QCD and Rescattering in Nuclear Targets

Lecture 5

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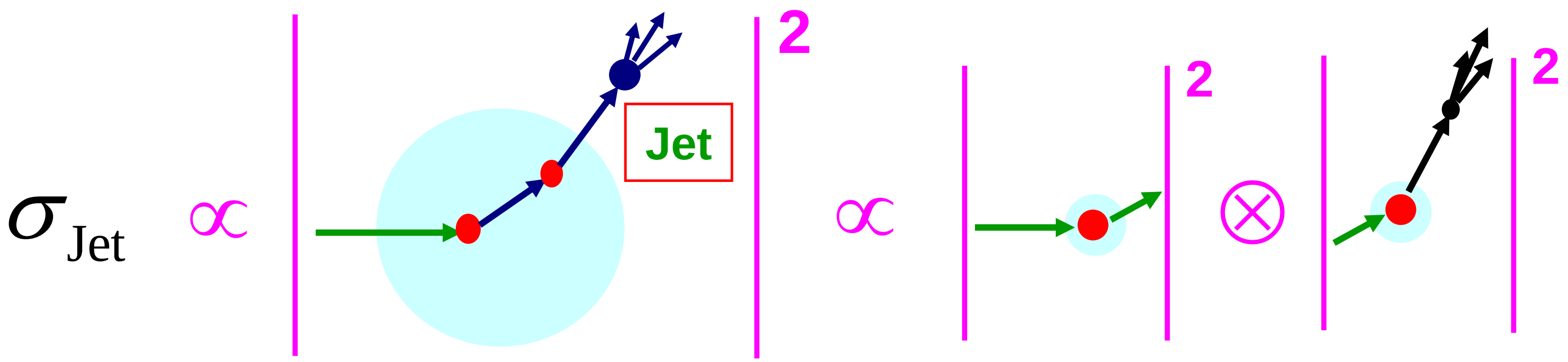
Incoherent Multiple Scattering

- ❑ Incoherent/independent rescattering
- ❑ Medium induced energy loss
- ❑ Jet quenching
- ❑ Important role of p(d)+A collision
- ❑ Calculation of Cronin effect
- ❑ Calculable transverse momentum broadening
- ❑ Summary and outlook

Incoherent/independent multiple scattering

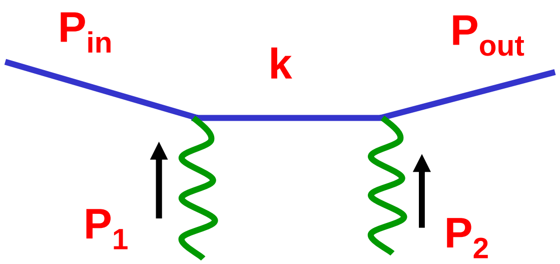
- **Weak quantum interference between scattering centers**



- **Modify jet spectrum without changing the total rate**
- **Nuclear dependence from the scattering centers'**
  - **density**
  - **number**
  - **momentum distribution and cut-off (new scale)**
  - **etc**

Parton level multiple scattering

❑ Classical multiple scattering – cross section level:



$\Rightarrow$ Kinematics fix only $P_1 + P_2$

either P_1 or P_2 can be ~ zero

$$d\sigma^{\text{Double}} \sim \sigma^{\text{single}}(p_{in}, p_1)\cdot\sigma^{\text{single}}(p_2, p_{out})dp_1 dp_2 \delta(p_1 + p_2 + p_{in} - p_{out})$$

❑ Parton level multiple scattering (incoherent/independent)

In massless pQCD, above $\sigma^{\text{single}} \to \infty$ as p_1 or $p_2 \to 0$

- ❖ parton distribution at x=0 is ill-defined
- ❖ pinch poles of k in above definition

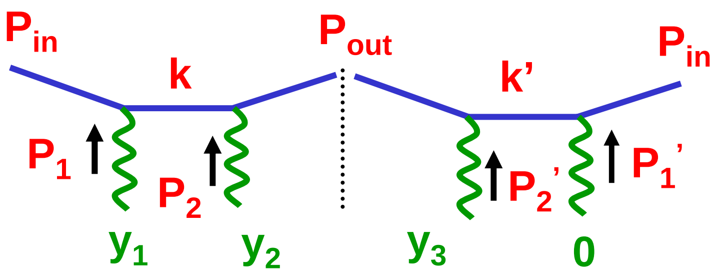
$$\Longrightarrow \quad d\sigma^{\text{double}} \to \infty \text{ as } p_1 \text{ or } p_2 \to 0$$

Need to include quantum interference diagrams

Multiple scattering in QCD

❑ Quantum mechanical multiple scattering

- Amplitude level



$$\langle A | \phi^+(0)\phi^+(y_3)\phi(y_2)\phi(y_1) | A \rangle$$

- ❖ 3-independent parton momenta
- ❖ no pinched poles
- ❖ depends on 4-parton correlation functions

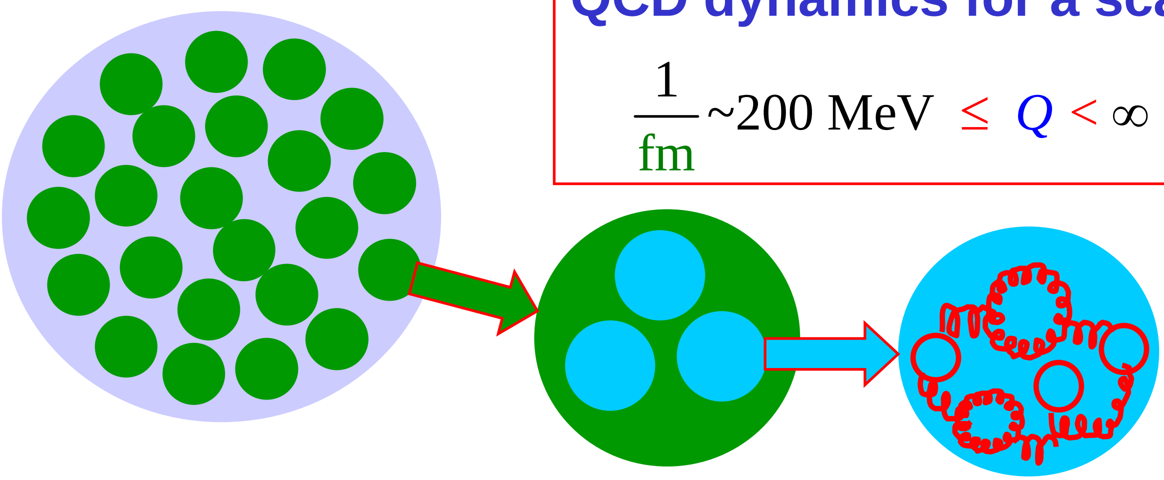
$$\Rightarrow \quad d\sigma^{\text{double}} \propto C^{\text{double}}(p_{in}, p_1, p_2, p_1', p_2', p_{out})T^{\text{double}}(p_1, p_2, p_1', p_2')$$
$$\times \; dp_1 dp_2 dp_1' dp_2' \delta(p_1 + p_2 + p_1' + p_2' + p_{in} - p_{out})$$

$$T^{\text{double}}(p_1, p_2, p_1', p_2') \propto \langle A | \phi^+(0)\phi^+(y_3)\phi(y_2)\phi(y_1) | A \rangle dy_1 dy_2 dy_3$$

It has no probability interpretation!

QCD Matter

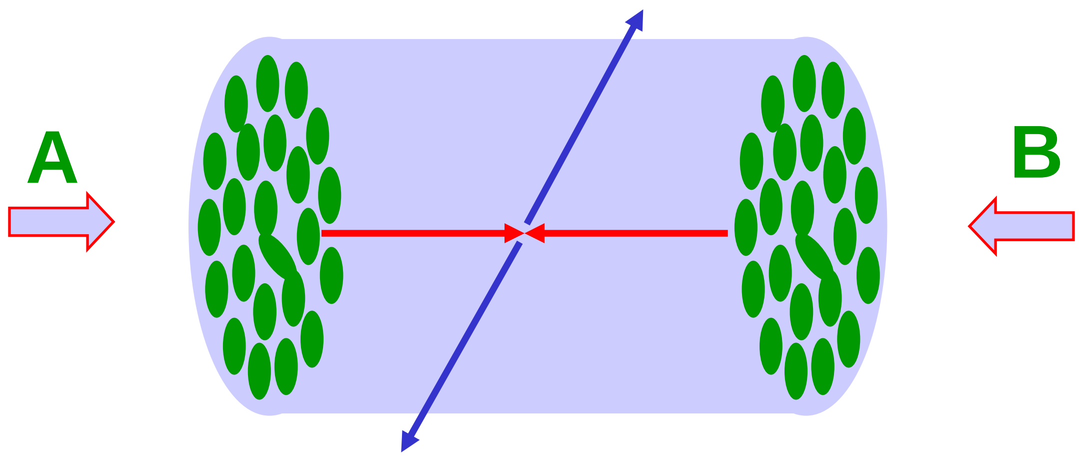
- **Hadronic matter:**
  - **nuclei**
  - **baryons**
  - **mesons**



QCD dynamics for a scale

$$\frac{1}{\text{fm}} \sim 200\text{ MeV} \leq Q < \infty$$

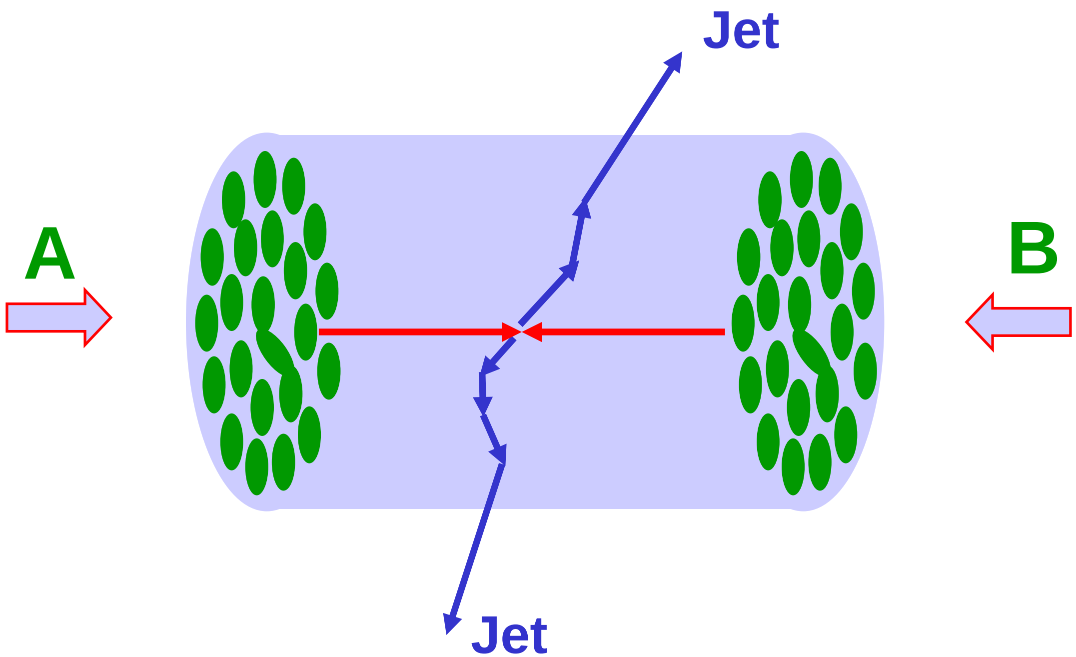
- **Hot matter produced in heavy ion collisions**



Initial hard collision should not be coherent with later multiple scattering of the high p_T jets

