QCD and Rescattering in Nuclear Targets

Lecture 2

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Fundamentals of perturbative QCD

- Infrared Safety
- Purely infrared safe cross sections
- Jets – “trace” of the partons
- Factorization – predictive power of pQCD
- Factorization for deeply inelastic scattering
- Evolution of parton distribution functions
- Factorization for hadronic collisions

Infrared Safety

❑ Running quark mass:

$$m(\mu_2) = m(\mu_1)\,\exp\left[-\int_{\mu_1}^{\mu_2}\frac{d\lambda}{\lambda}\left[1+\gamma_m(g(\lambda))\right]\right] \;\Rightarrow\; 0 \text{ as } \mu_2 \to \infty$$

Perturbation theory becomes a massless theory when $\mu \to \infty$

❑ Infrared safety:

$$\sigma_{\text{phy}}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\frac{m^2(\mu^2)}{\mu^2}\right) \;\Rightarrow\; \hat{\sigma}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)+O\left[\left(\frac{m^2(\mu^2)}{\mu^2}\right)^{\kappa}\right]$$

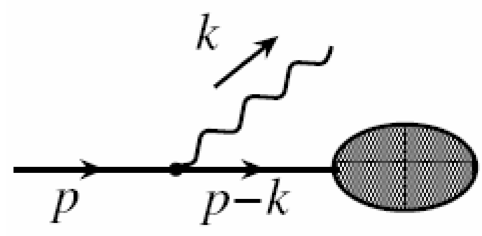
Infrared safe = $\kappa > 0$

Asymptotic freedom is useful for quantities that are infrared safe

QCD perturbation theory (Q>>Λ_{QCD}) is effectively a massless theory

Infrared and collinear divergence

Consider a general diagram:



$$p^2 = 0, \quad k^2 = 0 \quad \text{for a massless theory}$$

singularity

- $k^\mu \to 0 \;\; \Rightarrow \;\; (p-k)^2 \to p^2 = 0$

$\Longrightarrow$ **Infrared (IR) divergence**

- $k^\mu \parallel p^\mu \;\; \Rightarrow \;\; k = \lambda p \quad \text{with} \;\; 0 < \lambda < 1$

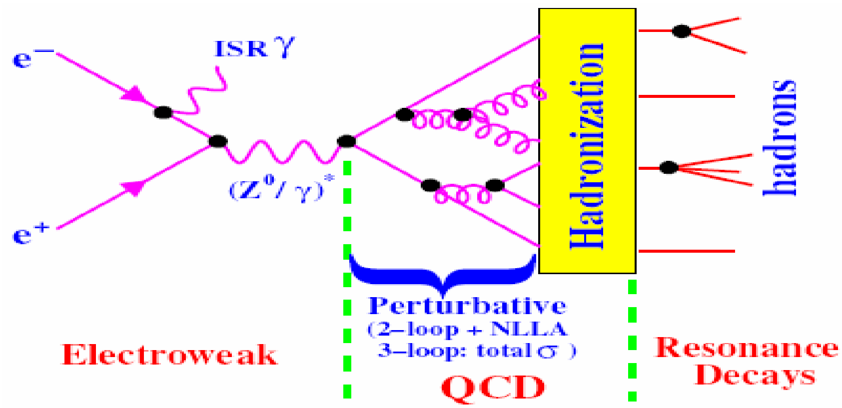
$$\Rightarrow \;\; (p-k)^2 \to (1-\lambda)^2 \, p^2 = 0$$

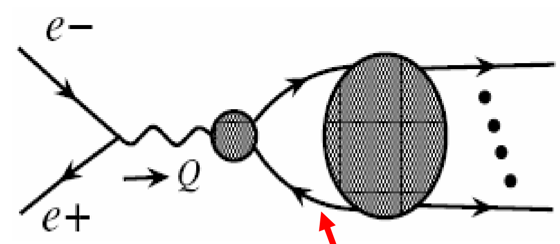
$\Longrightarrow$ **Collinear (CO) divergence**

IR and CO divergences are generic problems for massless perturbation theory

Purely Infrared safe cross sections

e+e- → hadron total cross section is infrared safe (IRS)





Hadrons "n"

Partons "m"

If there is no quantum interference between partons and hadrons,

$$\sigma^{\rm tot}_{e^+e^-\to \rm hadrons} \propto \sum_n P_{e^+e^-\to n} = \sum_n \boxed{\sum_m P_{e^+e^-\to m} P_{m\to n}} = \sum_m P_{e^+e^-\to m} \boxed{\sum_n P_{m\to n}} = 1$$

$$\sigma^{\rm tot}_{e^+e^-\to \rm partons} \propto \sum_m P_{e^+e^-\to m}$$

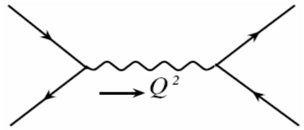
Unitarity

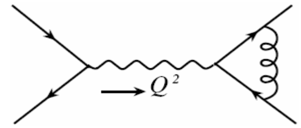
$$\Rightarrow \quad \boxed{\sigma^{\rm tot}_{e^+e^-\to \rm hadrons} = \sigma^{\rm tot}_{e^+e^-\to \rm partons}}$$

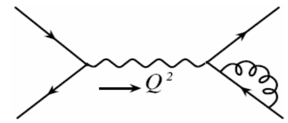
Finite in perturbation Theory – KLN theorem

"Local" – of order of 1/Q

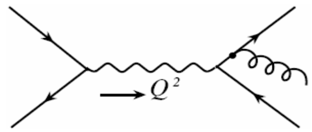
σ^{tot} for $e^+e^- \rightarrow$ hadrons in pQCD

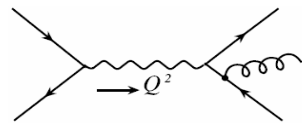
$\sigma^{tot} = \dfrac{1}{2s}\Big\{\Big|$ 

+ 

+ 

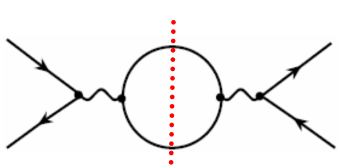
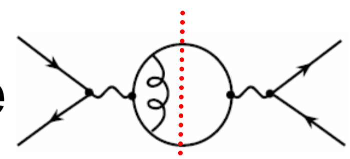
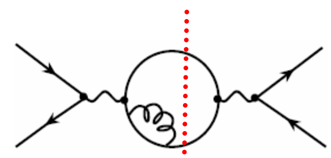
+ … $\Big|^2$ $\mathrm{PS}^{(2)}$

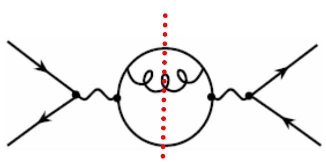
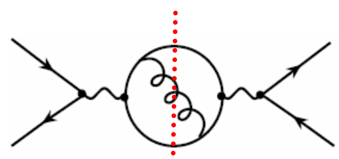
+ $\Big|$ 

+ 

+ … $\Big|^2$ $\mathrm{PS}^{(3)}$

+ … $\Big\}$ + UV counter-term

$= \dfrac{1}{2s}\Big\{$  + 2Re  + 2Re 

+ 2  + 2  + … $\Big\}$ + UV C.T.

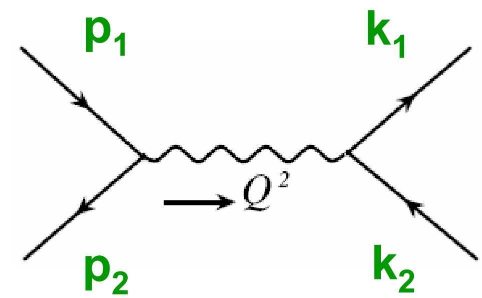
$$= \sigma_2^{(0)} + \sigma_2^{(1)} + \sigma_3^{(1)}$$

Born

O(α_s)

3-particle phase space

Lowest order contribution



- **Lowest order Feynman diagram:**
- **Invariant amplitude square:**

$$|\bar{M}_{e^+e^-\to Q\bar{Q}}|^2 = e^4 e_Q^2 N_c \frac{1}{s^2}\frac{1}{2^2}\mathrm{Tr}\left[\gamma\cdot p_2\gamma^\mu\gamma\cdot p_1\gamma^\nu\right]$$
$$\times\mathrm{Tr}\left[\left(\gamma\cdot k_1+m_Q\right)\gamma_\mu\left(\gamma\cdot k_2-m_Q\right)\gamma_\nu\right]$$
$$= e^4 e_Q^2 N_c \frac{2}{s^2}\left[(m_Q^2-t)^2+(m_Q^2-u)^2+2m_Q^2 s\right]$$

$$s=(p_1+p_2)^2 \quad t=(p_1-k_1)^2 \quad u=(p_2-k_1)^2$$

- **Lowest order cross section:**

$$\frac{d\sigma_{e^+e^-\to Q\bar{Q}}}{dt} = \frac{1}{16\pi s^2}|\bar{M}_{e^+e^-\to Q\bar{Q}}|^2 \qquad \text{where } s=Q^2$$

