Nucleon Polarizabilities; Color & Spin

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LECTURE IV

Outline

- Introduction
- Sum Rules (Bjorken sum rule, Gerasimov-Drell-Hearn sum rule and Burkhardt-Cottingham Sum Rule)
- At large Q^2: Color polarizabilities χ_E and χ_B
  - Experimental status of twist-3 d_2 and twist-4 f_2
- At low Q^2: Generalized Spin Polarizabilities γ_0 and $\delta_L T$
  - Experimental status
- Summary

Moments of spin structure functions

GDH
Sum Rule
$I_{GDH}(0)$

Generalized
GDH Integral
$I_{GDH}(Q^2)$

Bjorken
Sum Rule
$\Gamma_1^{p\text{-}n}$

Burkhardt-Cottingham
sum rule
$\Gamma_2(Q^2)$

Spin
Polarizabilities
$\gamma_0(Q^2), \delta_{LT}(Q^2)$

Higher twists & color
Polarizabilities
$d_2(Q^2), f_2(Q^2)$

0 1 10 ∞

Q^2

χPT *Lattice QCD* *OPE* *pQCD*

Sum Rules using Spin Structure Functions of the Nucleon

At large but finite Q^2 where pQCD is safe to use:

Bjorken Sum Rule:

$$\Gamma_1^p(Q^2) - \Gamma_1^n(Q^2) = \int_0^1 g_1^p(x,Q^2) - g_1^n(x,Q^2) \; dx \; = \; \frac{1}{6}\, g_A \, C_{NS}$$

Burkhardt-Cotthingham Sum Rule:

$$\int_0^1 g_2^p(x,Q^2)\; dx = 0 \quad ; \quad \int_0^1 g_2^n(x,Q^2)\, dx = 0$$

Gourdin-Ellis-Jaffe:

$$\Gamma_1^p(Q^2) \; = \; \int_0^1 g_1^p(x,Q^2)dx = \frac{1}{18}\left[C_{ns}(3F+D) + 2C_s(3F-D)\right]$$

$$\Gamma_1^n(Q^2) \; = \; \int_0^1 g_1^n(x,Q^2)dx = \frac{1}{9}\left[-DC_{ns} + C_s(3F-D)\right]$$

At $Q^2 = 0$, real photon case

Gerasimov-Drell-Hearn-Sum Rule:

$$I^{GDH} = \int_{\nu_{th}}^{\infty} \sigma_{TT}\frac{d\nu}{\nu} = \int_{\nu_{th}}^{\infty}(\sigma_{1/2} - \sigma_{3/2})\frac{d\nu}{\nu} = -\frac{2\pi^2\alpha}{M^2}\kappa^2$$

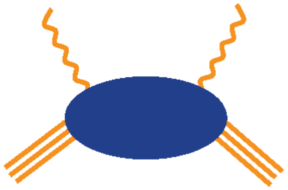
Intermediate region Q^2 (Confinement Region)

Generalized GDH Sum Rule:

$$\mathrm{Im}S_i(\nu,Q^2) = 2\pi G_i(\nu,Q^2)$$

$$S_1(\nu,Q^2) = 4\int_{Q^2/2M}^{\infty} \frac{\nu' G_1(\nu',Q^2)}{\nu'^2 - \nu^2} d\nu'$$

$$S_2(\nu,Q^2) = 4\int_{Q^2/2M}^{\infty} \frac{\nu' G_2(\nu',Q^2)}{\nu'^2 - \nu^2} d\nu'$$

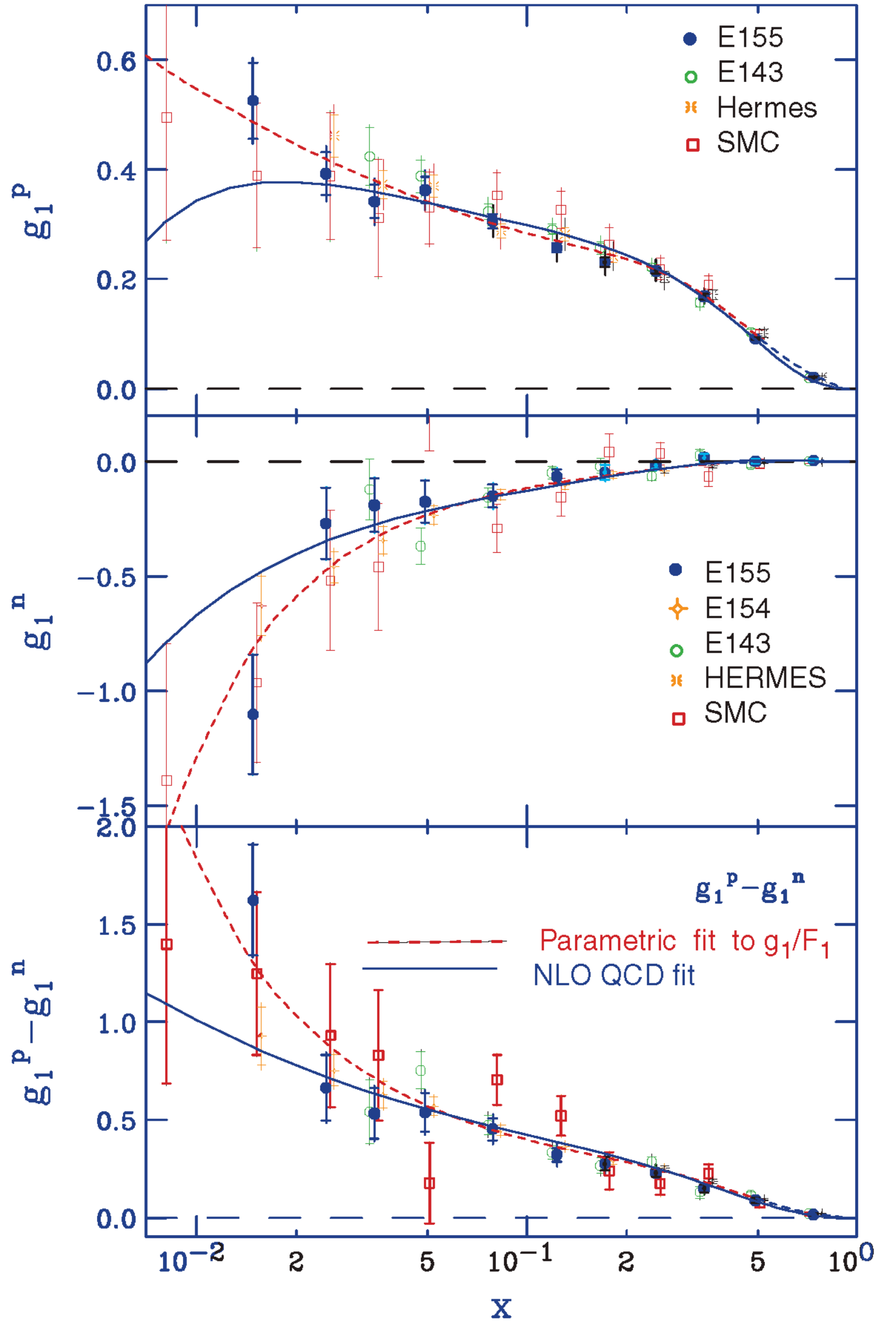


Virtual Compton Amplitude

Summary of High Energy Spin Structure Function Measurements

Lab	Exp.	Year	Beam	$\langle Q^2 \rangle$ GeV^2	x	P_B	Target	P_T	f	$\mathcal{L} \times 10^{-32}$ cm^{-2}-s
SLAC	E80	75	10-16 GeV e^-	2	$0.1 - 0.5$	85%	H-butanol	50%	0.13	400
	E130	80	16-23 GeV e^-	5	$0.1 - 0.6$	81%	H-butanol	58%	0.15	400
	E142	92	19-26 GeV e^-	2	$0.03 - 0.6$	39%	^{3}He	35%	0.35	2000
	E143	93	10-29 GeV e^-	3	$0.03 - 0.8$	85%	NH_3	70%	0.15	1000
							ND_3	25%	0.24	1000
	E154	95	48 GeV e^-	5	$0.01 - 0.7$	82%	^{3}He	38%	0.55	3000
	E155	97	48 GeV e^-	5	$0.01 - 0.9$	81%	NH_3	90%	0.15	1000
							LiD	22%	0.36	1000
	E155'	99	30 GeV e^-	3	$0.02 - 0.9$	83%	NH_3	75%	0.16	1000
							LiD	22%	0.36	1000
CERN	EMC	85	100-200 GeV μ^+	11	$0.01 - 0.7$	79%	NH_3	78%	0.16	0.3
	SMC	92	100 GeV μ^+	4.6	$0.006 - 0.6$	82%	D-butanol	35%	0.19	0.3
		93	190 GeV μ^+	10	$0.003 - 0.7$	80%	H-butanol	86%	0.12	0.6
		94-95				81%	D-butanol	50%	0.20	0.6
		96				77%	NH_3	89%	0.16	0.6
DESY	HERMES	95	28 GeV e^+	2.5	$0.02 - 0.6$	55%	^{3}He	46%	1.0	1
		96-97				55%	H	88%	1.0	0.1
		98	28 GeV e^-			55%	D	85%	1.0	0.2
		99-00	28 GeV e^+			55%	D	85%	1.0	0.2
CERN	COMPASS	01	190 GeV μ^+	10	$0.005 - 0.6$	80%	NH_3	90%	0.16	3
							LiD	40%	0.50	3
BNL	RHIC	02	200 GeV p - p	~ 100	$0.05 - 0.6$	70%	Collider	70%	1.0	2
DESY	ZEUS/H1	??	28×800 GeV e - p	22	$0.00006 - 0.6$	70%	Collider	70%	1.0	0.2

World Results on g_1



The Bjorken Sum Rule

- Isospin Symmetry
- Current Algebra or • OPE within QCD

$$\Gamma_1^p(Q^2) - \Gamma_1^n(Q^2) = \int \; g_1^p(x,Q^2) - g_1^n(x,Q^2) \; dx \; = \; \frac{1}{6}\, g_A \, C_{NS}$$

$g_A = 1.2601 \pm 0.0025$ Neutron beta decay coupling constant

C_{NS} Non-singlet QCD correction.

