# Through the looking glass: And what nucleons do there

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Lecture 1/2

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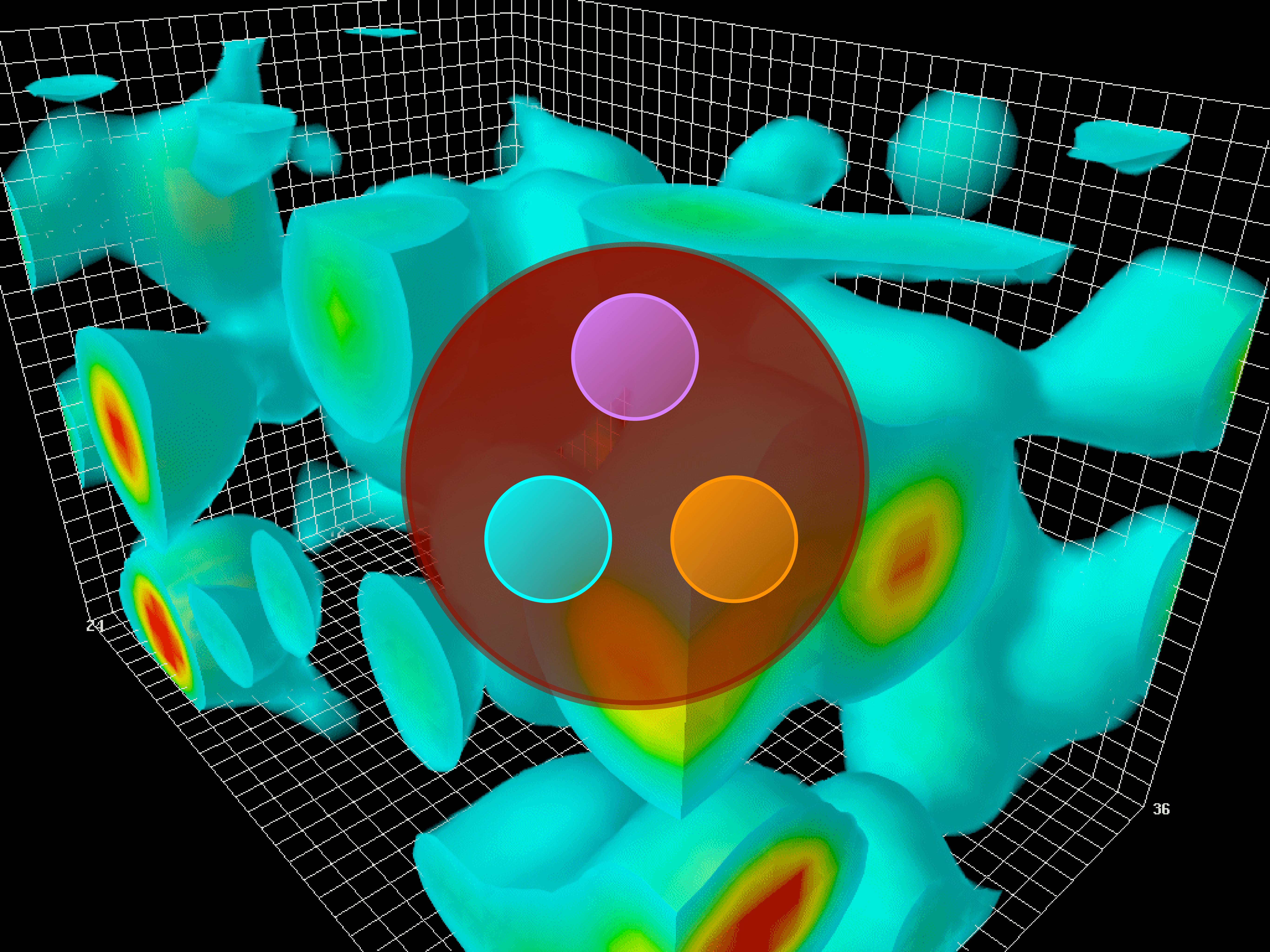
What does our world look like in the mirror?

