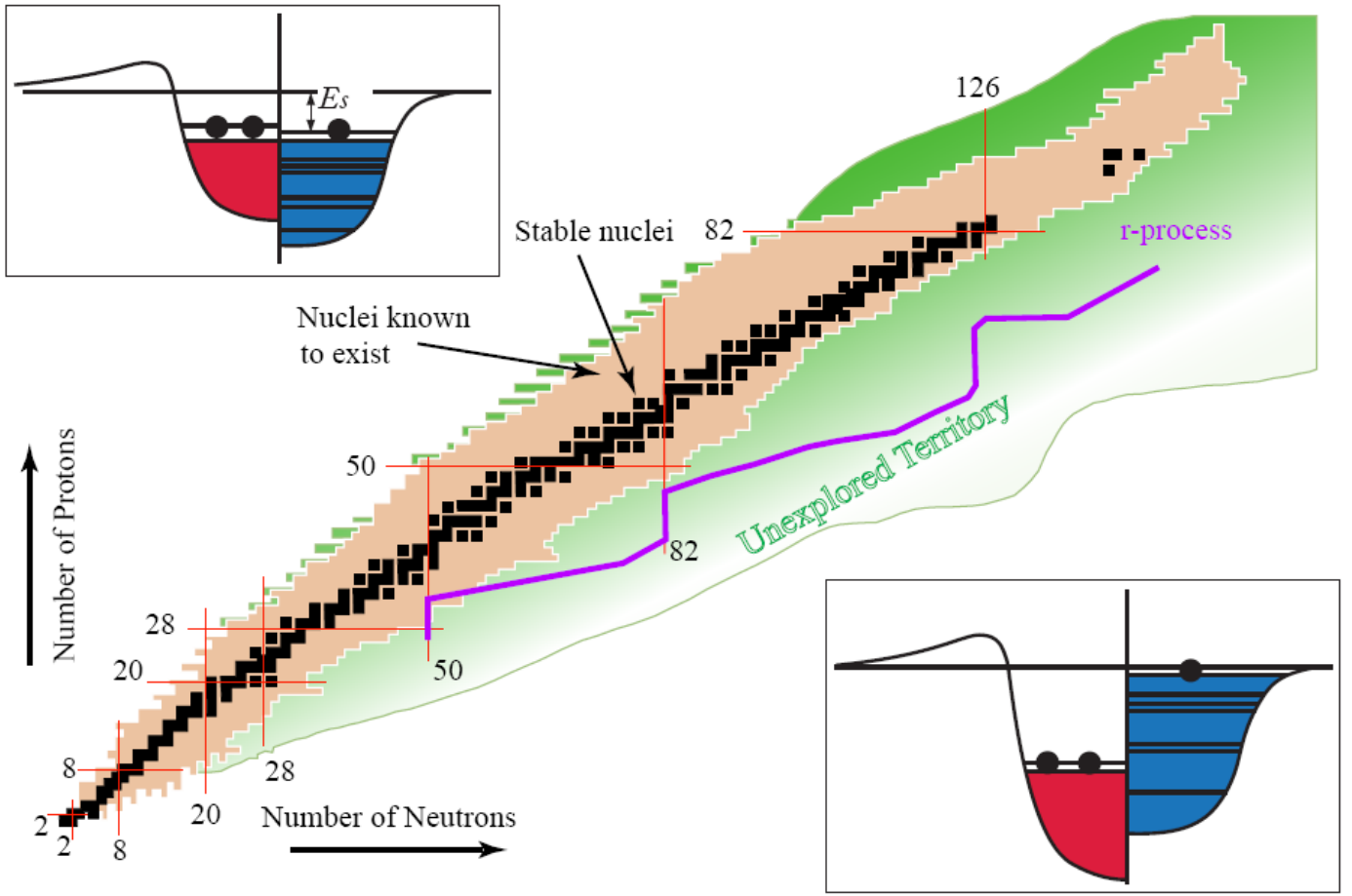
Nuclear landscape and consequences.

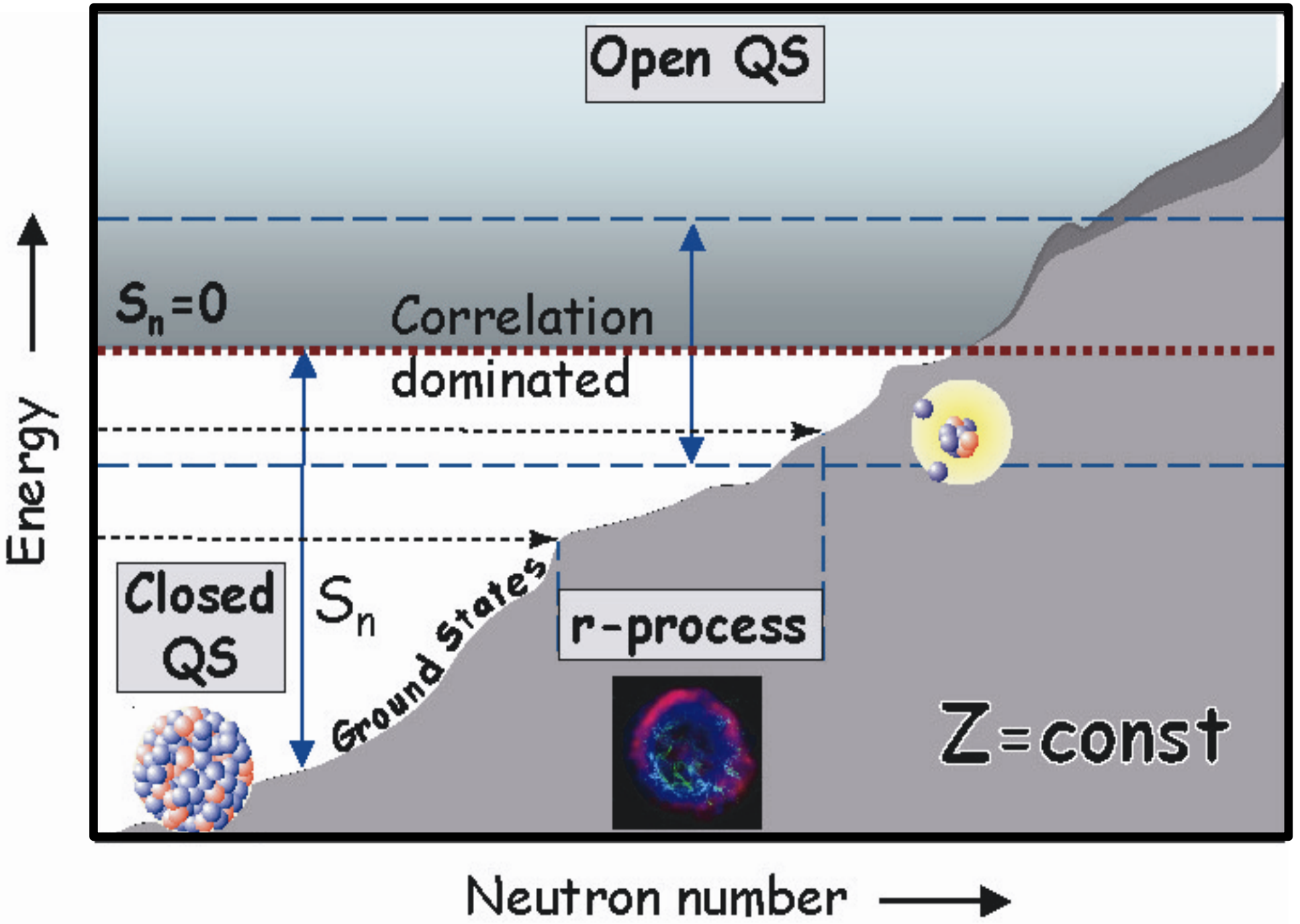
~ 300 stable nuclei

N/Z~1 for light nuclei

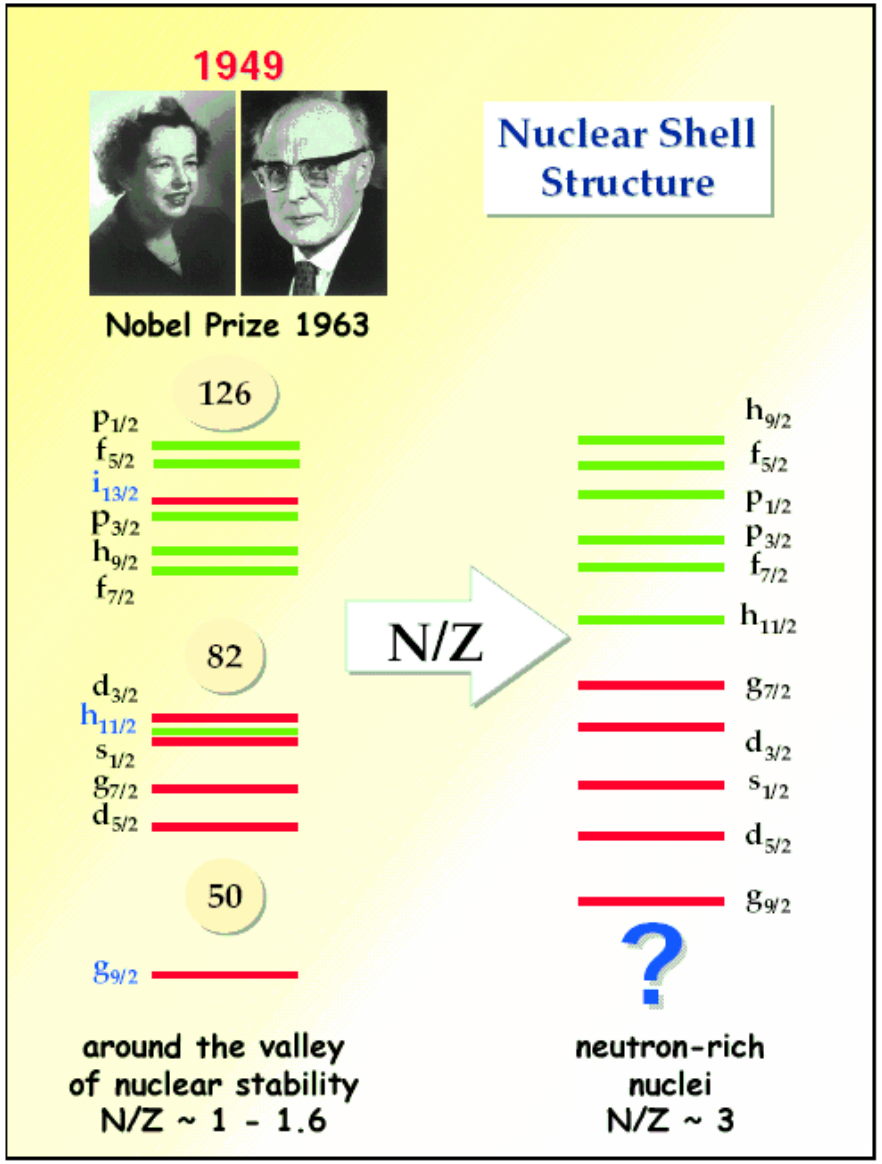
N/Z~1.5 for ^{208}Pb

~4000-6000 unstable nuclei
decay by α, β, 1p, 2p,
1n, cluster emission, fission…




Open QS
S_n=0
Correlation
dominated
Energy
Closed
QS
S_n
Ground States
r-process
Z=const
Neutron number

Modification and quenching of shell structure at the dripline.