Jet Tomography

- High p_T jet suffers multiple scattering in medium



Prerequisites

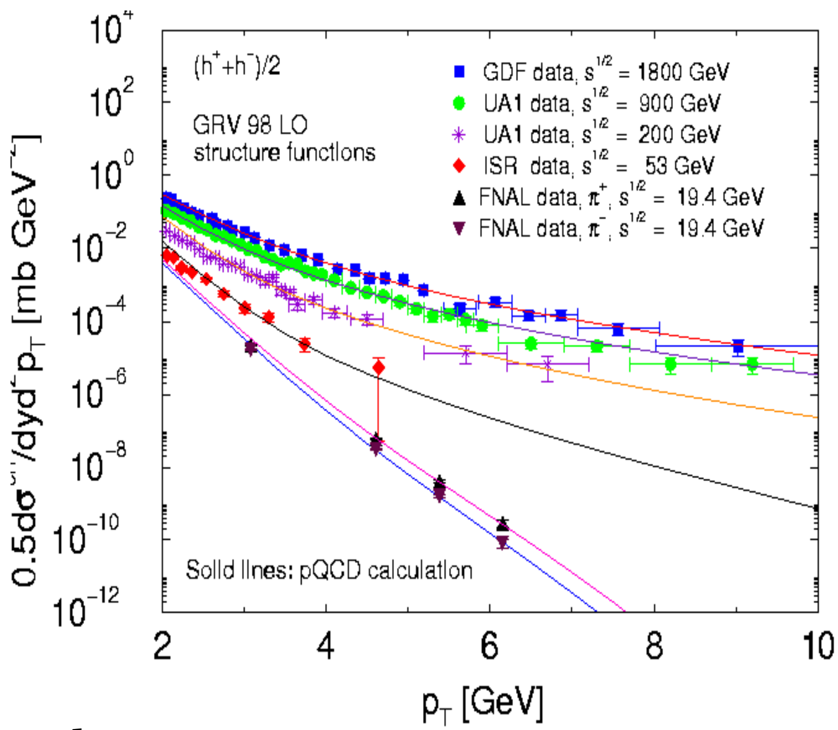
- Calibrated source
- Calculable absorption cross sections
- Interpretation of the results

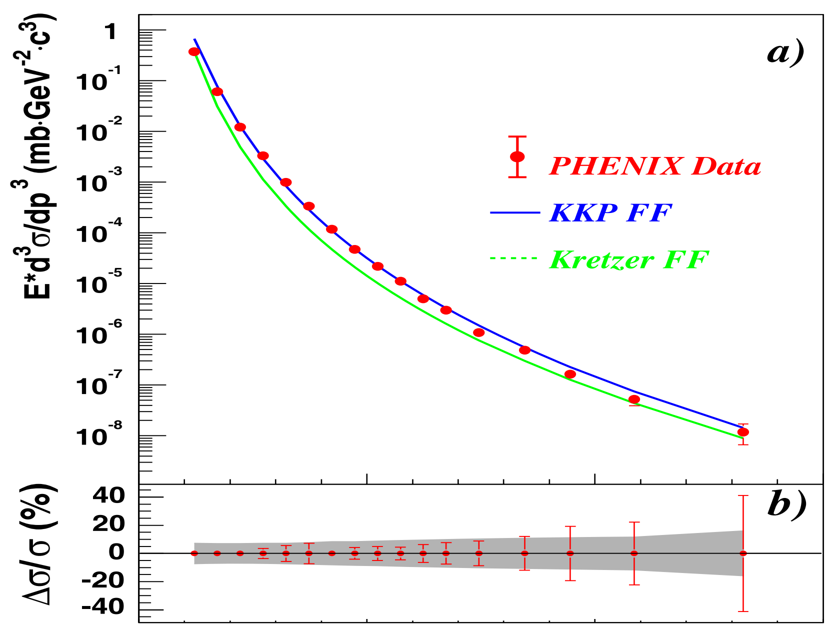
A change of single jet spectrum or two-jet correlation provides information on medium properties – tomography

Calibrated probes

- Inclusive hadron distribution – calculable in perturbative QCD

Leading order pQCD phenomenology | **Next-to-Leading order pQCD**

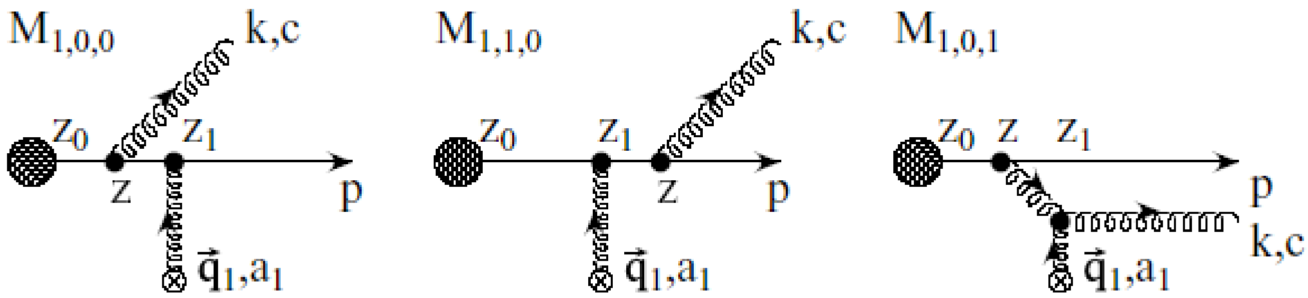




$$E_h \frac{d\sigma}{d^3p} \propto f_{a/A}(x_a, Q^2) \otimes f_{b/B}(x_b, Q^2) \otimes \frac{d\sigma^{ab\to cd}}{dt} \otimes D_{h/c}(z_c^*, Q^2)$$

Parton distribution functions | **Perturbative cross sections** | **Fragmentation functions**

Medium indenced rescattering



$$M_{1,1,0} = J(p) e^{ipx_0} (-i) \int \frac{d^2\mathbf{q}_1}{(2\pi)^2} v(0, \mathbf{q}_1)\, e^{-i\mathbf{q}_1 \cdot \mathbf{b}_1} \times$$
$$\times (-2ig_s) \frac{\epsilon \cdot \mathbf{k}}{\mathbf{k}^2} e^{i\omega_0 z_1} c a_1 T_{a_1} .$$

Answer depends on the choice of source potential $v(0, \vec{q}_1)$ and the scattering phases

$$M_{1,0,1} = J(p) e^{ipx_0} (-i) \int \frac{d^2\mathbf{q}_1}{(2\pi)^2} v(0, \mathbf{q}_1)\, e^{-i\mathbf{q}_1 \cdot \mathbf{b}_1} \times$$
$$\times 2ig_s \frac{\epsilon \cdot (\mathbf{k} - \mathbf{q}_1)}{(\mathbf{k} - \mathbf{q}_1)^2} e^{i(\omega_0 - \omega_1) z_1} (e^{i\omega_1 z_1} - e^{i\omega_1 z_0}) [c, a_1] T_{a_1}$$

Introduce approximation to make sense of the calculation

Medium induced QCD energy loss

❑ Approximation:

G.Bertsch and F.Gunion, PRD25 (1982)

$$\mathcal{M}_1 = -2ig_s e^{it_0\omega_0}\vec{\epsilon}_\perp \cdot \left\{ \cancel{\vec{H} a_1 c} + \vec{B}_1 e^{it_{10}\omega_0}\,[c, a_1] + \cancel{\vec{C}_1 e^{-it_{10}(\omega_1-\omega_0))}}\,[c, a_1] \right\} .$$

$$\mathbf{H} = \frac{\mathbf{k}}{\mathbf{k}^2}\,, \qquad \mathbf{C}_{(i_1 i_2 \cdots i_m)} = \frac{(\mathbf{k} - \mathbf{q}_{i_1} - \mathbf{q}_{i_2} - \cdots - \mathbf{q}_{i_m})}{(\mathbf{k} - \mathbf{q}_{i_1} - \mathbf{q}_{i_2} - \cdots - \mathbf{q}_{i_m})^2}\,,$$

$$\mathbf{B}_i = \mathbf{H} - \mathbf{C}_i\,, \qquad \mathbf{B}_{(i_1 i_2 \cdots i_m)(j_1 j_2 \cdots i_n)} = \mathbf{C}_{(i_1 i_2 \cdots j_m)} - \mathbf{C}_{(j_1 j_2 \cdots j_n)}\ .$$

❑ Induced gluon production:

$$\frac{dN_g^{(GB)}}{dy\,d^2\vec{k}_\perp} = C_A \frac{\alpha_s}{\pi^2} \frac{q_1^2}{k_\perp^2 (k - q_1)_\perp^2}$$

Where $y = \ln 1/x$ **is interpreted as rapidity**

❑ Gluon production in a medium:

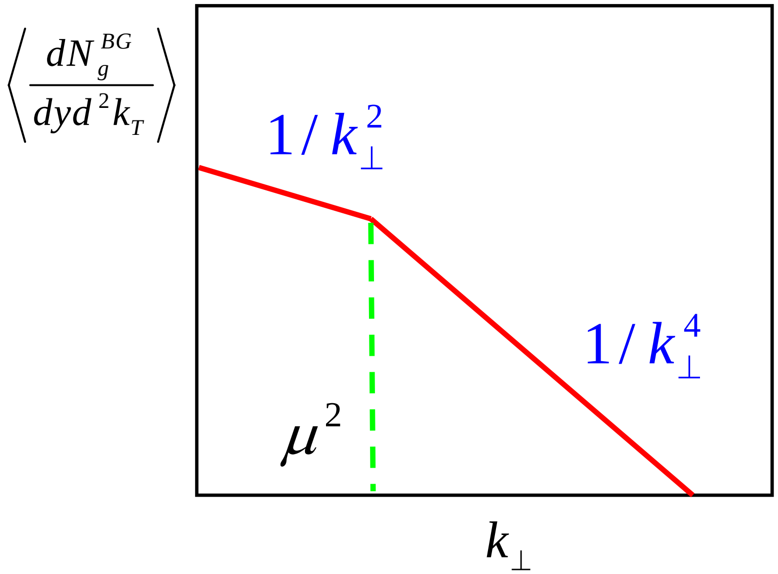
$$\left\langle \frac{dN_g^{BG}}{dy\,d^2k_T} \right\rangle_{q-transfer} = \int d^2q_T \left[\frac{\mu^2}{\pi (q_T^2 + \mu^2)^2} \right] \frac{C_A \alpha_s(k_T)}{\pi^2} \frac{q_T{}^2}{k_T^2 (k_T - q_T)^2}$$

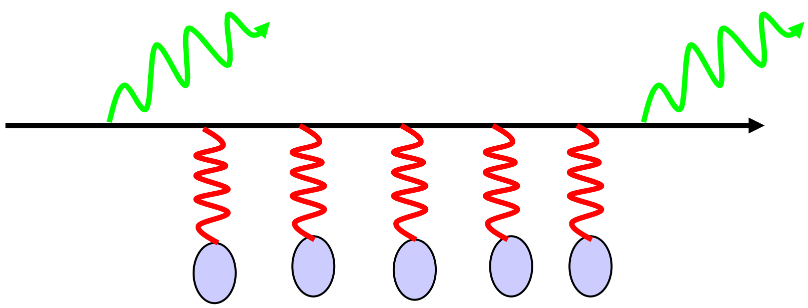
$$\sim \frac{C_A \alpha_s(k_T)}{\pi^2} \frac{\mu^2}{k_T^2 (k_T^2 + \mu^2)}$$