# What does our world look like in the mirror?


Nucleon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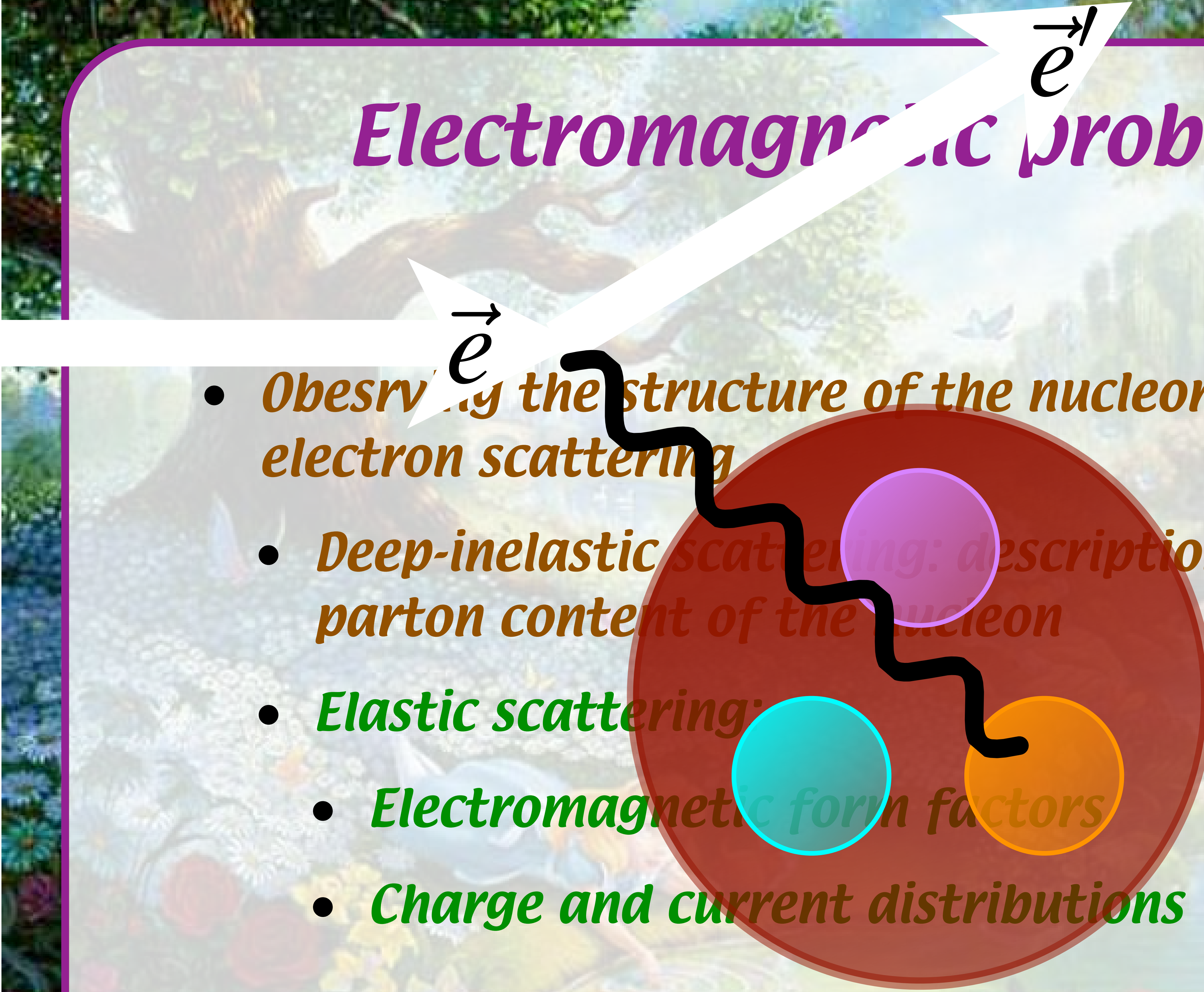
# Electromagnetic probe

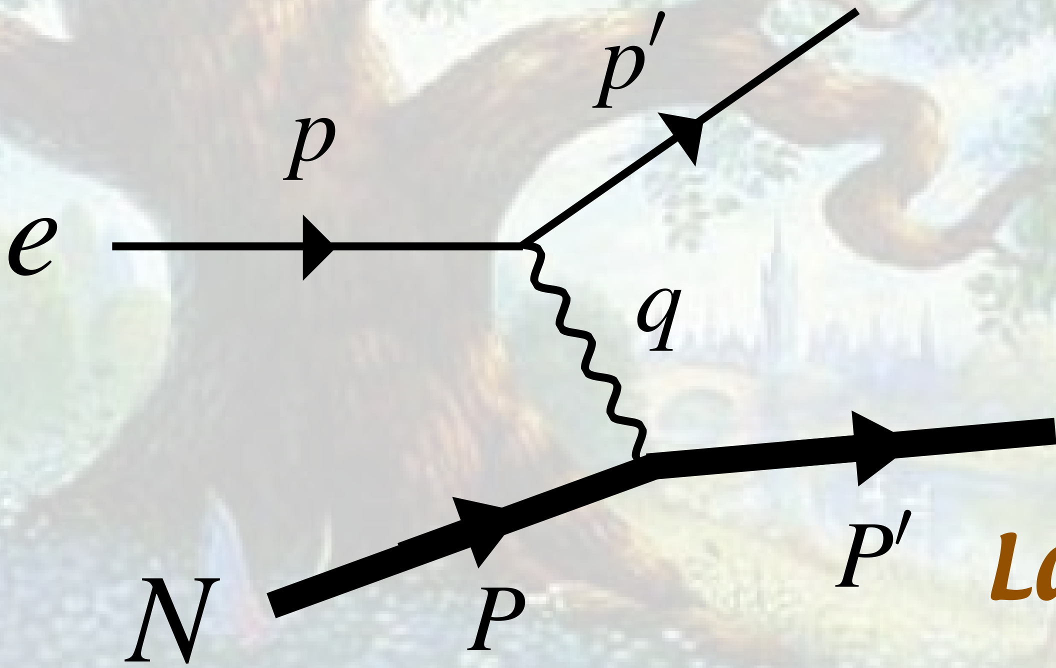
- Obesrving the structure of the nucleon in electron scattering
  - Deep-inelastic scattering: description of the parton content of the nucleon
  - Elastic scattering:
    - Electromagnetic form factors
    - Charge and current distributions