Threshold constraint

$$\sigma_2^{(0)} = \sum_Q \sigma_{e^+e^-\to Q\bar{Q}} = \sum_Q e_Q^2 N_c \frac{4\pi\alpha_{em}^2}{3s}\left[1+\frac{2m_Q^2}{s}\right]\sqrt{1-\frac{4m_Q^2}{s}}$$

One of the best tests for the number of colors

Dimensional regulation for IR and CO

❑ **n=4-2ε dimension:**

$$\sigma_{3,\varepsilon}^{(1)} = \sigma_{2,\varepsilon}^{(0)} \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left[\frac{\Gamma(1-\varepsilon)^2}{\Gamma(1-3\varepsilon)}\right]\left[\frac{1}{\varepsilon^2}+\frac{3}{2\varepsilon}+\frac{19}{4}\right]$$

$$\sigma_{2,\varepsilon}^{(1)} = \sigma_{2,\varepsilon}^{(0)} \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left[\frac{\Gamma(1-\varepsilon)^2\,\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}\right]\left[-\frac{1}{\varepsilon^2}-\frac{3}{2\varepsilon}+\frac{\pi^2}{2}-4\right]$$

$$\boxed{\sigma_{3,\varepsilon}^{(1)} + \sigma_{2,\varepsilon}^{(1)} = \sigma_2^{(0)}\left[\frac{\alpha_s}{\pi}+O(\varepsilon)\right]}$$

$$\sigma^{\text{tot}} = \sigma_2^{(0)} + \sigma_{3,\varepsilon}^{(1)} + \sigma_{2,\varepsilon}^{(1)} + O(\alpha_s^2) = \sigma_2^{(0)}\left[1+\frac{\alpha_s}{\pi}\right] + O(\alpha_s^2)$$

❑ **Lesson:**

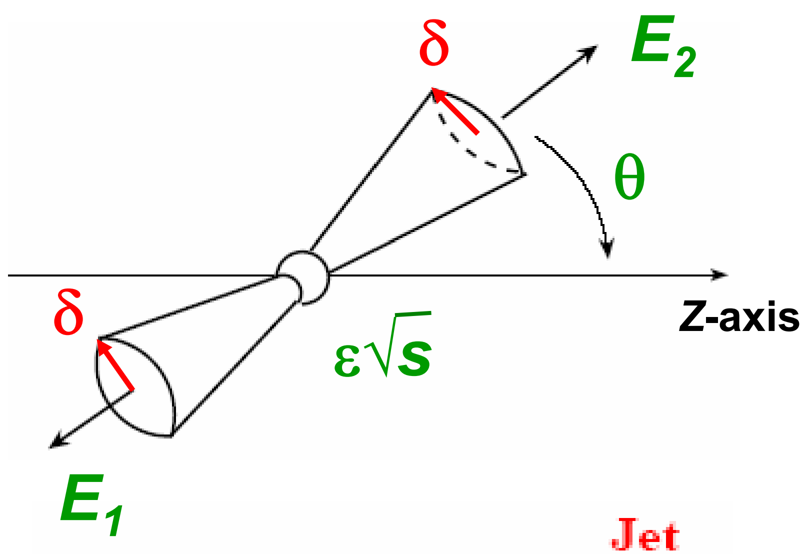
σ^{tot} is independent of the choice of IR and CO regularization

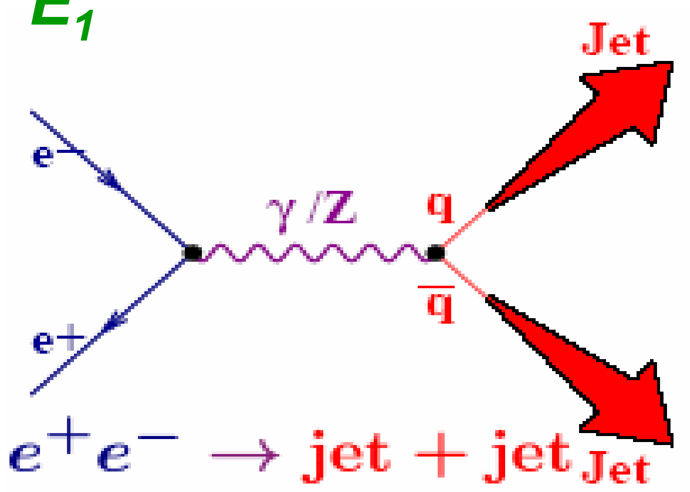
⇨ **σ^{tot} is Infrared safe!**

Jets in e^+e^- - trace of the partons

- Jets – Inclusive x-section with a limited phase-space
- Q: will IR cancellation be completed?
  - Leading partons are moving away from each other
  - Soft gluon interactions should not change the direction of an energetic parton $\rightarrow$ a "jet" – "trace" of a parton
  - Jet algorithm

Sterman-Weinberg Jet



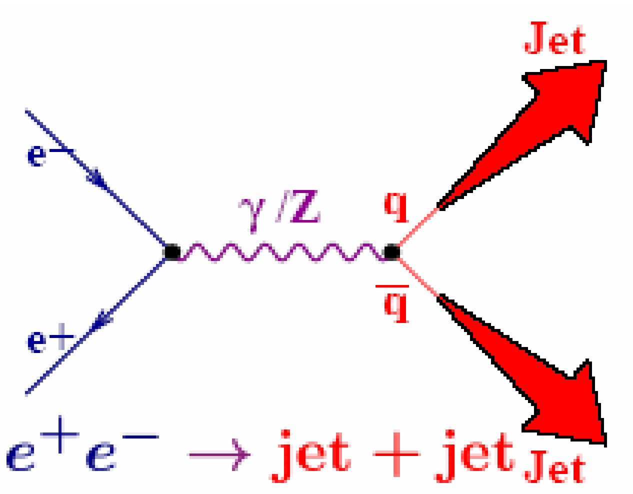


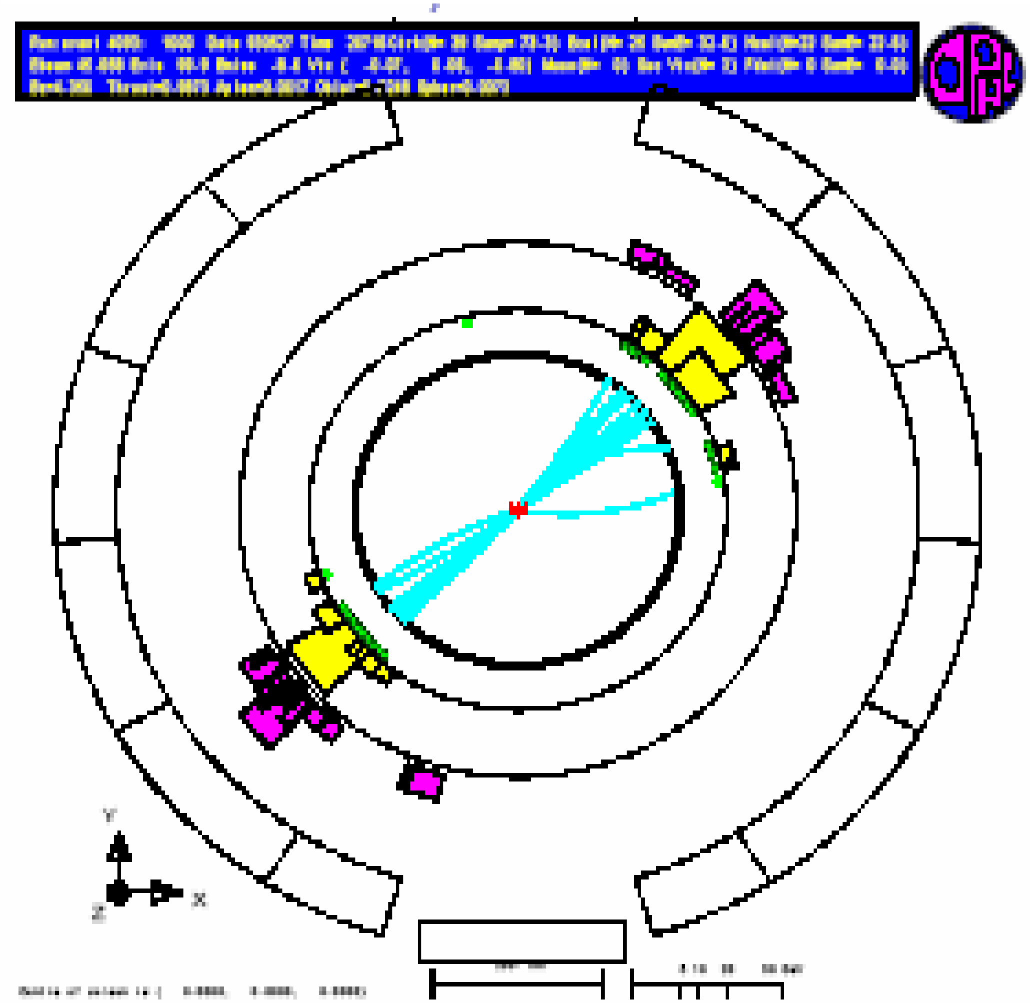
$e^+e^- \rightarrow \text{jet} + \text{jet}$

