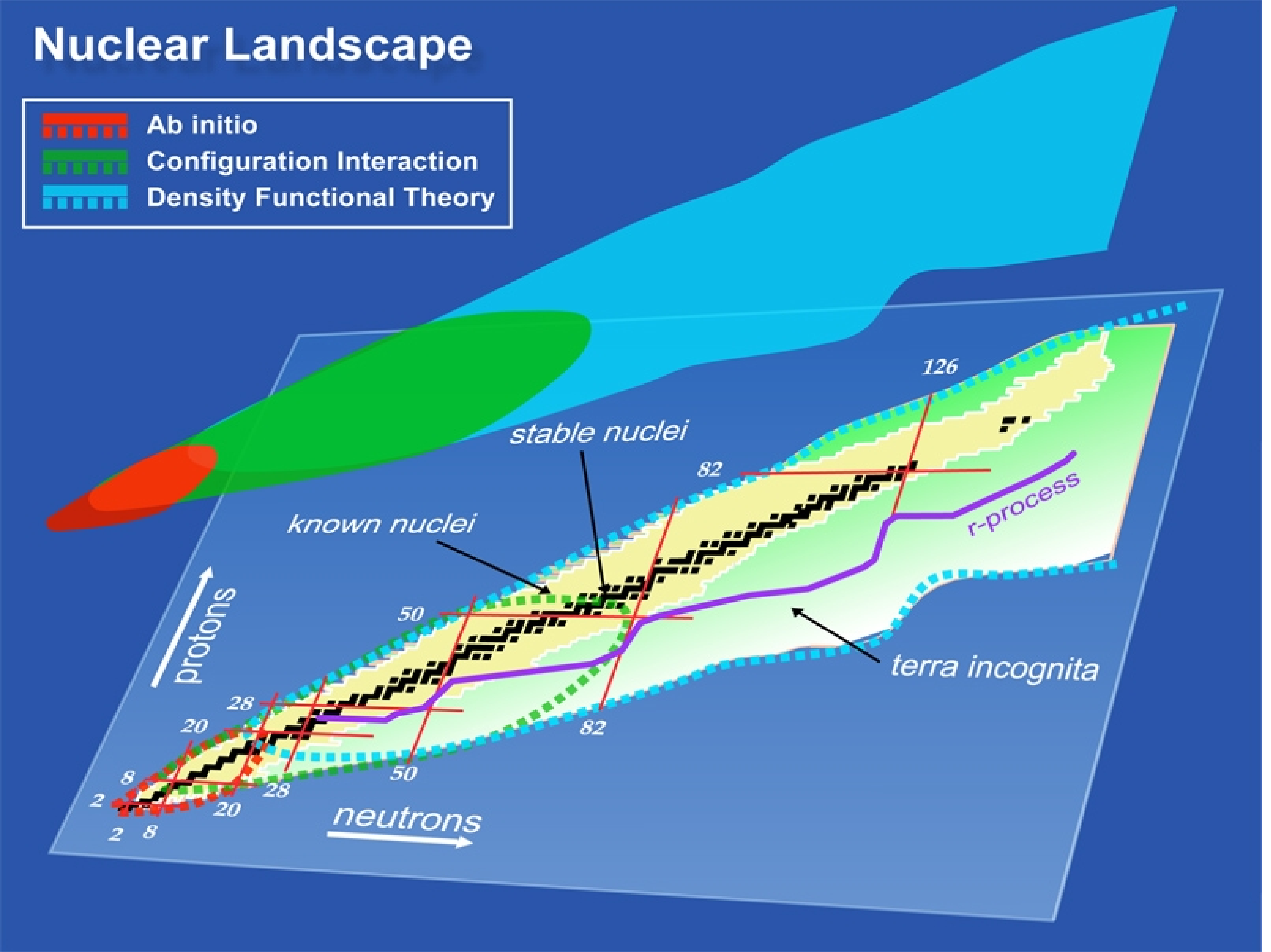
The nuclear many-body problem
Lecture 2

Renormalization of the nucleon-nucleon force.

- Aim: Construct the best nucleon-nucleon force applicable for light and medium mass nuclei.
- High momentum modes of the nucleon-nucleon force makes many-body wave-function expansions converge slowly.
- Construct effective interactions where high momentum modes are integrated out.



Nuclear Landscape
Ab initio
Configuration Interaction
Density Functional Theory
stable nuclei
known nuclei
r-process
terra incognita
protons
neutrons
126
82
50
28
20
8
2
82
50
28
20
8
2

Chiral Perturbation Theory.

"If you want more accuracy, you have to use more theory (more orders)"

Effective Lagrangian → obeys QCD symmetries (spin, isospin, chiral symmetry breaking)

Lagrangian
→ infinite sum of Feynman diagrams.