Assuming three quark flavors, and in the $\overline{MS}$ scheme to order three:

$$C_{NS} = 1 - \frac{\alpha_s(Q^2)}{\pi} - 3.58\left(\frac{\alpha_s(Q^2)}{\pi}\right)^2 - 20.22\left(\frac{\alpha_s(Q^2)}{\pi}\right)^3$$

where $\alpha_s(Q^2)$ is the strong coupling constant.

Test of the Sum Rule

E155 Global Analysis

At $Q^2 = 5\ (\mathrm{GeV/c})^2$

Theory: $\Gamma_1^p - \Gamma_1^n = 0.182 \pm 0.005$

Experiment: $\Gamma_1^p - \Gamma_1^n = 0.176 \pm 0.003 \pm 0.007$

Combined world data are consistent with the Bjorken sum rule at the one standard deviation level of 5%

Moments of Structure Functions

$$\Gamma_1(Q^2) = \int_0^1 g_1(x, Q^2)\, dx = \mu_2 + \frac{\mu_4}{Q^2} + \frac{\mu_6}{Q^4} + \cdots$$

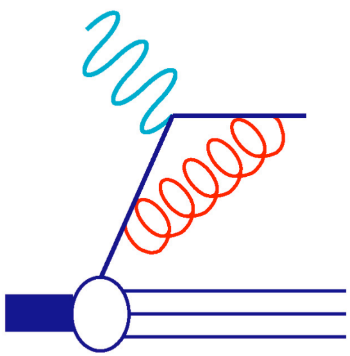
leading twist

higher twist

$$\mu_2^{p,n}(Q^2) = (\pm \frac{1}{12} g_A + \frac{1}{36} a_8) + \frac{1}{9}\Delta\Sigma$$ + pQCD corrections

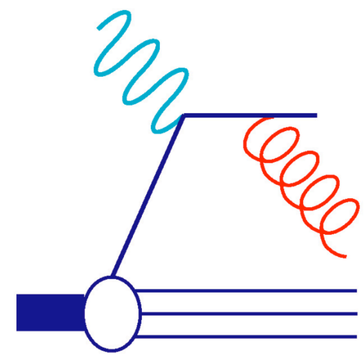
$g_A = 1.257$ and $a_8 = 0.579$ are the triplet and octet axial charge, respectively

$\Delta\Sigma$ = singlet axial charge



$$\begin{aligned} g_A &= \Delta u - \Delta d \\ a_8 &= \Delta u + \Delta d - 2\Delta s \\ \Delta\Sigma &= \Delta u + \Delta d + \Delta s \end{aligned}$$

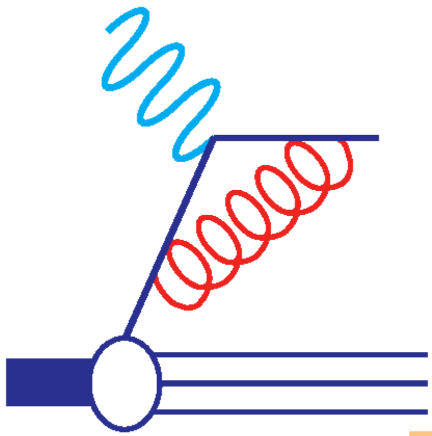
pQCD radiative corrections

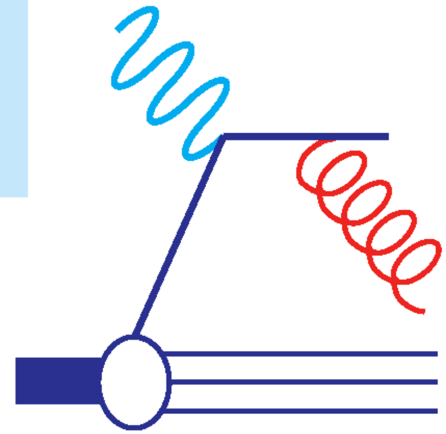


$g_A = 1.257$ and $a_8 = 0.579$ are the triplet and octet axial charge, respectively.

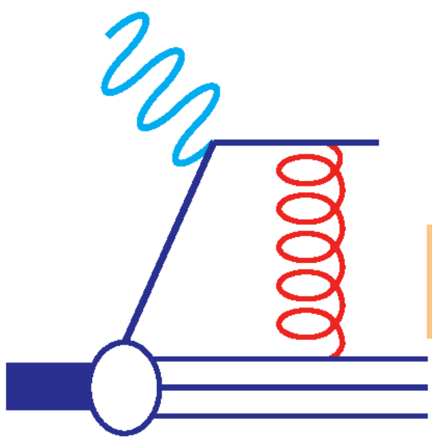
$\Delta\Sigma_\infty$ is defined as the renormalization group invariant nucleon matrix element of the singlet axial current.

$$\begin{aligned} g_\mathrm{A} &= \Delta u - \Delta d \\ a_8 &= \Delta u + \Delta d - 2\Delta s \\ \Delta\Sigma &= \Delta u + \Delta d + \Delta s \end{aligned}$$

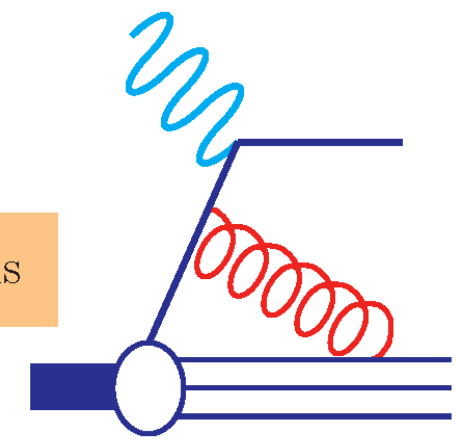




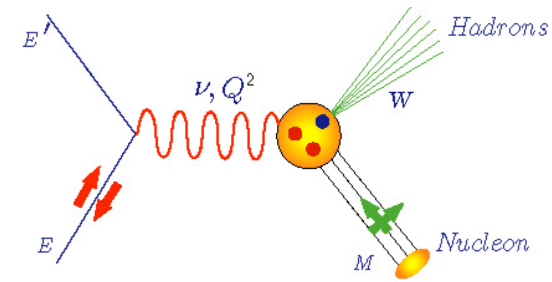
pQCD radiative corrections



Higher twist corrections



Spin Structure Functions



- Unpolarized structure functions $F_1(x,Q^2)$ and $F_2(x,Q^2)$

U $$\frac{d^2\sigma}{dE'd\Omega}(\downarrow\Uparrow + \uparrow\Uparrow) = \frac{8\alpha^2\cos^2(\theta/2)}{Q^4}\left[\frac{F_2(x,Q^2)}{\nu} + \frac{2F_1(x,Q^2)}{M}\tan^2(\theta/2)\right]$$

Q^2 :Four-momentum transfer
x : Bjorken variable
ν : Energy transfer
M : Nucleon mass
W : Final state hadrons mass

- Polarized structure functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$

L $$\frac{d^2\sigma}{dE'd\Omega}(\downarrow\Uparrow - \uparrow\Uparrow) = \frac{4\alpha^2}{MQ^2}\frac{E'}{\nu E}\left[(E+E'\cos\theta)g_1(x,Q^2) - \frac{Q^2}{\nu}g_2(x,Q^2)\right]$$

T $$\frac{d^2\sigma}{dE'd\Omega}(\downarrow\Rightarrow - \uparrow\Rightarrow) = \frac{4\alpha^2\sin\theta}{MQ^2}\frac{E'^2}{\nu^2 E}\left[\nu g_1(x,Q^2) + 2E g_2(x,Q^2)\right]$$

Virtual photon-nucleon asymmetries

Longitudinal

$$\frac{\sigma^{\downarrow\Uparrow} - \sigma^{\uparrow\Uparrow}}{\sigma^{\downarrow\Uparrow} + \sigma^{\uparrow\Uparrow}} = A_{\parallel} = D(A_1 + \eta A_2)$$

Transverse

$$\frac{\sigma^{\downarrow\Leftarrow} - \sigma^{\uparrow\Leftarrow}}{\sigma^{\downarrow\Leftarrow} + \sigma^{\uparrow\Leftarrow}} = A_{\perp} = d(A_1 - \xi A_2)$$