A clean two-jet event

Lowest order ($\mathcal{O}(\alpha^2\alpha_s^0)$):

LEP ($\sqrt{s} = 90 - 205$ GeV)

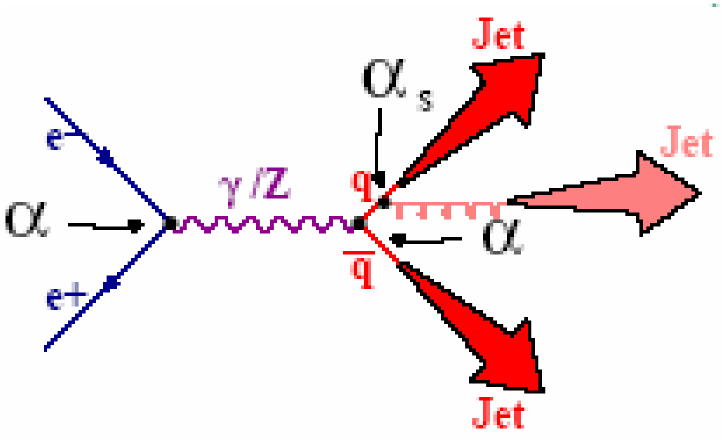




A clean trace of two partons – a pair of quark and antiquark

Discovery of a gluon jet

First order in QCD ($\mathcal{O}(\alpha^2\alpha_s^1)$):



Reputed to be the first three-jet event from TASSO

TASSO Collab., Phys. Lett. B86 (1979) 243

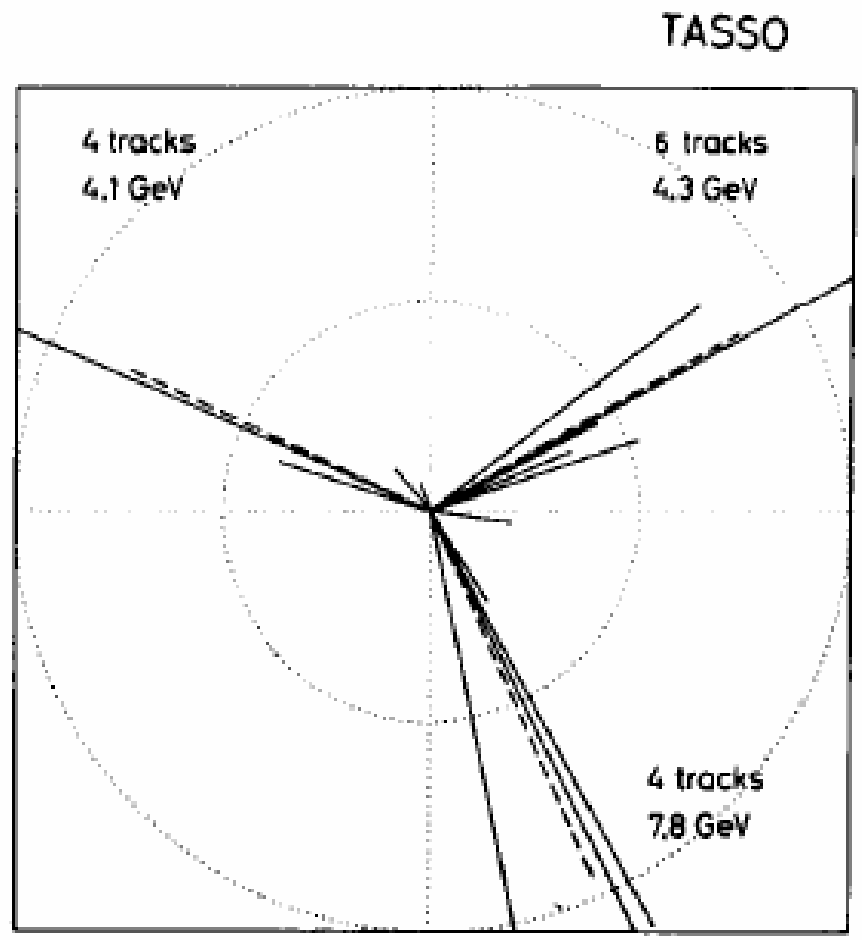
MARK-J Collab., Phys. Rev. Lett. 43 (1979) 830

PLUTO Collab., Phys. Lett. B86 (1979) 418

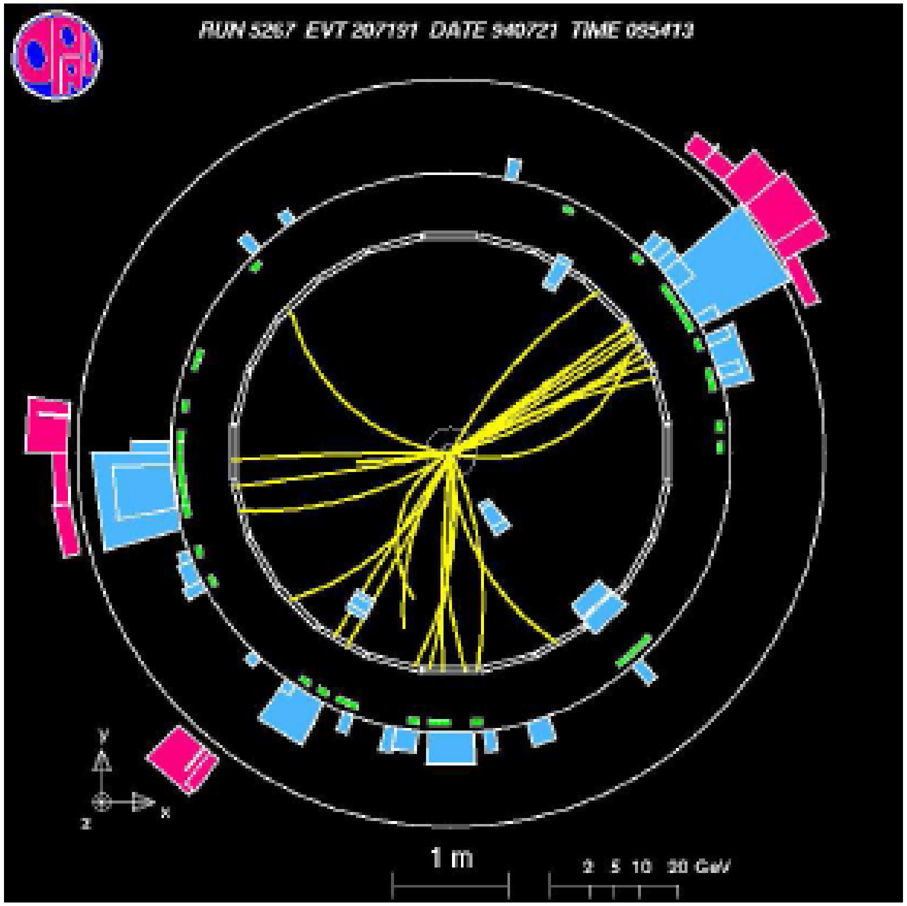
JADE Collab., Phys. Lett. B91 (1980) 142

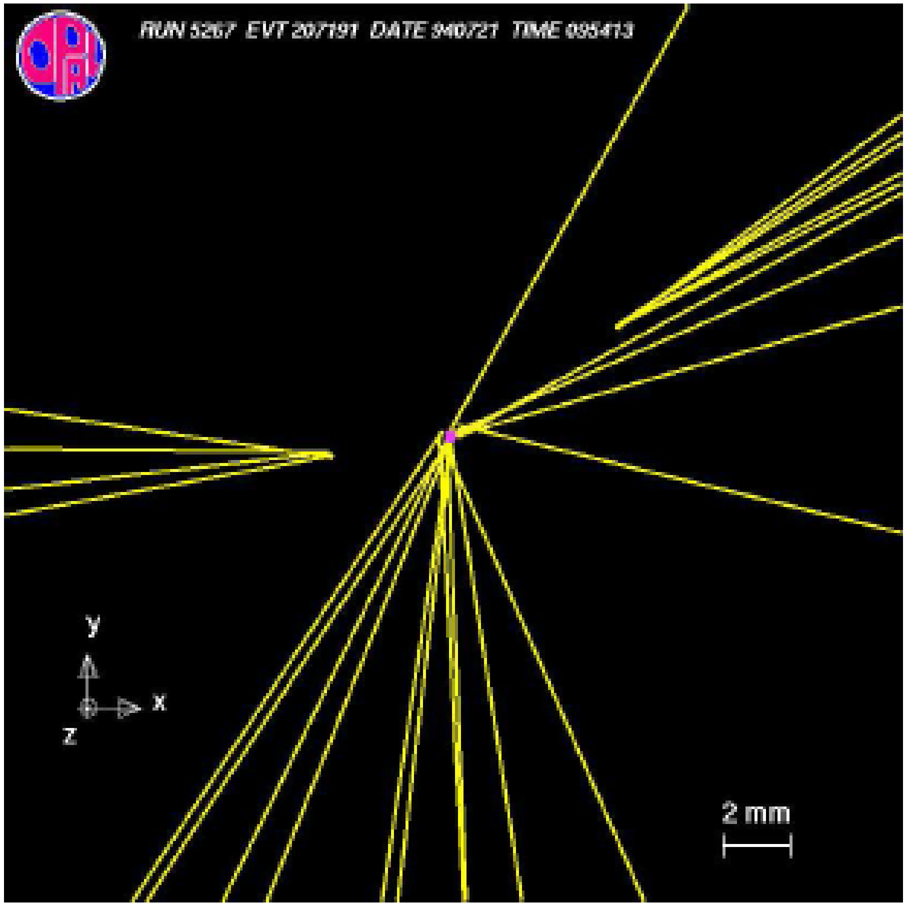
PETRA e^+e^- storage ring at DESY:

$E_{c.m.} \gtrsim 15$ GeV



Tagged 3-jet event from LEP





Gluon Jet

Basics of jet finding algorithms

❑ Recombination jet algorithms:

—— **almost universal choice at e+e- colliders**

- Recombination metric: $y_{ij} = \dfrac{M_{ij}^2}{E_{c.m.}^2}$

$\longrightarrow$ Combine the particle pair (i, j) with the smallest y_{ij}:

$$(i, j) \to k$$

<u>E scheme :</u> $p_k = p_i + p_j \quad \longrightarrow$ massive jets

<u>E₀ scheme :</u> $E_k = E_i + E_j$

$$\vec{p}_k = \frac{\vec{p}_i + \vec{p}_j}{|\vec{p}_i + \vec{p}_j|} E_k \quad \longrightarrow \text{massless jets}$$

- Iterate until all remaining pairs satisfy $y_{ij} > y_{cut}$

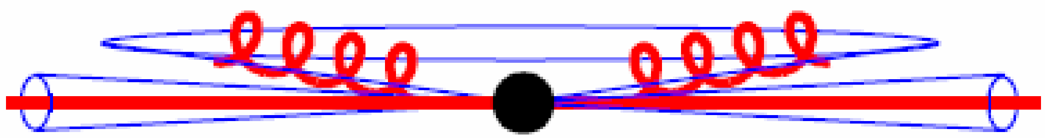
The JADE jet finder

[JADE Collab., Z. Phys. C33 (1986) 23]

⟶ The original recombination jet finder:

- $M_{ij}^2 = 2E_iE_j(1-\cos\theta_{ij}) \approx$ (invariant mass)^2
- Original version based on the E_0 scheme

Sometimes leads to the formation of "junk jets"



⟶ Two-jet events with ≥ 2 soft, colinear gluons can be classified, unnaturally, as three-jet events

⟶ Prevents re-summation techniques from being applied

The Durham k_T jet finder

[S. Catani et al., Phys. Lett. B269 (1991) 432]

$\longrightarrow$ Introduced to reduce the problem of junk jets

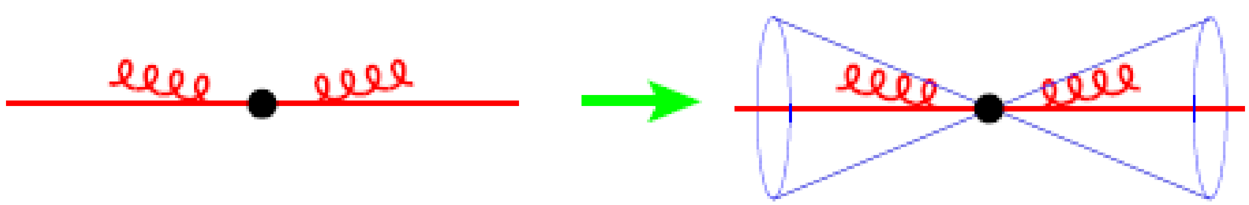
- $M_{ij}^2 = 2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij})$
- E scheme combination of particles: $(i, j) \to k$

$\longrightarrow$ Consider small emission angles θ_{ij}:

$$\begin{aligned} M_{ij}^2 &\approx 2\min(E_i^2, E_j^2)\left[1 - \left(1 - \theta_{ij}^2/2 + \cdots\right)\right] \\ &\approx \min(E_i^2, E_j^2)\,\theta_{ij}^2 \approx K_\perp^2 \end{aligned}$$

(min. transverse momenta of one particle w.r.t. the other)

$\longrightarrow$ Soft, colinear radiation is attached to the quark jet(s)

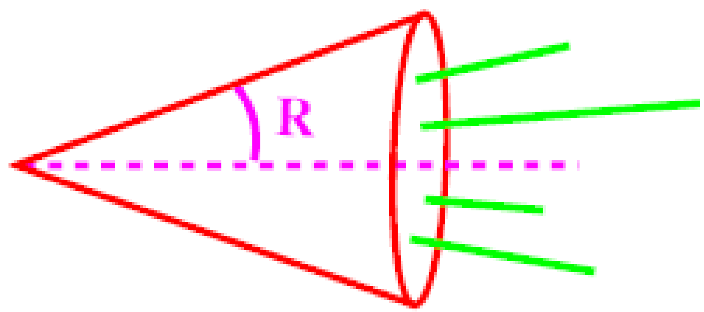


$\longrightarrow$ Permits re-summation

The Cone jet finder

CDF Collab., Phys. Rev. D45, 1448 (1992); OPAL Collab., Z. Phys. C63, 197 (1994)

- Cluster particles within a cone of half angle R into a jet



- Require a minimum visible jet energy: $E_{\text{jet}} \geq \epsilon$

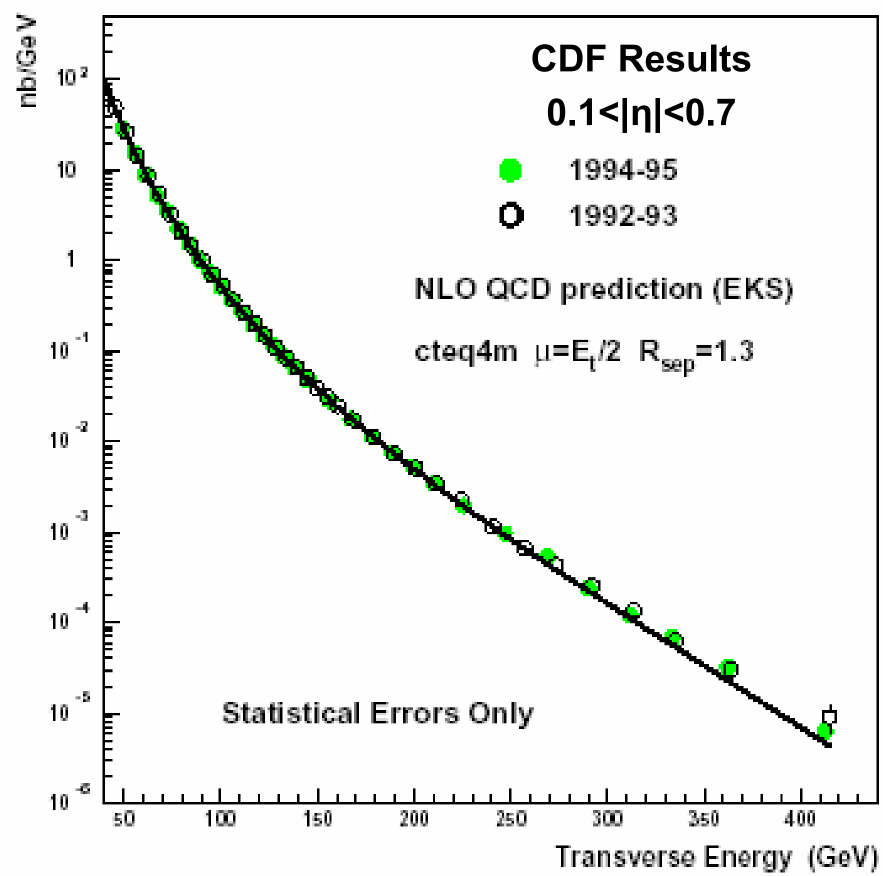
$\longrightarrow$ Two resolution parameters: R and ϵ , as opposed to re-combination algorithms which only have one (y_{cut})