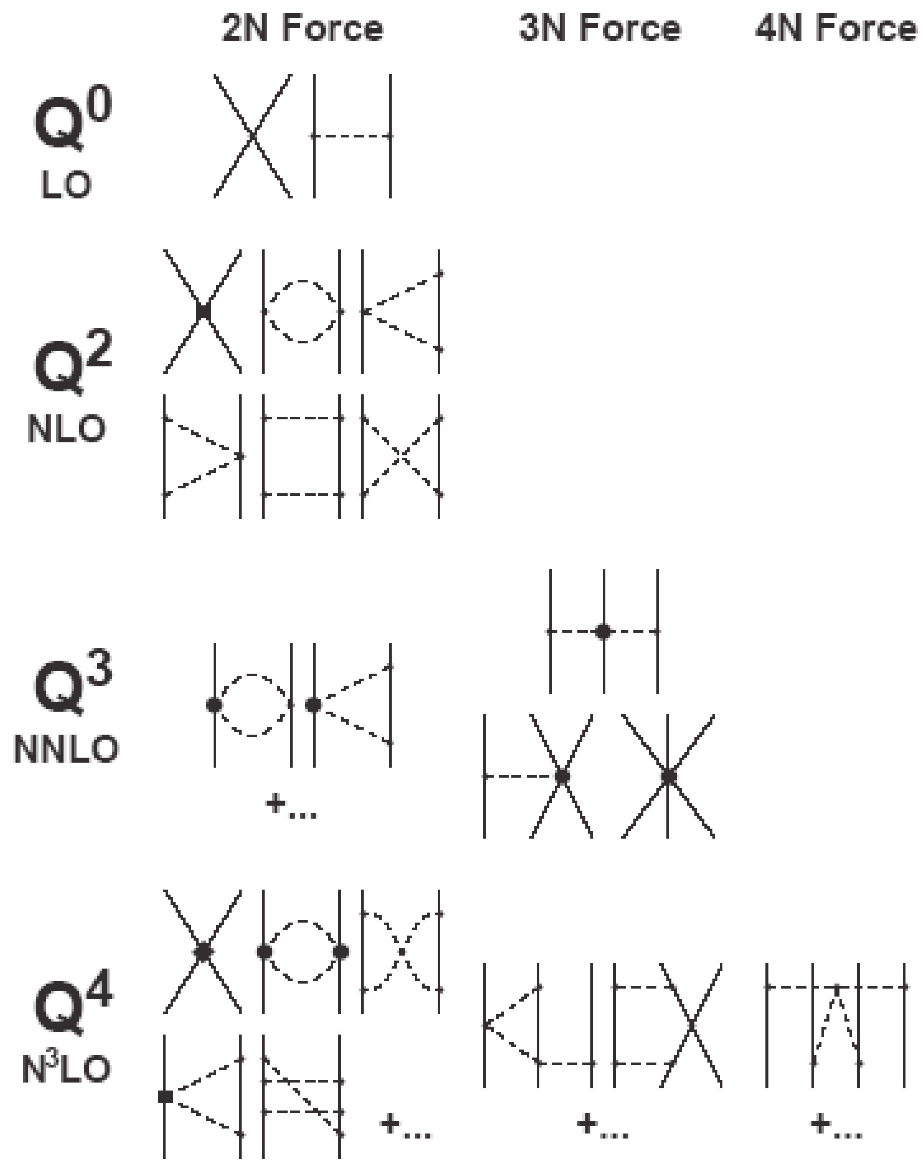
Expand in $O(Q/\Lambda_{QCD})$

Weinberg, Ordonez, Ray, van Kolck

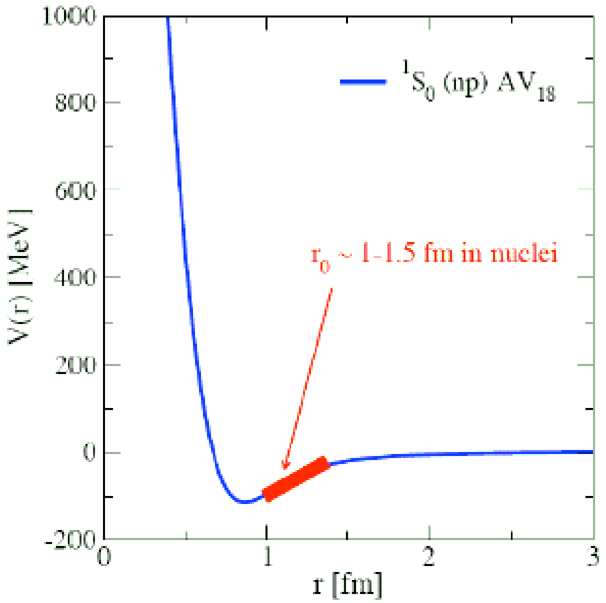
NN amplitude uniquely determined by two classes of contributions: contact terms and pion exchange diagrams.

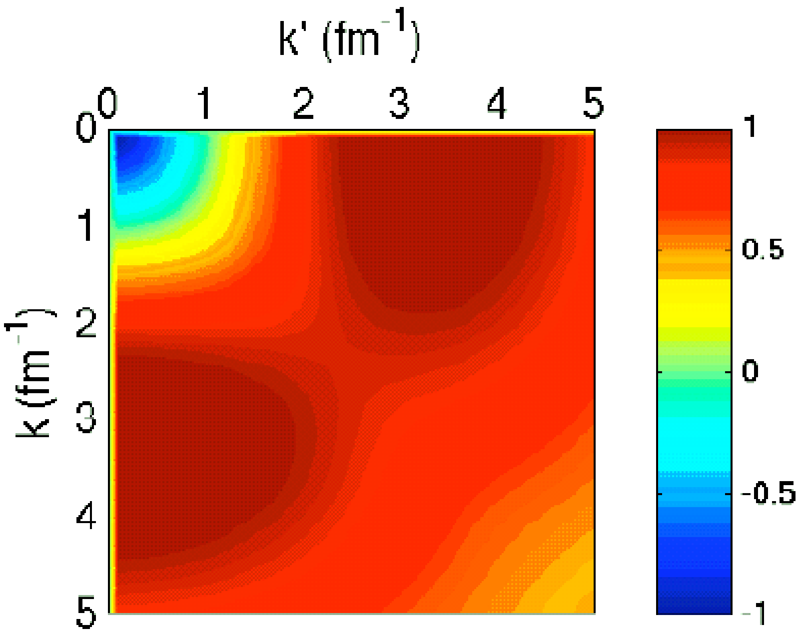
24 paramters (rather than 40 from meson theory) to describe 2400 data points with

$$\chi^2_{\text{dof}} \approx 1$$



“Resolution dependent” Sources for Non-perturbative Physics





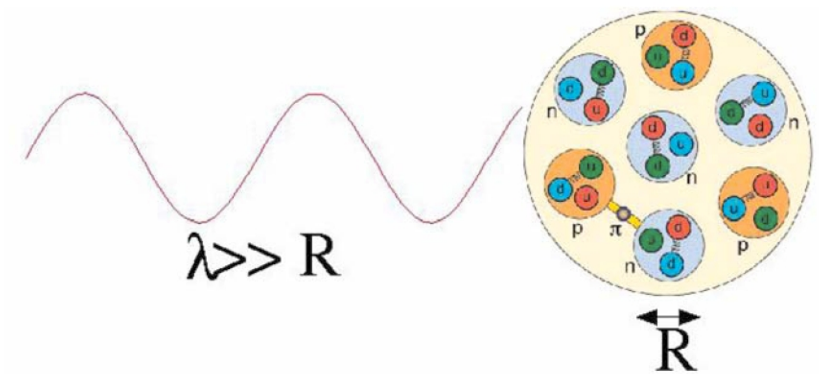
- short-ranged repulsive core
- strong tensor force

⇨ Strong coupling to high-momentum modes

BUT typical momentum in a large nucleus only ≈ 1 fm^{-1} (200 MeV)!

Principles of low energy effective theories

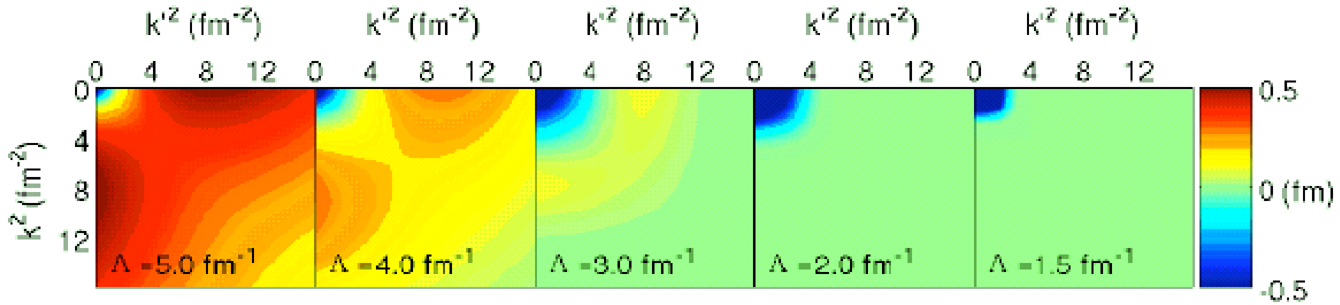
- High-precision potentials contain short-range (high momentum) physics that is not constrained by phase shifts.
- Is it necessary to know the NN interaction at short distances to understand long wavelength physics?



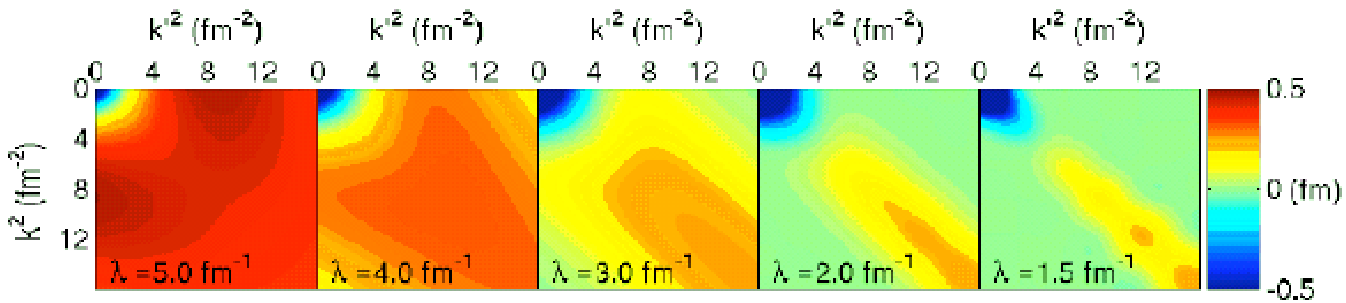
- Introduce momentum cutoff Λ and integrate out high momentum modes such that low-momentum observables are unchanged (Renormalization group transformation).
- Resulting low-momentum potential $V_{low\text{-}k}$.
- Recall: Fermi momentum at saturation density $k_F = 1.4\ \mathrm{fm}^{-1}$.