Screen mas μ

Vitev, QM2004

Momentum distribution of induced gluon





- The scattering scale does not follow from calculation

  = input screen mass

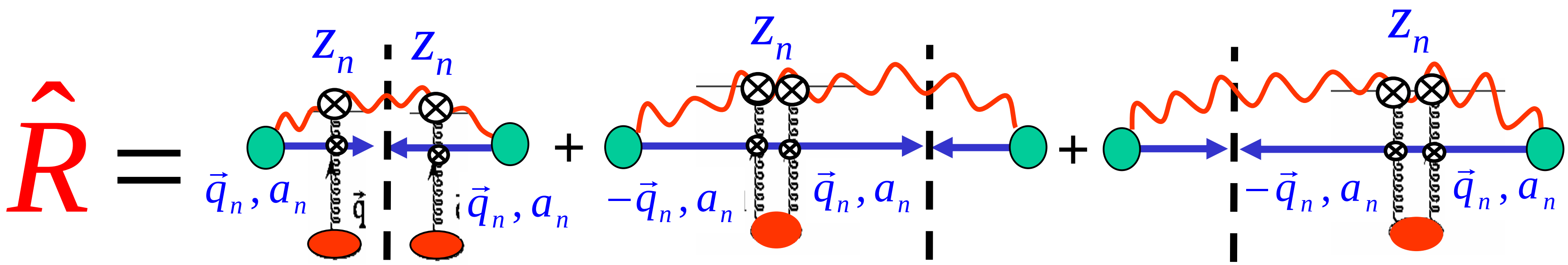
$$\langle k_T^{\,2} \rangle = \mu^2$$

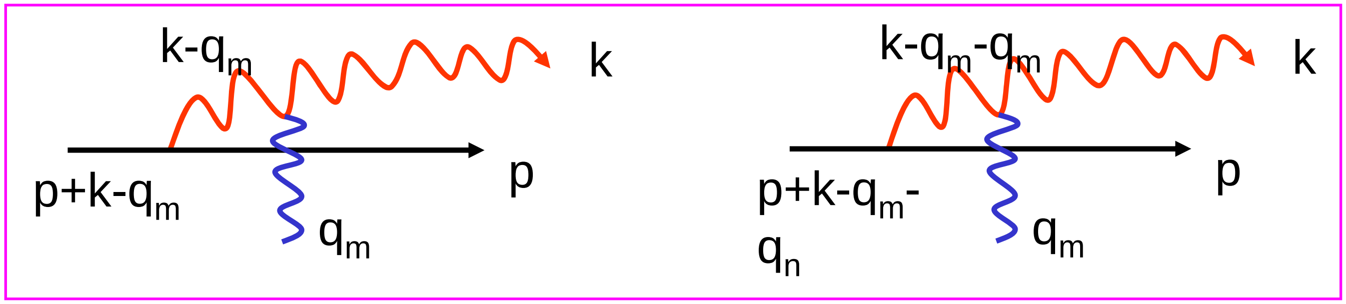
- the scale will enlarge by the nuclear size

$$\langle k_T^{\,2} \rangle = \mu^2 A^{1/3} = Q_s^2$$

Reaction operator approach

Gyulassy, Levai, Vitev, Nucl.Phys.B571 (2000); Nucl.Phys.B594 (2001)





$$\Gamma_m \equiv (2p+k-q_m)_\alpha \Gamma^\alpha(k;q_m) \approx 2E^+ (\epsilon\cdot(\mathbf{k}-\mathbf{q}_m)) \;,$$

$$\Gamma_{mn} \equiv (2p+k-q_m-q_n)_\alpha \Gamma^\alpha(k;q_n,q_m) \approx 2E^+ k^+ (\epsilon\cdot(\mathbf{k}-\mathbf{q}_m-\mathbf{q}_n))$$

Gluon **Gluon polarization** **Parent parton**

$$k = [xE^+, k^- \equiv \omega_0, \mathbf{k}] \;,\; \epsilon(k) = [0, 2\frac{\epsilon\cdot\mathbf{k}}{xE^+}, \epsilon] \;,\; p = [(1-x)E^+, p^-, \mathbf{p}]$$

Approximation: $E^+ \gg k^+ \gg \omega_{(i\cdots j)} \gg \frac{(\mathbf{p}+\mathbf{k})^2}{E^+}$

QCD radiative energy loss

- **Effective 2D Schroedinger equation**

R.Baier *et al.*, Nucl.Phys.B 483 (1997); *ibid.* 484 (1997)

- **Path integral formulation**

B.Zakharov, JETP Lett. 63 (1996)
U.Wiedemann, Nucl.Phys.B588 (2000

- **Reaction operator approach**

M.Gyulassy *et al.*, Nucl.Phys.B594 (2001); Phys.Rev.Lett.85 (2000

$$\Delta E^{(1)} \approx \frac{C_R \alpha_s}{4} \frac{\mu^2 L^2}{\lambda_g} \mathrm{Log} \frac{2E}{\mu^2(L)L} + \ldots ,$$

– Static medium

$$\Delta E^{(1)} \approx \frac{9\pi C_R \alpha_s^3}{4} L \frac{1}{A_\perp} \frac{dN^g}{dy} \mathrm{Log} \frac{2E}{\mu^2(L)L} + \ldots ,$$

– 1+1D Bjorken

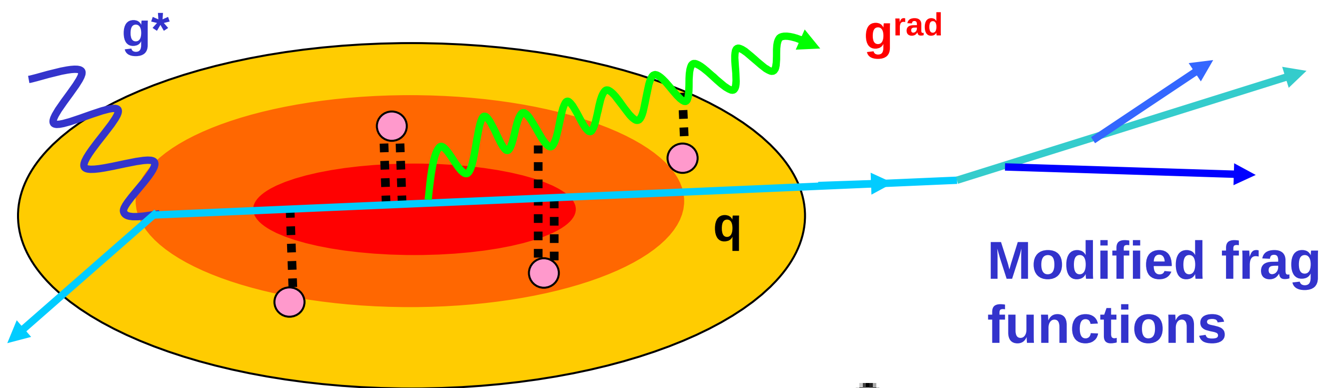
- **Elastic energy lose:**

J.D.Bjorken, SLAC preprint (1982) unpublished

$$\Delta E^{elastic} \approx 6\alpha_s^2 T^2 e^{-\mu/T} \left(1 + \frac{\mu}{T}\right) L \ln \frac{4E_{jet}T}{\mu^2}$$

Modified jet cross section – jet quenching

Vitev, QM2004



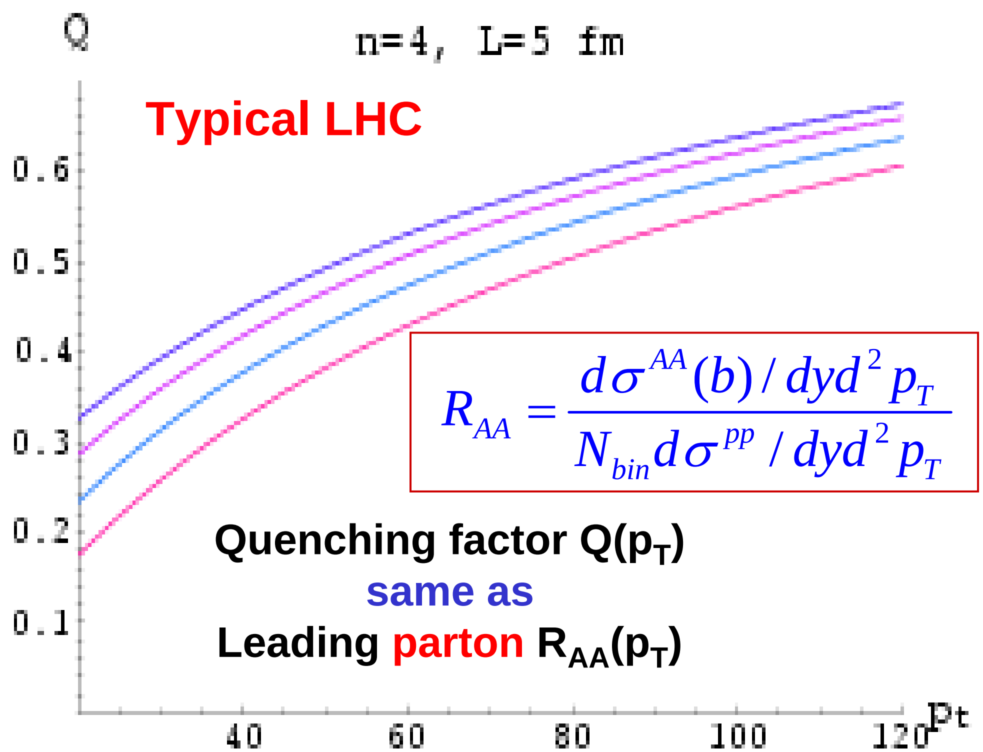
$$\pi^0 \quad d\sigma^{hadron}$$

Modified fragmentation functions

Reduced partonic cross sections

$$\frac{d\sigma^{vac}}{dp_T{}^2} \propto \frac{1}{(p_T)^n}, \quad \omega_c = \frac{\hat{q}L^2}{2} = \frac{\mu^2}{\lambda_g}\frac{L^2}{2}$$

$$Q(p_T) \simeq \exp\left(-2C_R\alpha_s\sqrt{\frac{2n\omega_c}{\pi\, p_T}}\right)$$



Gluon transport coefficient:

$$\hat{q}^{cold} \simeq 0.045\ GeV^2/fm, \quad \hat{q}^{hot} \simeq 1\ GeV^2/fm$$

R.Baier *et al.*, JHEP0109 , (2001)