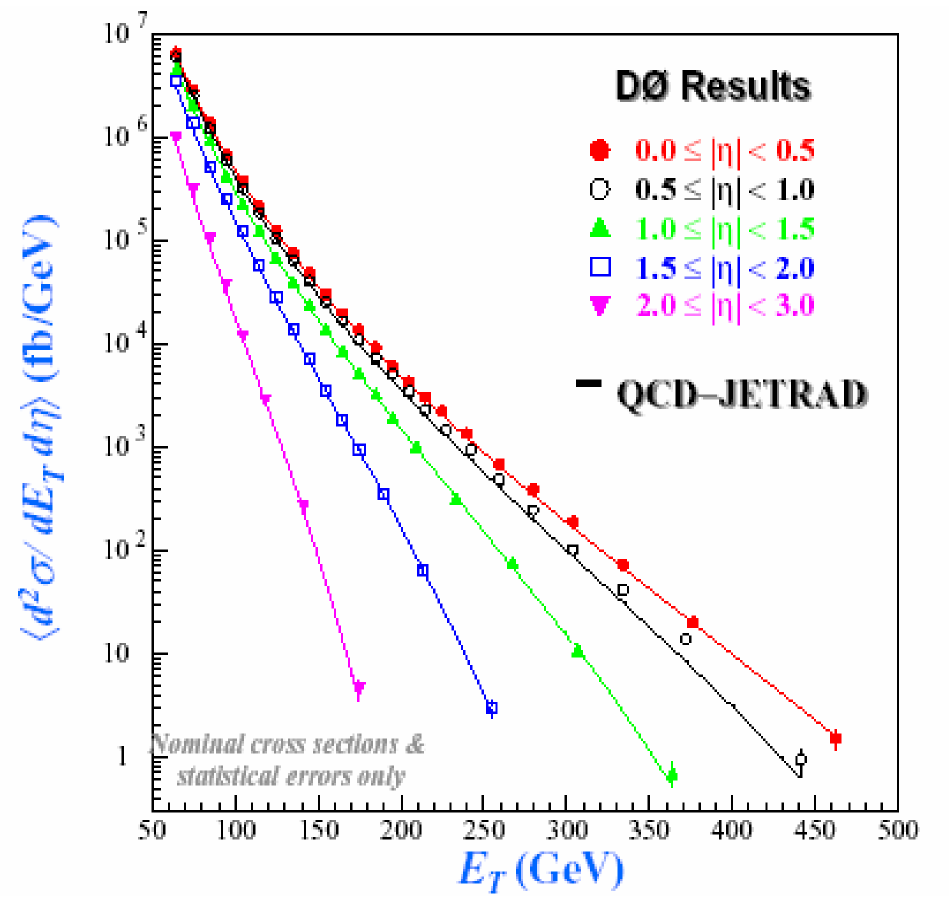
- Eliminate or merge overlapping or redundant jets

$\longrightarrow$ Unlike recombination algorithms, not all particles in an event are necessarily assigned to a jet

Inclusive jet cross section at Tevatron

Run – 1b results





Data and Predictions span 7 orders of magnitude!

Infrared safety for jet cross sections

- Jet cross section = inclusive cross section with a phase-space constraint
- For any observable with a phase space constraint, Γ,

$$
\begin{aligned}
d\sigma(\Gamma) \equiv & \frac{1}{2!}\int d\Omega_2 \frac{d\sigma^{(2)}}{d\Omega_2}\Gamma_2(k_1,k_2) \\
& + \frac{1}{3!}\int d\Omega_3 \frac{d\sigma^{(3)}}{d\Omega_3}\Gamma_3(k_1,k_2,k_3) \\
& + \ldots \\
& + \frac{1}{n!}\int d\Omega_n \frac{d\sigma^{(n)}}{d\Omega_n}\Gamma_n(k_1,k_2,\ldots,k_n) + \ldots
\end{aligned}
$$

Where $\Gamma_n(k_1,k_2,\ldots,k_n)$ are constraint functions and invariant under Interchange of n-particles

Measurement cannot distinguish a state with a zero momentum parton from a state without the parton

- Conditions for IRS of $d\sigma(\Gamma)$:

$$\Gamma_{n+1}\left(k_1,k_2,\ldots,(1-\lambda)k_n^\mu,\lambda k_n^\mu\right)=\Gamma_n\left(k_1,k_2,\ldots,k_n^\mu\right) \quad \text{with} \;\; 0\le\lambda\le 1$$

Special case: $\Gamma_n\left(k_1,k_2,\ldots,k_n\right)=1$ for all n $\Rightarrow$ $\sigma^{(\mathrm{tot})}$

PQCD Factorization

- Can pQCD calculate cross sections **with identified hadrons?**

Typical hadronic scale: $1/R \sim 1\ \mathrm{fm}^{-1} \sim \Lambda_{QCD}$

Energy exchange in hard collisions: $Q >> \Lambda_{QCD}$

⇨ pQCD works at $\alpha_s(Q)$, but not at $\alpha_s(1/R)$

- **PQCD can be useful iff quantum interference between perturbative and nonperturbative scales can be neglected**

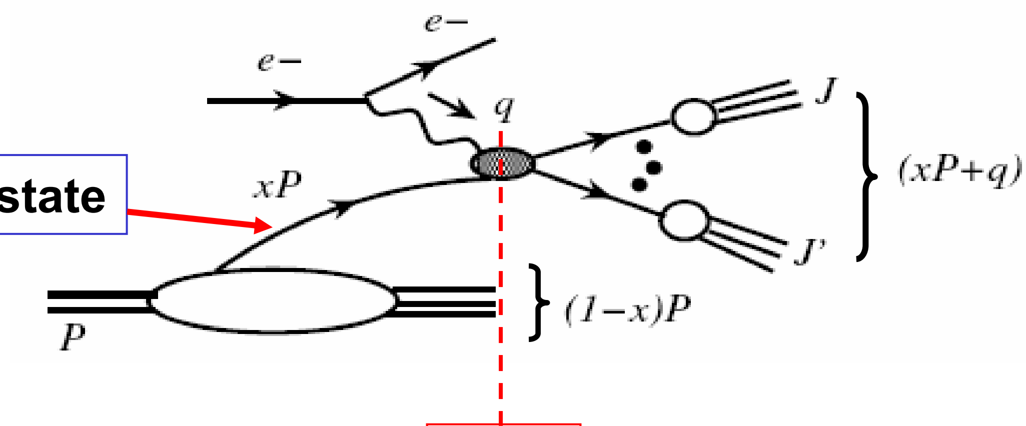
$$\underbrace{\sigma_{\text{phy}}}_{\text{Measured}}(Q,1/R) \sim \underbrace{\hat{\sigma}}_{\text{Short-distance}}(Q) \otimes \underbrace{\varphi}_{\text{Long-distance}}(1/R) + \underbrace{O(1/QR)}_{\text{Power corrections}}$$

⇨ **Factorization – "forgetting the past"**

Picture of factorization in DIS

- **Time evolution:**

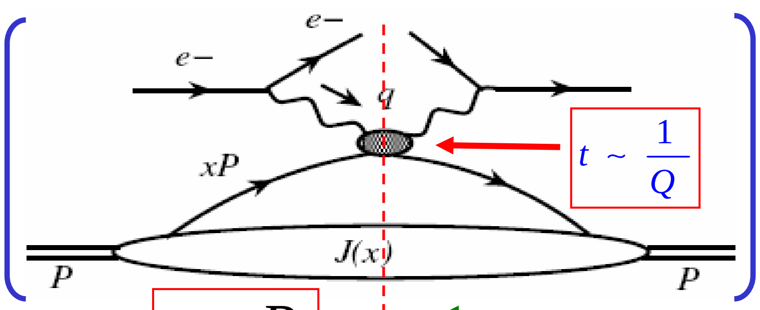
Long-lived parton state



Time: "Past" **Now** "Future"

- **Unitarity – summing over all hard jets:**

$$\sigma^{\text{DIS}}_{\text{tot}} \propto \text{Im}$$



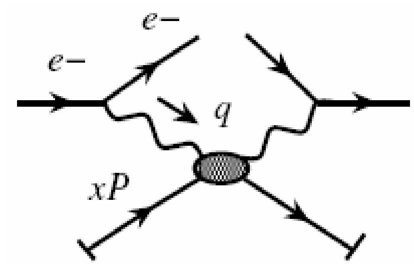
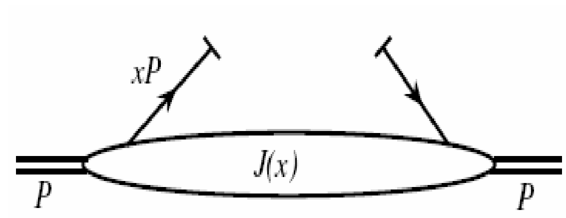
$t \sim \frac{1}{Q}$

$t \sim R$

Not IR safe

Interaction between the "past" and "now" are suppressed!