# Electromagnetic probe

- *Obesrving the structure of the nucleon in electron scattering*
  - *Deep-inelastic scattering: description of the parton content of the nucleon*
  - *Elastic scattering:*
    - *Electromagnetic form factors*
    - *Charge and current distributions*



# Elastic Scattering




$$q = p - p' = P' - P$$

$$p \equiv p^\mu = (\varepsilon, \vec{p})$$

$$P \equiv P^\mu = (E, \vec{P})$$

**Lab frame (proton target at rest)**

$$P^\mu = (M, \vec{0})$$

$\theta$ : *e scattering angle*

$$\frac{\varepsilon'}{\varepsilon} = \left(1 + \frac{2\varepsilon}{M}\sin^2\frac{\theta}{2}\right)^{-1}$$

$$Q^2 \equiv -q^2 = 4\varepsilon\varepsilon'\sin^2\frac{\theta}{2}$$

**Total scattering cross-section**

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4\varepsilon^2\sin^4\frac{\theta}{2}}\frac{\varepsilon'}{\varepsilon}\cos^2\frac{\theta}{2}\left\{F_1^2(Q^2) + \frac{Q^2}{4M^2}\left[F_2^2(Q^2) + 2\left(F_1(Q^2) + F_2(Q^2)\right)^2\tan^2\frac{\theta}{2}\right]\right\}$$

**Nucleon Form Factors**

# Form Factors

- ***Electromagnetic current***

$$j^{\mu}_{\mathrm{EM}} = Q\bar{\psi}\gamma^{\mu}\psi$$

- ***Structureless particle (spin-1/2)***

$$\langle l(p')|j^{\mu}(0)|l(p)\rangle = \bar{u}(p')\gamma^{\mu}u(p)$$

- ***Extended structure, like nucleon***

$$\langle N(P')|J^{\mu}(0)|N(P)\rangle = \bar{u}(P')\left[\gamma^{\mu}\underbrace{F_1^N}_{\text{Dirac}}(Q^2) + i\sigma^{\mu\nu}\frac{q_{\nu}}{2M}\underbrace{F_2^N}_{\text{Pauli}}(Q^2)\right]u(P)$$

$F_1(0) = Q$ ***Net charge***

$F_2(0) = \kappa$ ***Anomalous magnetic moment***

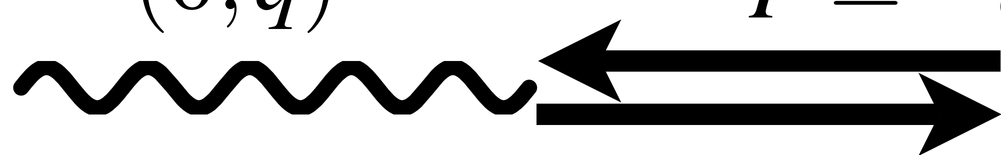
# The Breit Frame

## Sachs form factors

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

$$q^\mu = (0, \vec{q}) \qquad \vec{P} = -\vec{q}/2$$



$$\vec{P}' = \vec{q}/2$$

$$E = E'$$

**No energy transferred to target**

# The Breit Frame

## Sachs form factors

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

## Breit frame matrix elements

$$\langle N_{s'}(q/2)|J^0(0)|N_s(-\vec{q}/2)\rangle = 2M\, G_E(\vec{q}^2)\delta_{ss'}$$

$$\langle N_{s'}(q/2)|\vec{J}(0)|N_s(-\vec{q}/2)\rangle = G_M(\vec{q}^2)\chi^\dagger_{s'} i\vec{\sigma}\times\vec{q}\,\chi_s$$