Suppression of fragmentation functions

Independent Poisson approximation for multiple gluon emission

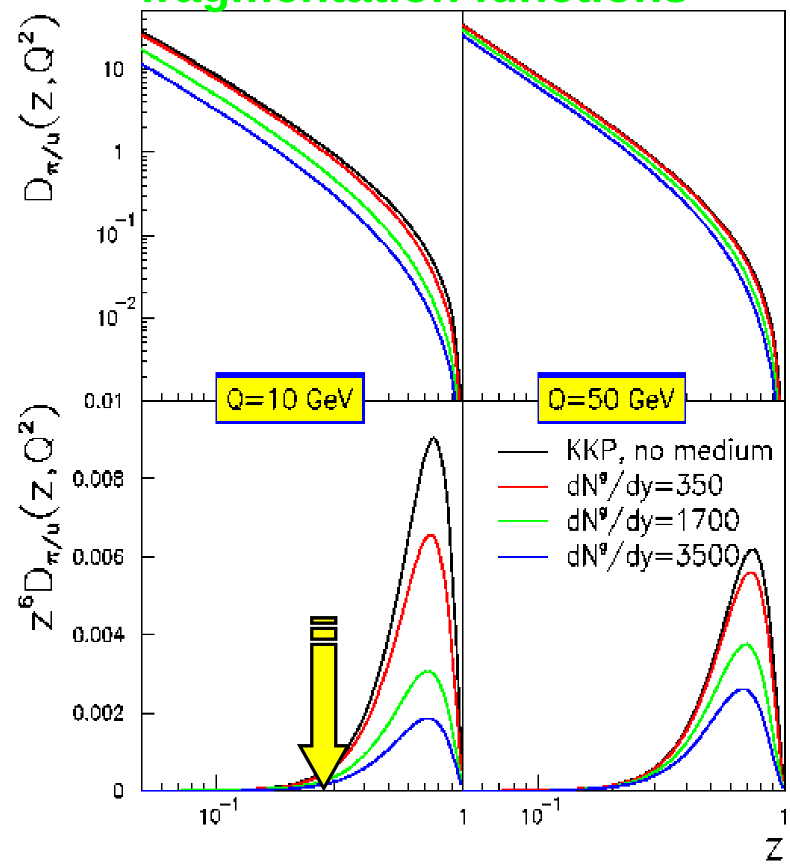
Probability for fractional energy loss $\varepsilon = \Delta E / E_{jet}$

$$P(\varepsilon) = \sum_{n=0}^{\infty} \frac{1}{n!} \left[\prod_{i=1}^{n} \int d\omega_i \frac{dN(\omega_i)}{d\omega} \right] \times e^{-\int d\omega \frac{dN}{d\omega}} \delta\left(\varepsilon - \sum_{i=1}^{n} \frac{\omega_i}{E_{jet}} \right)$$

Normalized for suppressed leading hadrons (no feedback)

$$D^{med}{}_{h/q}(z,Q^2) = \int_0^1 d\varepsilon\, P(\varepsilon) \frac{1}{1-\varepsilon} D^{vac}{}_{h/q}\left(\frac{z}{1-\varepsilon}, Q^2 \right)$$

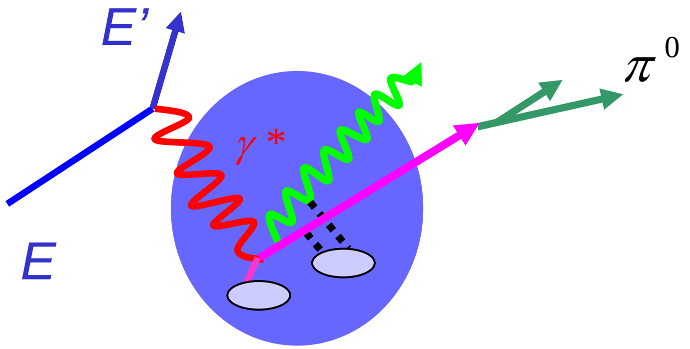
Kniehl, Kramer, Potter fragmentation functions



$z=p_h/p_c$

C.Salgado, U.Wiedemann, Phys.Rev.Lett. 89 (2002)

Semi-inclusive DIS



$\nu = E - E'$ - energy transfer

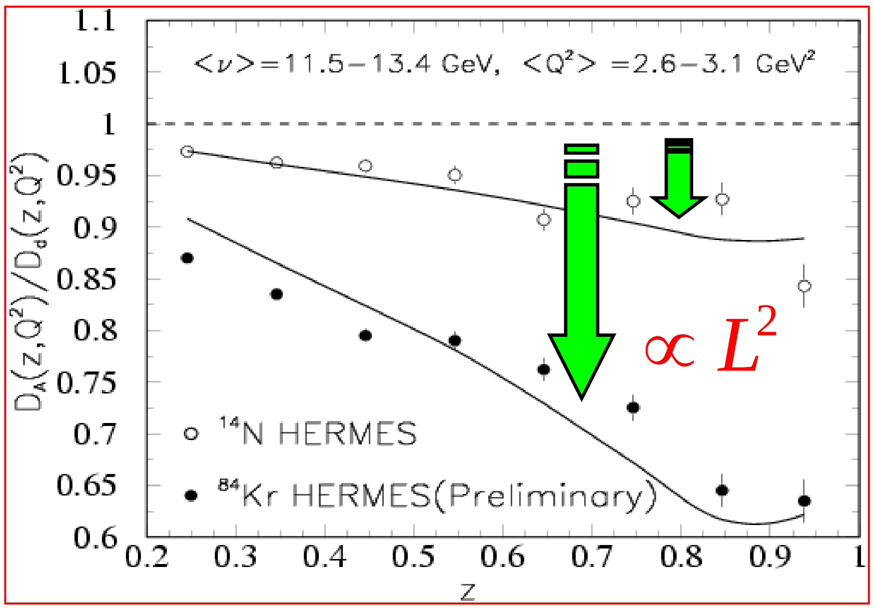
$\langle \Delta z \rangle$ - radiative energy loss fraction

$$\Delta E = \nu \langle \Delta z \rangle = (E - E')\langle \Delta z \rangle$$

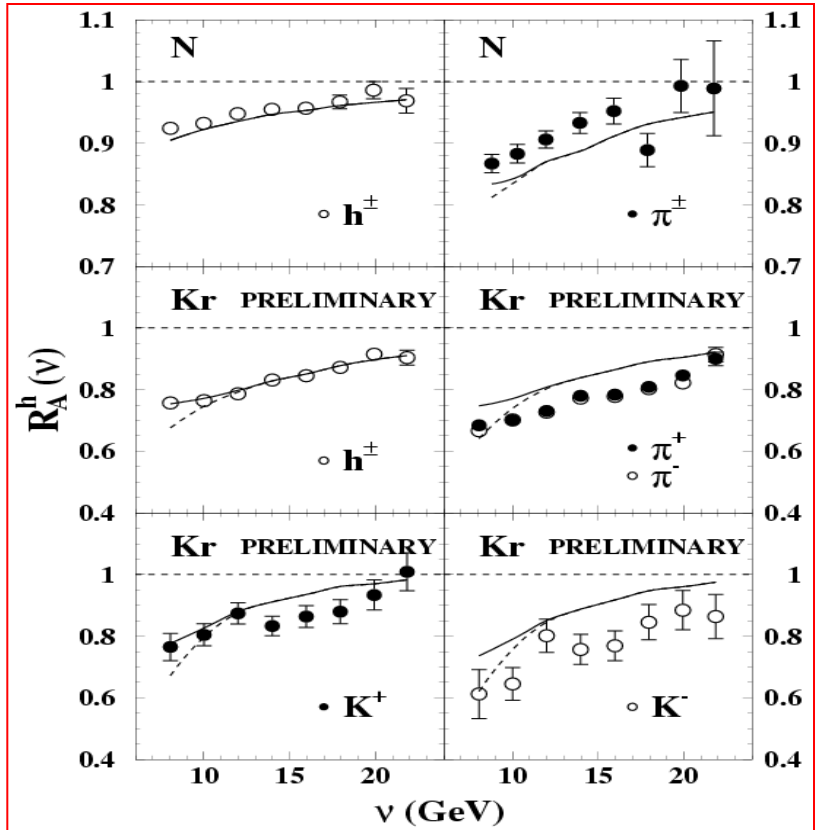
$$x_B = Q^2/(2p\cdot q), \quad x_A = 1/(m_N R_A)$$

$$\langle \Delta z \rangle = \tilde{C}(Q^2)\frac{C_A \alpha_s^2(Q^2)}{N_c}\frac{x_B}{Q^2 x_A^2} 6 \ln\frac{1}{2x_B}$$

$$\langle -dE/dL \rangle_{cold} \approx 0.5 - 0.6\ GeV/fm$$

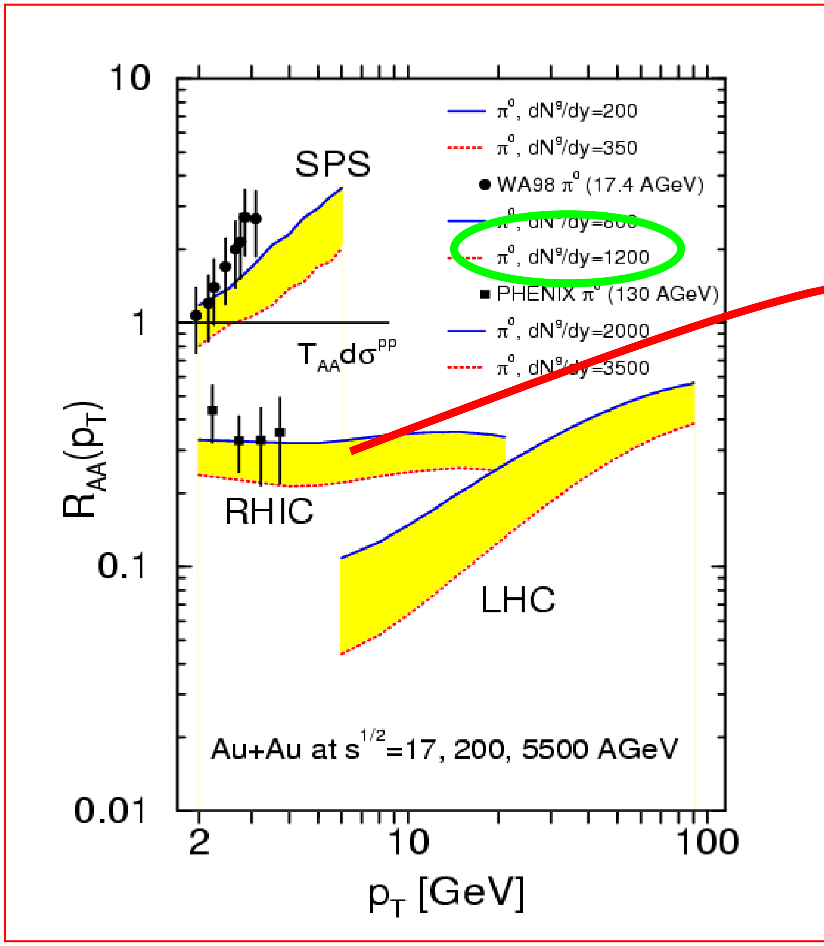


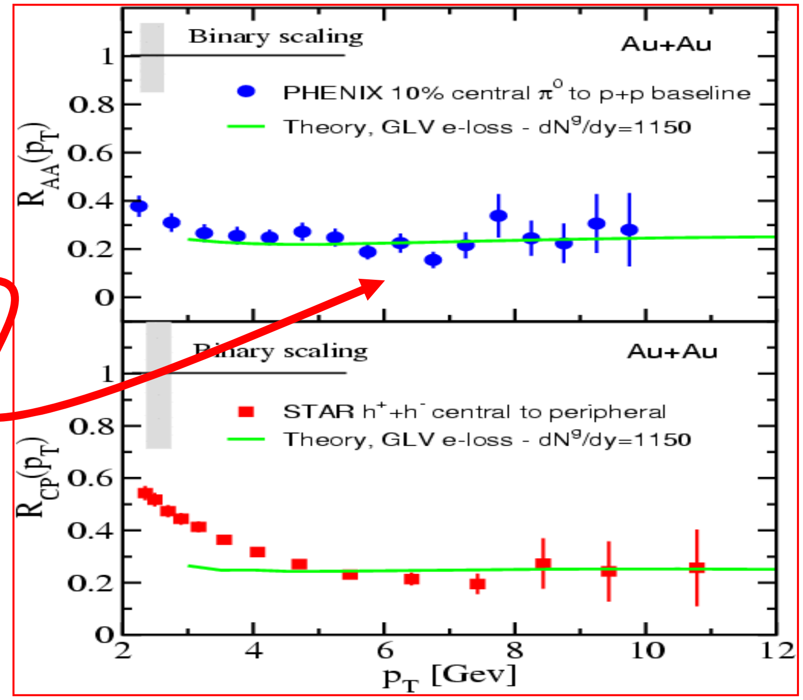
E.Wang, X.-N.Wang, PRL 89 (2002)