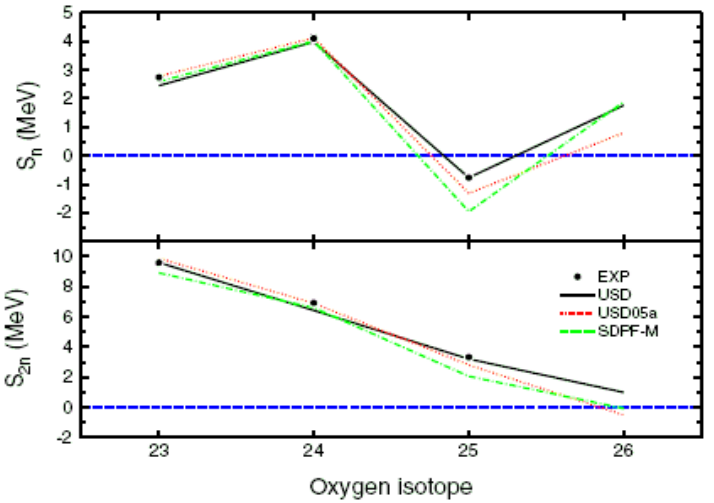


FIG. 4 (color online). The experimental [25,26] (data points) and theoretical [13–15] (lines) one- and two-neutron separation energies for the $N = 15$–18 oxygen isotopes. The experimental error is shown if it is larger than the symbol size.

25O neutron separation energy: -820 keV
the width was measured to be 90(30) keV
giving a lifetime of t ~ 7x10-21 sec

C. Hoffman PRL 100 (2008) 152502

Modification of shell structure at the drip lines!

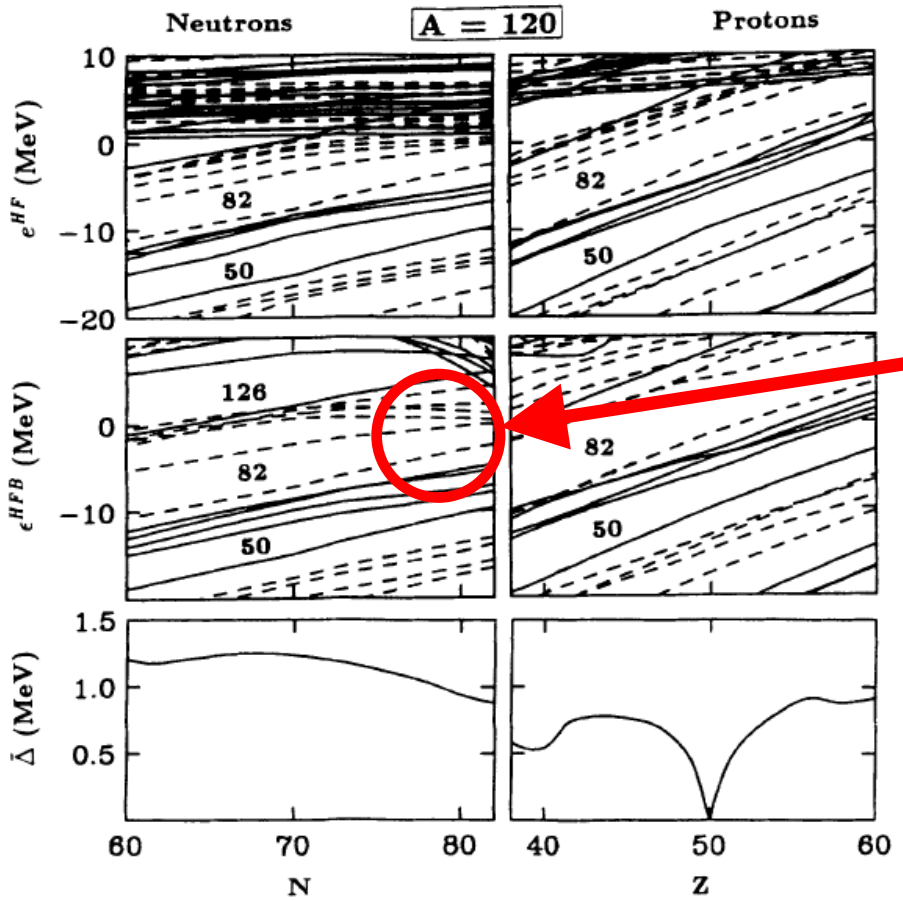


FIG. 3. Spherical single-particle levels for the A=120 isobars calculated in the SkP HF model (top) and SkP HFB model (middle) as a function of neutron number. The single-particle canonical HFB energies are given by $\epsilon_k = \langle \Psi_k | h | \Psi_k \rangle$. Solid (dashed) lines represent the orbitals with positive (negative) parity. The bottom portion shows the average neutron and proton gaps defined by $\bar{\Delta} = \int \Delta(\boldsymbol{r}) \rho(\boldsymbol{r}) d^3r / \int \rho(\boldsymbol{r}) d^3r$.

J. Dobaczewski et al, PRL 72 (1994) 981.

Quenching of 82 shell gap when neutron drip line is approached.

Also observed in lighter nuclei

Caution: Shell structure seen in many observables.

Superheavy hydrogen isotopes.

The most exotic system ever found !!

System with a N/Z =6 can exist !
Gives important information on the existence of a tetra neutron (4N).

Fitting to a Breit-Wigner distribution the extracted resonance value is 0.57 MeV above the 3H+4N threshold and with a width 0.09 MeV.

Need for theory!!

M. Caamano PRL 99, 062502 (2007)

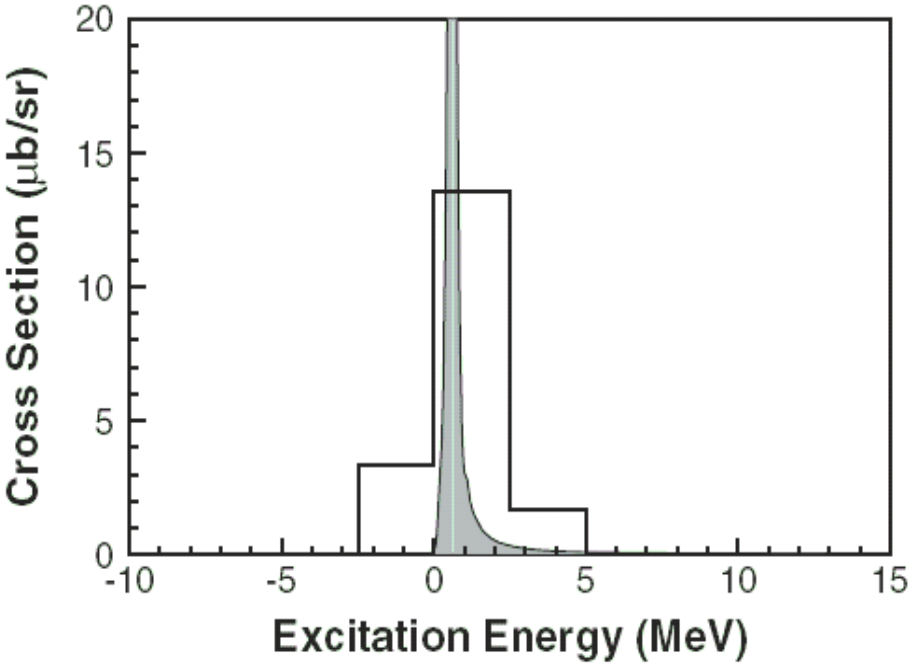
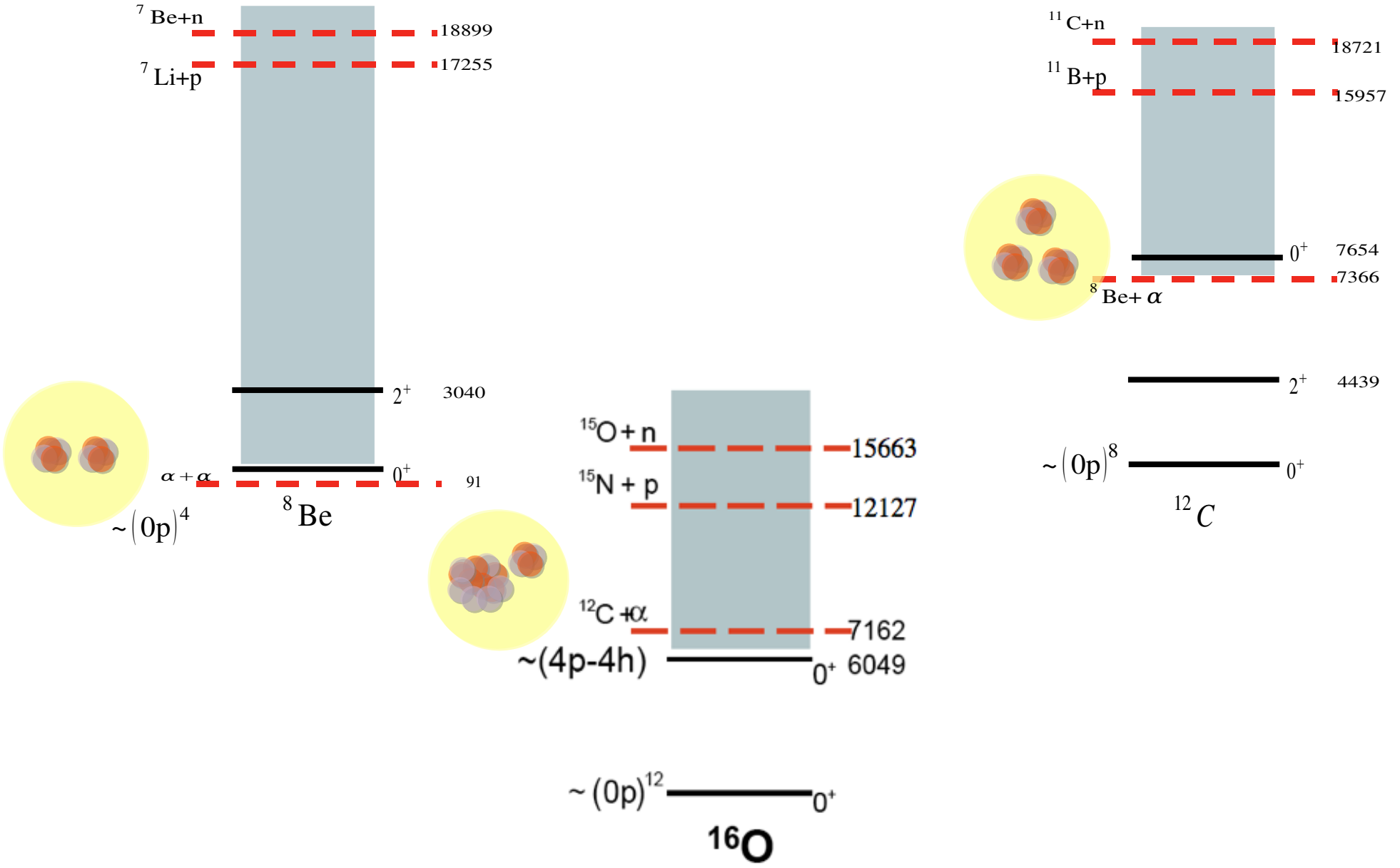
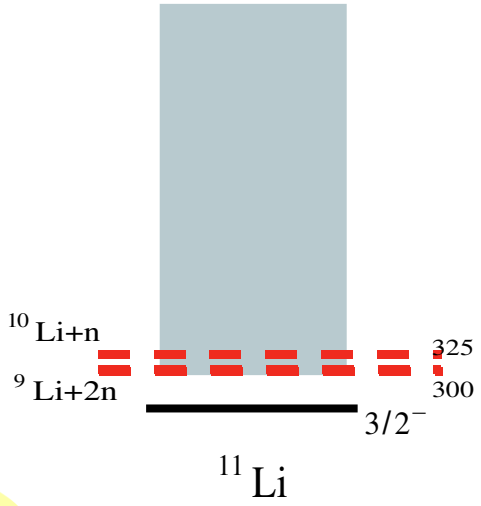