2-Types of Renormalization Group Transformations

- "$V_{low\ k}$" => lowers a cutoff in k′,k



- SRG => drives Hamiltonian towards the diagonal



Both decouple the high momentum modes ***leaving low E NN observables unchanged.***

Similarity Renormalization Group applied to nuclear structure

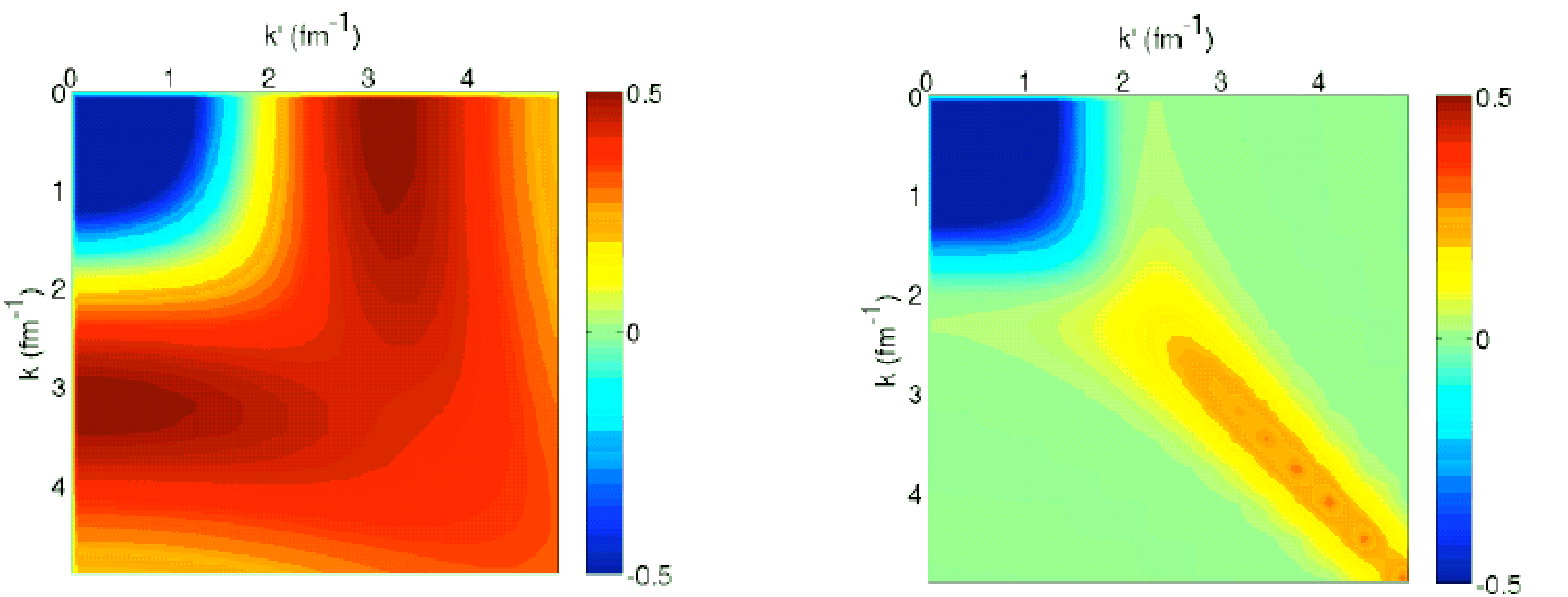
The evolution (flow) of the Hamiltonian to block diagonal form:

$$H_s = U(s) H U^\dagger(s) \equiv T_{\rm rel} + V_s\,, \qquad \frac{dH_s}{ds} = [\eta(s), H_s]\,, \qquad \eta(s) = [T_{\rm rel}, H_s]\,,$$

Differentiating with respect to the flow parameter s we get:

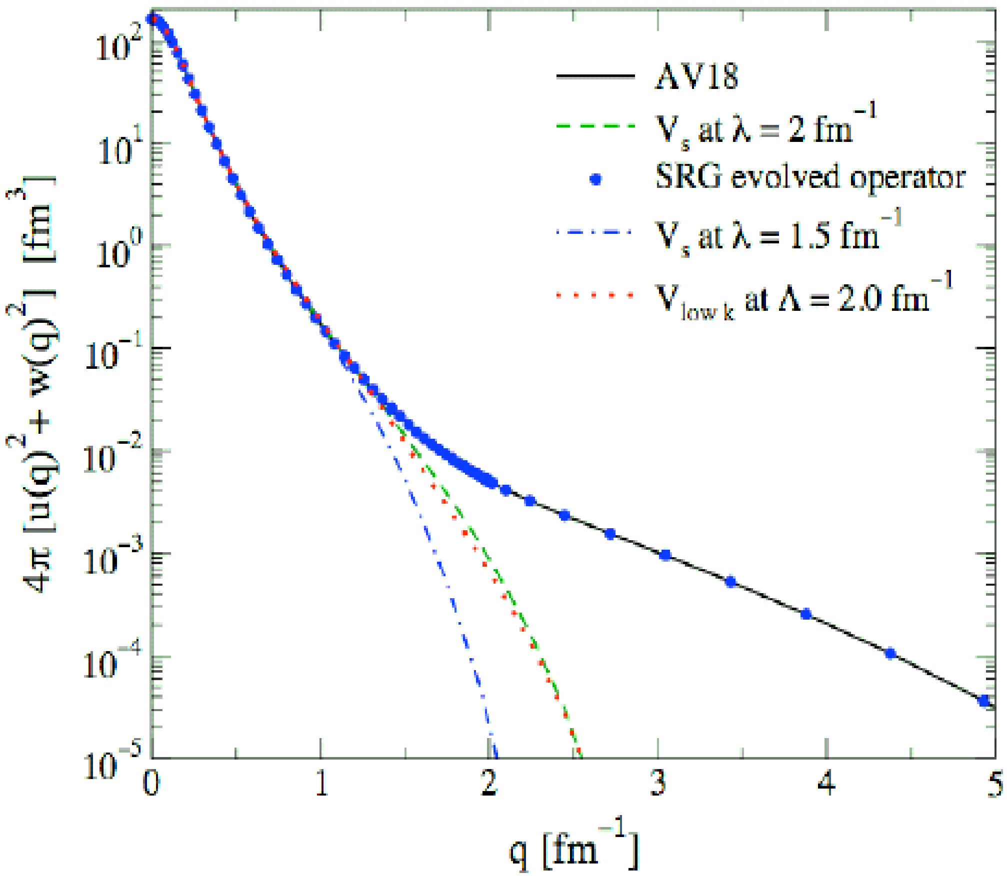
$$\frac{dH_s}{ds} = [[T_{\rm rel}, H_s], H_s] = [[T_{\rm rel}, V_s], H_s]\,.$$

$$\frac{dV_s(k,k')}{ds} = -(k^2 - k'^2)^2\, V_s(k,k') + \frac{2}{\pi}\int_0^\infty q^2\, dq\, (k^2 + k'^2 - 2q^2)\, V_s(k,q)\, V_s(q,k')\,.$$



Effective operators in the SRG

$$\frac{d\mathcal{O}_s}{ds} = [\eta(s), \mathcal{O}_s]$$



From S. Bogner

Integrating out high-momentum modes by similarity transformations.