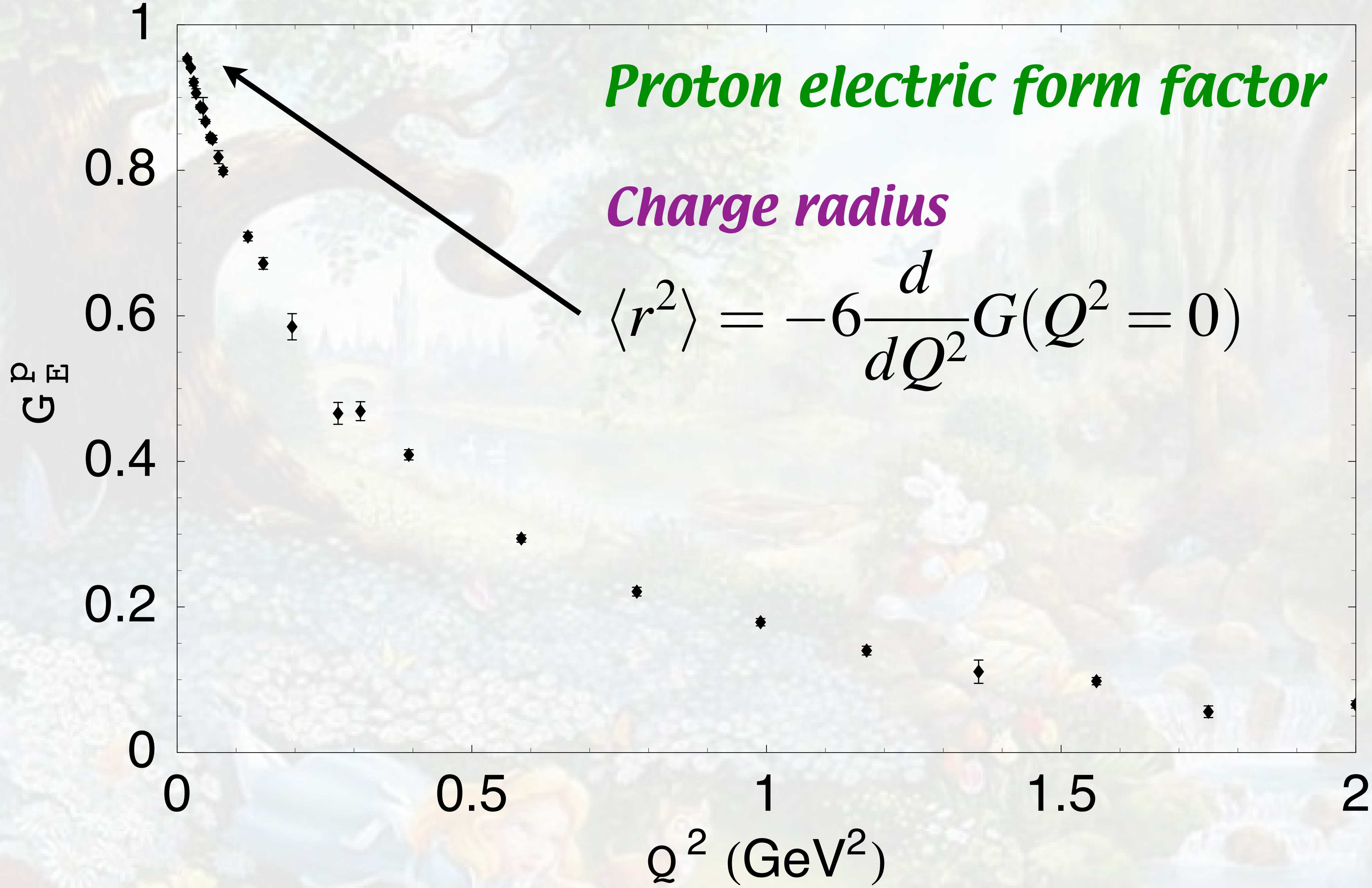
## Fourier transform gives classical charge distribution