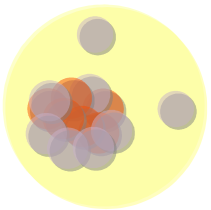
FIG. 4. Excitation energy distribution for the identified ^{7}H events. The solid function is the Breit-Wigner distribution resulting from the fit to the experimental events. The data are represented with the empty histogram merely as a guide to the eye, with a 2.5 MeV binning corresponding to the average estimated uncertainty.

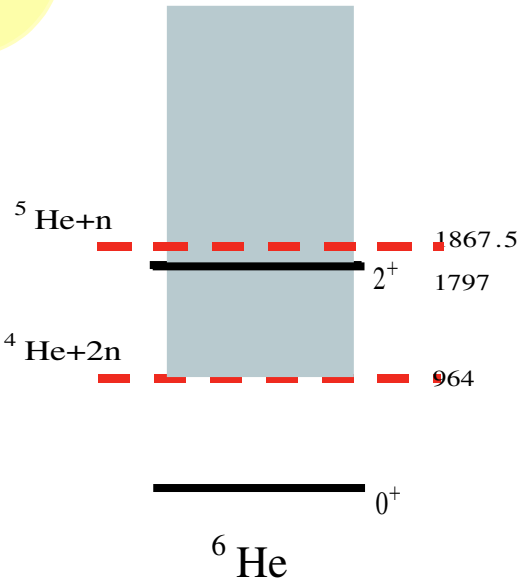
Cluster states near threshold.



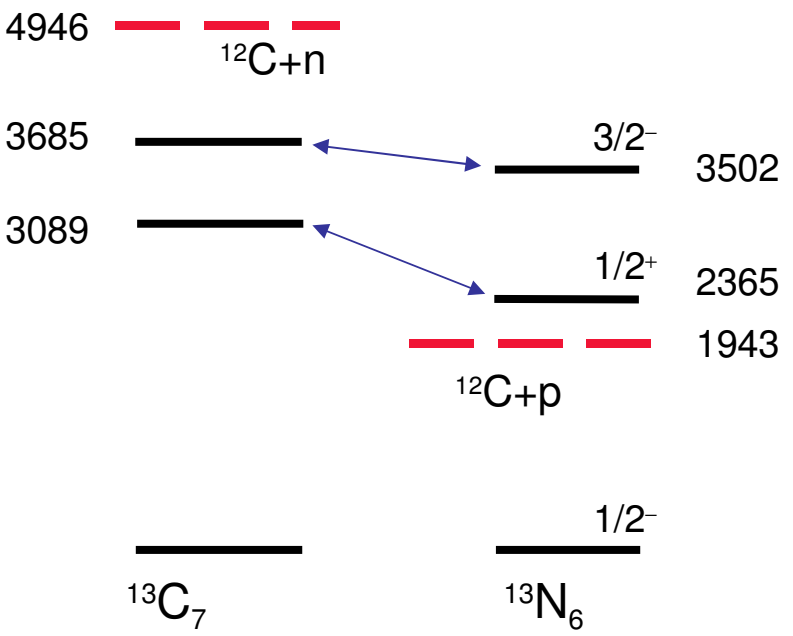
Halo structures







Thomas-Ehrmann effect

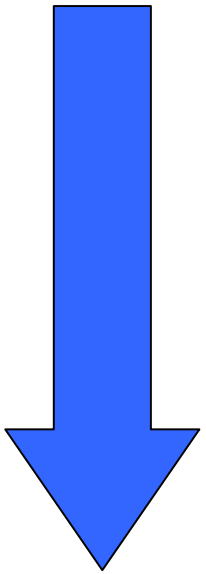


Spectra and matter distribution modified by the proximity of scattering continuum

Unification of structure and reaction

(i) Spectra and matter distribution modified by the proximity of scattering continuum

(ii) Structure of nuclei impact on scattering and reaction properties



Need for unification of 'structure' and 'reaction'

Major challenges for nuclear theory

Mathematical formulation within the Hilbert space of nuclear states embedded in the continuum of decay channels goes back to H. Feshbach (1958-1962), U. Fano (1961), and C. Mahaux and H. Weidenmüller (1969)

- develop theories and algorithms that would allow to understand properties of those exotic physical systems

- unify structure and reaction aspects of weakly-bound or unbound nuclei based on the open quantum system (OQS) formalism

Open quantum system many-body framework

Continuum (real-energy) Shell Model
(1977 - 1999 - 2005)

H.W.Bartz et al, NP A275 (1977) 111
R.J. Philpott, NP A289 (1977) 109
K. Bennaceur et al, NP A651 (1999) 289
J. Rotureau et al, PRL 95 (2005) 042503

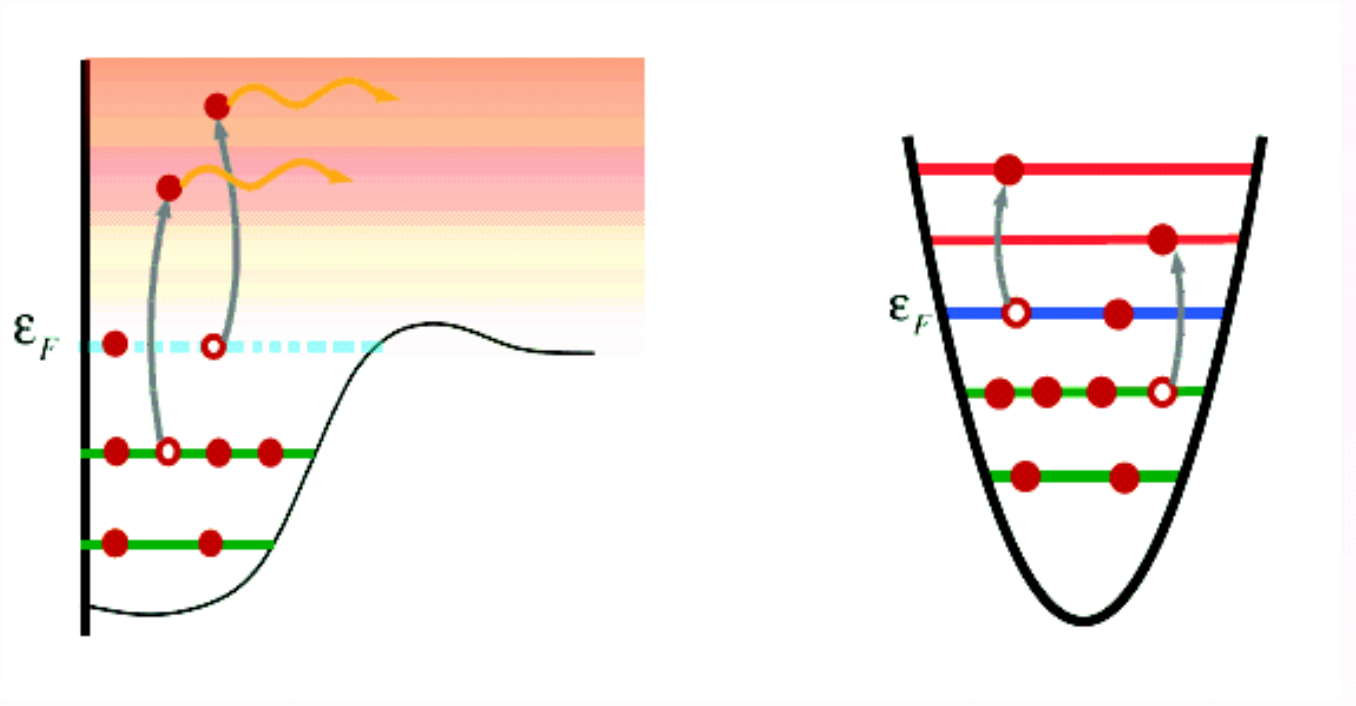
Gamow (complex-energy) Shell Model
(2002 -)

N. Michel et al, PRL 89 (2002) 042502
R. Id Betan et al, PRL 89 (2002) 042501
N. Michel et al, PRC 70 (2004) 064311
G. Hagen et al, PRC 71 (2005) 044314
J. Rotureau et al, PRL 97 (2006) 110603

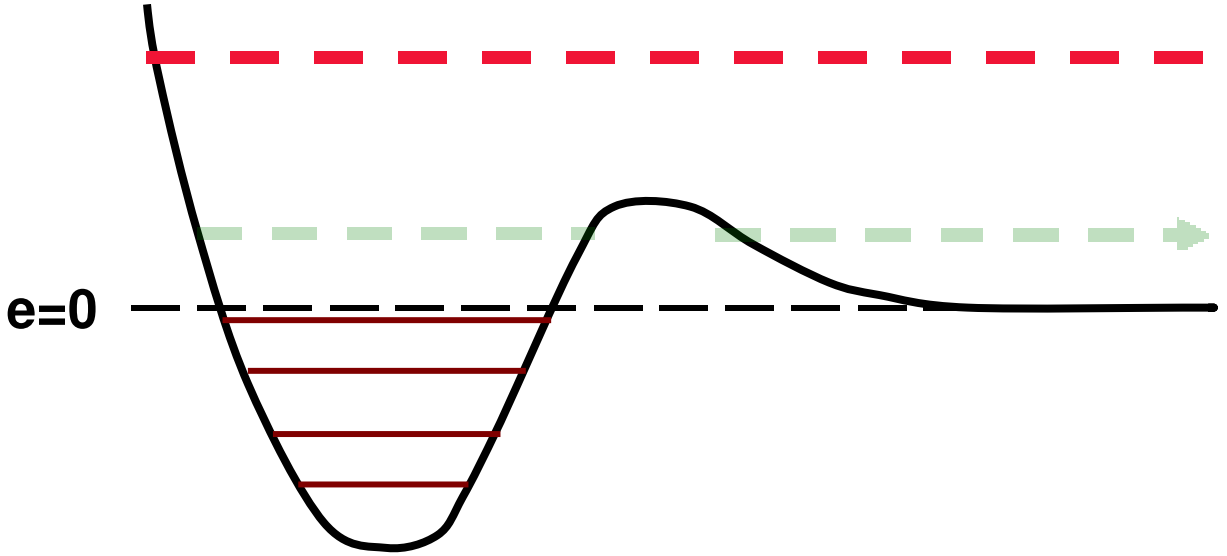
Open vs. closed quantum systems.