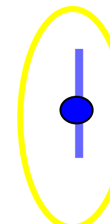


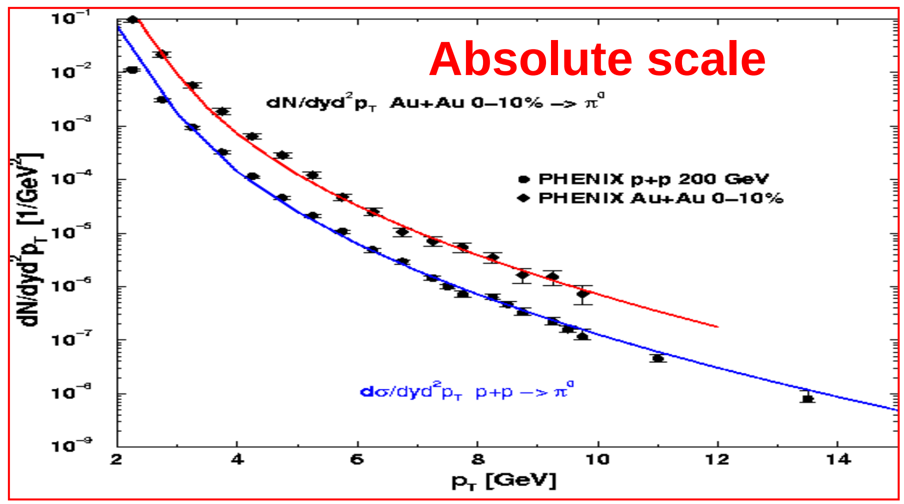
F.Arleo, Eur.Phys.J. C30 (2003)

Jet quenching in a dense medium









Vitev and Gyulassy, Phys.Rev.Lett. 89 (2002)

$$\mu^2 \sim \omega_{pl}{}^2 \propto T^2,\ \ \rho \propto T^3,\ \ \varepsilon \propto T^4$$

PHENIX Collab., Phys.Rev.Lett. 91 (2003)

STAR Collab., Phys.Rev.Lett. 91 (2003)

Centrality Dependence

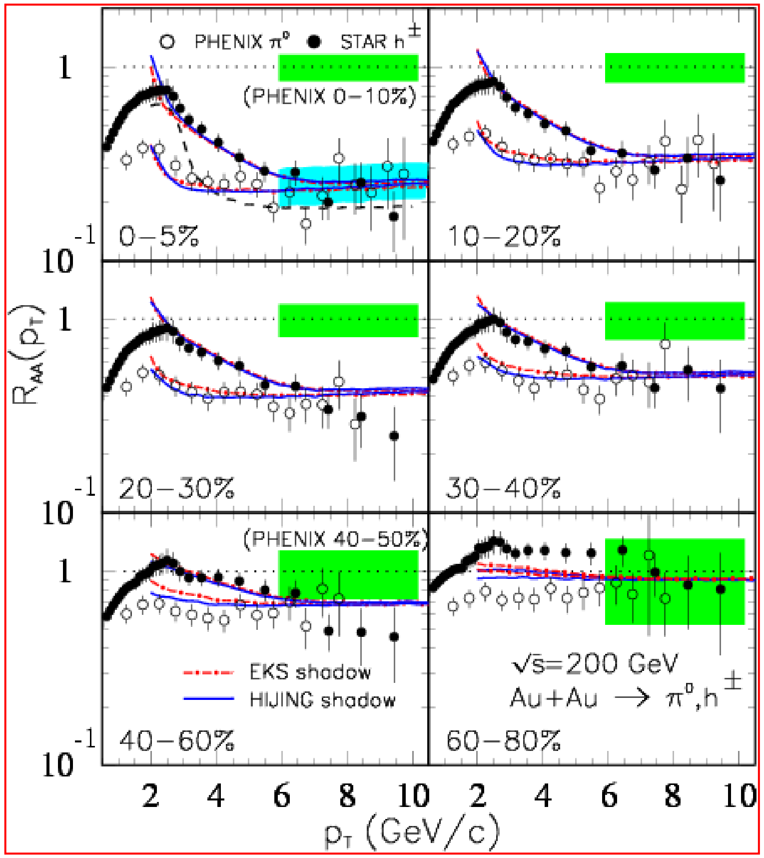
$$R_{AA}(p_T) = (1 + c \cdot \Delta p_T / p_T)^{-n}$$

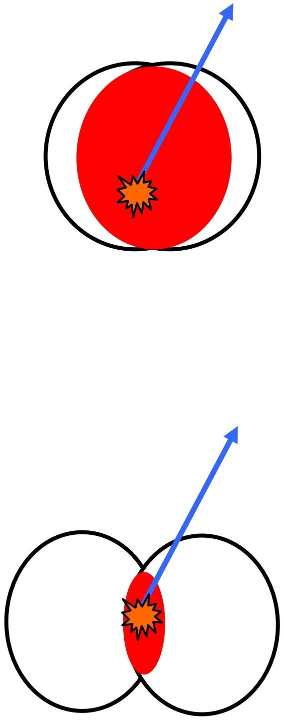
$$\frac{\Delta p_T}{p_T} \approx \frac{\Delta E}{E} \propto N_{part}^{2/3}$$

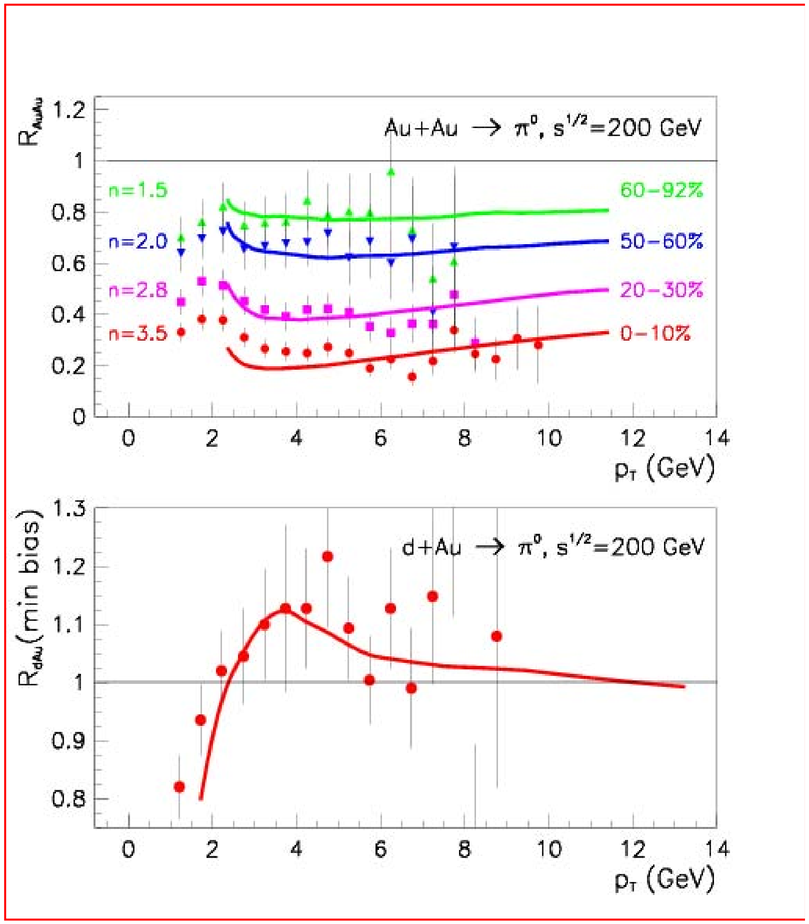
1+1D GLV

Small number of semihard scatterings

$$n_{scat} = 1.5(peripheral) - 3.5(central)$$







X.-N.Wang, nucl-th/0305010

G.G.Barnafoldi *et al.*, hep-ph/0311343

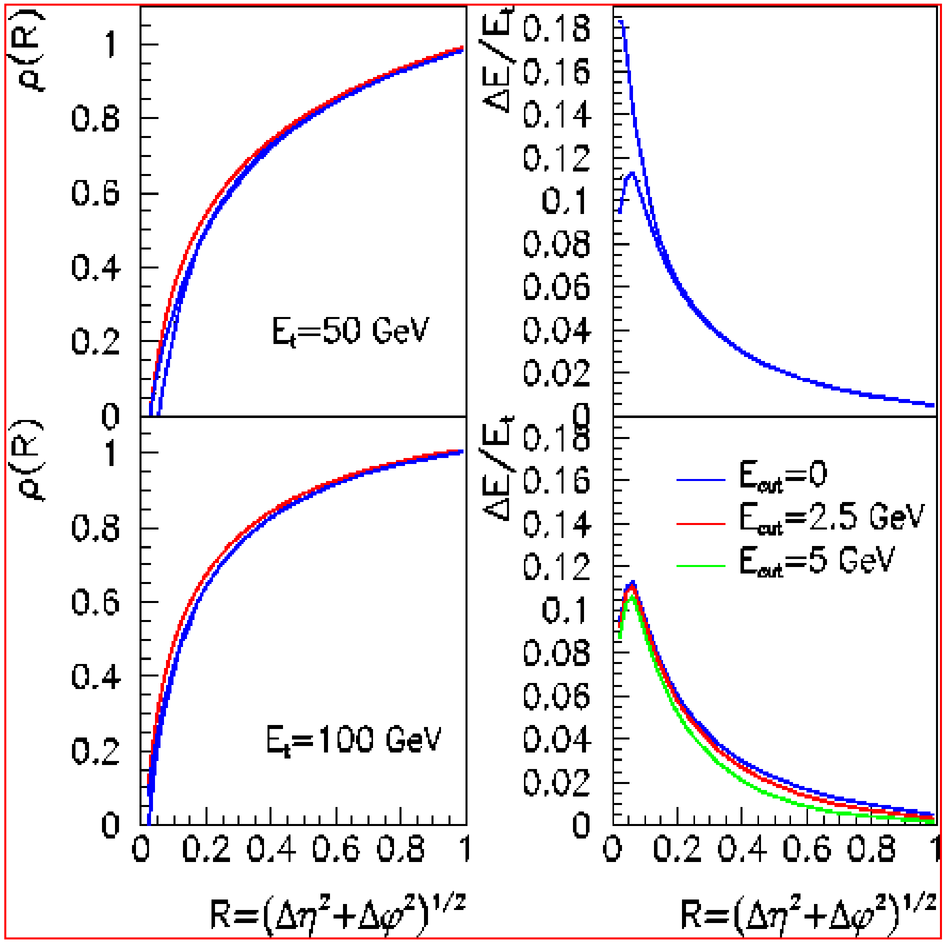
Broadening of the Jet Cone

- **Jet cone opening angle:** $R=\sqrt{\Delta\phi^2+\Delta\eta^2}$

$\rho(R)$ - **fraction of the total energy within a jet subcone**

$$\rho_{vac}(R)=\frac{1}{N_{jets}}\sum_{jets}\frac{E_t(R)}{E_t(R=1)}$$

$$\rho_{med}(R)=\rho_{vac}(R)-\frac{\Delta E_t(R)}{E_t}+\frac{\Delta E_t}{E_t}\left(1-\rho_{vac}(R)\right)$$