D, d, η and ξ are kinematic factors

D depends on $R(x,Q^2) = \sigma_L/\sigma_T$

$$A_1 = \frac{g_1(x,Q^2) - \gamma^2 g_2(x,Q^2)}{F_1(x,Q^2)}$$

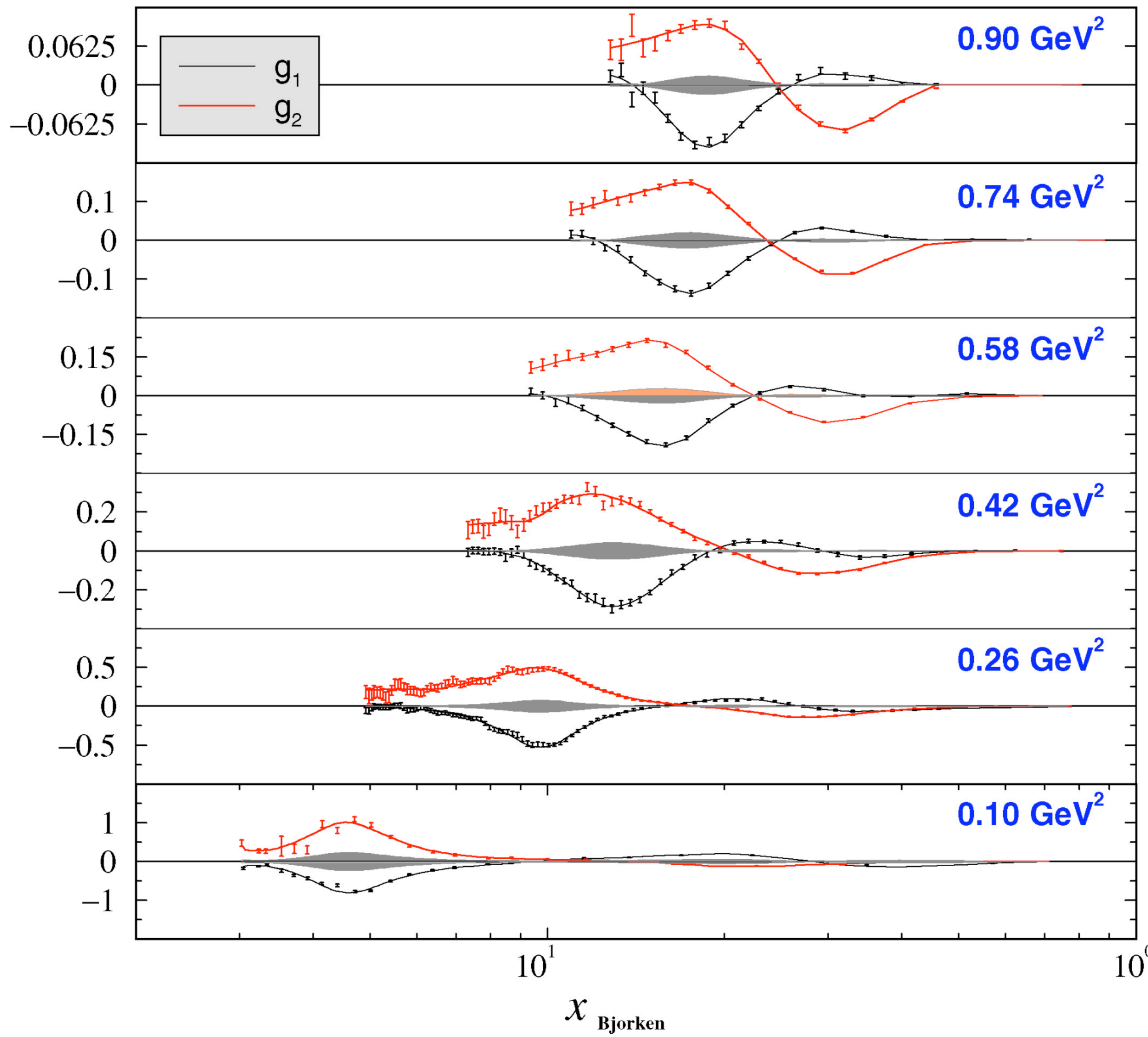
$$A_2 = \frac{\gamma[g_1(x,Q^2) + g_2(x,Q^2)]}{F_1(x,Q^2)}$$

where $\gamma = \sqrt{Q^2}/\nu$

- Positivity constraints

$|A_1| \leq 1$ and $|A_2| \leq \sqrt{R(1+A_1)/2}$

g_1 and g_2 extracted at constant Q^2



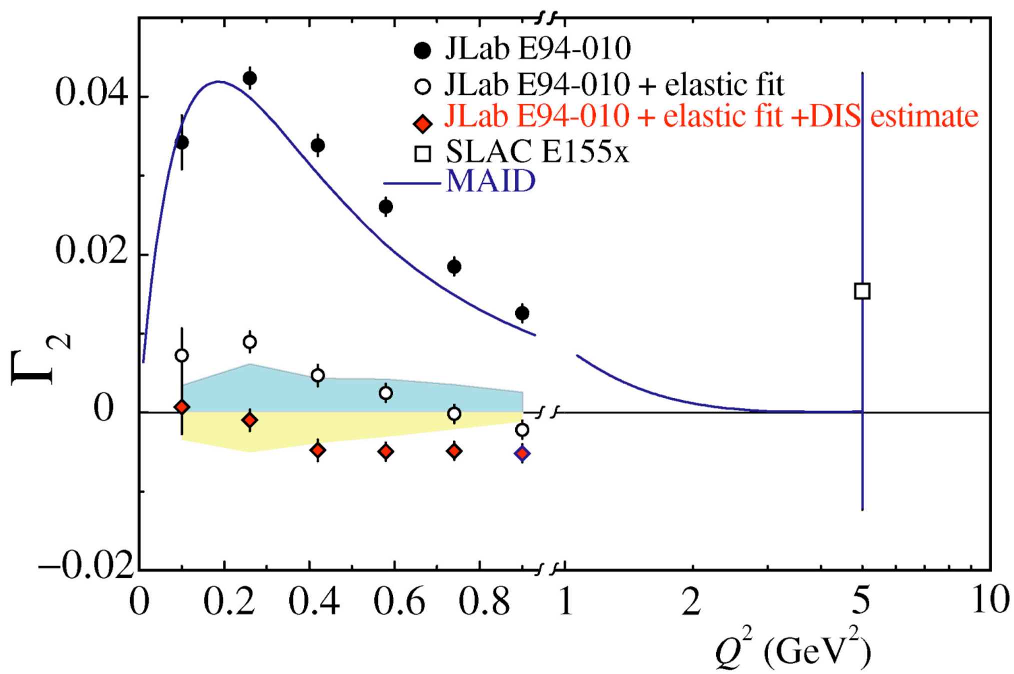
Burkhardt-Cottingham Sum Rule (1965-1966)

$$\Gamma_2(Q^2) = \int_0^1 g_2(x, Q^2)\, dx = 0$$

- Dispersion relation for a spin-flip Compton amplitude
  - Causality
  - Analyticity
  - Absence of a J=0 pole with non polynomial residue
- Doesn't follow from Operator Product Expansion and is valid at all Q^2
- Many scenarios of g_2's low x behavior which would invalidate the sum rule are discussed in the literature.

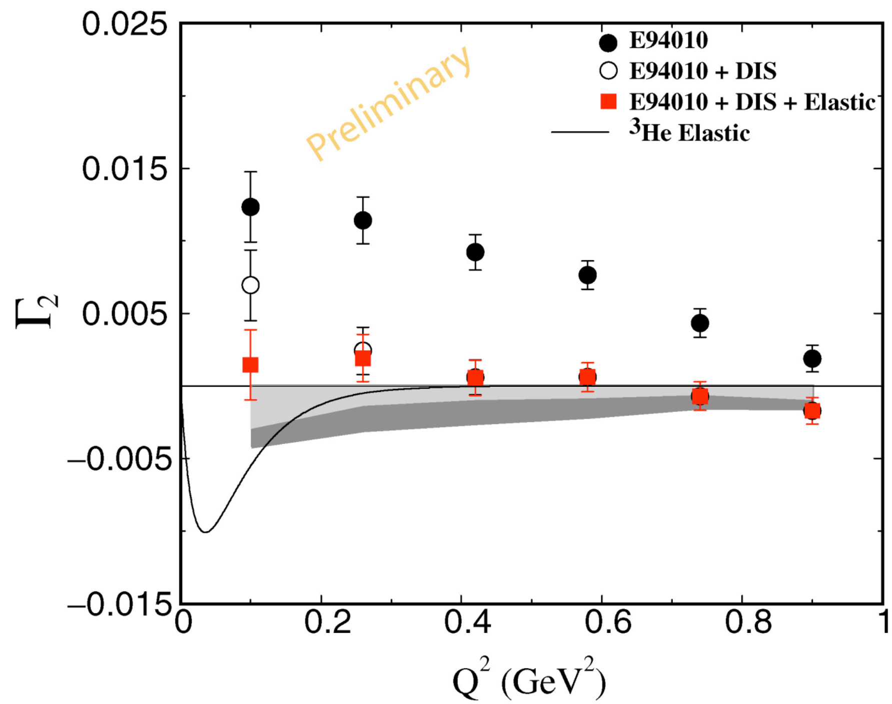
E94-010 B-C sum rule; results

Neutron



M. Amarian et al., Phys. Rev. Lett. 92, 022301 (2004)

^{3}He



M. Amarian et al., in preparation

Color "Polarizabilities"