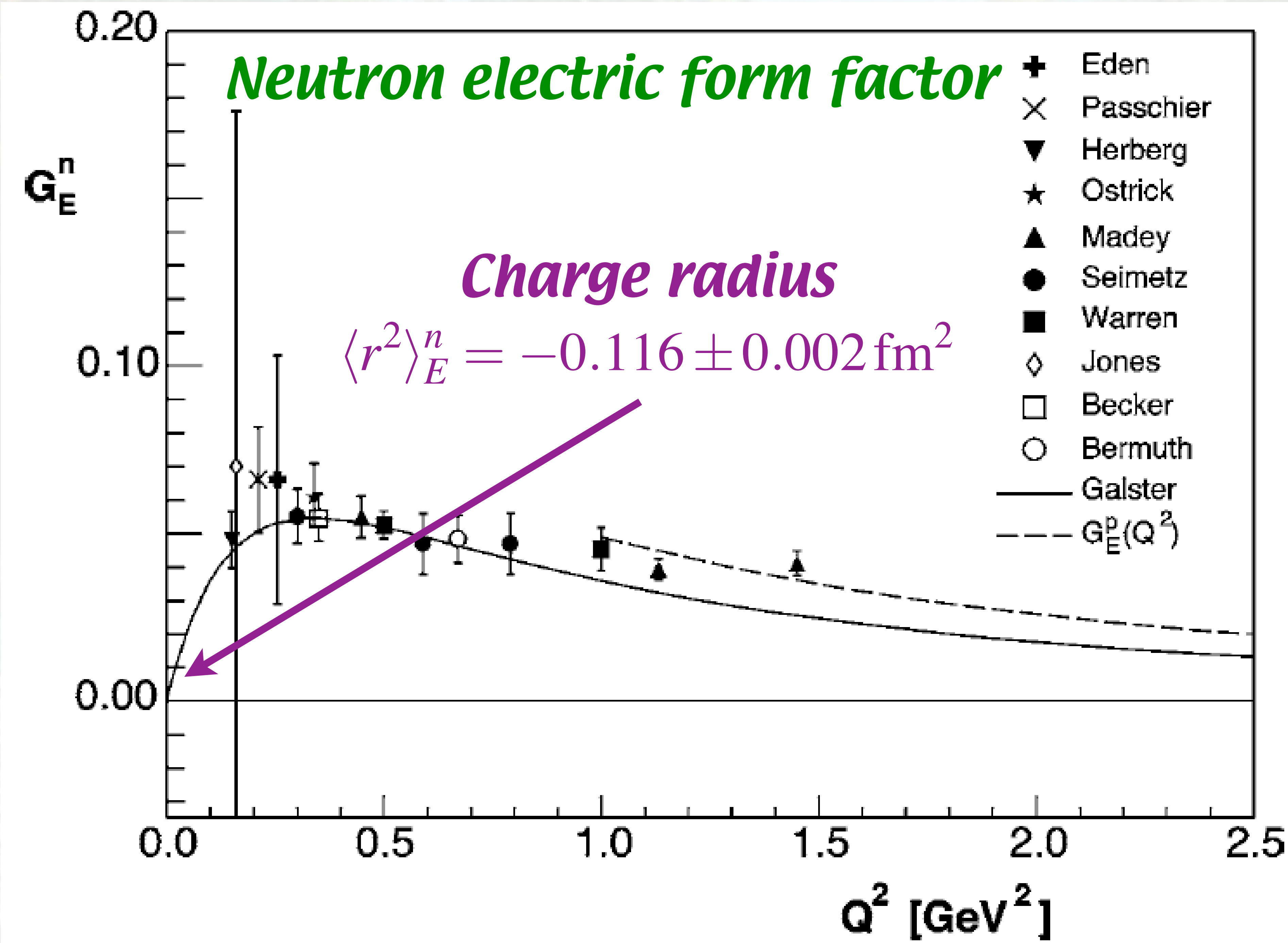
$$\rho_E(\vec{r}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} \frac{M}{E(\vec{q})} G_E(\vec{q}^2)$$

## Proton electric form factor

### Charge radius

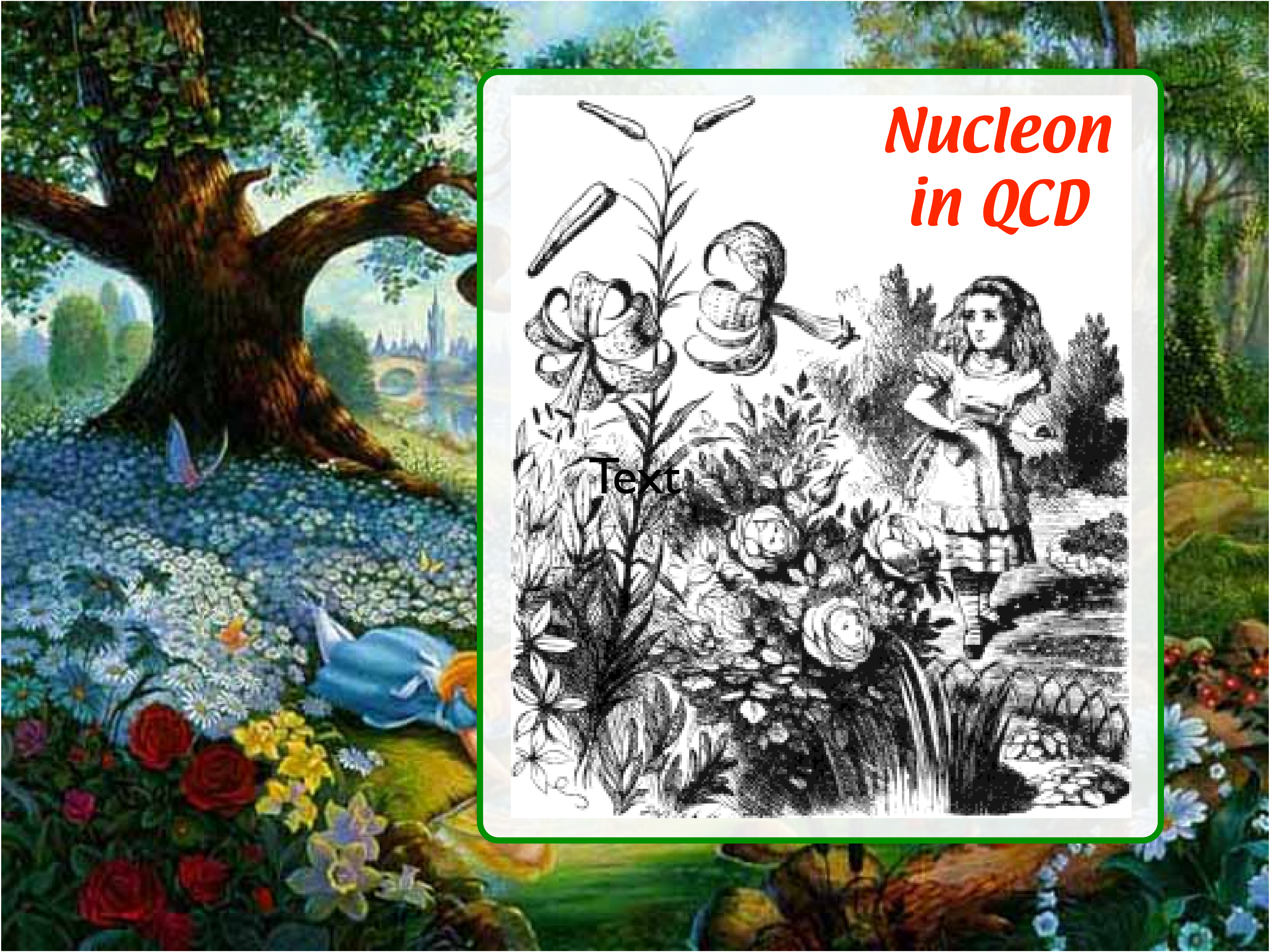
$$\langle r^2 \rangle = -6 \frac{d}{dQ^2} G(Q^2 = 0)$$