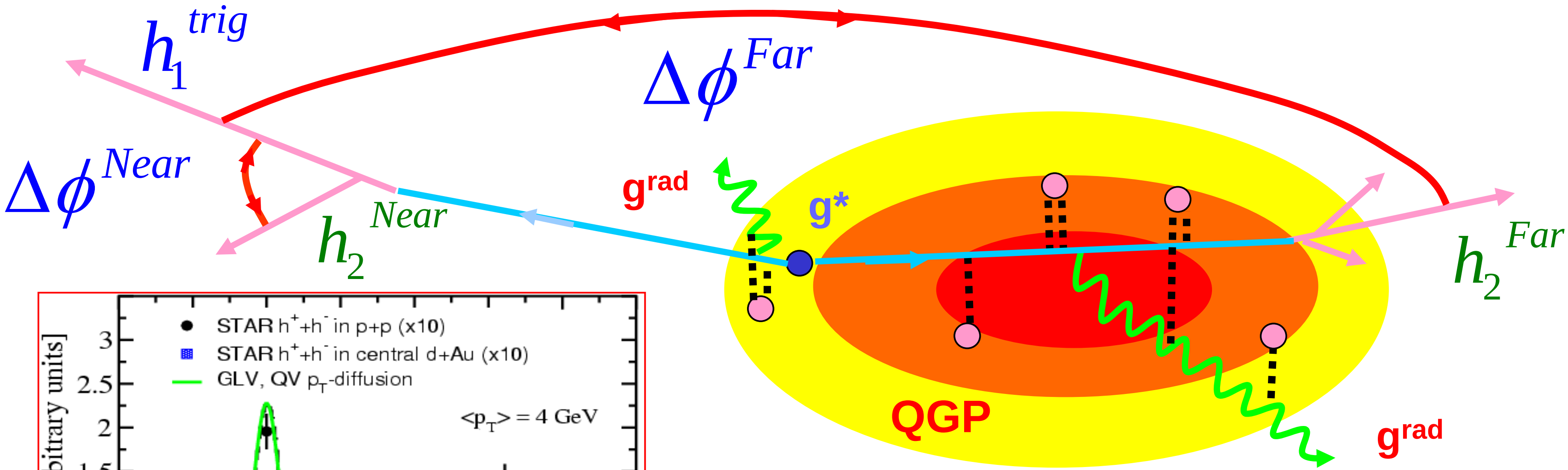
Very small effect even at the LHC

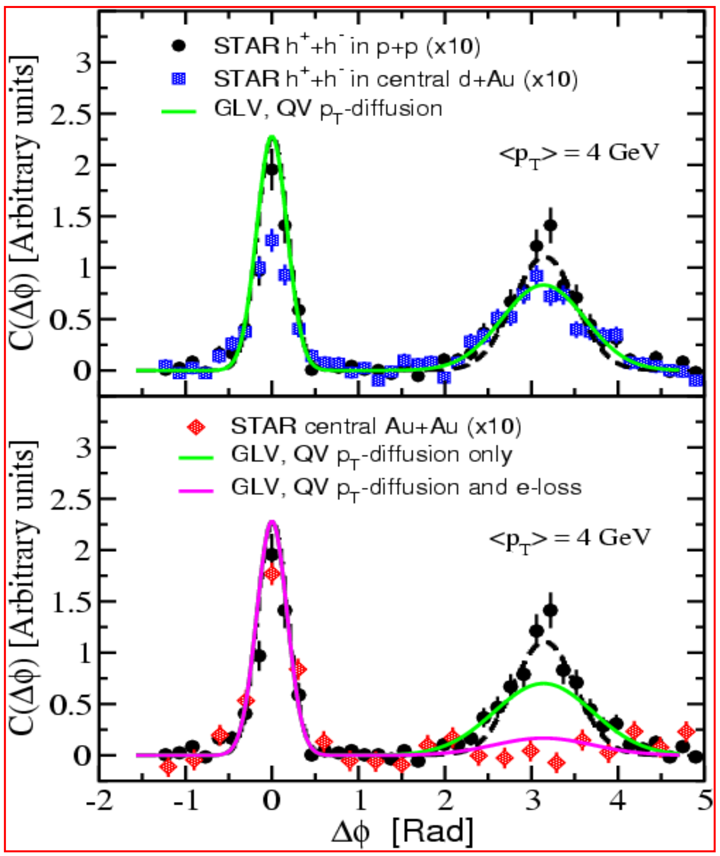
C.Salgado, U.Wiedemann, hep-ph/0310079

$R=0.3$ $\quad E_t=50\ GeV,$ 5% effect

$E_t=100\ GeV,$ 3% effect

Di-hadron Correlation





$$E_{h_1} E_{h_2} \frac{d\sigma}{d^3 p_1 d^3 p_2} \propto f_{a/A}(x_a, Q^2) \otimes f_{b/B}(x_b, Q^2)$$
$$\otimes \, d\sigma^{ab\to cd}/dt$$
$$\otimes \, D_{h_1/c}(z_c^*, j_{Tc}, Q^2) \otimes D_{h_2/d}(z_d^*, j_{Td}, Q^2)$$
$$\otimes \, \delta^4\left(p_a + p_b - p_c - p_d\right)$$

- **Di-jet acoplanarity (insufficient)**
- **Away-side quenching**

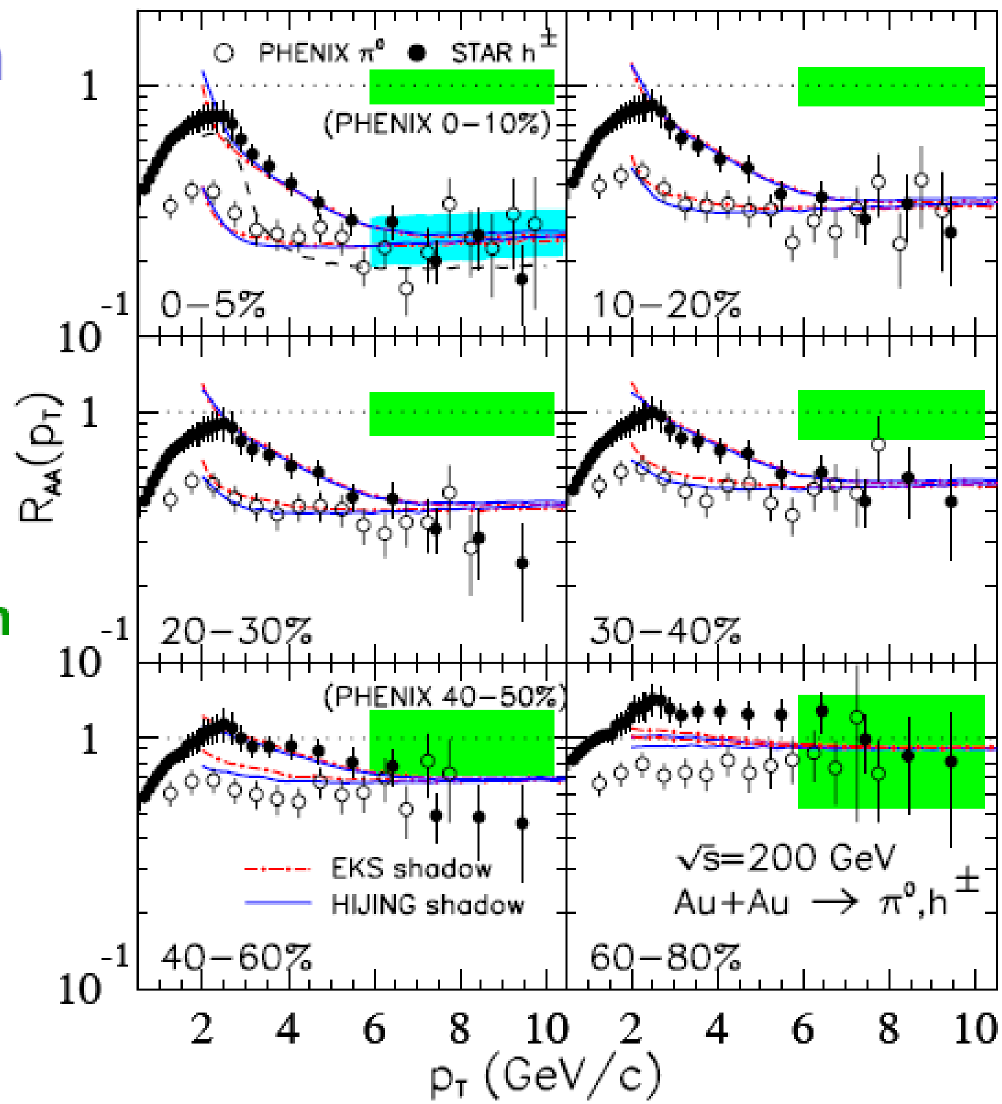
 Qiu, Vitev, Phys.Lett.B570 (2003)

Important role of *dA* at RHIC

- Single inclusive hadron in *AA* collision

$$R^{AB} = \frac{\sigma_{AB}}{\langle N_{\rm binary}\rangle_{AB}\langle\sigma_{NN}\rangle}$$

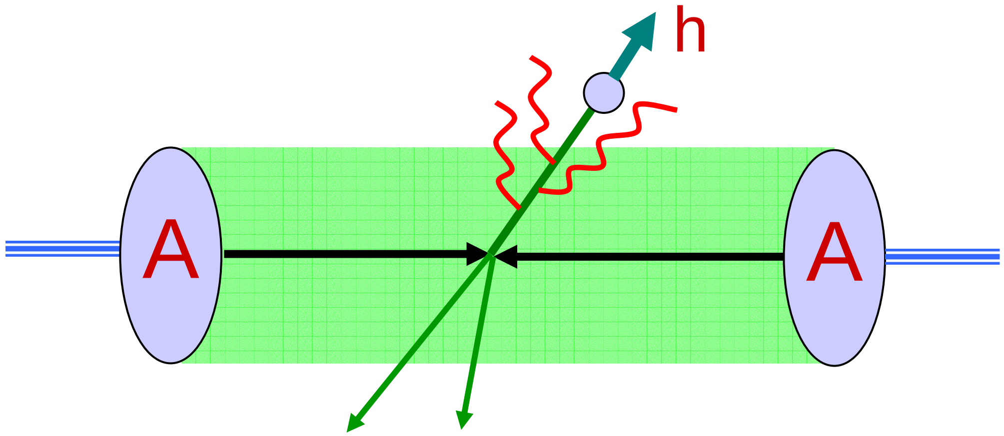
- What cause the suppression or enhancement?
- What should we expect for dA?