Open Quantum System.
Coupling with continuum taken into account.

Closed Quantum System.
No coupling with external continuum.



Formation of single particle resonances.



$$h\psi = (e - i\frac{\Gamma}{2})\psi$$

bound state : $k_n = i\kappa_n$

- **Gamow, Z. Phys. 51, 204 (1928)**
- Siegert, Phys. Rev. **36**, 750 (1939)
- Humblet and Rosenfeld, Nucl. Phys. **26**, 529 (1961)

resonance : $k_n = \gamma_n - i\kappa_n$

$$u''(r) = \left[\frac{l(l+1)}{r^2} + \frac{2\mu}{\hbar^2}V(r) - k^2\right] u(r)$$

$u(r) \sim C_0\, r^{l+1}$, $r \to 0$

$u(r) \sim C_+\, H^+_{l,\eta}(kr)$, $r \to +\infty$ (bound,resonant)

$u(r) \sim C_+\, H^+_{l,\eta}(kr) + C_-\, H^-_{l,\eta}(kr)$, $r \to +\infty$ (scattering)

The contour deformation method in momentum space.

The Schrödinger equation in Momentum space representation,

$$\int_0^\infty dk'\, k'^2 \langle k|T+V|k'\rangle\langle k'|\psi_\alpha\rangle = E_\alpha \langle k|\psi_\alpha\rangle,$$

Analytic continuation in the complex k-plane may be achieved by deforming the integral giving, (G Hagen et al 2004 J. Phys. A: Math. Gen. 37 8991-9021.)

$$\int_{L^+} dk'\, k'^2 \langle k|T+V|k'\rangle\langle k'|\psi_\alpha\rangle = E_\alpha \langle k|\psi_\alpha\rangle.$$

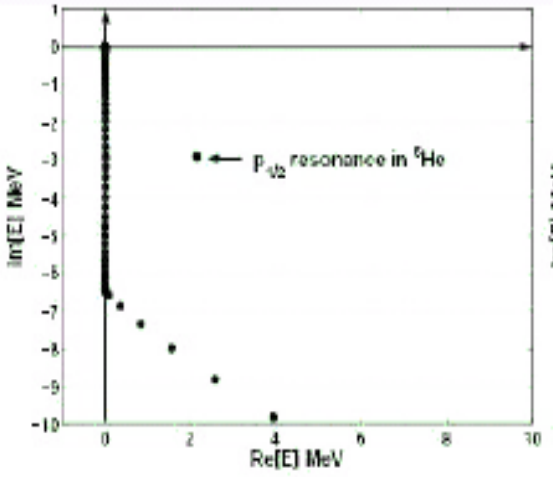
This is analog with the complex scaling method $r \to r\exp(i\theta), k \to k\exp(-i\theta)$.

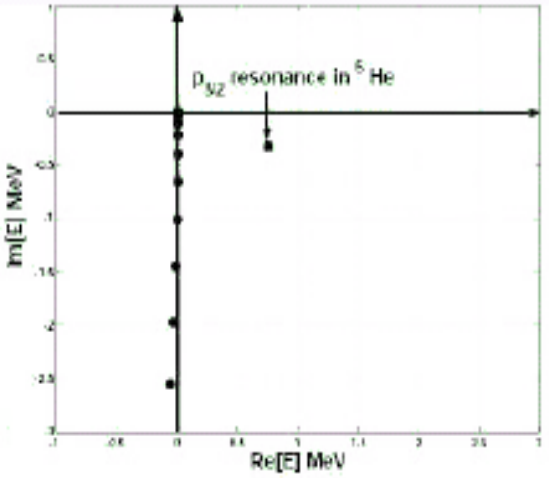
Discretizing L^+, a complete basis within the discretization space is obtained:

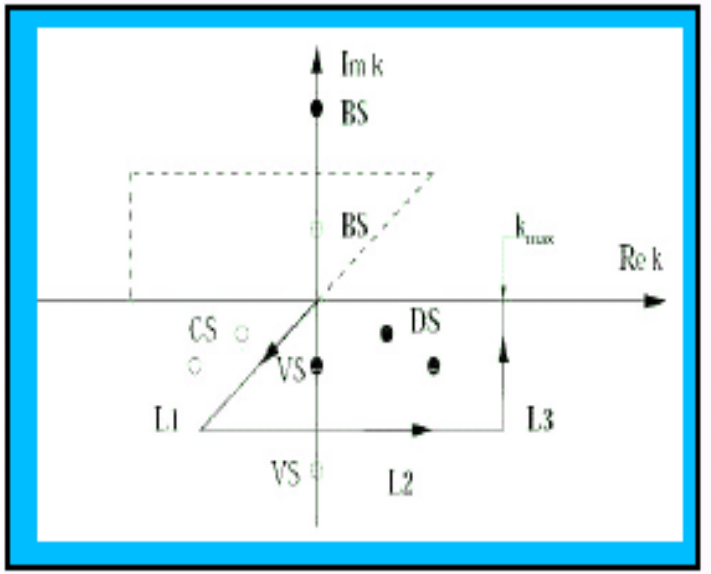
$$\mathbf{1} = \sum_{p=1}^{\#poles} \psi_p(\vec{k})\psi_p^T(\vec{k}) + \sum_{c=1}^{N-\#poles} \psi_c(\vec{k})\psi_c^T(\vec{k}), \;\; \vec{k} = [k_1, ..., k_N]\,.$$

Here p denotes pole states and c denotes the non-resonant continuum states.

A model example: 5He







n_R	n_T	Re[E]	Im[E]	Re[E]	Im[E]
10	10	0.752321	-0.329830	2.148476	-2.912522
12	12	0.752495	-0.327963	2.152992	-2.913609
20	20	0.752476	-0.328033	2.154139	-2.912148
30	30	0.752476	-0.328033	2.154147	-2.912162
40	40	0.752476	-0.328033	2.154147	-2.912162

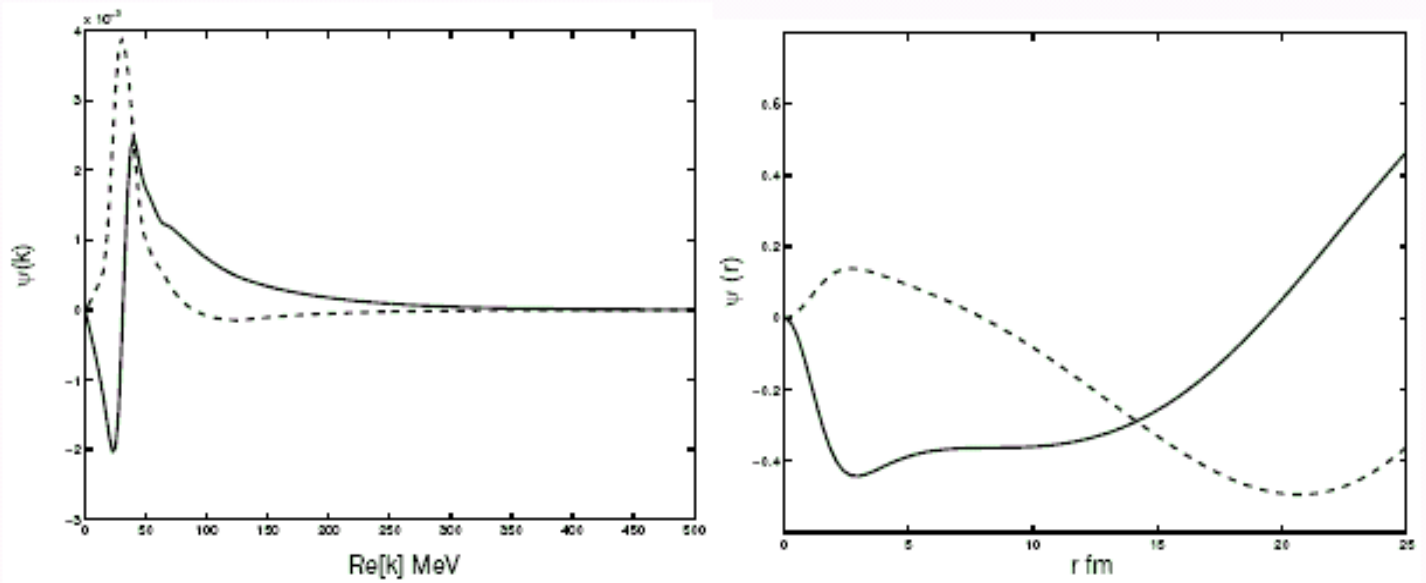
Contour used in calculations of He5 single particle resonances.

Resonances in momentum space vs. position space

The position space wave functions are given by the Fourier-Bessel transform

$$\psi_{nl}(r) = \int_{L+} dk\, k^2 j_l(kr)\tilde{\psi}_{nl}(k) \approx \sum_{i=1}^{N} w_i k_i^2 j_l(k_i r)\tilde{\psi}_{nl}(k_i)$$

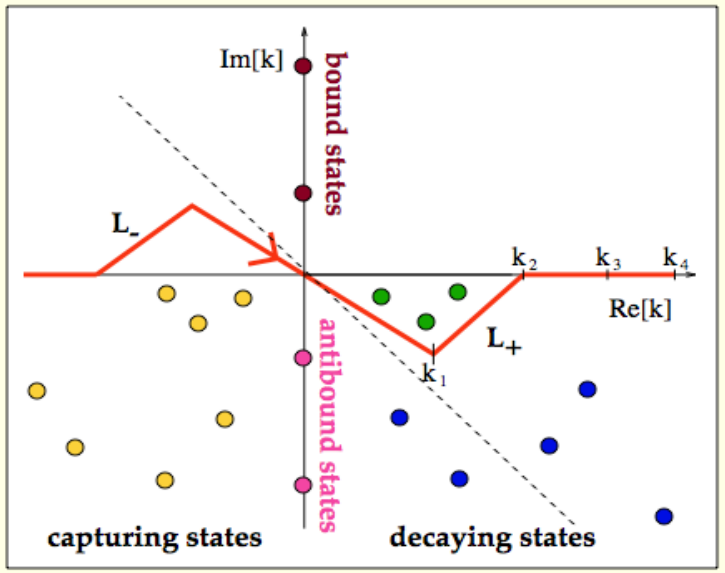
Example: $p_{3/2}$ resonance in a Woods-Saxon potential; $E = 0.74 - 0.35i$.