Define a model space P and a complement Q-space given by a cutoff Λ in momentum space

$$P = \frac{2}{\pi} \int_0^\Lambda p^2 dp \, |p\rangle\langle p| \quad \text{and} \quad Q = \frac{2}{\pi} \int_\Lambda^\infty q^2 dq \, |q\rangle\langle q| .$$

The Hamiltonian can then be written in Block form as

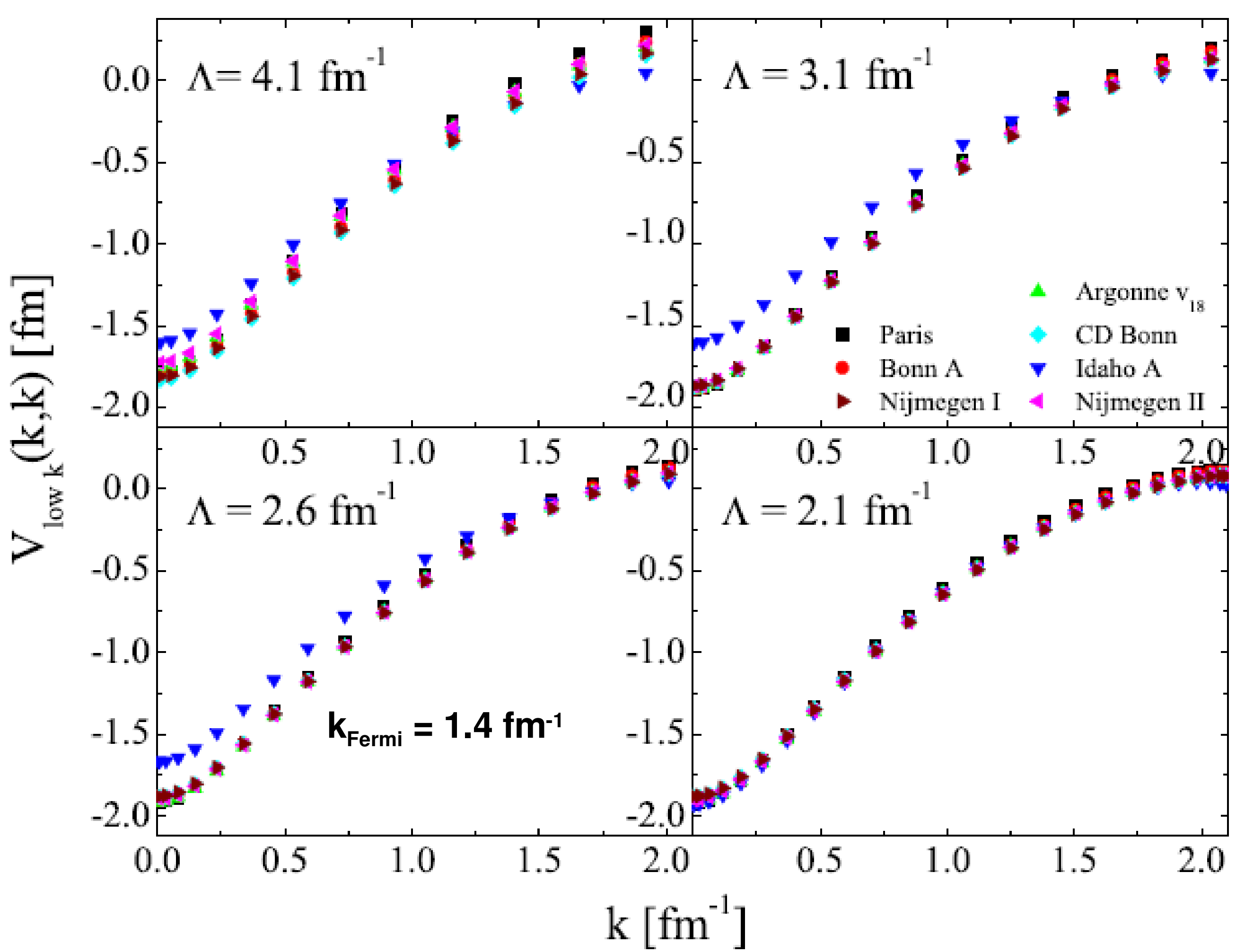
$$\begin{pmatrix} PHP & PHQ \\ QHP & QHQ \end{pmatrix} \begin{pmatrix} P\,|\Psi\rangle \\ Q\,|\Psi\rangle \end{pmatrix} = E \begin{pmatrix} P\,|\Psi\rangle \\ Q\,|\Psi\rangle \end{pmatrix}$$

The Lee-Suzuki method finds a similarity transformation that brings the Hamiltonian to the block structure

$$\Theta^{-1}\, H\, \Theta = \mathcal{H}^{\mathrm{LS}}_{\mathrm{low\ k}} = \begin{pmatrix} P\mathcal{H}P & P\mathcal{H}Q \\ 0 & Q\mathcal{H}Q \end{pmatrix} .$$

Low momentum potential $V_{\text{low-k}}$

$$\frac{d}{d\Lambda} V_{\text{low k}}(k', k) = \frac{2}{\pi} \frac{V_{\text{low k}}(k', \Lambda)\, T(\Lambda, k; \Lambda^2)}{1 - (k/\Lambda)^2}$$



Different high-precision potentials

Universal low-momentum potential

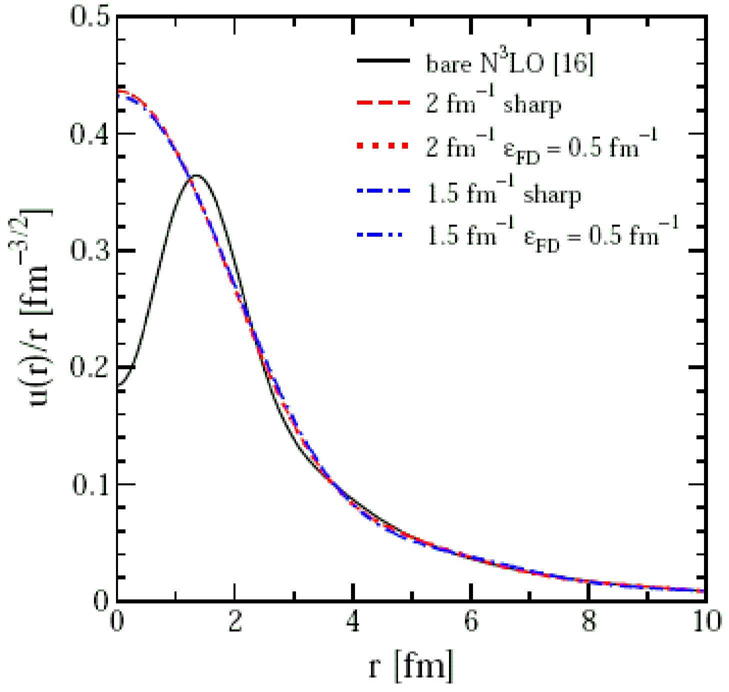
Properties of $V_{\text{low-k}}$:

- No hard core
- Nonlocal
- Hartree-Fock already yields bound nuclei.

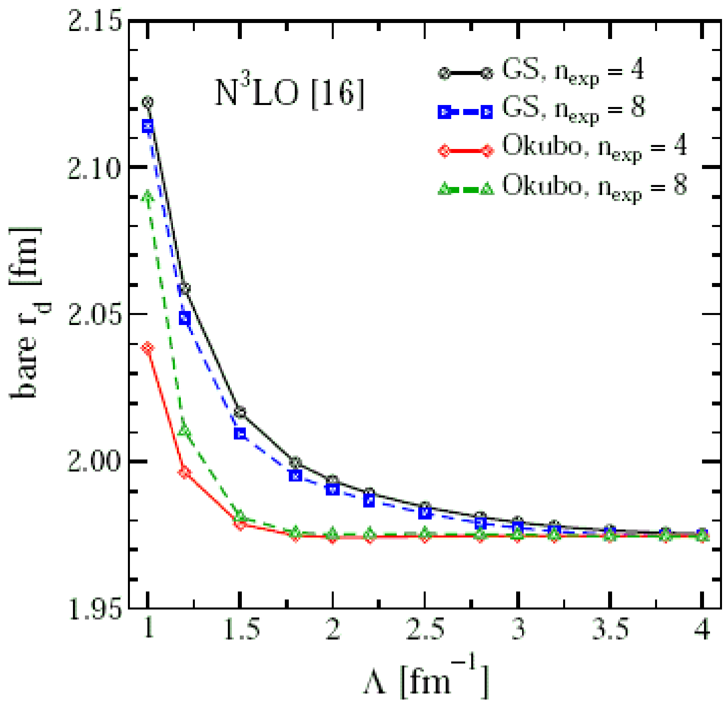
S. K. Bogner, T. T. S. Kuo, and A. Schwenk, Phys. Rep. 386 (2003) 1