Jet quenching

❑ Assumptions:

- ❖ **Soft interactions between the ions does not change the effective PDF's**



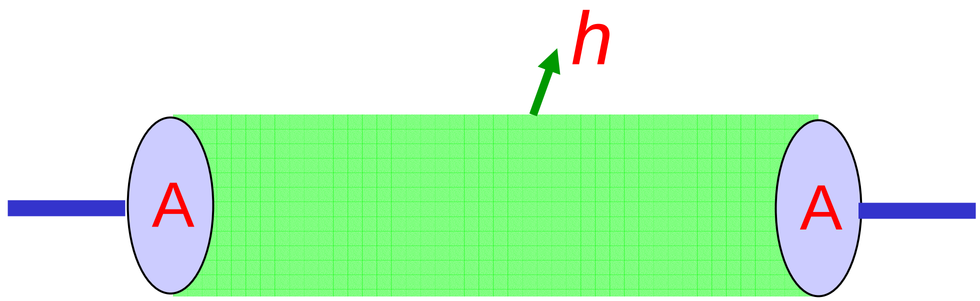
- ❖ **Multiple scattering with the medium leads to energy loss**
- ❖ **Reduction of leading hadron momentum leads to suppression at high p_T**

Suppression is a final-state effect

No suppression expected for dA

Saturation and CGC

- Soft interactions between the ions **alter** the PDF's



- Convolution of two universal saturated distributions at a saturation scale: $Q_s \sim$ GeV; or
- Solve classical Yang-Mills Equation

  ⟹ gluon density from the *AA* collisions, then convert the gluons to the observed hadron

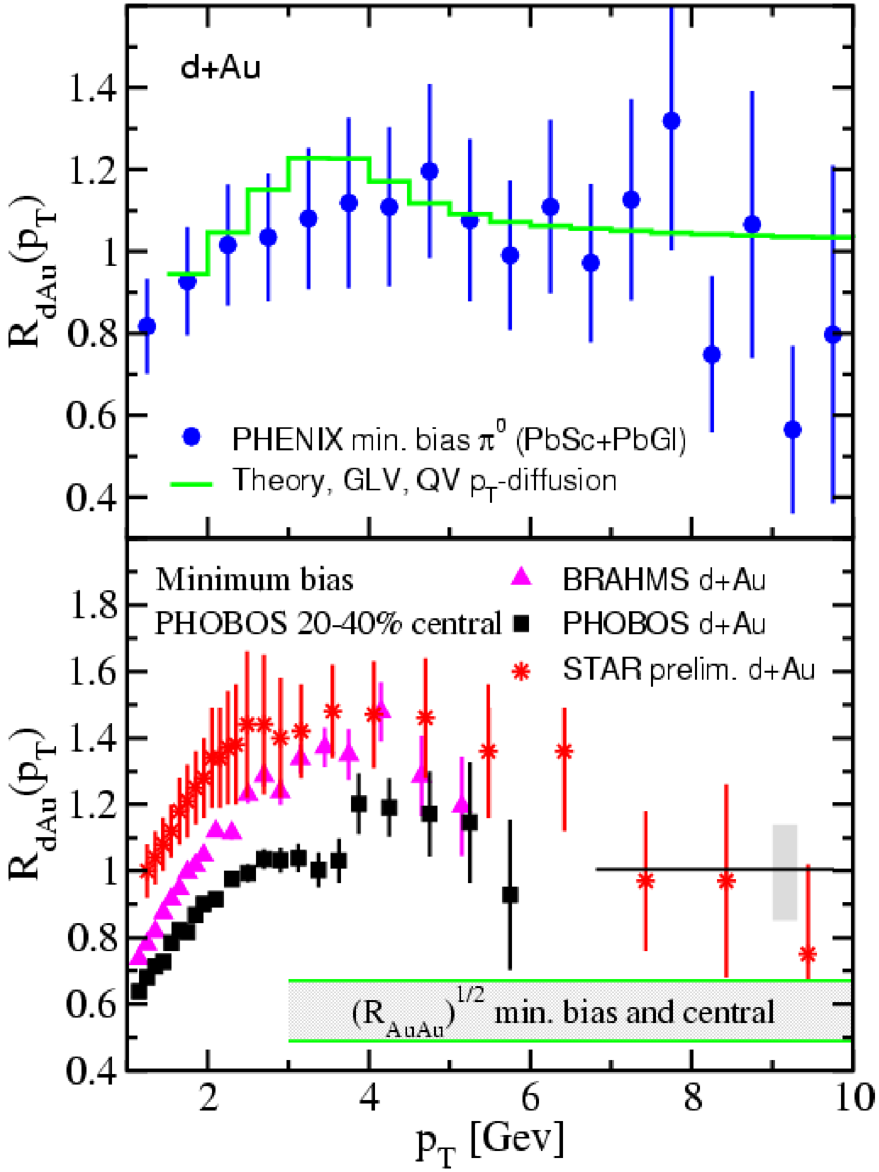
- Momentum of the GeV hadron *h* is balanced by many soft particles ⟹ no back-to-back hadron correlation

Suppression is a initial-state effect

Large suppression expected for dA

$$R_{dAu} = \sqrt{R_{AuAu}} \ll 1, \quad R_{AuAu} \simeq 0.2 - 0.4$$

Comparison to the d+Au Data



RHIC Data from:

B.Back *et al*. [PHOBOS], Phys.Rev.Lett. 91 (2003)

J.Adams *et al*. [STAR], Phys.Rev.Lett. 91 (2003)

S.Adler *et al*. [PHENIX], Phys.Rev.Lett. 91 (2003)

I.Arsene *et al*. [BRAHMS], Phys.Rev.Lett. 91 (2003)

Theoretical predictions:

I.Vitev and M.Gyulassy, Phys.Rev.Lett. 89 (2002)

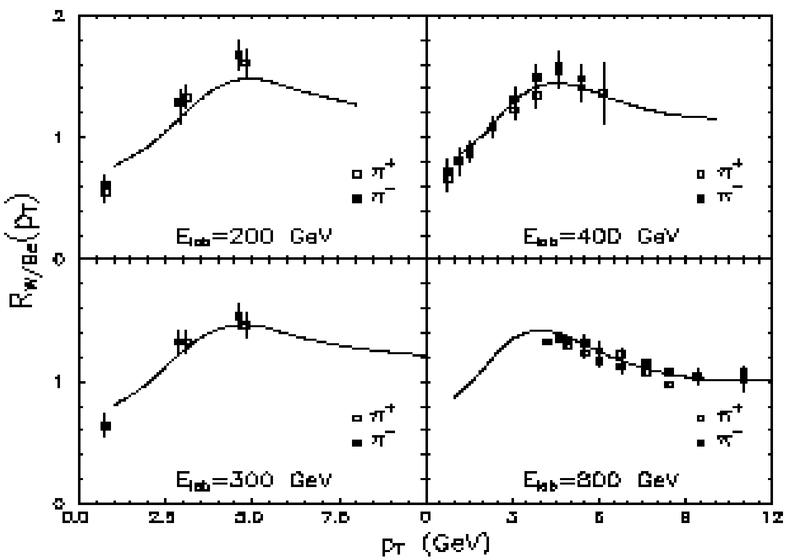
I.Vitev, Phys.Lett. B562 (2003)

D.Kharzeev, E.Levin,L.McLerran, Phys.Lett. B 561 (2003)

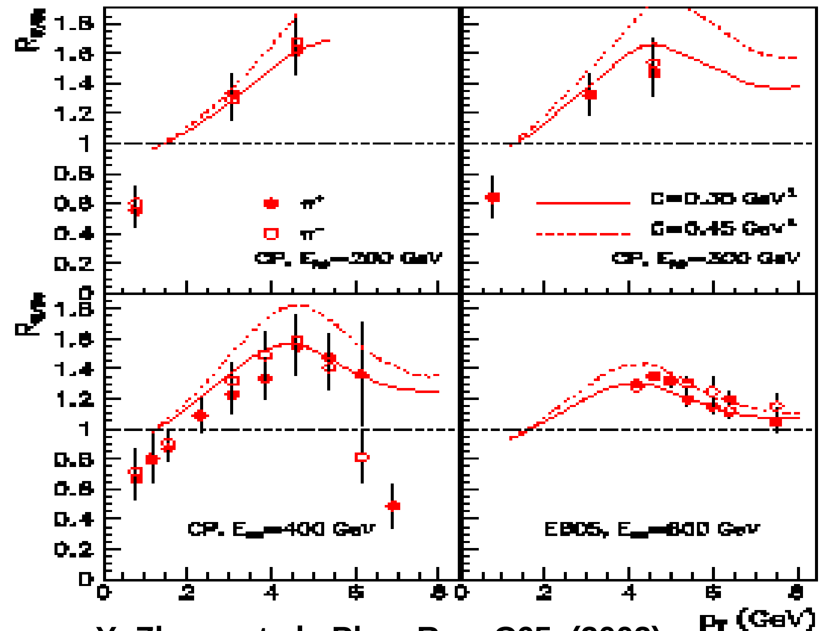
Current Data from RHIC:
- support Cronin type effect in d+Au
- disfavor the saturation picture in d+Au

Parton *x* is not small enough:
- Increases collision energy – the LHC
- moves to the forward region – lower *x*

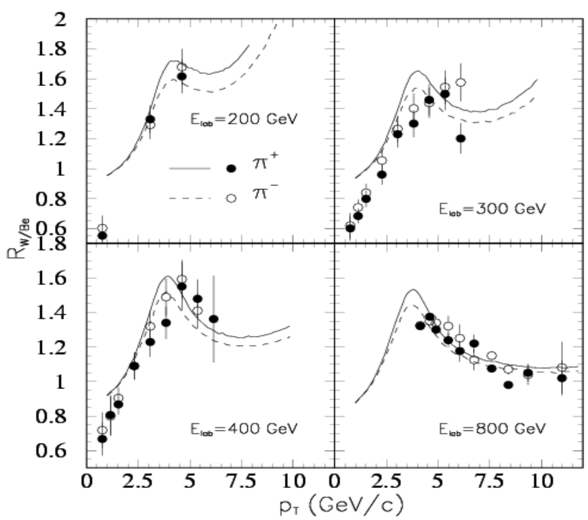
Calculations of Cronin Effect



B. Kopeliovich *et al.*, Phys.Rev.Lett. 88, (2002)



Y. Zhang et al., Phys.Rev. C65, (2002)



X.N. Wang, Phys.Rev. C 61, (2000)

$$\left\langle k_T{}^2 \right\rangle_{pA} = \left\langle k_T{}^2 \right\rangle_{pp} + \left\langle \Delta k_T{}^2 = \mu^2 L / \lambda \right\rangle_{pA}$$

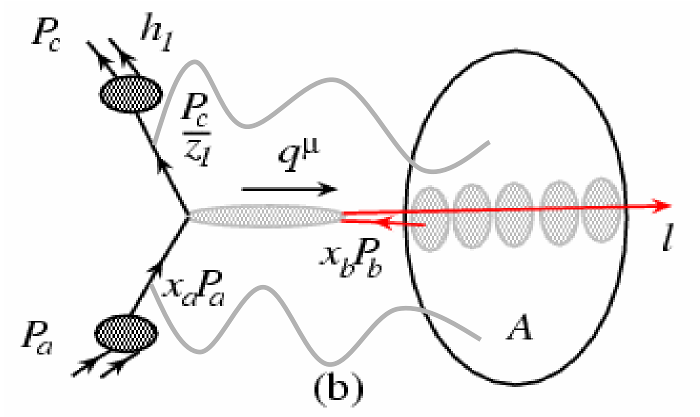
These calculations have addressed the Y=0 total inclusive Cronin Effect at RHIC

All of the above find: $R_{dAu} \simeq 1.1 - 1.4$
at RHIC

Power Corrections in p(d)+A Collisions

- ❑ Hadronic factorization fails for power corrections of the order of $1/Q^4$ and beyond
- ❑ Medium size enhanced dynamical power corrections in p+A are factorizable

⇨ to make predictions for p+A collisions, but, multiple hard scales, s, t, u.