X.Ji 95, E. Stein et al. 95

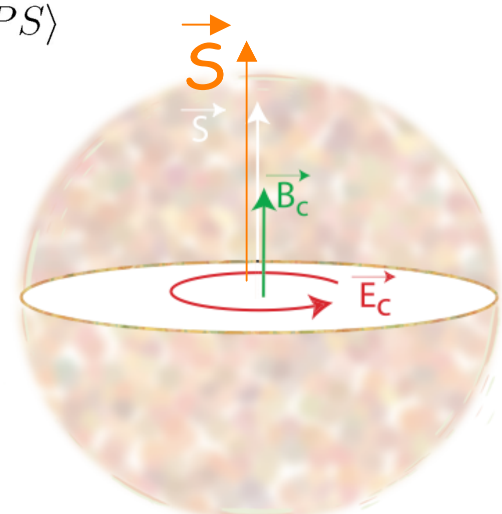
How does the gluon field respond when a nucleon is polarized ?

Define color magnetic and electric polarizabilities (in nucleon rest frame):

$$\chi_{B,E}\, 2M^2 \vec{S} = \langle PS | \vec{O}_{B,E} | PS \rangle$$

where $\vec{O}_B = \psi^\dagger g \vec{\boldsymbol{B}} \psi$

$$\vec{O}_E = \psi^\dagger \vec{\alpha} \times g \vec{\boldsymbol{E}} \psi$$

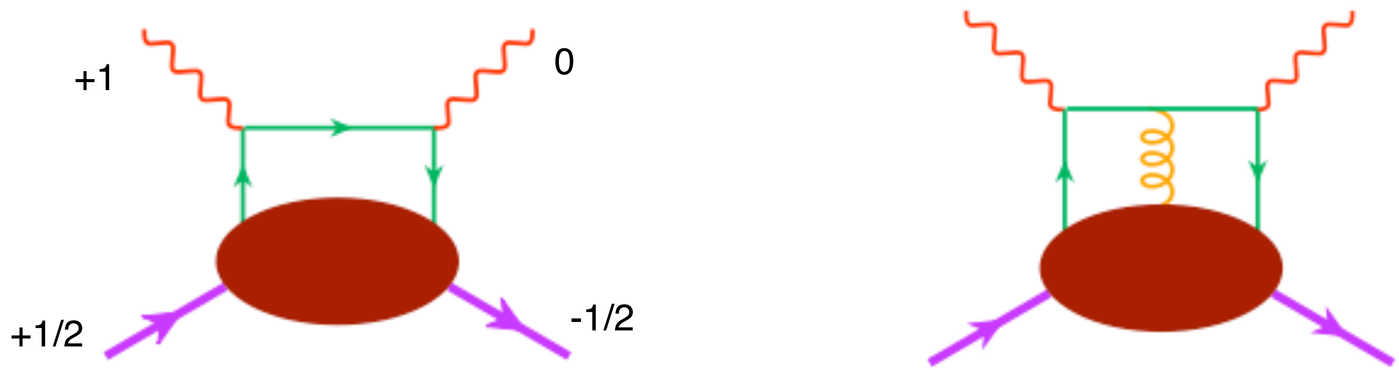


$$d_2 = (\chi_E + 2\chi_B)/8$$

$$f_2 = (\chi_E - \chi_B)/2$$

d_2 and f_2 represent the response of the color $\vec{B}$ & $\vec{E}$ fields to the nucleon polarization

g_2 and Quark-Gluon Correlations



$$g_2(x, Q^2) = g_2^{WW}(x, Q^2) + \bar{g}_2(x, Q^2)$$

- a twist-2 term (Wandzura & Wilczek, 1977):

$$g_2^{WW}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 g_1(x, Q^2)\frac{dy}{y}$$

- a twist-3 term with a suppressed twist-2 piece (Cortes, Pire & Ralston, 1992):

$$\bar{g}_2(x, Q^2) = -\int_x^1 \frac{\partial}{\partial y}\left[\frac{m_q}{M}\boxed{h_T(y, Q^2)} + \boxed{\xi(y, Q^2)}\right]\frac{dy}{y}$$

Transversity

qg correlations

Moments of Structure Functions

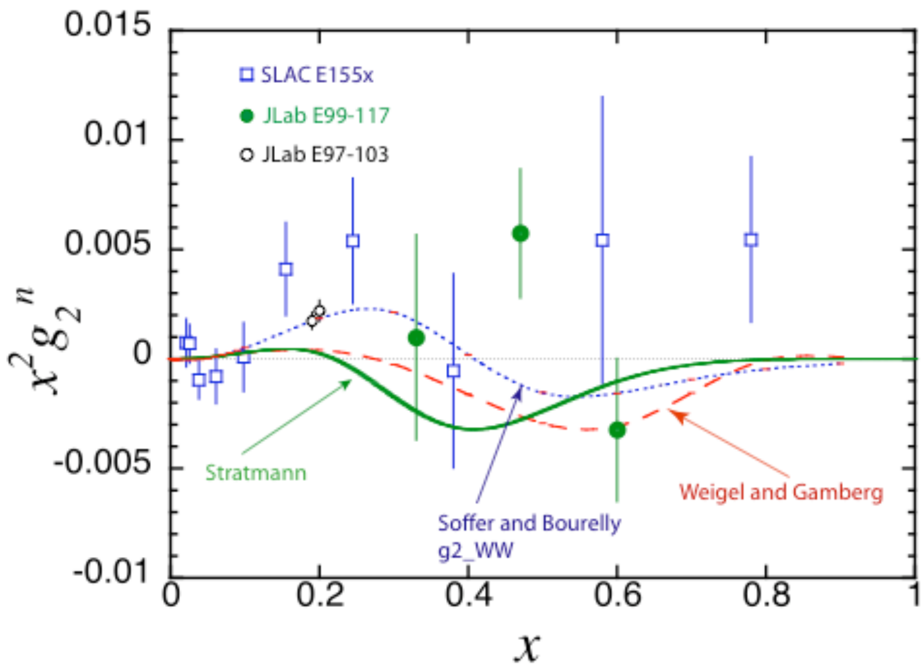
$d_2(Q^2)$ → dynamical twist-3 matrix element

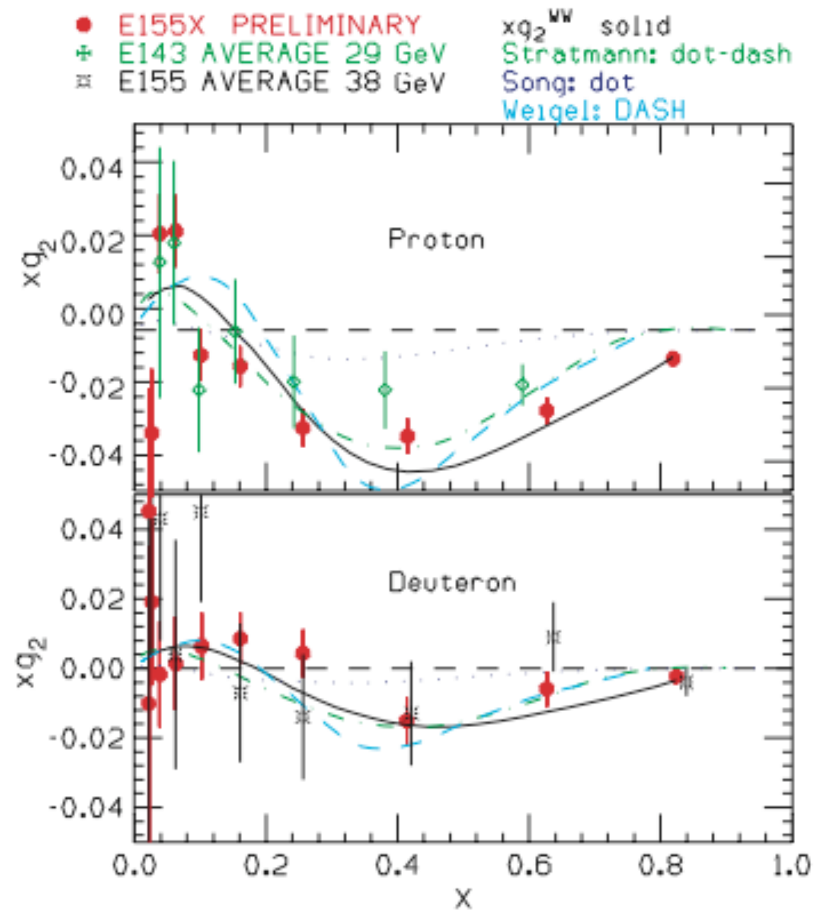
$$d_2 S^{[\mu} P^{\{\nu]} P^{\lambda\}} = \frac{1}{8} \sum_q \langle P, S | \bar{\psi}_q \; g \tilde{F}^{\{\mu\nu} \gamma^{\lambda\}} \psi_q | P, S \rangle$$

$$d_2(Q^2) = 3 \int_0^1 x^2 \left(g_2(x, Q^2) - g_2^{WW}(x, Q^2) \right) dx$$

$$\boxed{d_2(Q^2) = \int_0^1 dx \; x^2 \left[2 g_1(x, Q^2) + 3 g_2(x, Q^2) \right]}$$