Factorization in DIS

$\sigma_{\text{tot}}^{\text{DIS}} \sim$  $\otimes$  $+\, O\left(\dfrac{1}{QR}\right)$

Now | Past | Connection

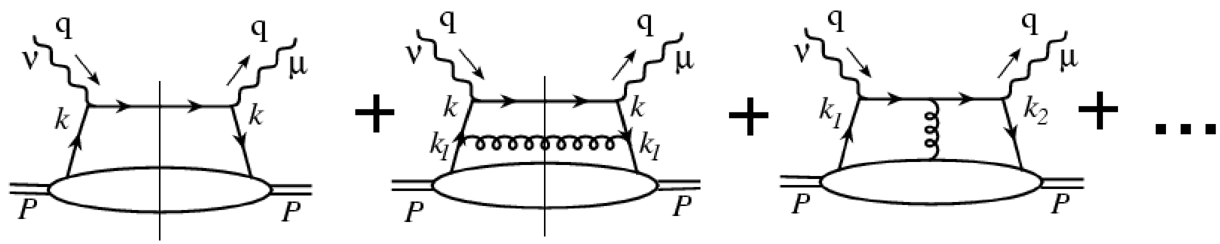
Predictive power of pQCD

- ❖ short-distance and long-distance are separately gauge invariant
- ❖ short-distance part is Infrared-Safe, and calculable
- ❖ long-distance part can be defined to be Universal

Long-lived parton states

❑ Feynman diagram representation:

$$W^{\mu\nu} \propto$$



❑ Perturbative pinched poles:

$$\int d^4k \; \mathrm{H}(Q,k)\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}(k,\frac{1}{r_0}) \;\Rightarrow\; \infty \;\; \text{perturbatively}$$

❑ Perturbative factorization:

$$k^\mu = xp^\mu + \frac{k^2+k_T^2}{2xp\cdot n}n^\mu + k_T^\mu$$

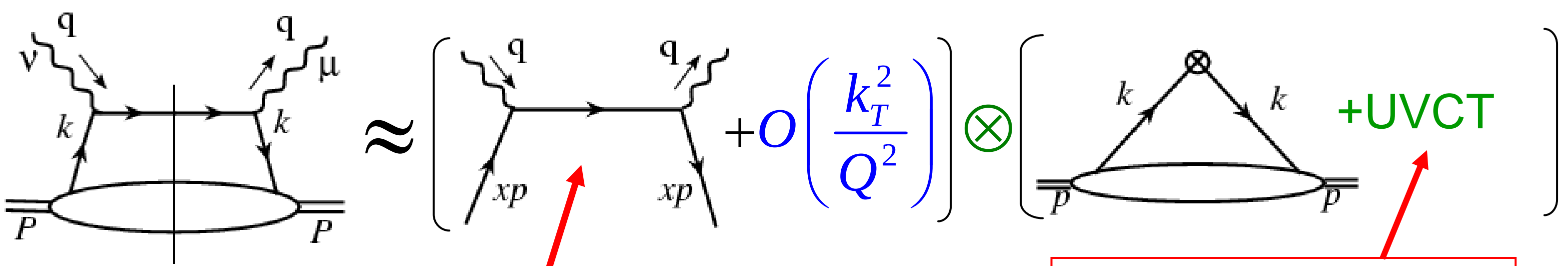
Nonperturbative matrix element

$$\int \frac{dx}{x}d^2k_T \; \mathrm{H}(Q,k^2=0)\;\boxed{\int dk^2\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}(k,\frac{1}{r_0})}$$

Short-distance

Collinear factorization

- **Collinear approximation, if** $Q \sim xp\cdot n \gg k_T, \sqrt{k^2}$



Same as elastic x-section

Scheme dependence

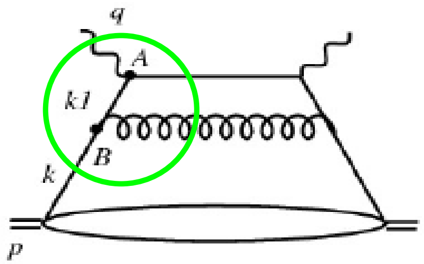
- **DIS limit:** $\nu, Q^2 \to \infty$, while x_B fixed

⇨ **Feynman's parton model and Bjorken scaling**

$$F_2(x_B, Q^2) = x_B \sum_f e_f^2\, \varphi_f(x_B) + O(\alpha_s) + O\left(\frac{\Lambda^2_{\rm QCD}}{\rm Q^2}\right)$$

Scaling violation and factorization

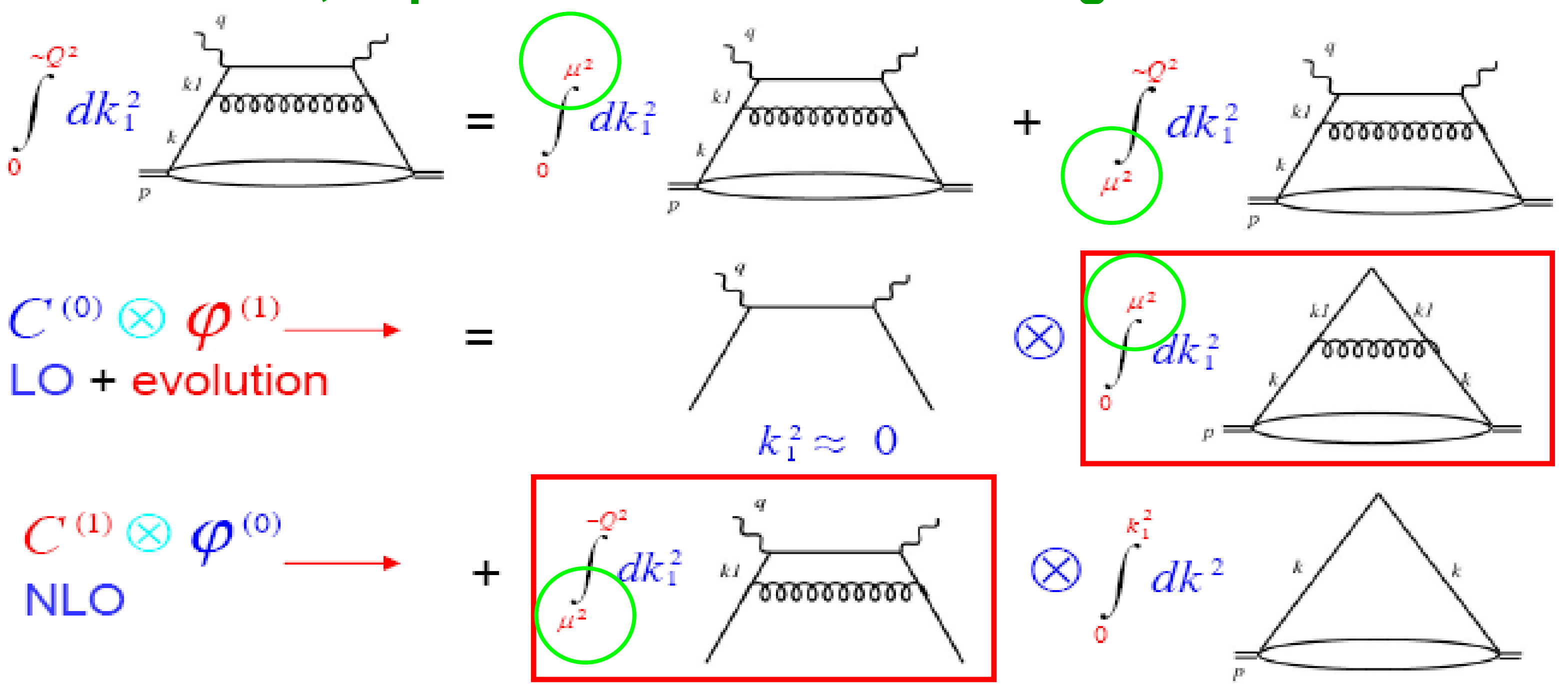
- **NLO partonic diagram to structure functions:**



$$\propto \int_0^{-Q^2} \frac{dk_1^2}{k_1^2} \qquad \textbf{Dominated by} \quad \begin{cases} k_1^2 \sim 0 \\ t_{AB} \to \infty \end{cases}$$

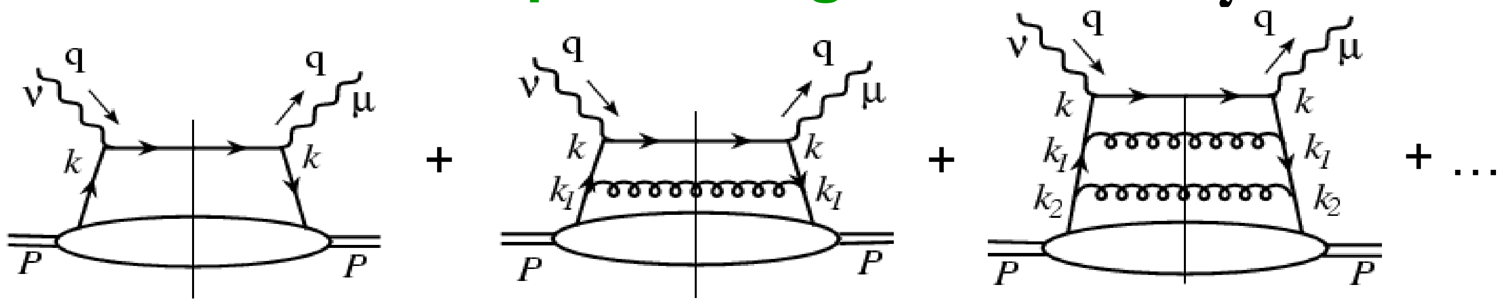
Diagram has both long- and short-distance physics