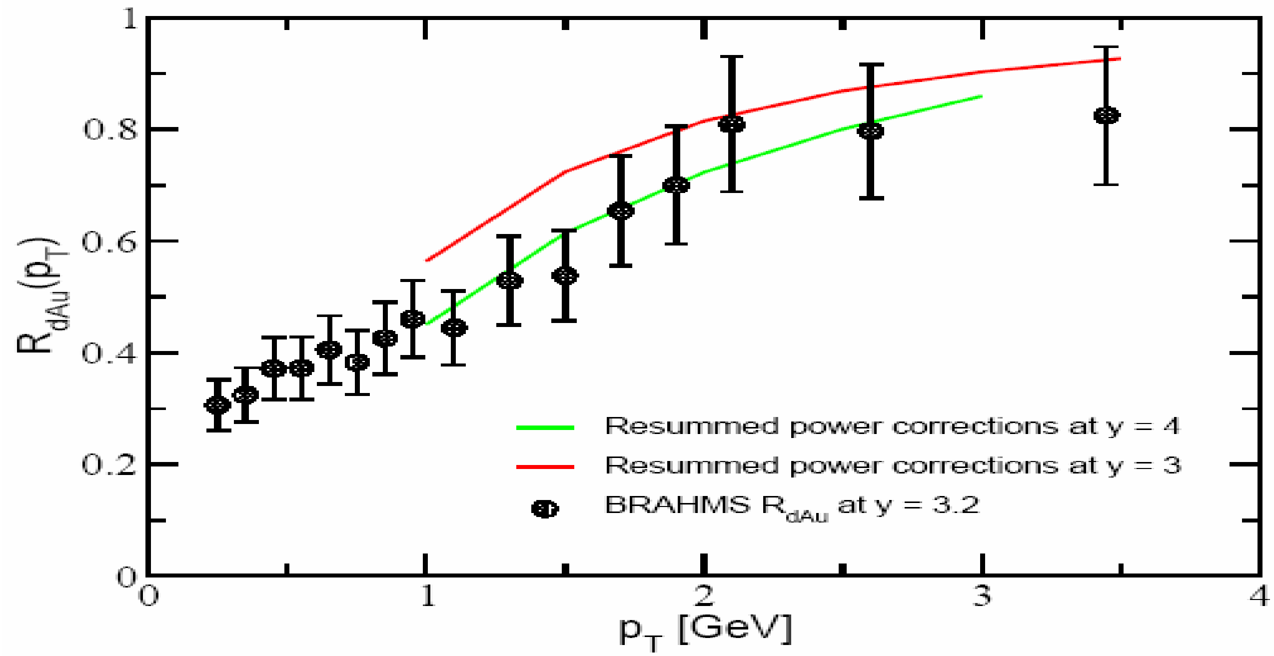
- ❑ Single hadron inclusive production:

Once we fix the incoming parton momentum from the beam and outgoing fragmentation parton, we uniquely fix the momentum exchange, q^μ, and the probe size

⇔ coherence along the direction of $q^\mu - p^\mu$

⇔ only t relevant. smaller t, larger coherent effect

Numerical results for power corrections



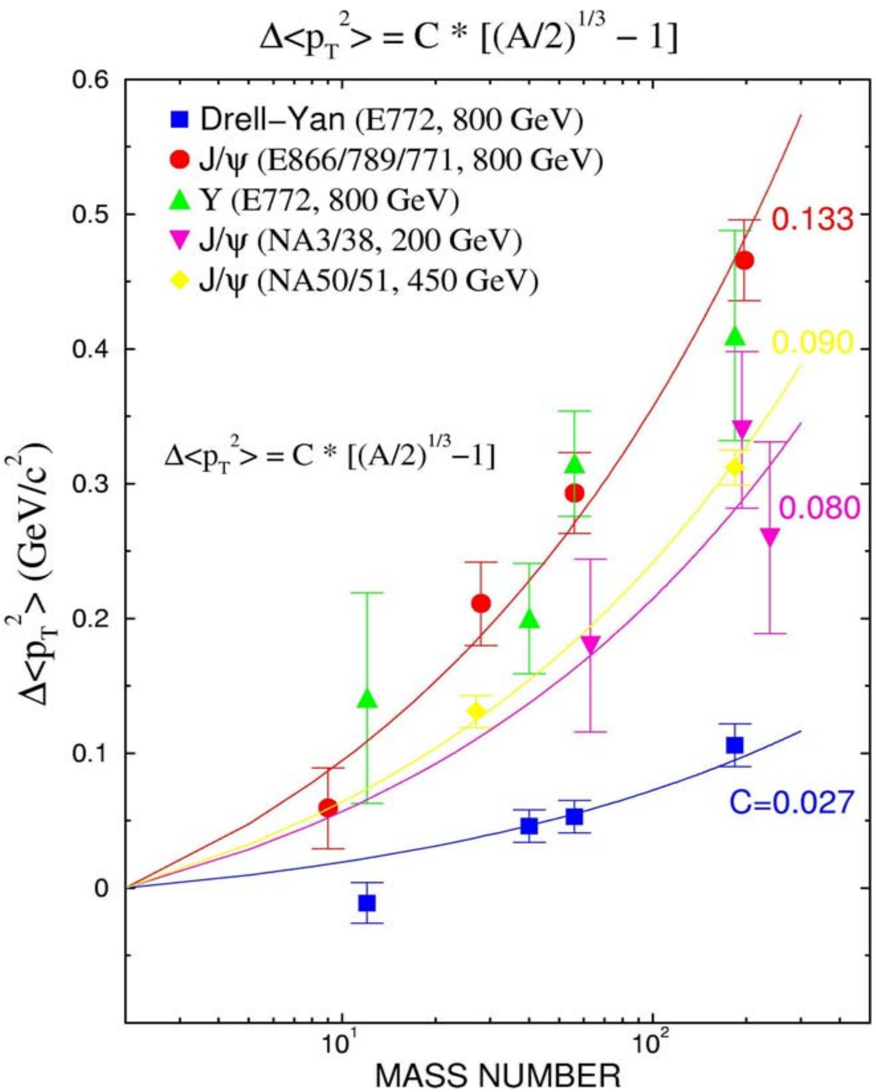
Qiu and Vitev, hep-ph/0405068

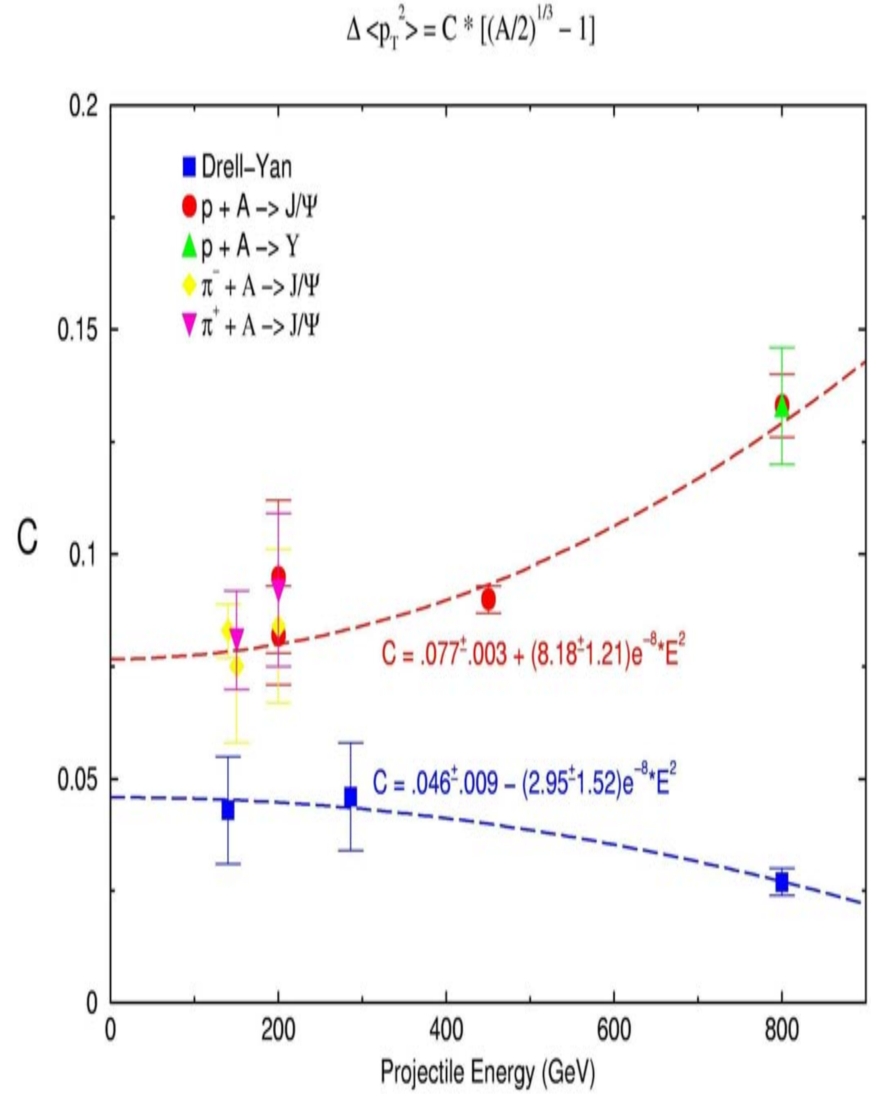
- ❖ **Similar power correction modification to single and double inclusive hadron production**

- ❖ **increases with centrality and increase with rapidity**
- ❖ **disappears at high p_T because of the power suppression**

Power corrections to evolution equations will flatten the p_T dependence – slower evolution

Systematics of P_T broadening

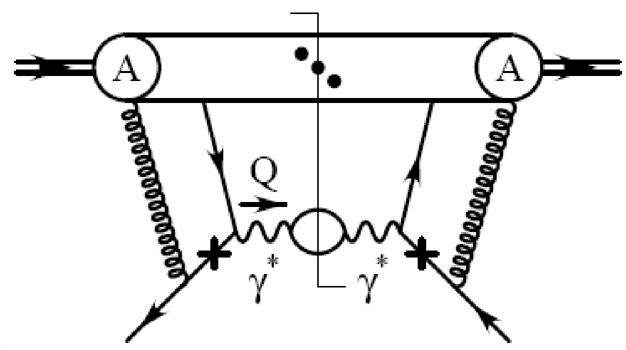




Total Q_T broadening

- **Direct Q_T from multiple scattering is not perturbative:**

$$\frac{d\sigma}{dQ^2 dQ_T^2} \Big/ \frac{d\sigma}{dQ^2} \propto \frac{\alpha_s}{Q_T^2} T_q(x, A)$$



- **Drell-Yan Q_T average is perturbative:**

$$\left\langle Q_T^2 \right\rangle \equiv \int dQ_T^2 \left(Q_T^2\right)\left(\frac{d\sigma}{dQ^2 dQ_T^2}\right) \Big/ \int dQ_T^2 \left(\frac{d\sigma}{dQ^2 dQ_T^2}\right)$$

Single scale Q

- **Drell-Yan Q_T broadening:** $\Delta\left\langle Q_T^2 \right\rangle \equiv \left\langle Q_T^2 \right\rangle^{hA} - A\left\langle Q_T^2 \right\rangle^{hN} \propto \sigma^{(D)}$

- **Four-parton correlation:**

$$T_q(x, A) = \int \frac{dy^-}{2\pi} \mathrm{e}^{ixp^+y^-} \int dy_1^- dy_2^- \theta\left(y^- - y_1^-\right)\theta\left(-y_2^-\right)$$

$$\times \left\langle p_A \left| F_\alpha^+\left(y_2^-\right)\bar{\psi}(0)\frac{\gamma^+}{2}\psi\left(y^-\right)F^{+\alpha}\left(y_1^-\right)\right| p_A \right\rangle \approx \frac{9A^{1/3}}{16\pi R^2}\left\langle F^{+\alpha}F_\alpha^+ \right\rangle q_A(x)$$

- **Characteristic scale:**

$$\left\langle F^{+\alpha}F_\alpha^+ \right\rangle \equiv \frac{1}{p^+}\int dy_1^- \left\langle N \left| F^{+\alpha}(0) F_\alpha^+\left(y_1^-\right)\right| N \right\rangle \theta\left(y_1^-\right)$$

Guo, PRD 58 (1998)

Summary and outlook

With all the data becoming available

QCD and strong interaction physics

will have a new exciting era

Thanks!