Gamow Shell Model (2002)

Gamow states and completeness relations

T. Berggren, Nucl. Phys. A109, 265 (1968); Nucl. Phys. A389, 261 (1982)
T. Lind, Phys. Rev. C47, 1903 (1993)

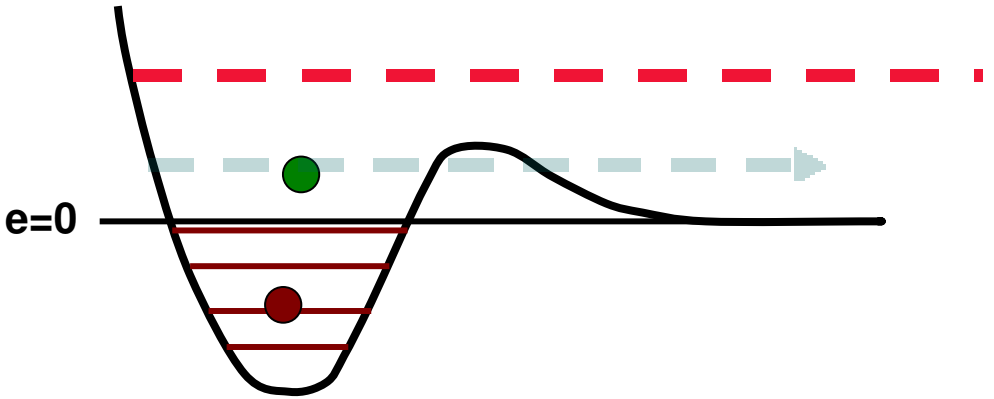


$$\sum_{n=b,r} |u_n\rangle\langle\tilde{u}_n| + \frac{1}{\pi}\int_{L_+} |u(k)\rangle\langle u(k^*)|\, dk = 1$$

particular case: Newton completeness relation

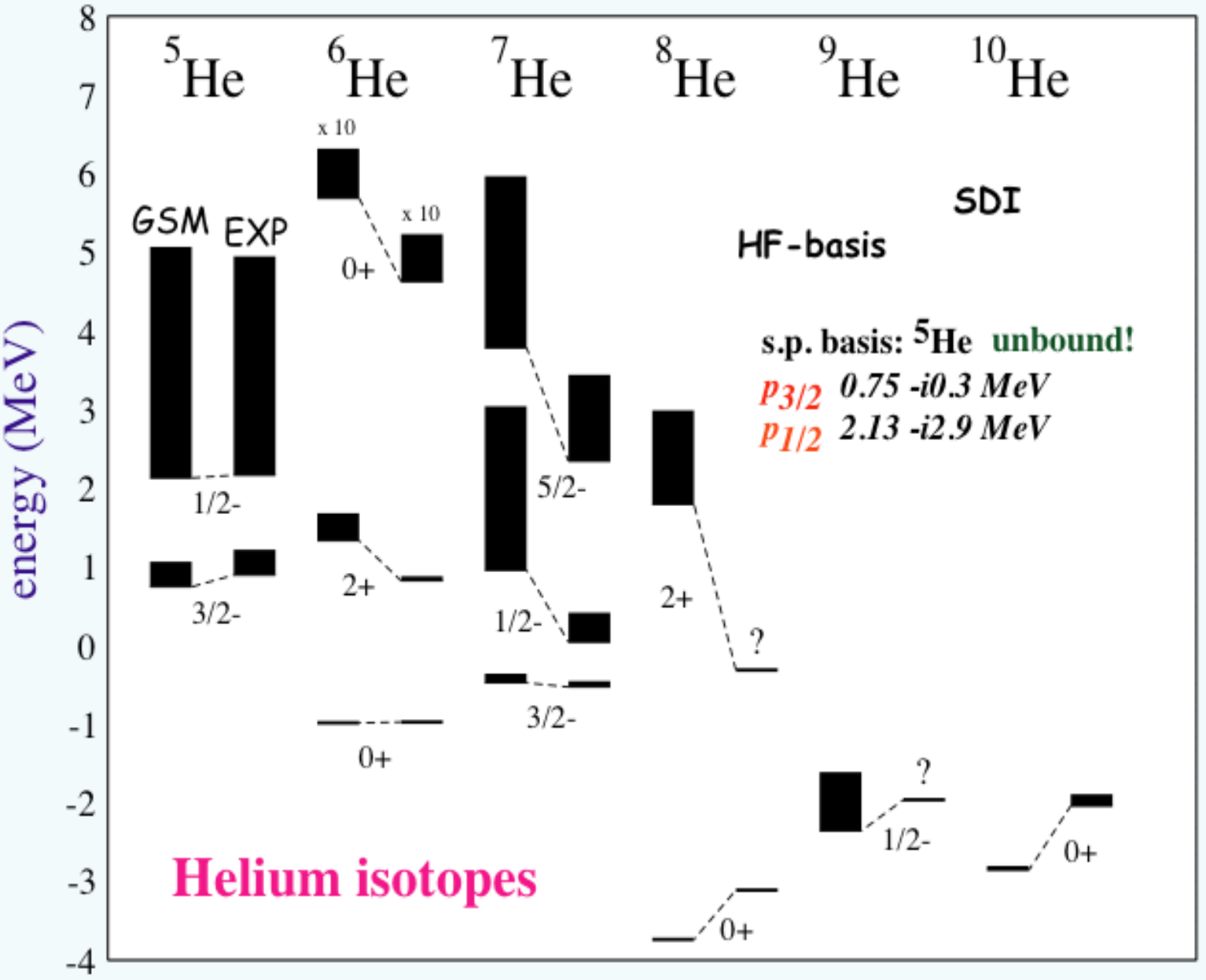
$$\sum_{n=b} |u_n\rangle\langle\tilde{u}_n| + \frac{1}{\pi}\int_{R} |u(k)\rangle\langle u(k^*)|\, dk = 1$$

(N. Michel *et al*, PRL 89 (2002) 042502)

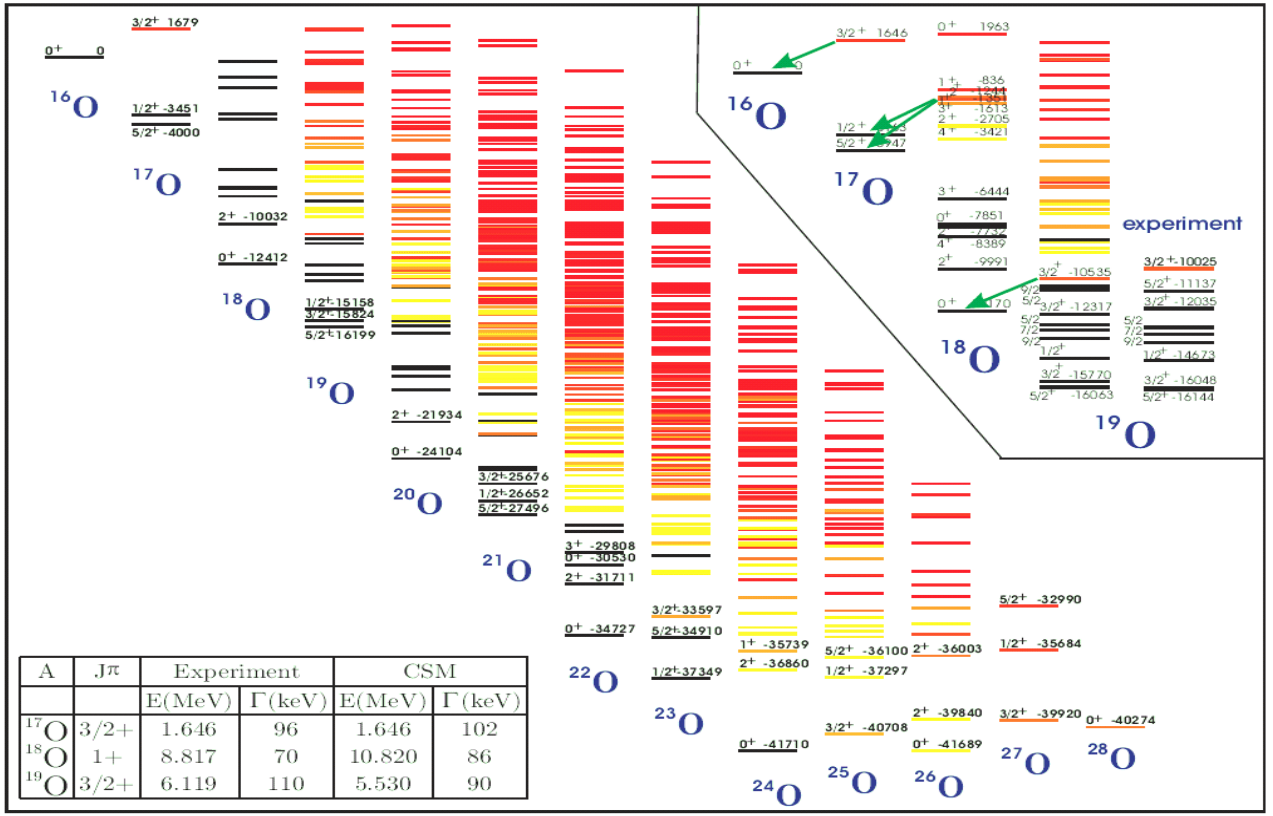


complex-symmetric eigenvalue problem for hermitian hamiltonian

GSM: N. Michel et al., Phys.Rev.Lett. 89, 042502 (2002)



The Continuum Shell-Model



A	Jπ	Experiment		CSM	
		E(MeV)	Γ(keV)	E(MeV)	Γ(keV)
^{17}O	3/2+	1.646	96	1.646	102
^{18}O	1+	8.817	70	10.820	86
^{19}O	3/2+	6.119	110	5.530	90

FIG. 2: CSM calculations for oxygen isotopes with the HBUSD interaction. States from yellow (long lifetime) to red (short lifetime) are resonance states. The insert on the upper right shows a more detailed picture for the lightest ^{16}O to ^{19}O isotopes. Decays from all states that are experimentally measured are shown with arrows. A full comparison between available data and the calculation is given for ^{19}O. Energies are expressed in units of keV. Comparison of widths with available data is given in the table in lower-left corner. For both inserts the interaction USD was used that works better in this mass region.

A. Volya et al., PRL 94 (2005) 052501

Nucleon-nucleon interactions in Berggren representation.

Separable expansion of $V_{\mathrm{low}-k}$ in a finite oscillator basis.
Hagen et. al., Phys.Rev. C73 (2006) 064307

$$\langle ab|V_{\mathrm{osc}}|cd\rangle \approx \sum_{\alpha\leq\beta}^{N}\sum_{\gamma\leq\delta}^{N}\langle ab|\alpha\beta\rangle\langle\alpha\beta|V_{\mathrm{low-k}}|\gamma\delta\rangle\langle\gamma\delta|cd\rangle,$$

- Two-body states $|ab\rangle$ in lab-system via vector-bracket transformation $\rightarrow$ complicated !
- Expand the interaction in a two-body oscillator basis - simple analytical continuation in the complex plane.
- Expansion coefficients from Brody-Moshinsky transformation.
- Two-body overlaps $\langle ab|\alpha\beta\rangle$ needed. Converge everywhere in the complex plane of phsyical importance.

Construction of Berggren many-body basis.