Nucleon world results of g_2

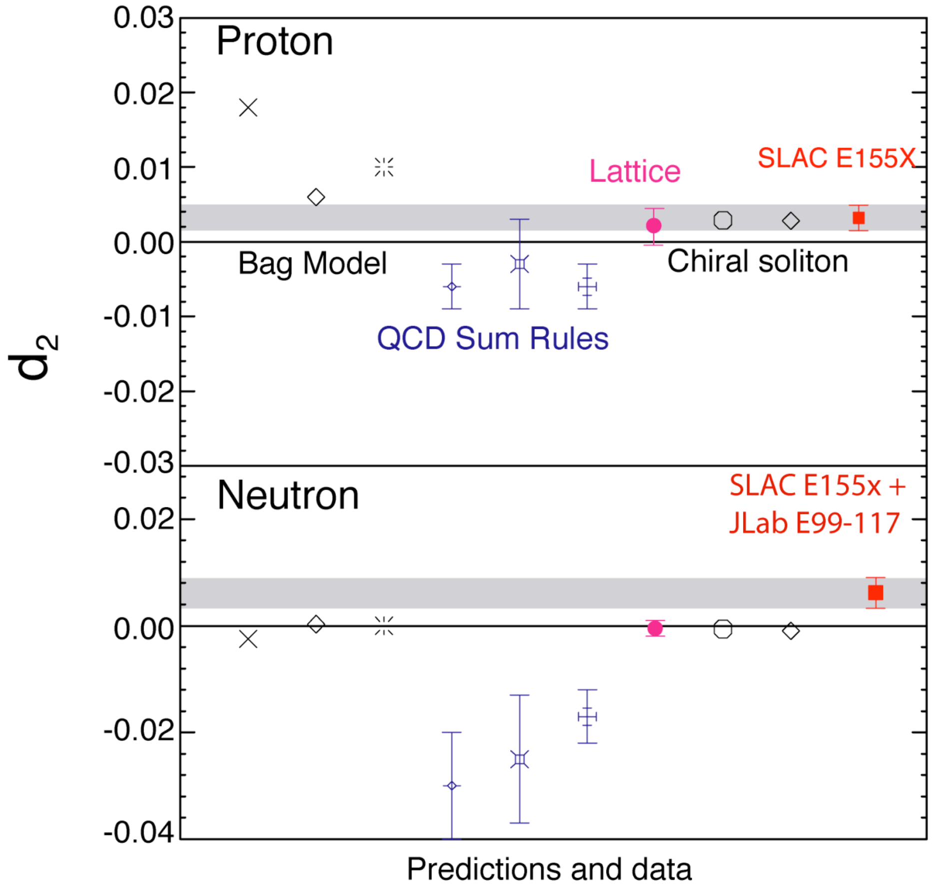




- SLAC E155x (proton and deuteron)
- JLab E99-117(helium-3), A_1^n in DIS
- Jlab E97-103 (helium 3) Q^2 dependence below 1 GeV^2

SLAC E155x (proton and deuteron)

Models and Lattice evaluations of d_2



Quark Bag Models

M.Stratmann, Z.Phys.C60,763(1993).
X.Song,Phys.Rev.D54,1955(1996).
X.Ji and P.Unrau, Phys.Lett.B333,228(1994).

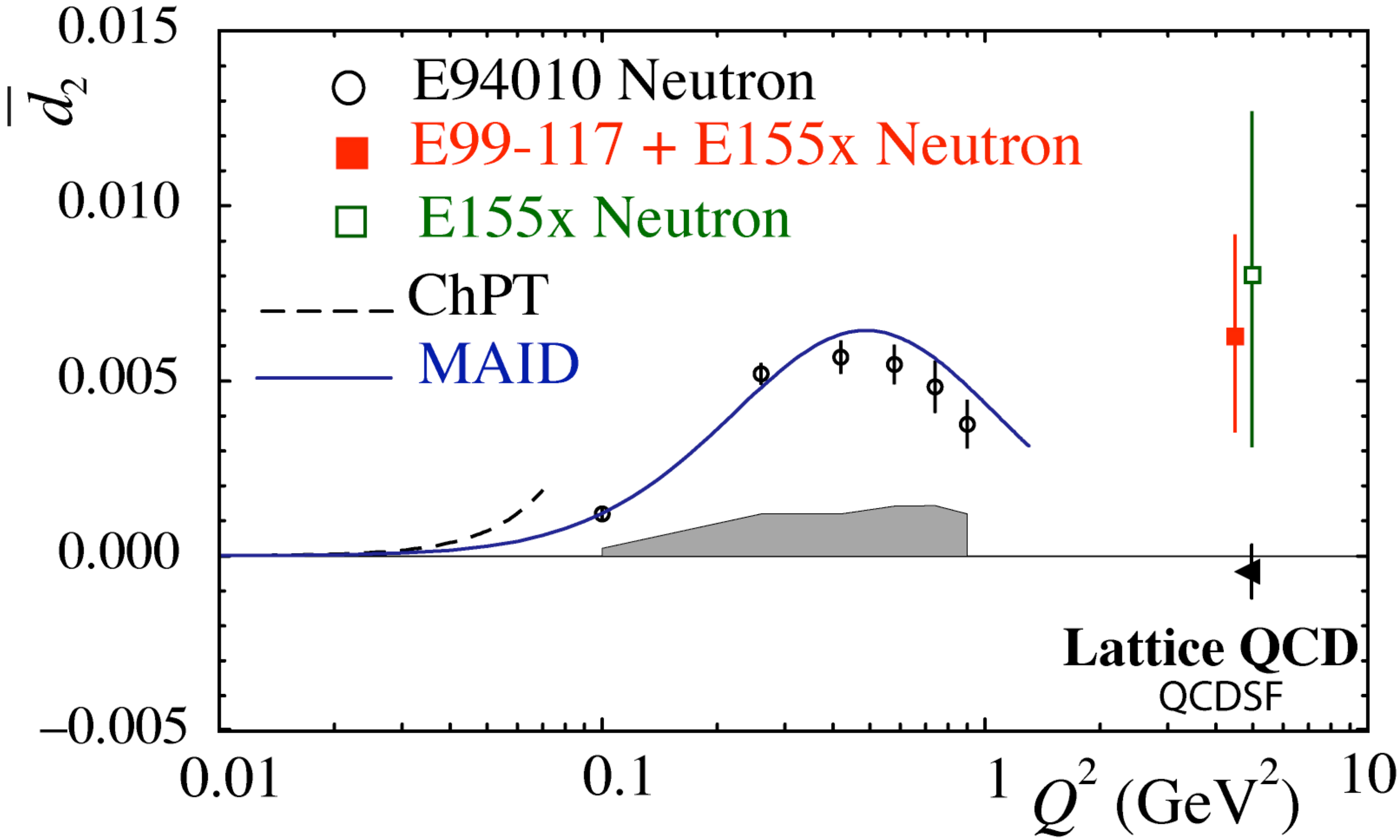
Chiral Soliton Model

H.Weigel and L.Gamberg,
Nucl. Phys. A680, 48 (2000).
M.Wakamatsu, Phys. Lett. B487,118(2000).

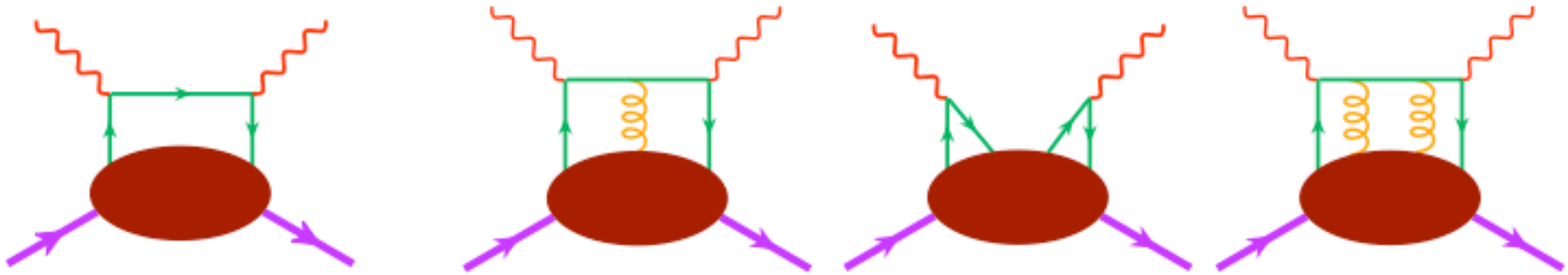
Lattice QCD

M.Gockeler et al.,
Phys.Rev.D72:054507,(2005)

Q^2 evolution of "d_2"