Deuteron wave function

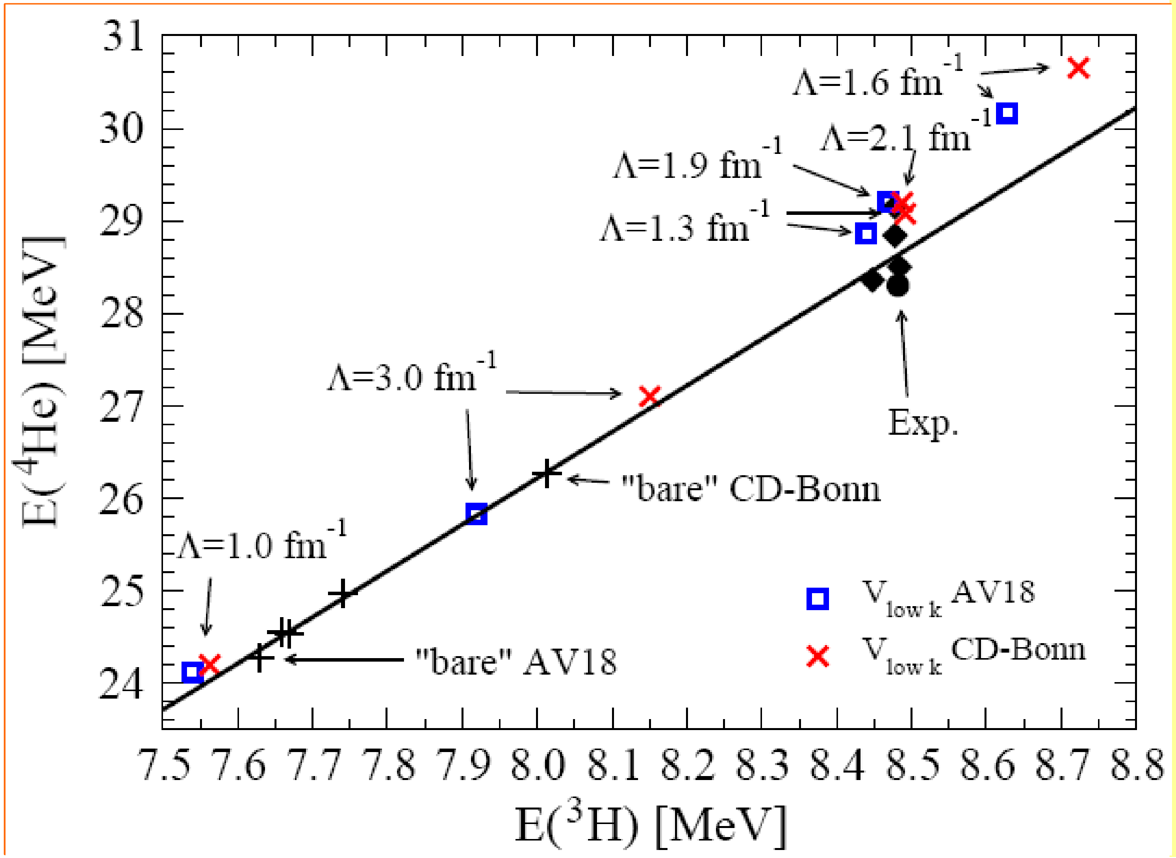
Deuteron wave function for different cutoffs



Deuteron RMS radii for different cutoffs



Light nuclei with $V_{low\text{-}k}$



As cutoff Λ is varied, motion along Tjon line.

Addition of Λ-dependent three-nucleon force yields agreement with experiment.

•Three-nucleon force necessary.

•There is no “best” potential & TNF. Choose the most convenient.

A. Nogga, S. K. Bogner, and A. Schwenk, Phys.Rev. C70 (2004) 061002

Q: Can we understand this more systematically?

A: Yes, resort to a model-independent approach via effective field theory (EFT).

Vlow-k filters out low-energy physics, while EFT starts from low-energy modes.

Convergence in light systems with v-lowk

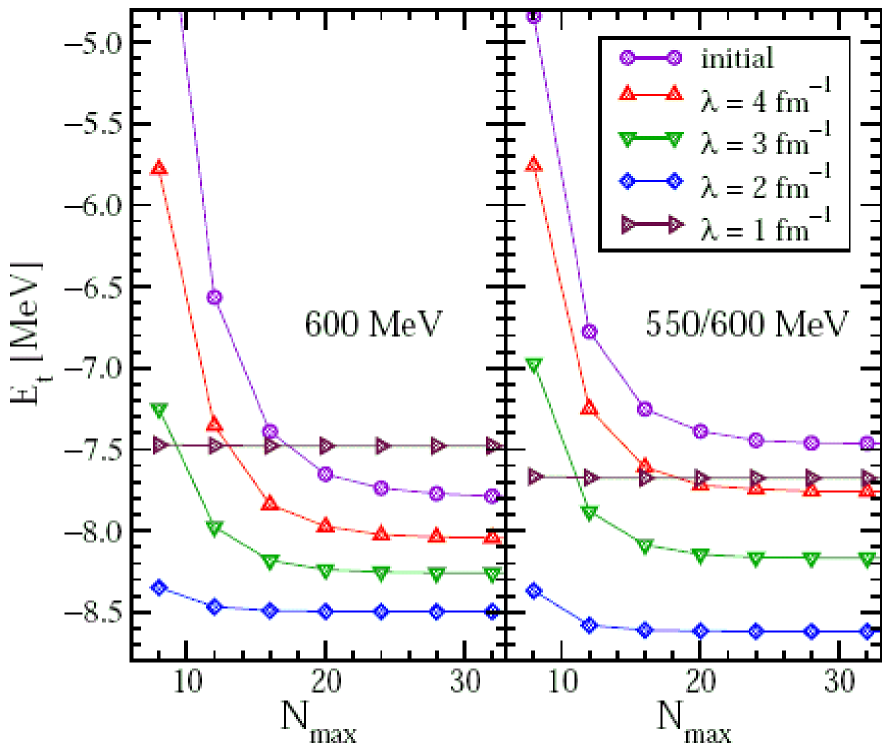


FIG. 6: (Color online) The variational binding energy for selected λ of the triton with two-nucleon interactions only, as a function of the size of the harmonic oscillator space ($N_{\max}\hbar\omega$ excitations), for the same initial potentials as in Fig. 1.

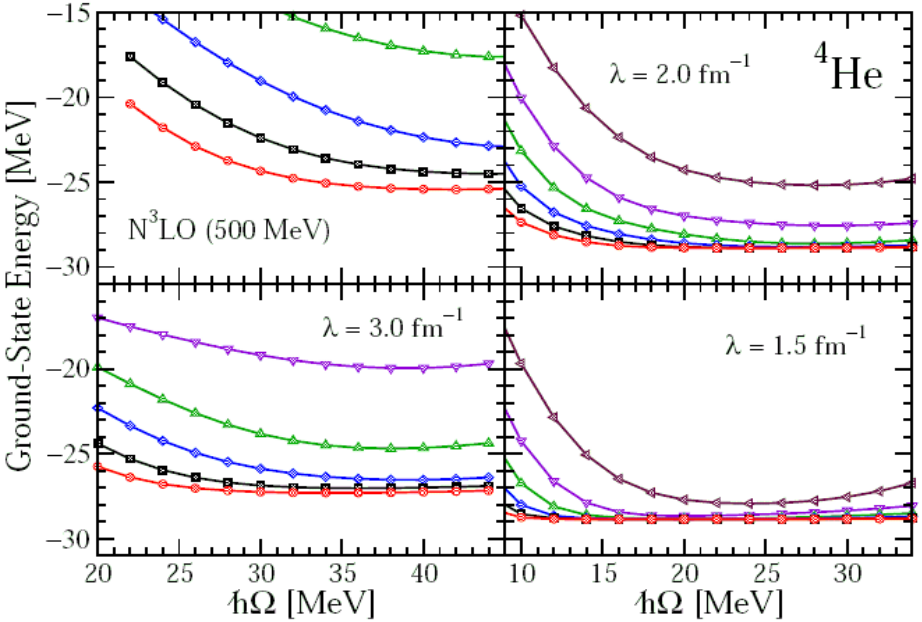
Convergence of triton binding energy for different cutoffs used in the renormalization.