Neutron electric form factor
Charge radius
$\langle r^2 \rangle_E^n = -0.116 \pm 0.002\,\mathrm{fm}^2$
$G_E^n$
0.20
0.10
0.00
0.0
0.5
1.0
1.5
2.0
2.5
$Q^2$ [GeV$^2$]
Eden
Passchier
Herberg
Ostrick
Madey
Seimetz
Warren
Jones
Becker
Bermuth
Galster
$G_E^p(Q^2)$

# Nucleon in QCD



# QCD is Nonperturbative

- At low energy, strong coupling constant becomes large; **perturbation theory doesn't make sense**
- Require nonperturbative definition of path integral
  - Numerical evaluation in **lattice QCD** only known technique
  - eg. Nucleon mass:

fermion determinant

$$C_N(\tau) = \sum_{\vec{x}} \sum_{\text{gluons}} \overbrace{\chi_N(\vec{x},\tau)\bar{\chi}_N(\vec{0},0)}\,(\det M_f)^2 \exp\left[-S_G\right]$$

nucleon 2-pt correlator

gluon action

large $\tau$ $$C_N(\tau) = \lambda_0^2 e^{-E_0\tau} + \cancel{\lambda_1^2 e^{-E_1\tau}} + \ldots$$

nucleon mass

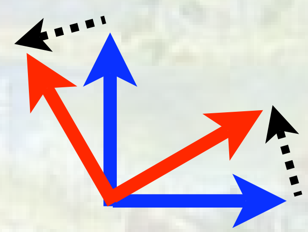
- *Lattice calculations are notoriously time consuming…*
- *What else can we learn while we wait?*

# Chiral symmetry

- *Free Lagrangian for 2 species of massless quarks*

$$\mathcal{L} = \bar{u}\partial\!\!\!/ u + \bar{d}\partial\!\!\!/ d = (\bar{u}\ \bar{d})\, \partial\!\!\!/ \begin{pmatrix} u \\ d \end{pmatrix} \qquad \psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

**Rotation** $\mathrm{SO}(2)$



$$\psi \to \exp\left[ i\phi \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \right] \psi = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \psi$$

**Unitary transformation** $\mathrm{SU}(2)$

$$\psi \to \exp\left[ i\vec{\phi}\cdot\vec{\sigma} \right] \psi$$

**Pauli matrices** $\vec{\sigma} = \left\{ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\}$

$$\sigma_i^\dagger = \sigma_i \qquad \{\sigma_i, \sigma_j\} = 2\delta_{ij}$$

# Chiral symmetry... 2

$$\mathcal{L} = \bar{u}\partial\!\!\!/ u + \bar{d}\partial\!\!\!/ d = \left(\bar{u}\ \bar{d}\right)\partial\!\!\!/ \begin{pmatrix} u \\ d \end{pmatrix}$$

$$\mathcal{L} = \bar{\psi}\partial\!\!\!/ \psi$$

**Transformation (vector)**

$$\psi \to \exp\left[i\vec{\phi}\cdot\vec{\sigma}\right]\psi$$

$$\bar{\psi} \to \bar{\psi}\exp\left[-i\vec{\phi}\cdot\vec{\sigma}^{\dagger}\right]$$

$$\mathcal{L} \to \mathcal{L}$$

$$m\bar{\psi}\psi \to m\bar{\psi}\psi$$

**Transformation (axial)**

$$\psi \to \exp\left[i\vec{\phi}\cdot\vec{\sigma}\gamma_5\right]\psi$$

$$\bar{\psi} \to \bar{\psi}\exp\left[i\vec{\phi}\cdot\vec{\sigma}\gamma_5\right]$$

$$\mathcal{L} \to \mathcal{L}$$

$$m\bar{\psi}\psi \to m\bar{\psi}\exp\left[2i\vec{\phi}\cdot\vec{\sigma}\gamma_5\right]\psi$$