Moments of Structure Functions (continued)



$\tau = 2$

single quark scattering

$\tau > 2$

qq and qg correlations

$$\Rightarrow \quad \Gamma_1(Q^2) \equiv \int_0^1 \mathrm{d}x \; g_1(x, Q^2)$$

$$= \Gamma_1^{\mathrm{twist}-2}(Q^2) + \frac{M_N^2}{9\,Q^2}\left[a_2(Q^2) + 4\,d_2(Q^2) + 4\,f_2(Q^2)\right] + \mathcal{O}\left(\frac{M_N^4}{Q^4}\right)$$

Moments of Structure Functions (continued)

$a_2(Q^2) \equiv 2\int_0^1 \mathrm{d}x\, x^2\, g_1^{\mathrm{twist-2}}(x,Q^2) \rightarrow$ target mass correction term

$d_2(Q^2) \rightarrow$ dynamical twist-3 matrix element

$$\boxed{d_2(Q^2) = \int_0^1 dx\ x^2 \left[2g_1(x,Q^2) + 3g_2(x,Q^2)\right]}$$

$f_2(Q^2) \rightarrow$ dynamical twist-4 matrix element

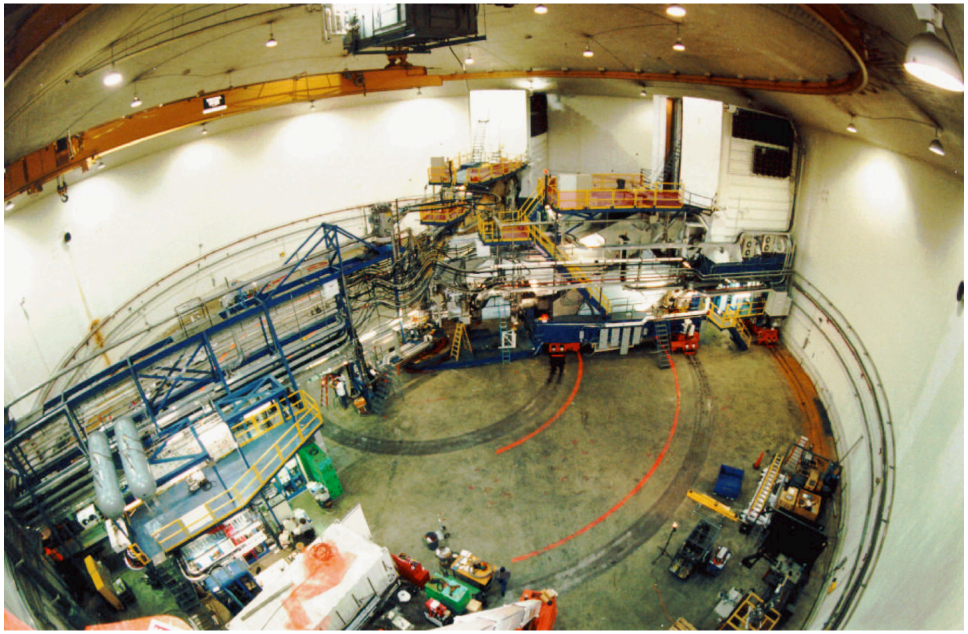
$$\boxed{f_2(Q^2) = \frac{1}{2}\int_0^1 dx\ x^2 \left[7g_1(x,Q^2) + 12g_2(x,Q^2) - 9g_3(x,Q^2)\right]}$$

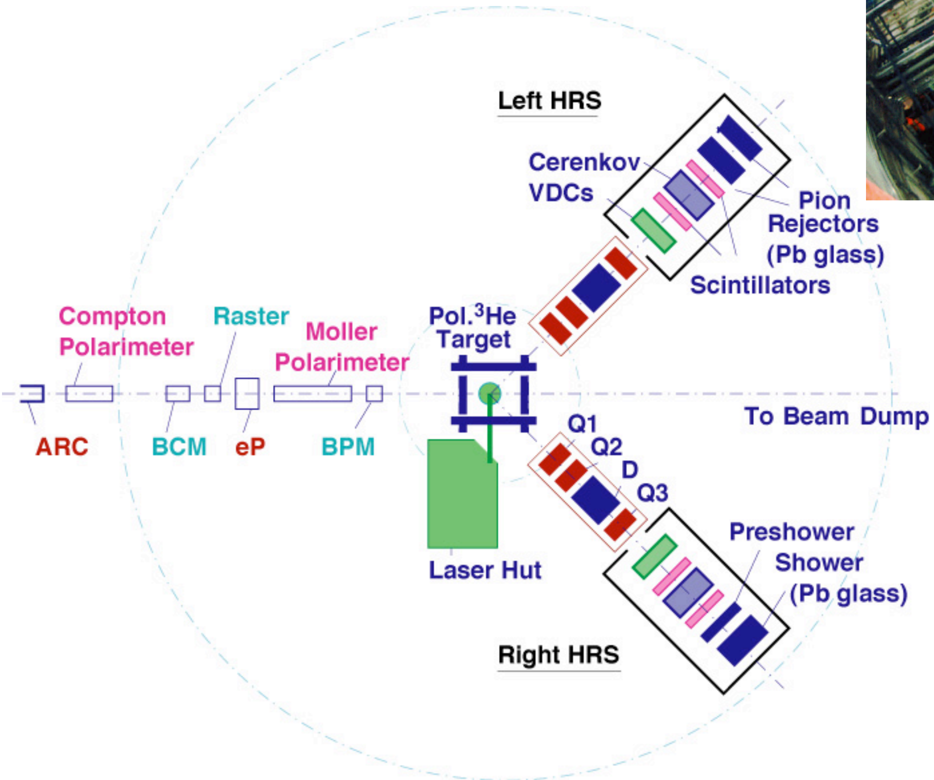
$$f_2 M^2 S^\mu = \frac{1}{2}\sum_q e_q^2 < N|\bar{\psi}_q g\widetilde{F}^{\mu\nu}\gamma_\nu\psi_q|N >$$