Having constructed a single-particle Berggren basis, a many-body Berggren basis may be constructed in a completely analogous way as when harmonic oscillator states are used. We construct a complete anti-symmetric N-body basis from the Slater determinants consisting of the Berggren orbitals φ_{nljm} i.e.

$$\Phi_{\alpha_1,\ldots,\alpha_N}(1,\ldots,N) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \varphi_{\alpha_1}(1) & \ldots & \varphi_{\alpha_1}(N) \\ \vdots & & \vdots \\ \varphi_{\alpha_N}(1) & \ldots & \varphi_{\alpha_N}(N) \end{vmatrix}, \tag{1}$$

where α_i labels the single-particle quantum numbers $(n_i l_i j_i m_i)$. We then have at hand a complete set of Slater determinants, i.e.

$$1 = \sum_i^d |\Phi_i\rangle\langle\Phi_i^*|, \tag{2}$$

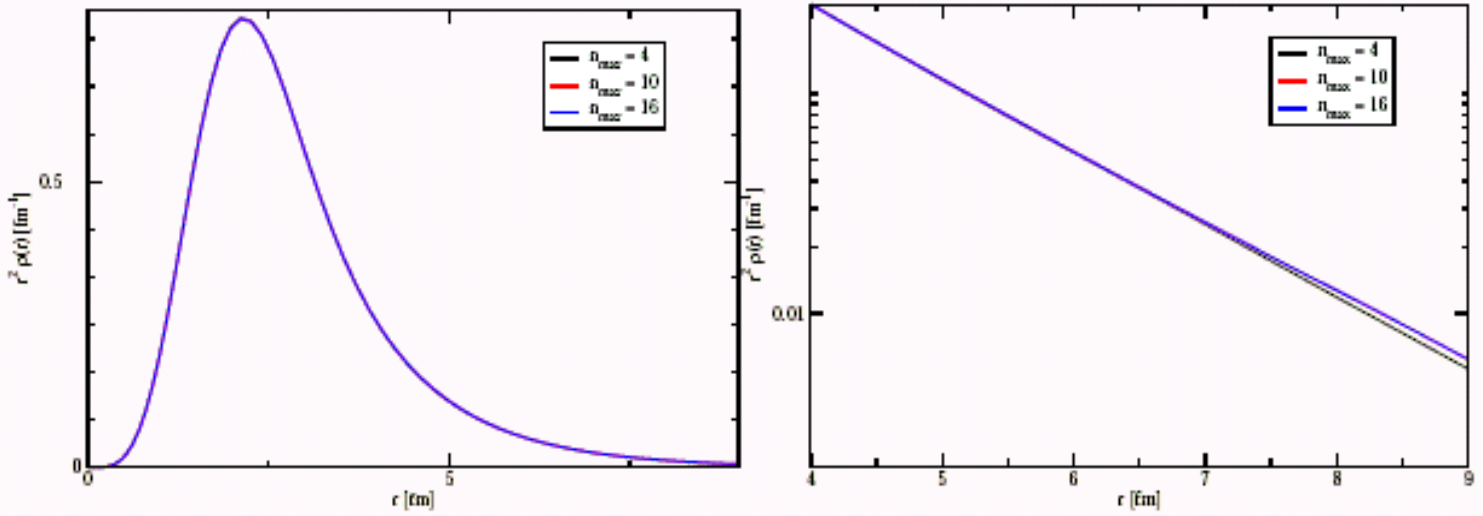
which the exact many-body wave function may be expanded in. The Shell Model problem then requires the solution of a complex symmetric $N \times N$ matrix eigenvalue equation,

$$H|\Psi_i\rangle = E_i|\Psi_i\rangle, \tag{3}$$

Gamow-Shell-Model calculations of 6He using realistic interactions.

- Calculations done in a Gamow-Hartree-Fock basis.
- $V_{\mathrm{low}-k}$ from N^{3}LO with $\Lambda = 1.9\mathrm{fm}^{-1}$.
- Energies/density converge with $n_{\max} \sim 4 - 6$.

	$J^\pi = 0_1{}^+$		$J^\pi = 2_1{}^+$	
$n_{\max}$	Re[E]	Im[E]	Re[E]	Im[E]
4	-0.4760	0.0000	0.9504	-0.0467
8	-0.4719	0.0000	0.9597	-0.0453
12	-0.4721	0.0000	0.9600	-0.0452
16	-0.4721	0.0000	0.9601	-0.0453
Exp.	-0.98	0.00	0.82	-0.06



Ab-Initio approach to dripline nuclei

Wish to solve to A-body nuclear Hamiltonian

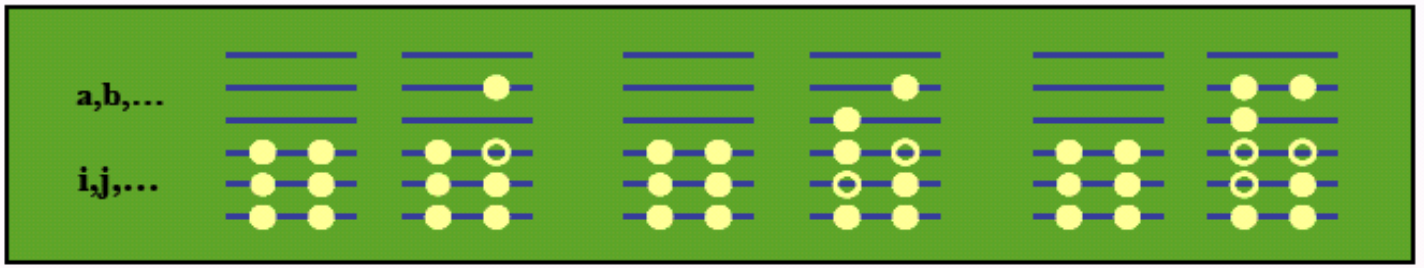
$$H = T - T_{CoM} + V = \left(1 - \frac{1}{A}\right) \sum_{i=1}^{A} \frac{\mathbf{k}_i^2}{2m} + \sum_{i<j}^{A} \left(V(i,j) - \frac{\mathbf{k}_i \cdot \mathbf{k}_j}{mA} \right) + \sum_{i<j<k}^{A} V(i,j,k)$$

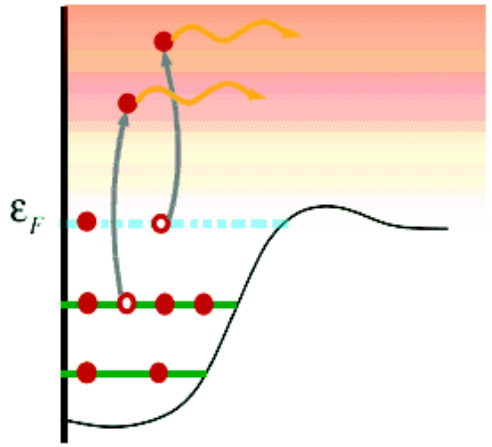
- Ground states embedded in the continuum - short lifetimes.
- Melting and reorganizing of shell structures.
- Extreme matter clusterizations - halo densities.
- ...

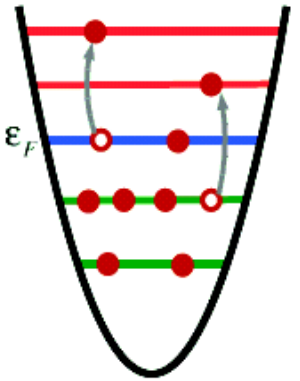
Coupled Cluster approach to many-body open quantum systems.

$$|\Psi\rangle = e^{T^{(A)}}|\Phi\rangle, \quad T^{(A)} = \sum_{k=1}^{m_A} T_k$$

$$T_1 = \sum_{\substack{i \\ a}} t_i^a \left|\Phi\, {}^{a}_{i}\right\rangle, \quad T_2 = \sum_{\substack{i>j \\ a>b}} t_{ij}^{ab} \left|\Phi\, {}^{ab}_{ij}\right\rangle, \quad T_3 = \sum_{\substack{i>j>k \\ a>b>c}} t_{ijk}^{abc} \left|\Phi\, {}^{abc}_{ijk}\right\rangle$$





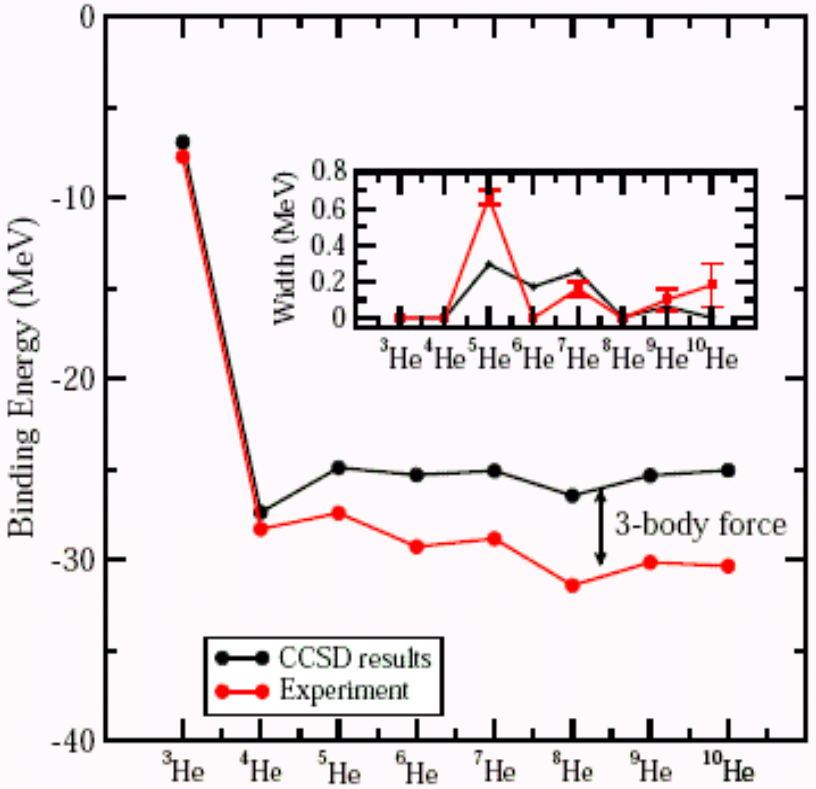


How well does single-reference Coupled-Cluster theory perform for open-shell nuclei ?

Various Coupled Cluster approaches to the $^{3-6}$He ground states. Single reference Coupled-Cluster methods works!