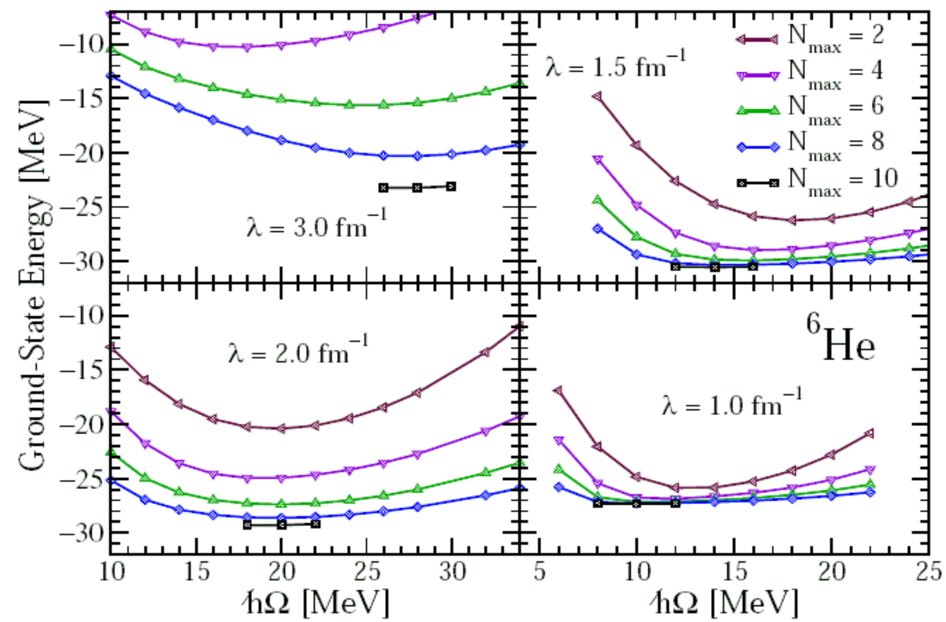
- Converges fast with smaller Λ
- Binding energy depends on Λ
- Must include three-body force!

S Bogner et al. nucl-th/0701013

$$H^A = T - T_{CM} + V_2(\Lambda) + V_3(\Lambda) + \cdots V_A(\Lambda) \approx T - T_{CM} + V_2(\Lambda) + V_3(\Lambda)??$$

Convergence in light systems with vlowk



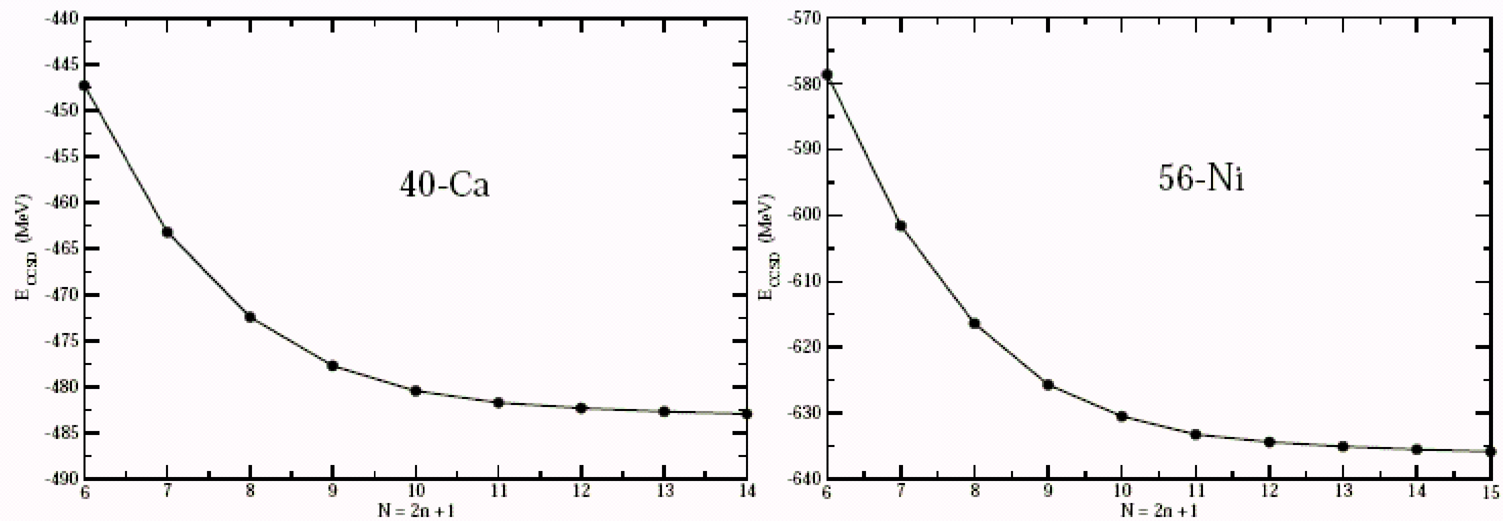


1. Running the interaction to lower cutoff Λ increases the convergence
2. Different cutoff L gives different converged ground state energies, and consequently different three-nucleon forces.

Question: $H^A = T - T_{CM} + V_2(\Lambda) + V_3(\Lambda) + \cdots V_A(\Lambda) \approx T - T_{CM} + V_2(\Lambda) + V_3(\Lambda)??$

What happens in medium sized nuclei ?

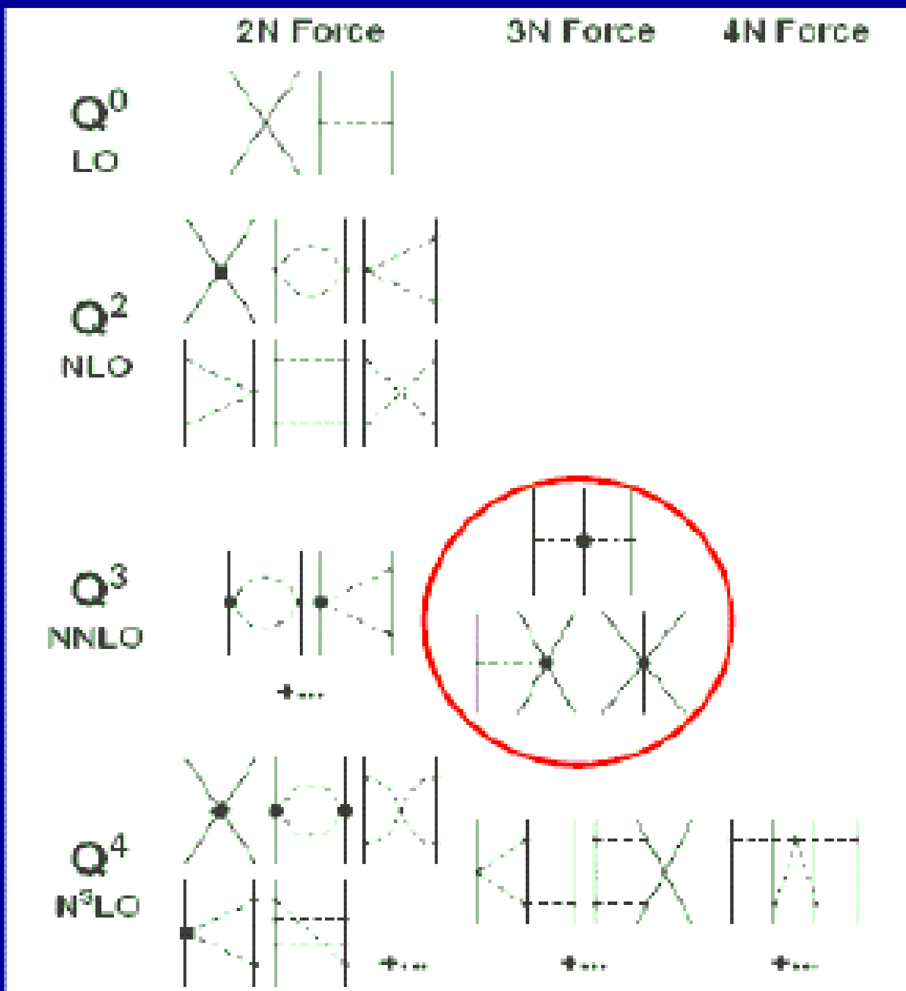
Converged Coupled-Cluster ground state energies
for 40Ca and 56Ni using Vsrg with Λ = 2.5fm-1



Several hundreds of MeV's overbinding !
Three-nucleon force must be largely repulsive !
Is three-nucleon forces enough ??