Jlab Hall A Experimental Setup

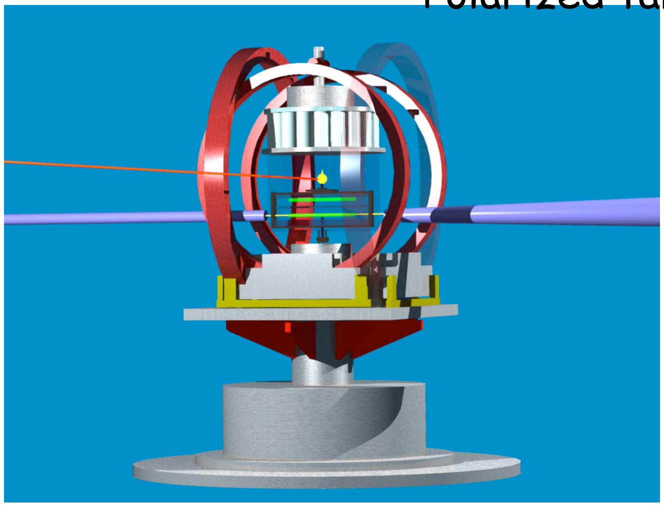
75-80% polarized beam at 15μA

35-40% polarized target in beam

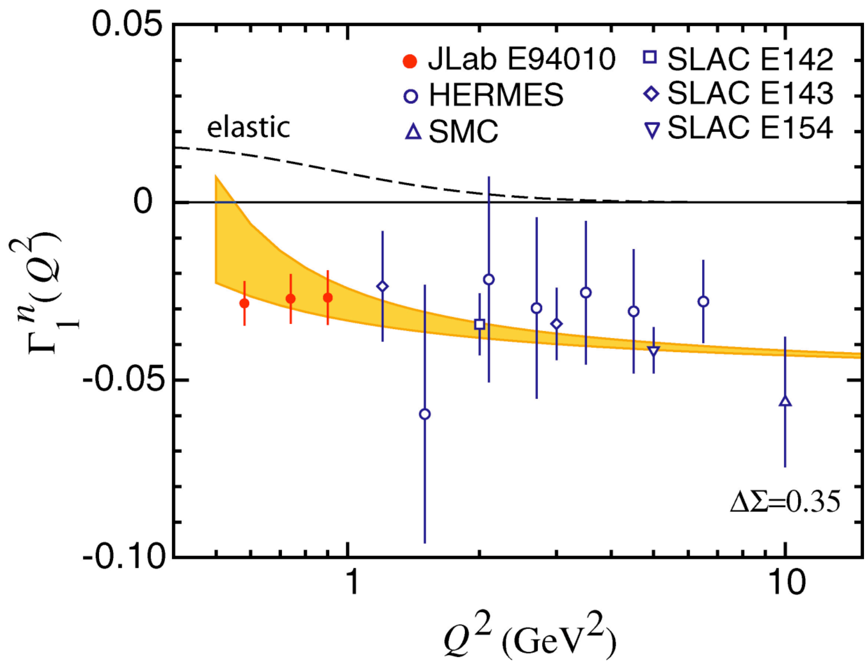




Polarized target



Twist-4 Matrix element f_2

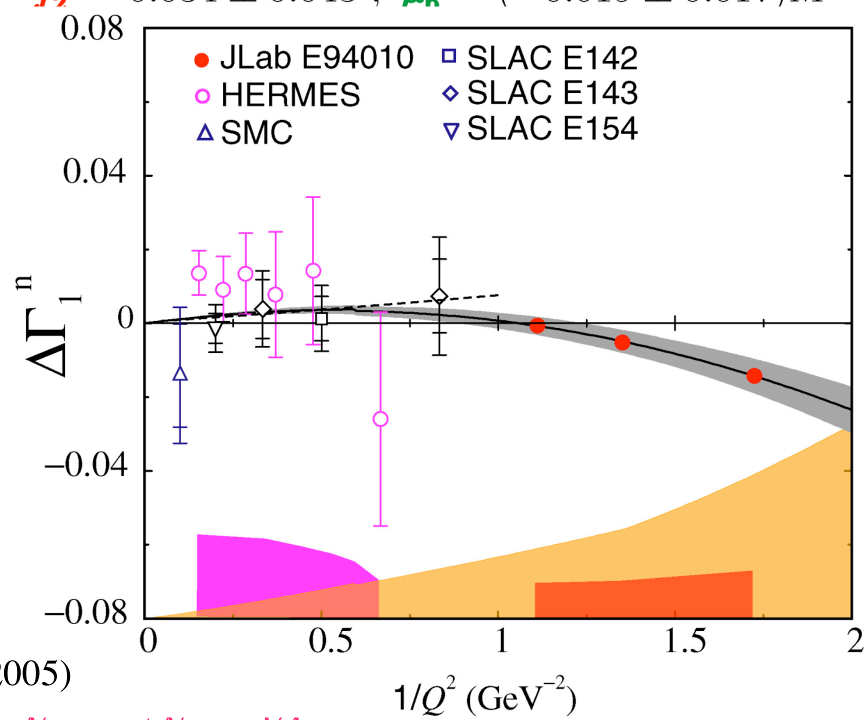


$$\Delta\Gamma_1^n(Q^2) \equiv \Gamma_1^n(Q^2) - \mu_2^n(Q^2) = \frac{\mu_4^n(Q^2)}{Q^2} + \frac{\mu_6^n(Q^2)}{Q^4} + \mathcal{O}\left(\frac{1}{Q^6}\right)$$

$$\mu_4^n = \frac{1}{9}M^2\left(a_2^n + 4d_2^n + 4f_2^n\right)$$

$f_2^n = 0.033 \pm 0.005$, $\mu_6^n = (-0.019 \pm 0.002)M^4$

$f_2^n = 0.034 \pm 0.043$, $\mu_6^n = (-0.019 \pm 0.017)M^4$



Adding $1/Q^6$ term gives the same f_2 and μ_6 with $\mu_8 = (0.00 \pm 0.03)M^2$

If one performs a one parameter fit down to $Q^2 = 0.5$ GeV^2 ; $f_2 = -0.014 \pm 0.010$

If one performs a one parameter fit for $Q^2 > 1$ GeV^2 ; $f_2 = 0.012 \pm 0.029$

Z.-E. M, W. Melnitchouk et al., Phys. Lett. B613,148 (2005)

Determination of f_2 for the proton

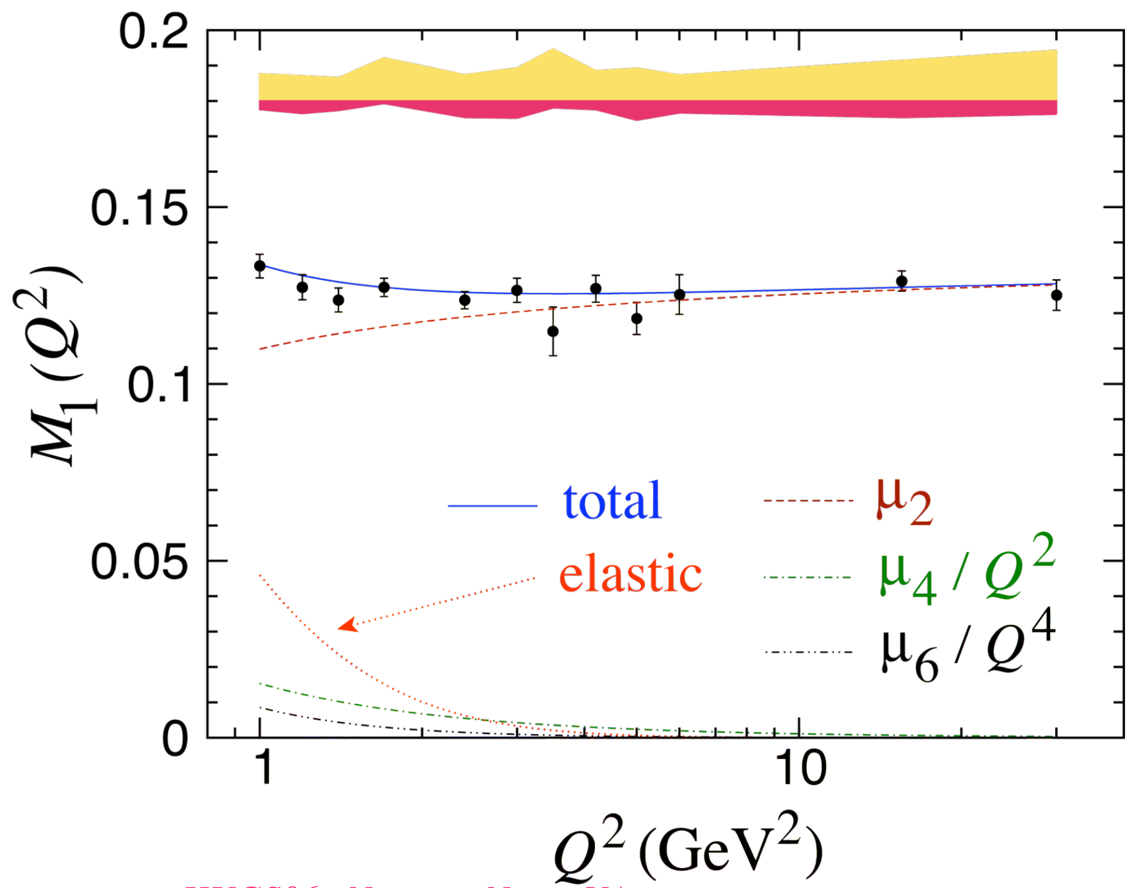
World data +

EG1a data: R. Fatemi et al., PRL, 91 222002 (2003)

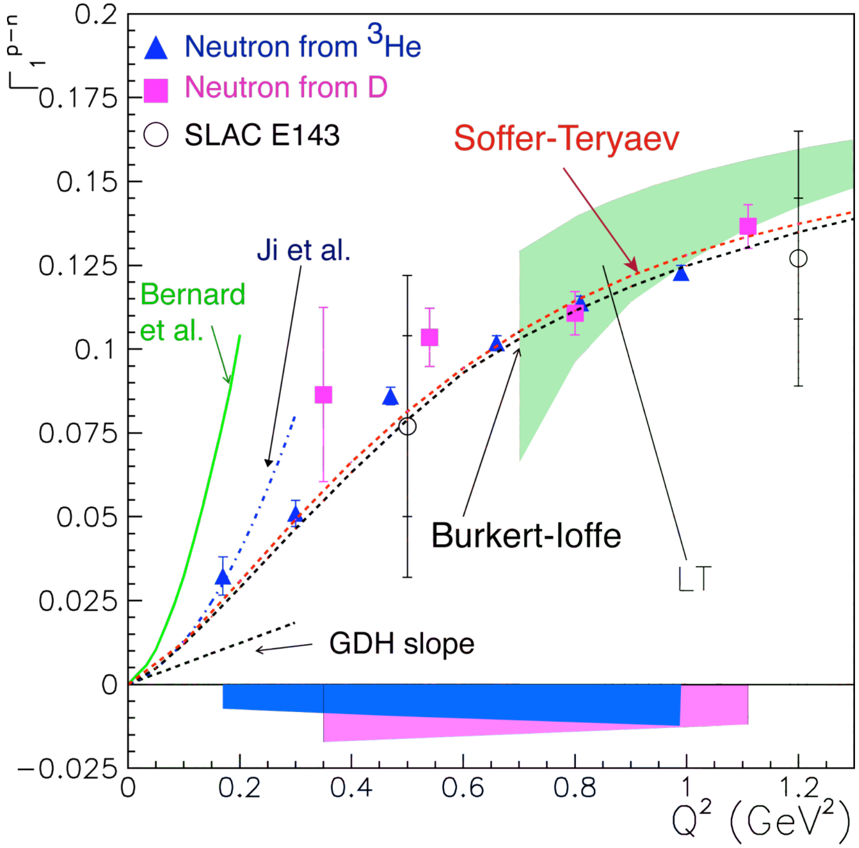
Osipenko et al. Phys. Lett. B 609, 258 (2005)