**In chiral limit (m=0), QCD invariant under vector and axial vector transformations**

# Chiral symmetry... 3

- Noether's theorem: Continuous symmetry implies conserved current

**Vector**

$$\vec{V}^{\mu} = \bar{\psi}\gamma^{\mu}\vec{\sigma}\psi$$

**Axial**

$$\vec{A}^{\mu} = \bar{\psi}\gamma^{\mu}\gamma_5\vec{\sigma}\psi$$

- Chiral currents simply R=V+A and L=V-A
  - defining right- and left-handed currents
  - chirality~helicity: $\vec{p}.\vec{s}$

Under parity transformation:

$$V \to -V, \quad A \to A$$
$$R \to -L, \quad L \to -R$$

# Chiral symmetry… 3

- Noether's theorem: Continuous symmetry implies conserved current

For fermions with mass, helicity cannot be a good quantum number: One can always boost to a frame where the helicity is reversed

- Cl
  - defining right- and left-handed currents
  - chirality~helicity: $\vec{p}.\vec{s}$

Under parity transformation: $V \to -V, \quad A \to A$

$R \to -L, \quad L \to -R$

# Chiral symmetry in QCD

- Light quark masses are very close to the chiral limit

$$m_{u,d} \sim 5-10\,\mathrm{MeV} \qquad m_N \sim 1000\,\mathrm{MeV}$$

- Real world should be almost chirally symmetric
  - Axial charge

$$\vec{A}^{\mu} = \bar{\psi}\gamma^{\mu}\gamma_5\vec{\sigma}\psi$$

$$\vec{Q}_A = \int d^3x \vec{A}^0$$

Expect $[H_{QCD}, Q_A] \sim m_q$

Nucleon mass

$$H_{QCD}|N^+\rangle = M_N|N^+\rangle$$

Should see degenerate parity partner $|N^-\rangle \equiv Q_A|N^+\rangle$

$$H_{QCD}|N^-\rangle = M_{N*}|N^-\rangle \qquad M_{N*} - M_N \sim 600\,\mathrm{MeV}$$

# Chiral symmetry in QCD... 2

- Resolution of apparent problem
  - Chiral symmetry is spontaneously broken
    Goldstone mechanism:

"... if there is a continuous symmetry transformation under which the Lagrangian is invariant, then either the vacuum state is also invariant under the transformation, or there must exist particles of zero mass."

Pion is an approximate "Goldstone boson", with mass vanishing in the chiral limit

Gell-Mann, Oakes, Renner relation $m_\pi^2 \propto m_q$

Negative-parity partner state of the nucleon $|N^-\rangle = |N^+, \pi\rangle$

# An EFT for QCD

At low energies, quarks and gluons are not directly observed

Explicit quark/gluon degrees of freedom frozen into effective fields – hadrons

Weinberg: An effective Lagrangian, containing these effective fields, describes all the physics of the underlying field theory, provided all terms consistent with underlying symmetries are included.

Since near the chiral limit the pion is vanishingly light then the pion is the only relevant dynamical degree of freedom

At low-energies, QCD Lagrangian can be substituted for an effective Lagrangian of pions, constructed to be consistent with QCD

# An EFT for QCD… 2

- A power-counting scheme in effective field theory (EFT) allows a systematic expansion in the small energy scales of the system

$$\varepsilon \sim \frac{p^2}{\Lambda_\chi^2} \sim \frac{m_\pi^2}{\Lambda_\chi^2}$$

$\Lambda_\chi$ scale of chiral symmetry breaking $\sim 1\,\mathrm{GeV}$

Expansion in strong coupling replaced by perturbative energy expansion

with a set of low-energy constants to be determined empirically

# The nucleon in chiral EFT

- **Leading-order nucleon Lagrangian** $\quad \Psi = \begin{pmatrix} p \\ n \end{pmatrix}$

$$\mathcal{L}_{N\pi} = \bar{\Psi} \left\{ \not\partial - (M_0 + c_2 m_\pi^2) - \frac{g_A}{2 f_\pi} \gamma_\mu \gamma_5 \vec{\tau} (\partial^\mu \vec{\pi}) \right\} \Psi$$