Method	^{3}He	^{4}He	^{5}He	^{6}He	$\langle J \rangle$, ^{6}He
CCSD	-6.21	-26.19	-21.53	-20.96	0.61
CCSD(T)	-6.40	-26.27	-21.88	-22.60	0.65
CCSDT-1	-6.41	-28.27	-21.89	-22.85	0.29
CCSDT-2	-6.41	-28.26	-21.89	-22.78	0.25
CCSDT-3	-6.42	-26.27	-21.92	-22.90	0.26
CCSDT	-6.45	-26.28	-22.01	-22.52	0.04
Exact	-6.45	-26.3	-22.1	-22.7	0.00

Coupled-Cluster Approach to Weakly bound and unbound nuclear states

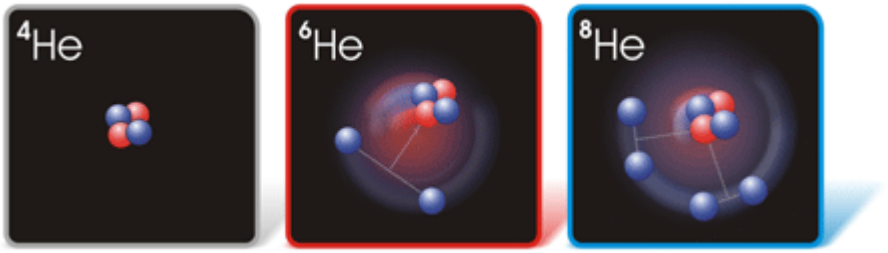


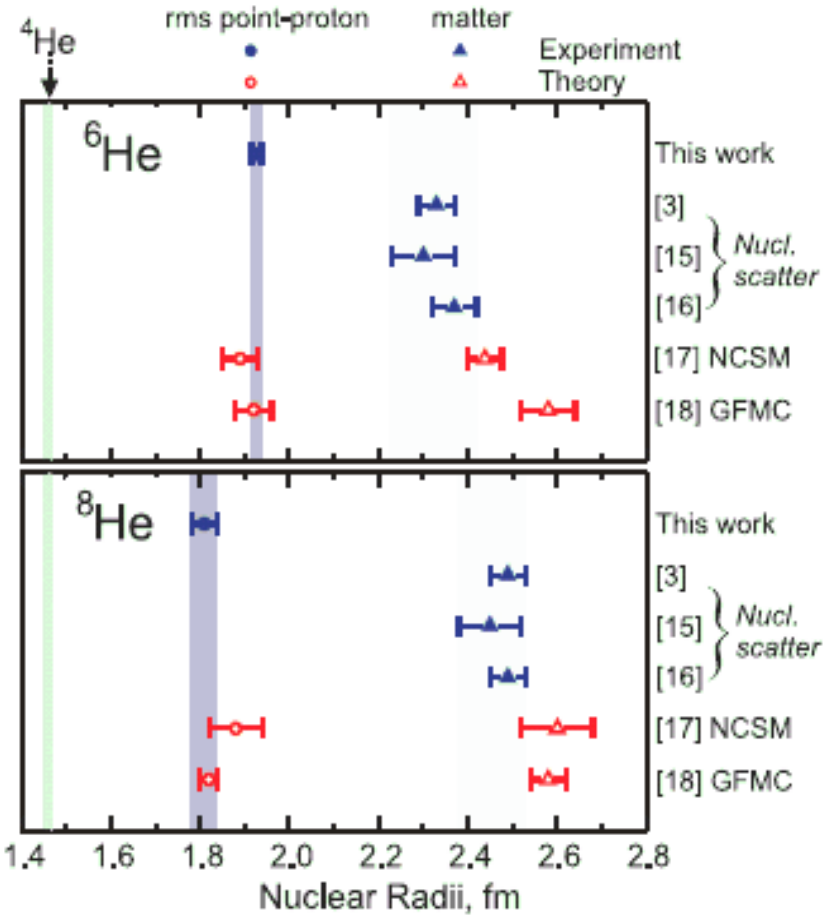
- $V_{\mathrm{low}-k}$ from N3LO with $\Lambda = 1.9\mathrm{fm}^{-1}$.
- G. Hagen et al., Phys. Lett. B 656, 169 (2007). arXiv:nucl-th/0610072.

- First *ab-initio* calculation of decay widths !
- CCM unique method for dripline nuclei.
- $\sim$ 1000 active orbitals
- Underbinding hints at missing 3NF

8He charge radii: theory vs. experiment

8He charge radii measurements using the measured isotope shift with the help of precision atomic theory calculations.

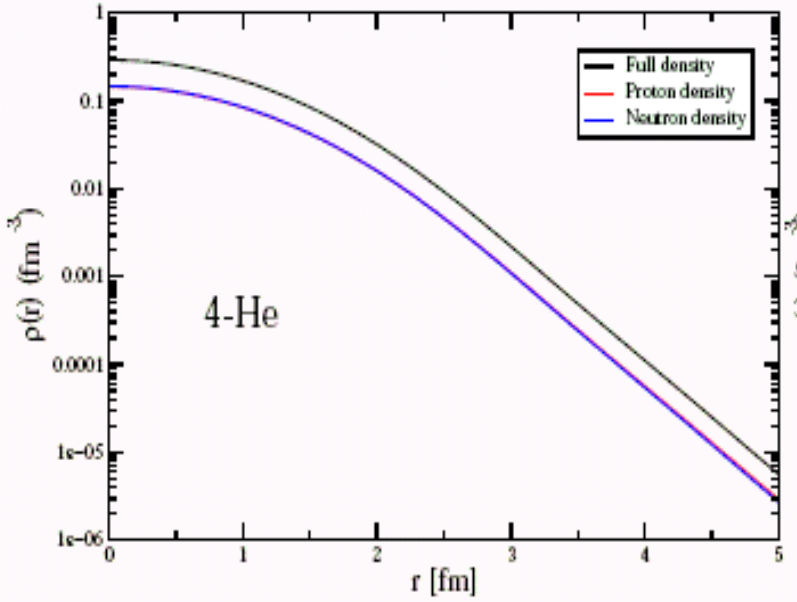


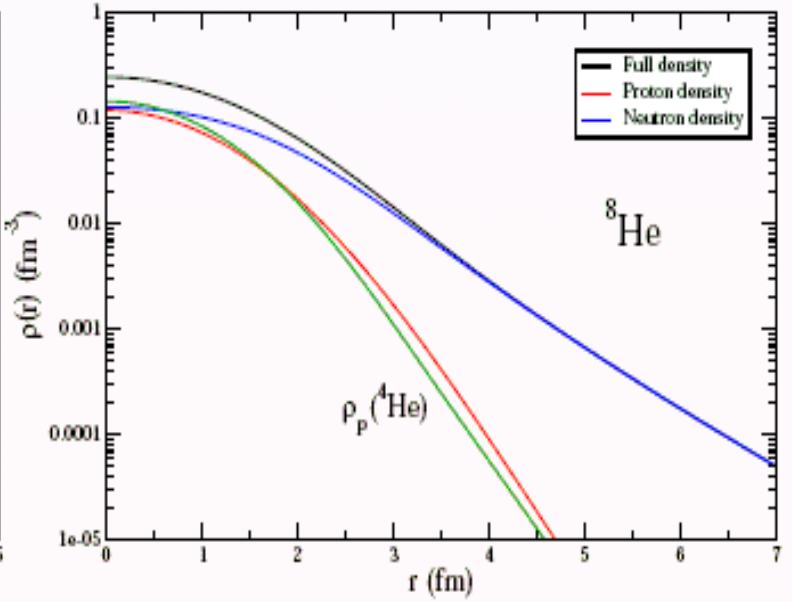


P. Mueller et al., PRL 99, 252501 (2007)

4He and 8He single particle density distributions with V-srg

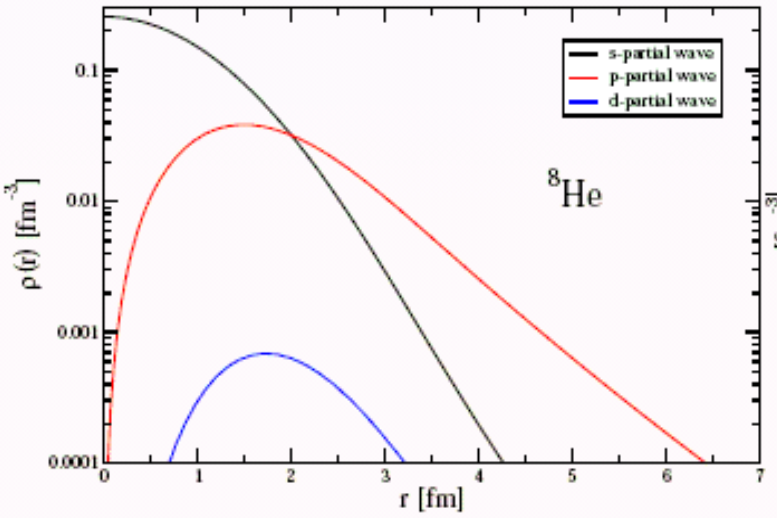
- Single-particle density in ^{4}He and ^{8}He.
- Gamow-Hartree-Fock basis has correct asymptotics.
- N^{3}LO evolved down to $\lambda = 2.0\mathrm{fm}^{-1}$ from similarity renormalization group theory.

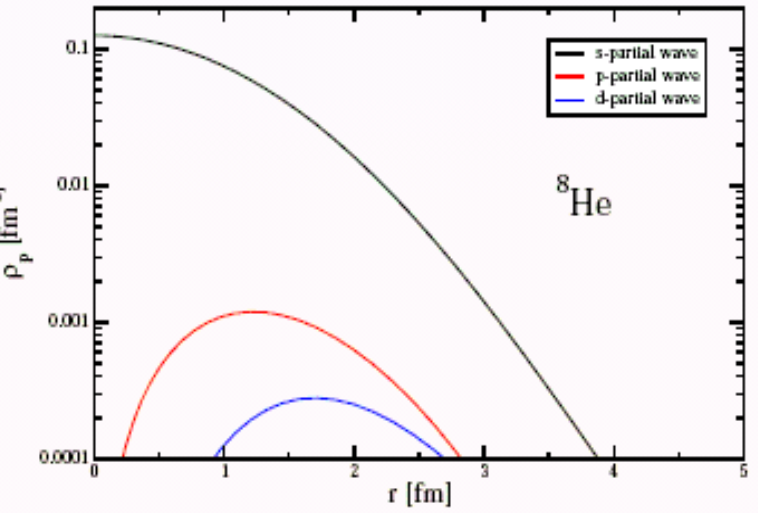




Partial wave decomposition of 8He single particle density.