3NF from Chiral EFT

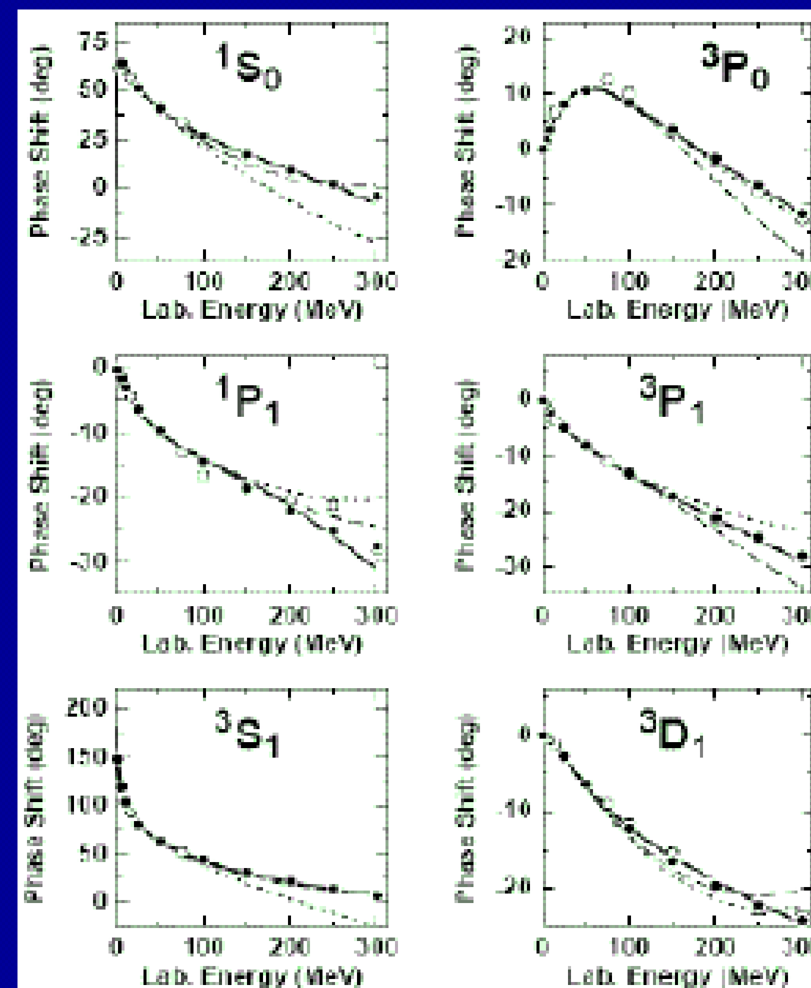
Feynman diagrams



R. Machleidt and D. R. Entem, J. Phys. G 31 (2005) S1235

Phase shifts reproduced to χ^2/datum=1

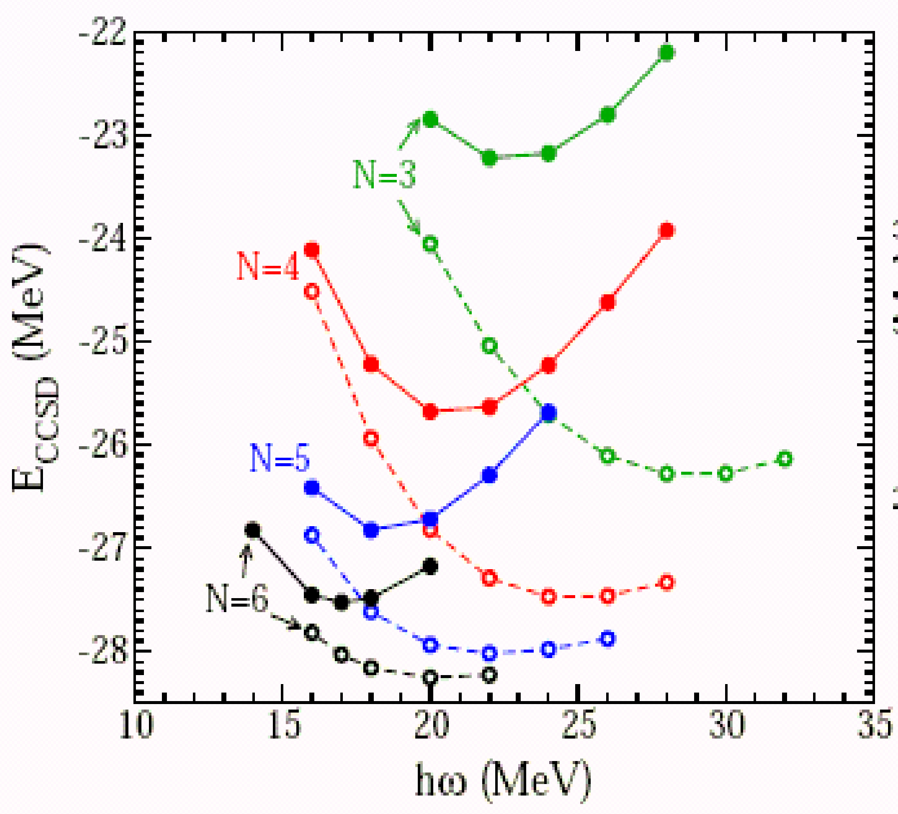
About 24+ parameters

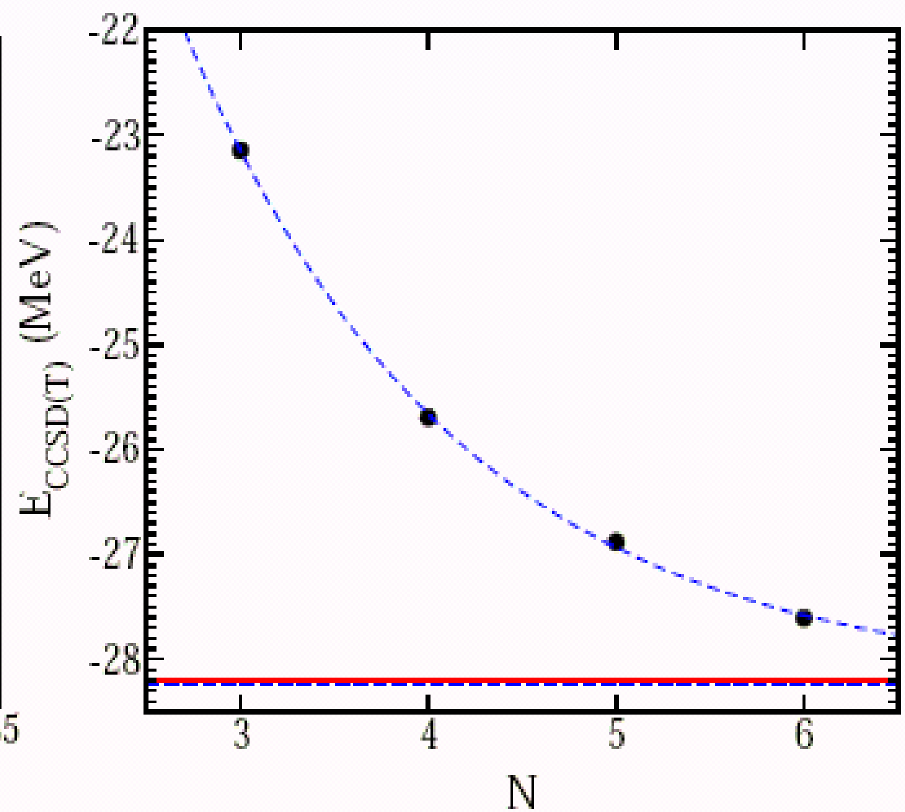


D. R. Entem and R. Machleidt, Phys.Rev. C68 (2003) 041001

Coupled Cluster results for He4 with 3NF

- $V_{\mathrm{low}-k}$ from AV18 with $\Lambda = 1.9\mathrm{fm}^{-1}$.
- 3NF brings in repulsion as expected !
- CCSD and CCSD(T) with 3NF meets Faddeev-Yakubovsky benchmark ! $E_{\mathrm{CCSD(T)}} \approx -28.24$ MeV. F-Y $E = -28.20(5)$MeV.