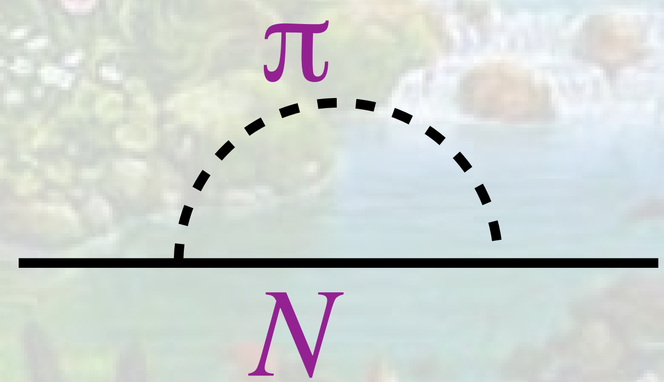
**Nucleon mass:**

$O(p^0): \quad M_0 \leftarrow$ chiral limit value

$O(p^2): \quad M_0 + c_2 m_\pi^2$

$m_\pi^2 \propto m_q$

**What about quantum fluctuations?**

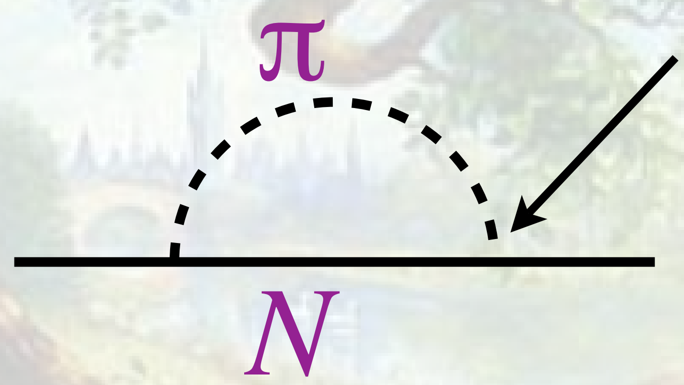


# Chiral correction

$$m_\pi^2 \propto m_q$$

$$O(p^2): \quad M_0 + c_2 m_\pi^2$$

$$O(p^3):$$



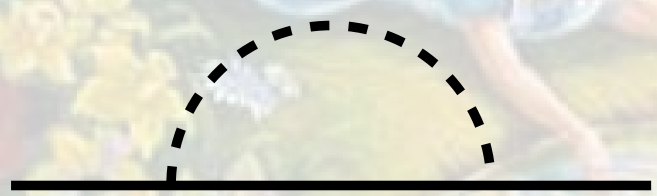

$$\sim \frac{g_A}{f_\pi}$$

***Nucleon axial charge***

***Pion decay constant***

***Quantum fluctuation, integrate over all intermediate pion energies***

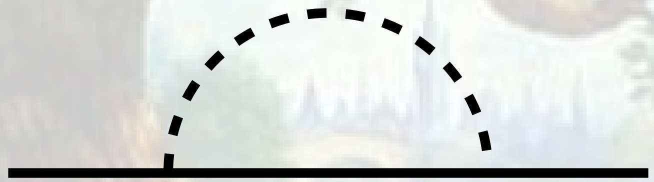
***Do some spin algegra, isospin sum, heavy-nucleon limit, and temporal and angular integration:***



$$= -\frac{3g_A^2}{16\pi^2 f_\pi^2} \int_0^\infty dk \frac{k^4}{k^2 + m_\pi^2}$$

# Poor-man renormalisation

$$O(p^2): \quad M_0 + c_2 m_\pi^2$$



$$= -\frac{3g_A^2}{16\pi^2 f_\pi^2}\int_0^\infty dk\,\frac{k^4}{k^2+m_\pi^2}$$

cubic divergence!

$$\int_0^\infty dk\,\frac{k^4}{k^2+m_\pi^2} = \int_0^\infty dk\,\frac{k^4-m_\pi^4+m_\pi^4}{k^2+m_\pi^2}$$

$$= \int dk\,\left(k^2-m_\pi^2\right) + \int dk\,\frac{m_\pi^4}{k^2+m_\pi^2} = \frac{\pi}{2}m_\pi^3$$

$$\boxed{M_N = M_0 + c_2 m_\pi^2 - \frac{3g_A^2}{16\pi^2 f_\pi^2}\left[\int dk\,k^2 - m_\pi^2\int dk + \frac{\pi}{2}m_\pi^3\right]}$$

# Renormalised expansion

$$\boxed{M_N = M_0 + c_2 m_\pi^2 - \frac{3g_A^2}{16\pi^2 f_\pi^2}\left[\int dk\, k^2 - m_\pi^2 \int dk + \frac{\pi}{2} m_\pi^3\right]}$$

$$M_0^{ren} = M_0 - \frac{3g_A^2}{16\pi^2 f_\pi^2}\int dk\, k^2$$

$$c_2^{ren} = c_2 + \frac{3g_A^2}{16\pi^2 f_\pi^2}\int dk$$

Absorb infinities into redefinition of expansion constants

$$\boxed{M_N = M_0^{ren} + c_2^{ren} m_\pi^2 - \frac{3g_A^2}{32\pi f_\pi^2} m_\pi^3}$$

Nonanalytic term: model-independent

# Recap

- Electromagnetic form factors encode the charge and current distributions in the nucleon

- Chiral symmetry can provide nontrivial constraints on the low-energy structure of the nucleon

- Important to incorporate these features in building a picture of the nucleon

# Tomorrow

- Getting on to parity violation
  - The weak interaction and nucleon structure
- Describing the strange-quark content of the nucleon