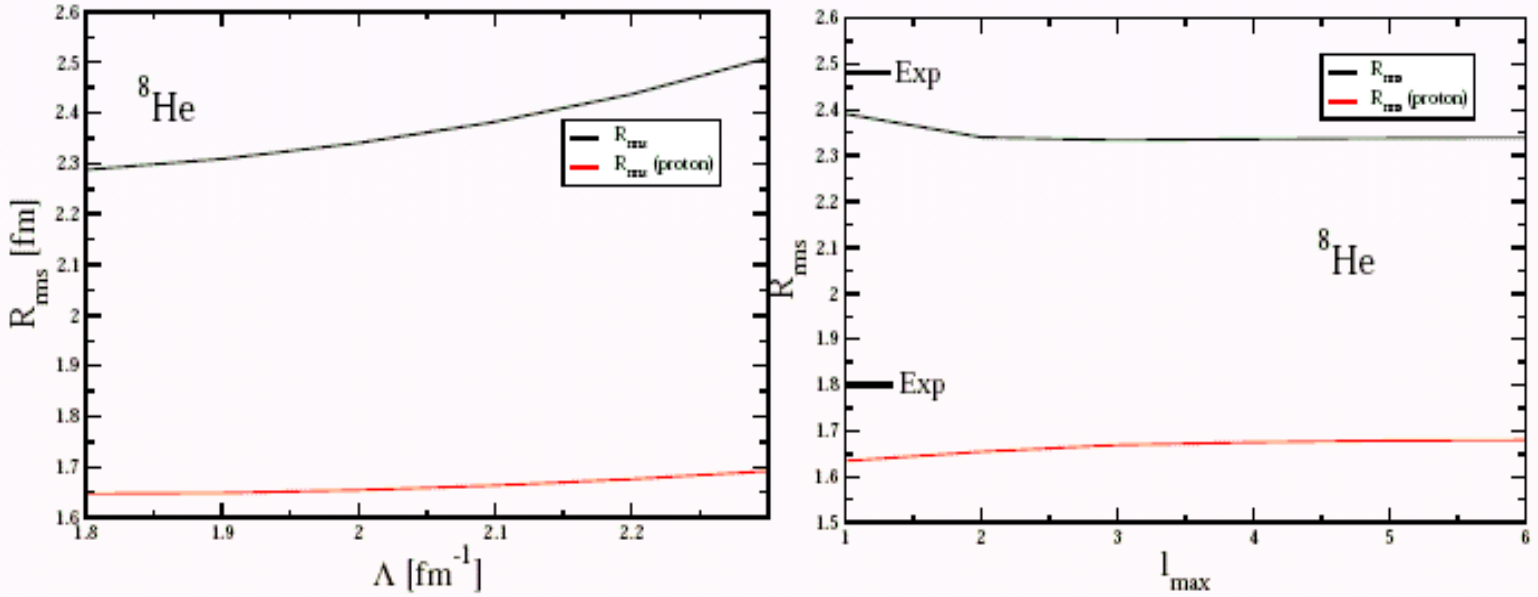
- N^{3}LO evolved down to $\lambda = 2.0\mathrm{fm}^{-1}$ from similarity renormalization group theory.
- Neutron skin in ^{8}He is mainly built from $s-$ and $p-$partial waves. Protons are mainly occupying $s-$ partial waves.





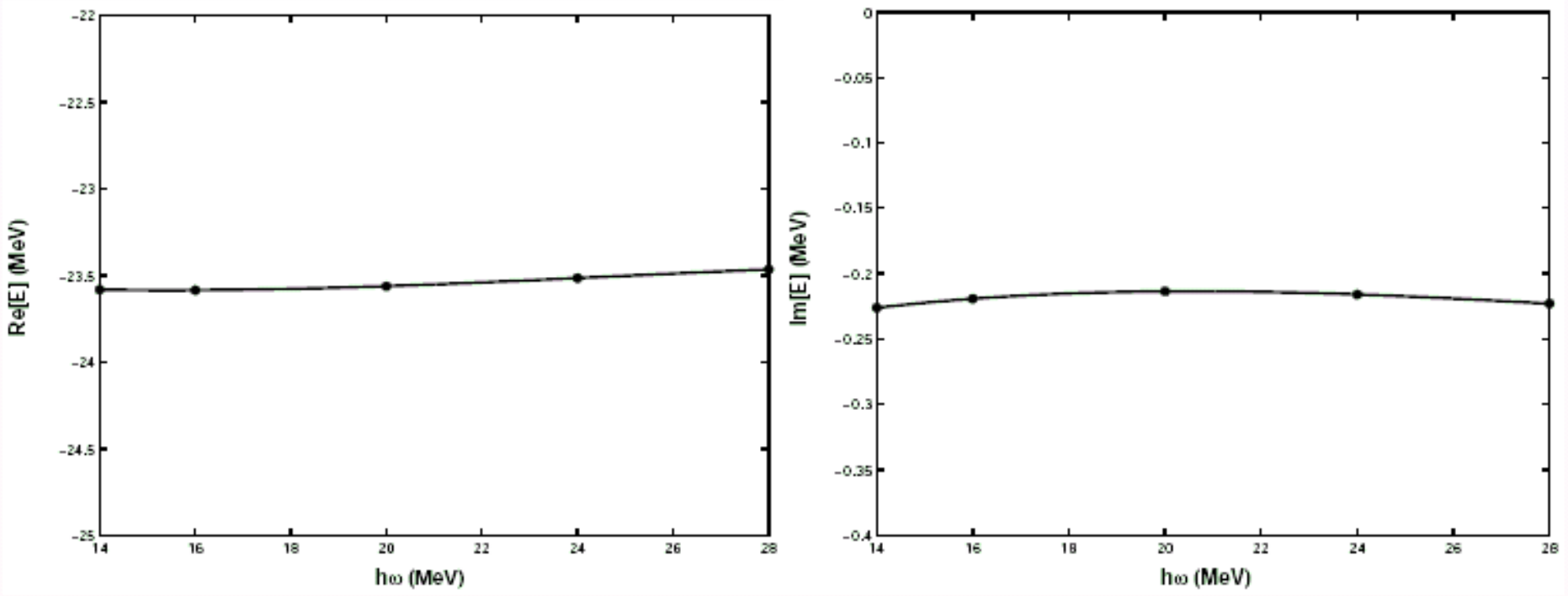
Matter and charge radii of 8He using V-srg

- Λ dependence on ^{8}He charge and matter radii indicates missing 3NF.
- Hamiltonians with two-body renormalized interactions (SRG/low-k) underestimates matter and charge radii.



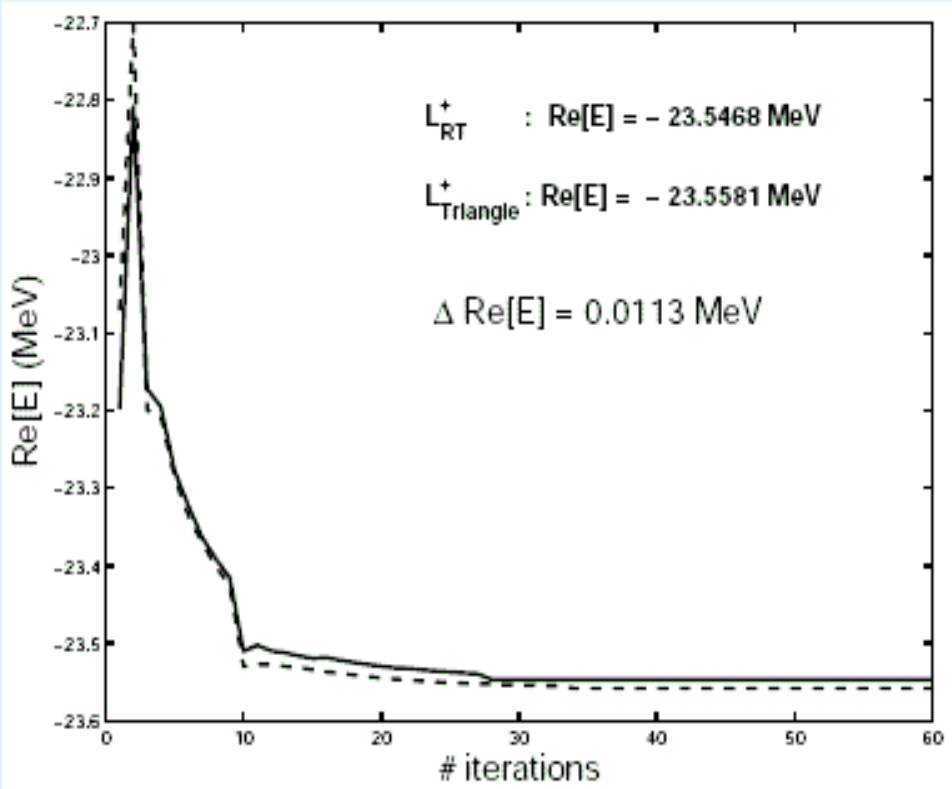
Convergence of CCSD energy.

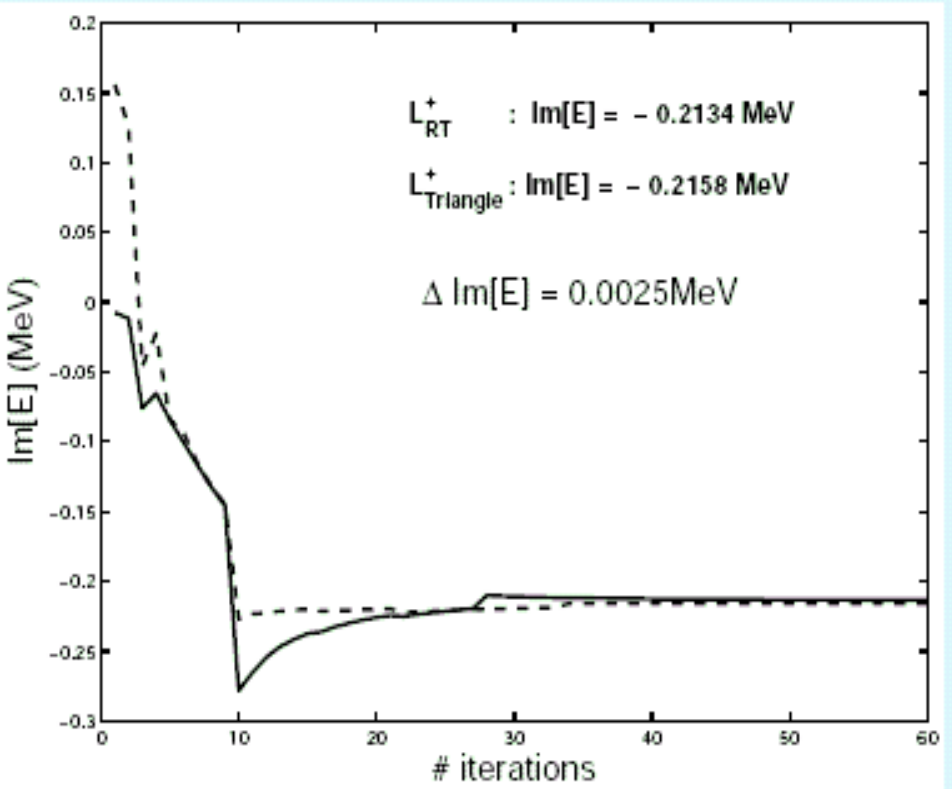
- ^{5}He ground state energy starting with oscillator bases given for different $\hbar\omega$ values.
- Weak $\hbar\omega$ dependence, Results are well converged.
  $\Delta Re[E] \sim 0.1$MeV, $\Delta Im[E] \sim 0.01$MeV



Convergence of CCSD energy

CCSD convergence of ^{5}He ground state energy for the $s-d$ space (300 orbitals) using $n = 20$ discretization points for L^+. The calculation where performed using two very different L^+ contours





Summary of Open-Quantum Systems

- Weakly bound and unbound nuclear states poses a great challenge for nuclear theory.
- Berggren showed that a basis can be generated which treats bound and unbound states on equal footing.
- Explosion of many-body configurations requires methods which scales softly with system size.
- Coupled-Cluster theory a promising approach for the study of exotic dripline physics.
- Have presented the first ab-initio calculation of lifetimes of a whole isotopic chain.