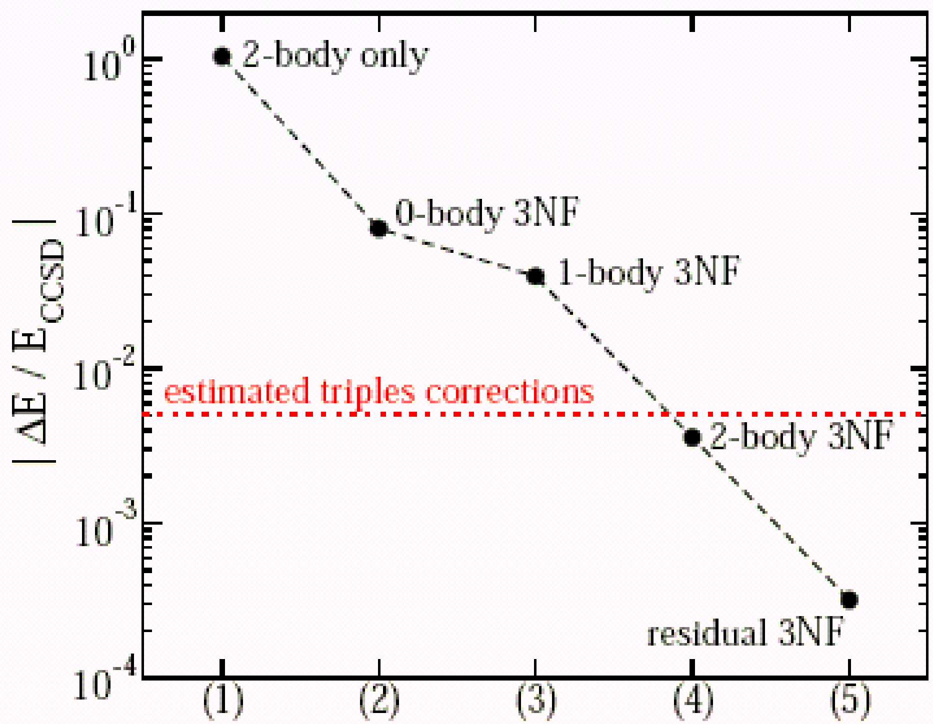




Different contributions to E(CCSD) from 3NF in He4

Three-body Hamiltonian in normal ordered form:

$$\hat{H}_3 = \frac{1}{6}\sum_{ijk}\langle ijk||ijk\rangle + \frac{1}{2}\sum_{ijpq}\langle ijp||ijq\rangle\{\hat{a}_p^\dagger\hat{a}_q\} + \frac{1}{4}\sum_{ipqrs}\langle ipq||irs\rangle\{\hat{a}_p^\dagger\hat{a}_q^\dagger\hat{a}_s\hat{a}_r\} + \hat{h}_3 \ ,$$



Really good news!

- The "density dependent" terms of 3NF are dominant!
- ϵ from residual 3NF costs $1-\epsilon$ of work !
- "2-body" machinery can be used.
- **Residual three-nucleon force can be neglected!**

What happens in nuclear matter calculations ?

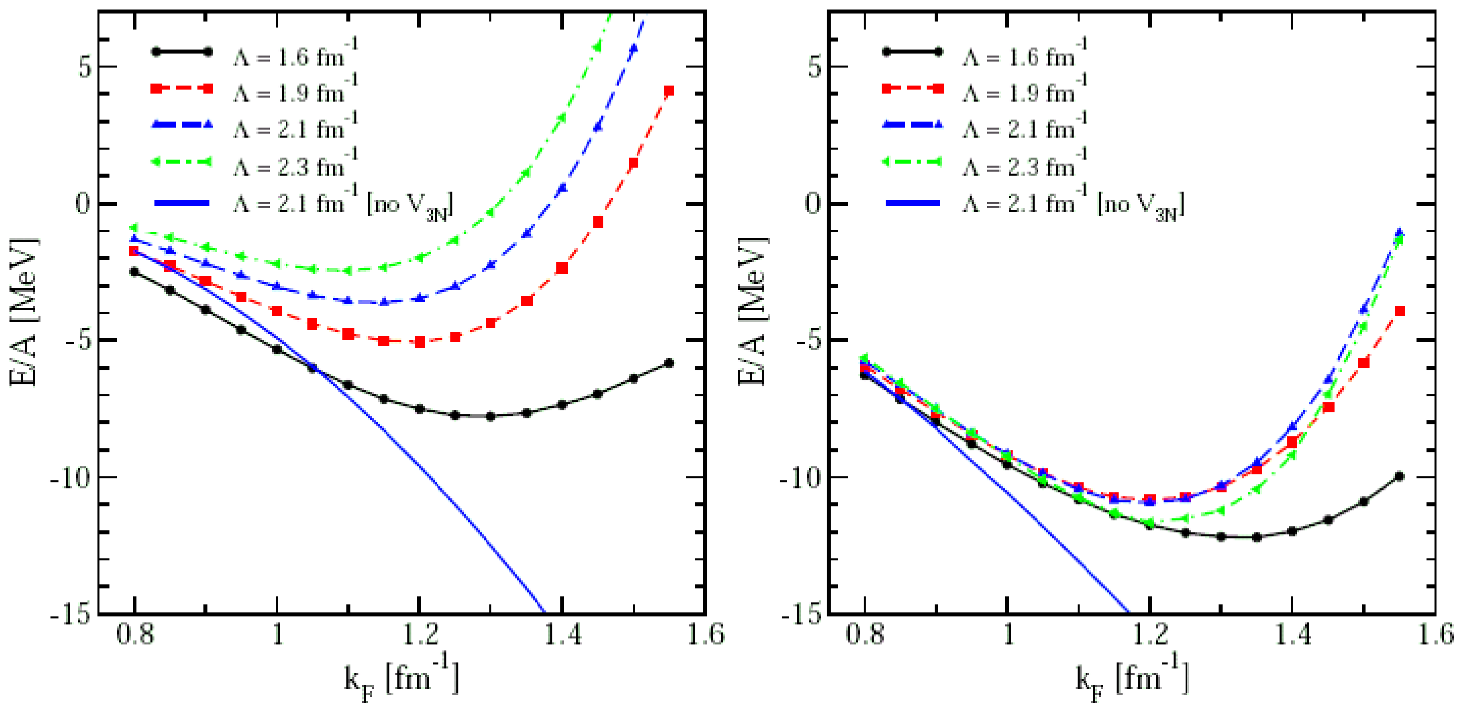


Fig. 6. Hartree-Fock (left figure) and Hartree-Fock plus dominant second-order contributions (right figure) calculated from $V_{\text{low }k}$ and V_{3N} for various cutoffs. Details of the approximate second-order calculations are given in the text.

Summary on low-momentum interactions

- The hard core of the nucleon-nucleon interaction makes many-body approaches difficult. Non-perturbative.
- By use of renormalization group theory or similarity transformations, high momentum modes can be integrated out, while preserving all two-body observables.
- However this procedure induces many-body forces since we remove degrees of freedom for the many-body system.
- Will effective three-body forces be sufficient to overcome the large overbinding seen in medium size nuclei ?
- Is there a systematic way of generating higher body forces as in the Chiral EFT approach ?
- Want a theory which minimizes the effect of many-body forces.