- **Factorization, separation of short- from long-distance:**

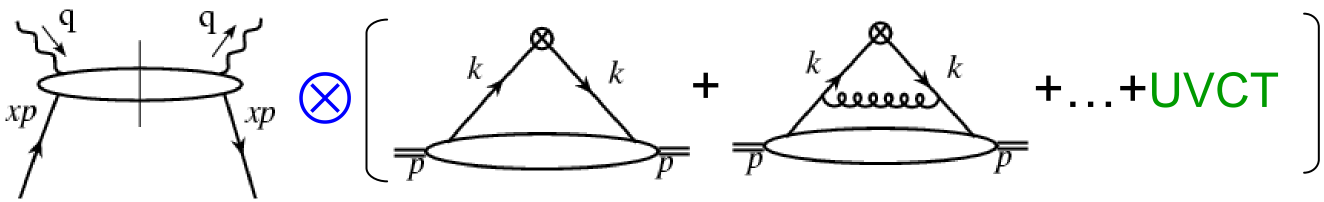


Leading power QCD formula

- ❑ **QCD corrections: pinch singularities in** $\int d^4 k_i$



- ❑ **Logarithmic contributions into parton distributions**



$$\Longrightarrow \quad F_2(x_B, Q^2) = \sum_f C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \otimes \varphi_f\left(x, \mu_F^2\right) + O\left(\frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}\right)$$

- ❑ **Factorization scale:** μ_F^2

⟶ **To separate collinear from non-collinear contribution**

Recall: renormalization scale to separate local from non-local contribution

Dependence on factorization scale

- Physical cross sections should not depend on the factorization scale

$$\mu_F^2 \frac{d}{d\mu_F^2} F_2(x_B, Q^2) = 0$$

→ **Evolution (differential-integral) equation for PDFs**

$$\sum_f \left[\mu_F^2 \frac{d}{d\mu_F^2} C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right)\right] \otimes \varphi_f\left(x, \mu_F^2\right) + \sum_f C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \otimes \mu_F^2 \frac{d}{d\mu_F^2} \varphi_f\left(x, \mu_F^2\right) = 0$$

- PDFs and coefficient functions share the same logarithms

PDFs: $\log\left(\mu_F^2/\mu_0^2\right)$ or $\log\left(\mu_F^2/\Lambda_{\text{QCD}}^2\right)$

Coefficient functions: $\log\left(Q^2/\mu_F^2\right)$ or $\log\left(Q^2/\mu^2\right)$

→ **DGLAP evolution equation:**

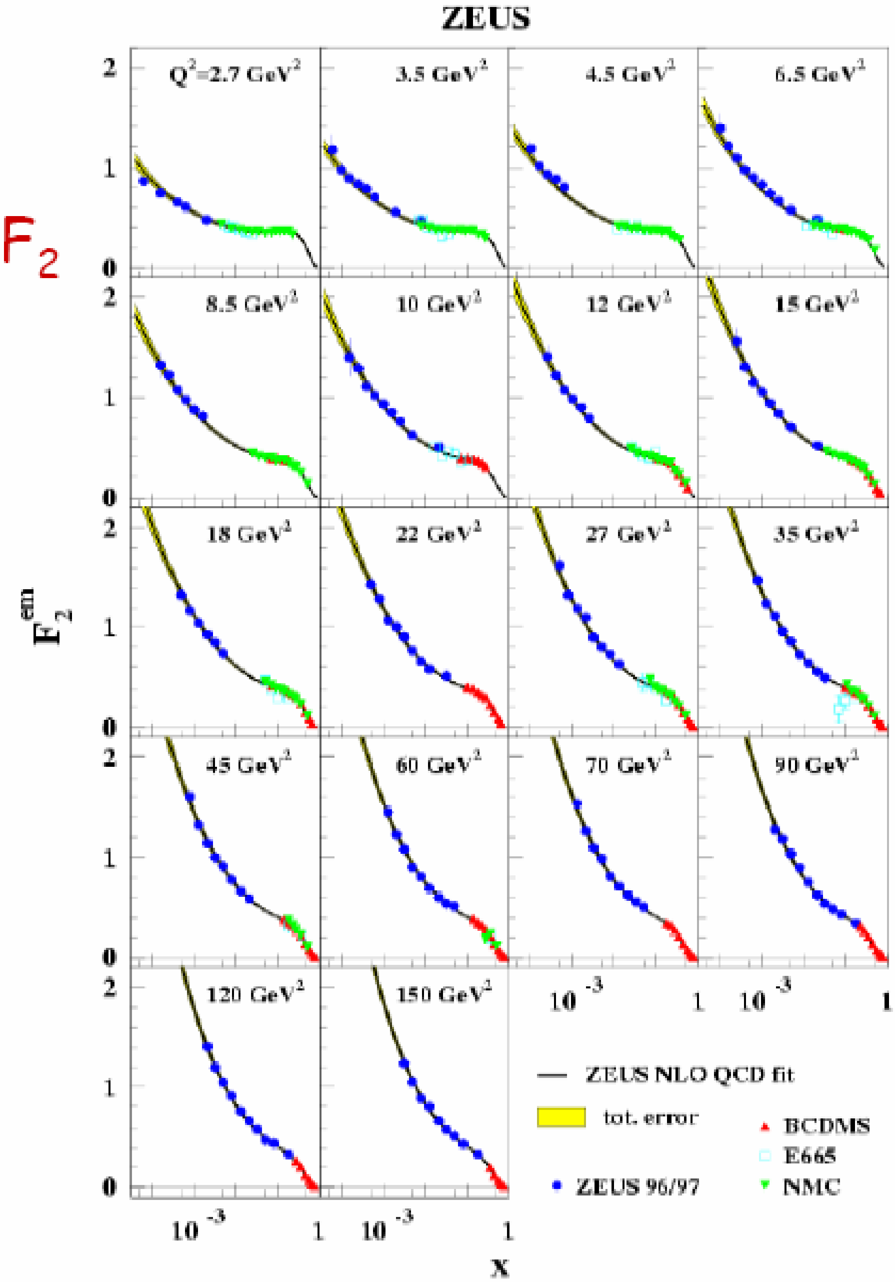
$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \varphi_i(x, \mu_F^2) = \sum_j P_{i/j}\left(\frac{x}{x'}, \alpha_s\right) \otimes \varphi_j(x', \mu_F^2)$$

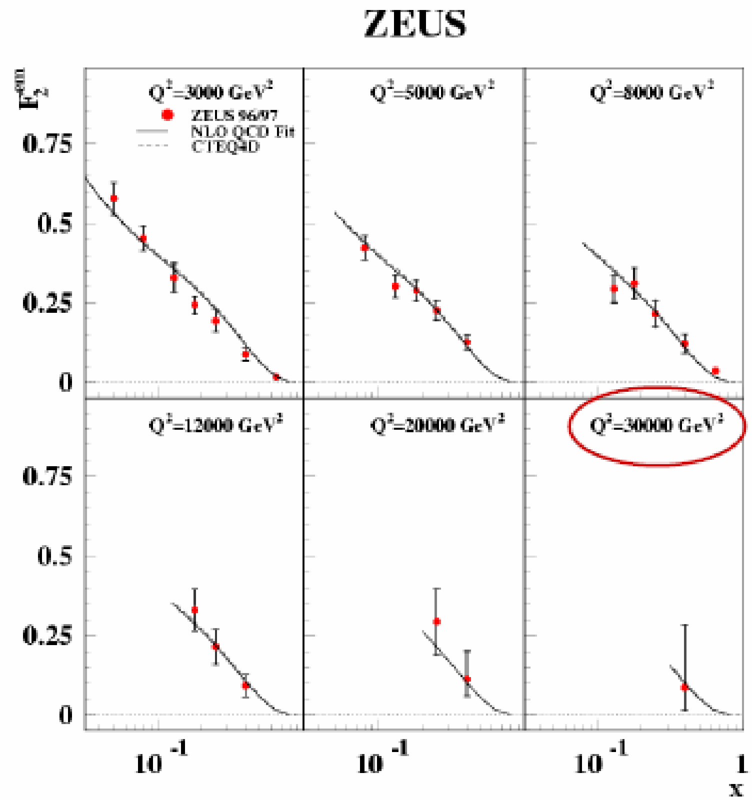
- Predictive power of pQCD:

Universality of PDFs and their scale dependence

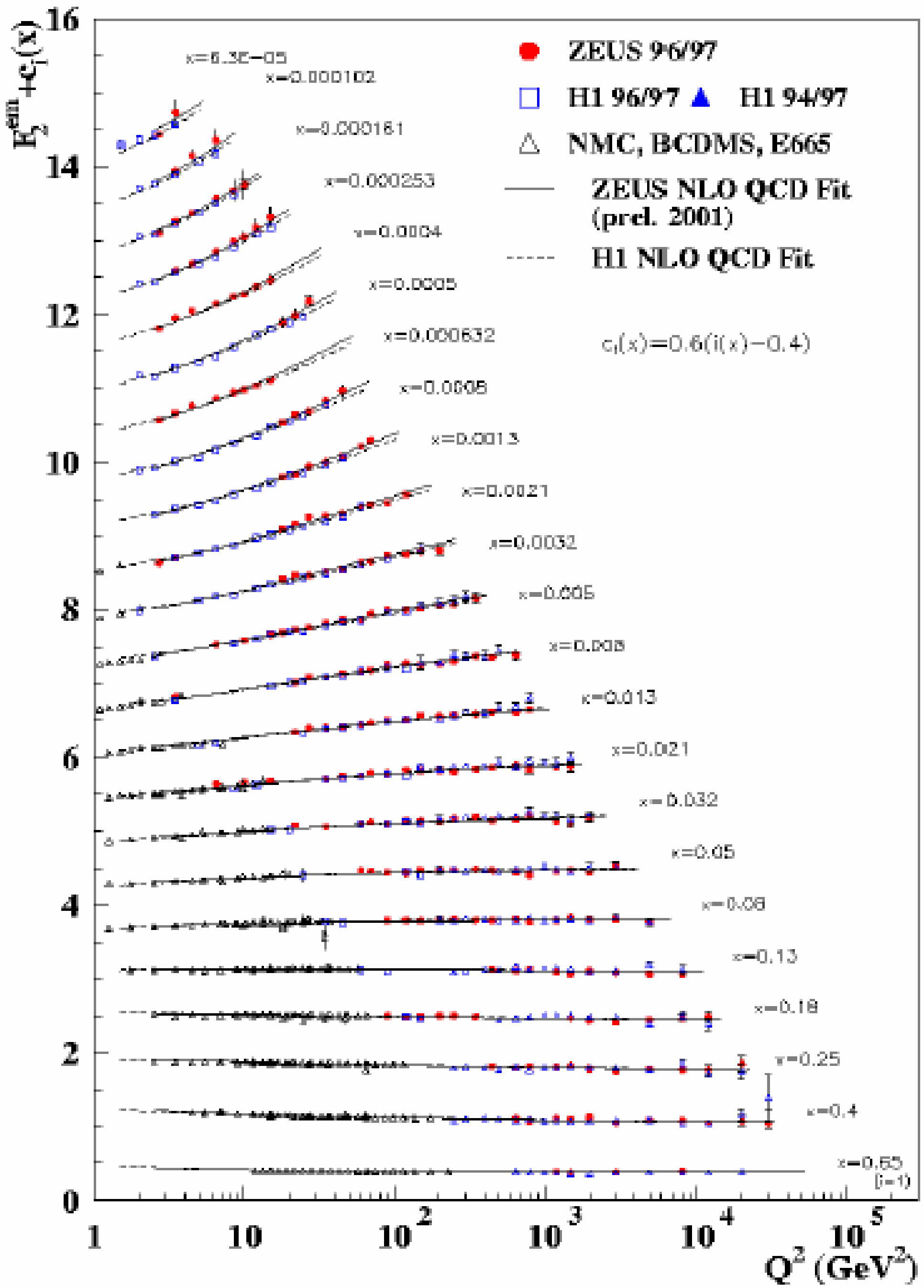
→ Q^2– **dependence of physical observables**

F_2





At high-Q^2 still statistics limited… → priority to the measurements at high-Q^2



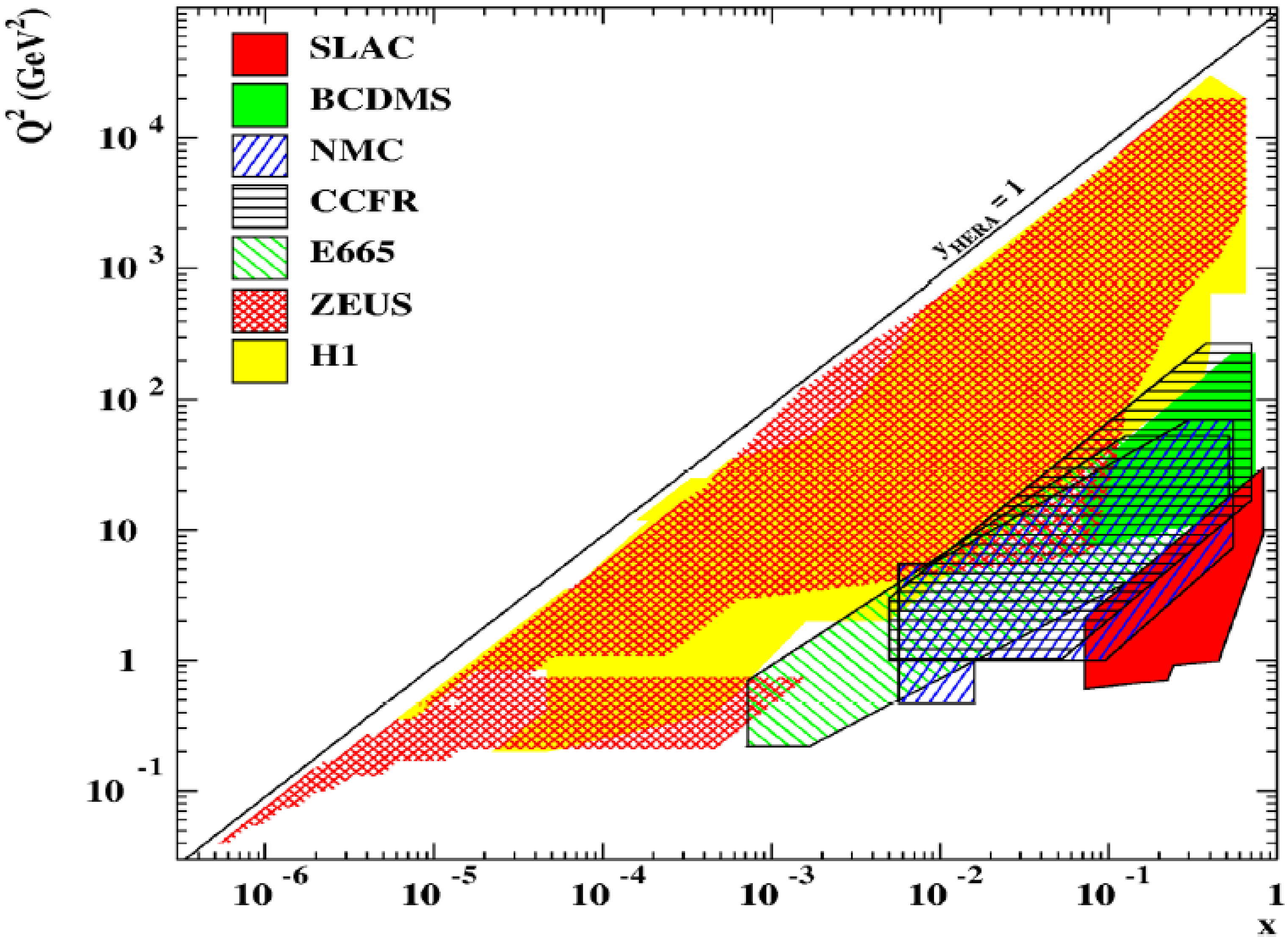
Measurement of F_2^{γ}

When $Q^2 \ll M_Z^2$, we can neglect the Z^0-contribution

Precision test of QCD:

as good as 2-3% error for such difficult measurement

Kinematic Regions of DIS



Improvement from the fixed order

- Beyond the Born term (lowest order), partonic hard-parts are **NOT** unique, due to renormalization of parton distributions

- Once $\varphi(x,\mu^2)$ is fixed in one scheme, same $\varphi(x,\mu^2)$ should be used for all calculations of partonic parts

- Coefficient has the $P_{qq}(x)\ell n\left(\frac{Q^2}{\mu_F^2}\right)$

  Suggests to choose the scale: $\mu_F^2 \sim Q^2$

- Coefficient has potentially large logarithms:

$$\ell n(x), \quad \frac{1}{(1-x)_+}, \quad \left(\frac{\ell n(1-x)}{1-x}\right)_+$$

$\Rightarrow$ Resummation of the large logarithms

Recover the effect of non-vanishing k_T

❑ Sources of power corrections:

- ❖ Parton transverse momentum: $\langle k_\perp^2 \rangle / Q^2 \sim \langle k^2 \rangle / Q^2$
- ❖ Target and parton masses: m^2/Q^2
- ❖ Coherent multiple scattering: $\left[\left(1/Q^2 \right) / R^2 \right] \langle F^+{}_\perp F^{+\perp} \rangle \langle \text{Medium length} \rangle$

❑ Systematics of power corrections:

Leading Twist

$$\sigma^h_{phys} = \boxed{\hat{\sigma}^i_2 \otimes [1 + \alpha_s + \alpha_s^2 + \ldots] \otimes T_2^{i/h}(x)}$$
$$+ \frac{\hat{\sigma}^i_4}{Q^2} \otimes [1 + \alpha_s + \alpha_s^2 + \ldots] \otimes T_4^{i/h}(x)$$
$$+ \frac{\hat{\sigma}^i_6}{Q^4} \otimes [1 + \alpha_s + \alpha_s^2 + \ldots] \otimes T_6^{i/h}(x)$$
$$+ \ldots$$

perturbative

Power corrections

Factorization may not be true for power corrections!

Need to be proved for any given process

Summary

- ❑ QCD is a SU(3) color non-Abelian gauge theory of quark and gluon fields
- ❑ QCD perturbation theory works at high energy because of the asymptotic freedom
- ❑ Perturbative QCD calculations make sense only for infrared safe (IRS) quantities
- ❑ Jets in high energy collisions provide us the “trace” of energetic quarks and gluons
- ❑ We can actually “see” and “count” the quarks and gluons?