$f_2=0.039^{+0.037}_{-0.043}$

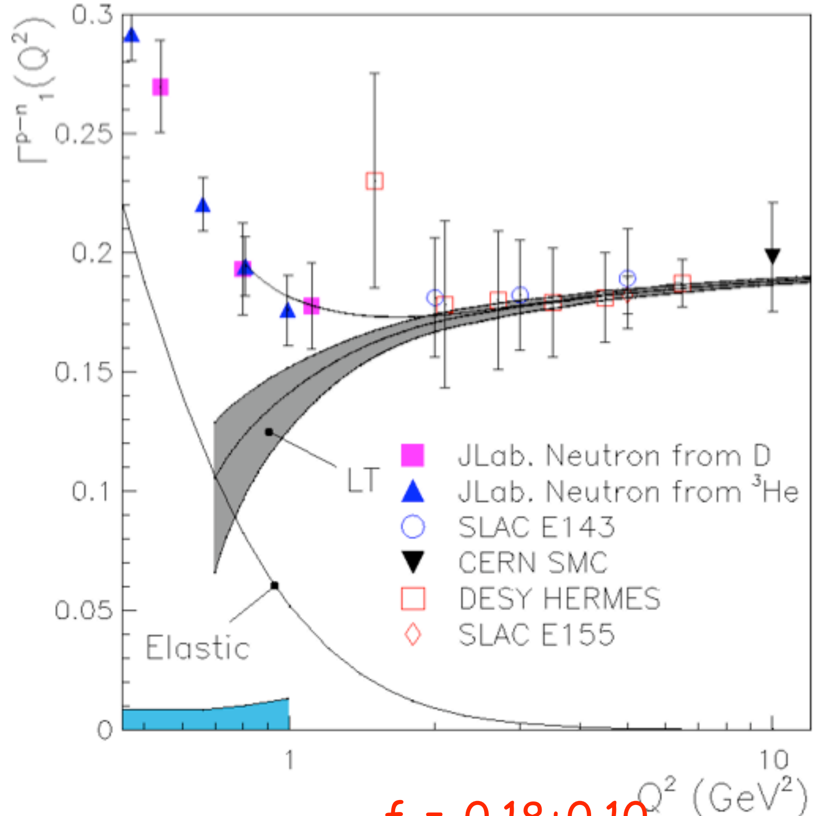
$\mu_6/M^4=0.011^{+0.017}_{-0.013}$



Bjorken Sum Q^2 evolution and higher twists



eg1a + E94-010 data
A. Deur et al. PRL 93, 212001 (2004)



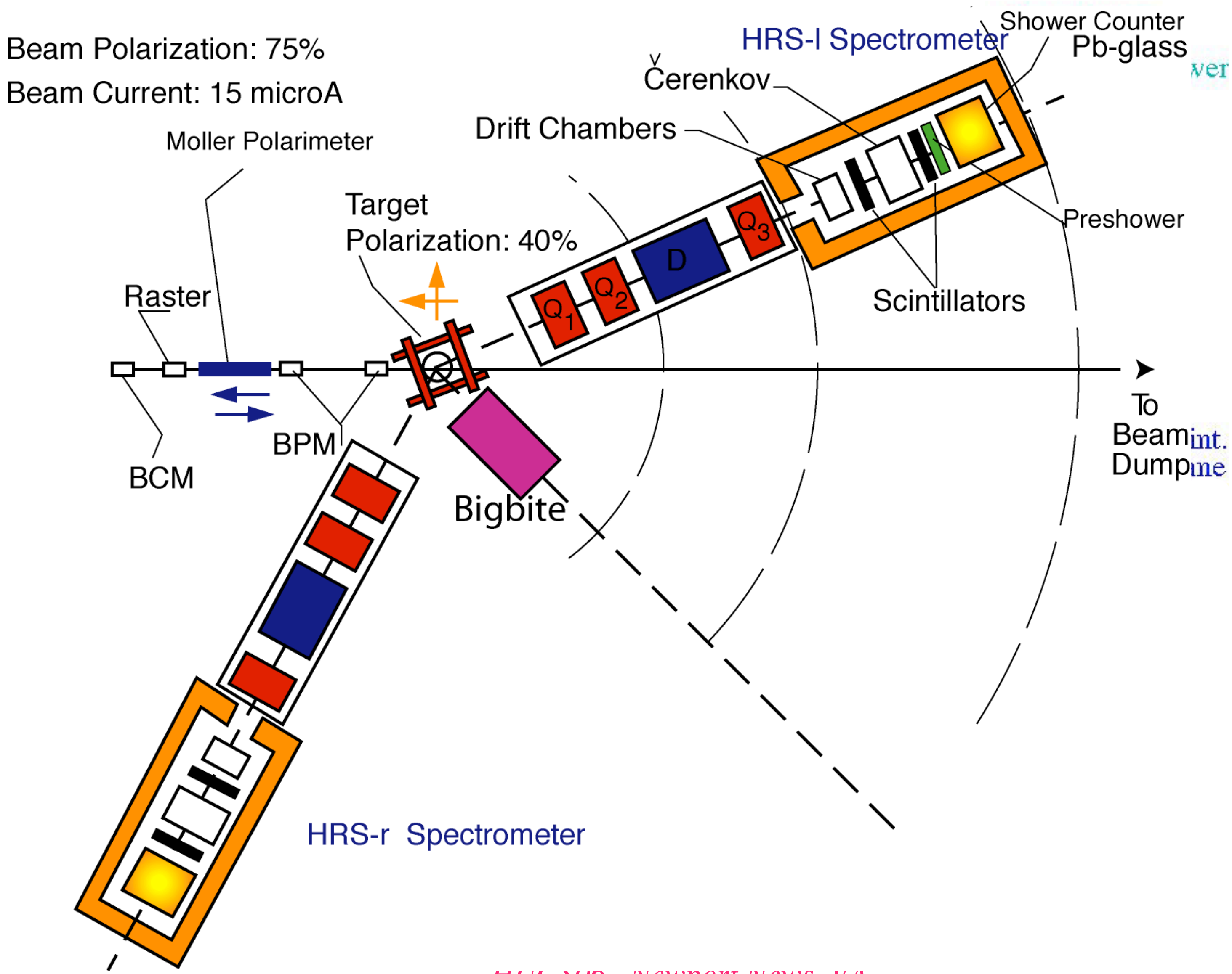
f_2=-0.18±0.10
μ_4/M^2=-0.06±0.02
μ_6/M^4=0.09±0.03

- Good quantity to test Chiral P. T. at low Q^2
  - Little or no contribution from the Delta
  - A well defined sum rule at large Q^2

New proposals

Hall A E06-114 and Hall C Sane experiment

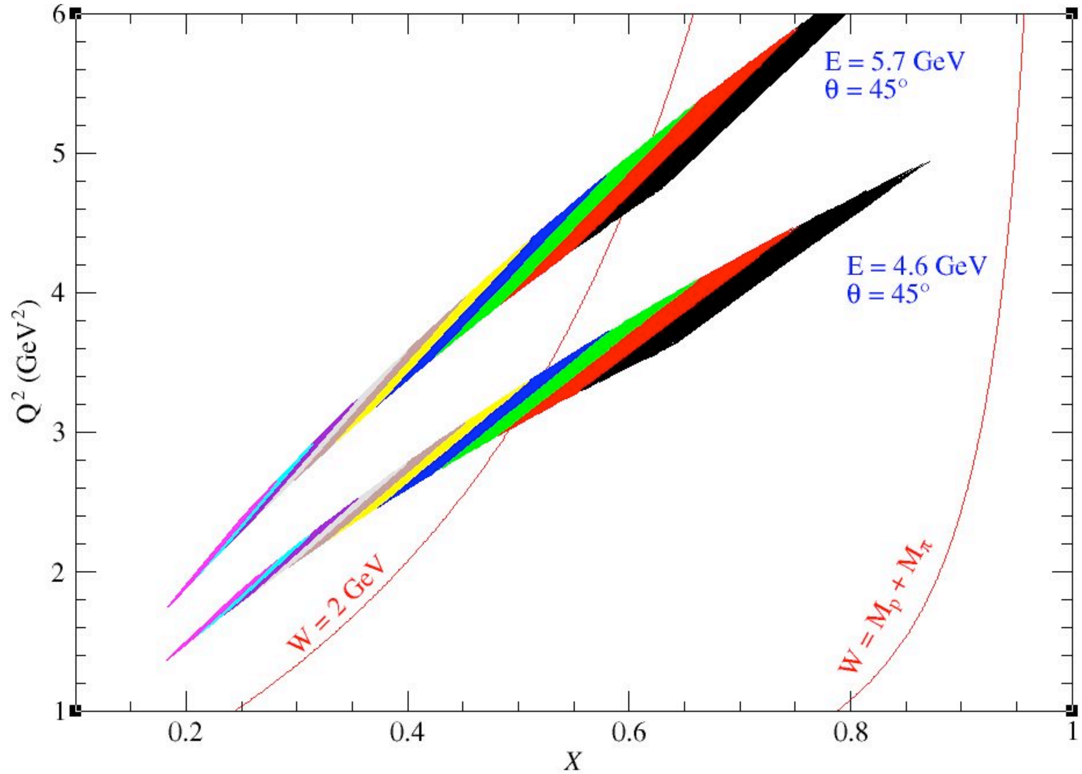
Experiment E06-114 Floor Setup



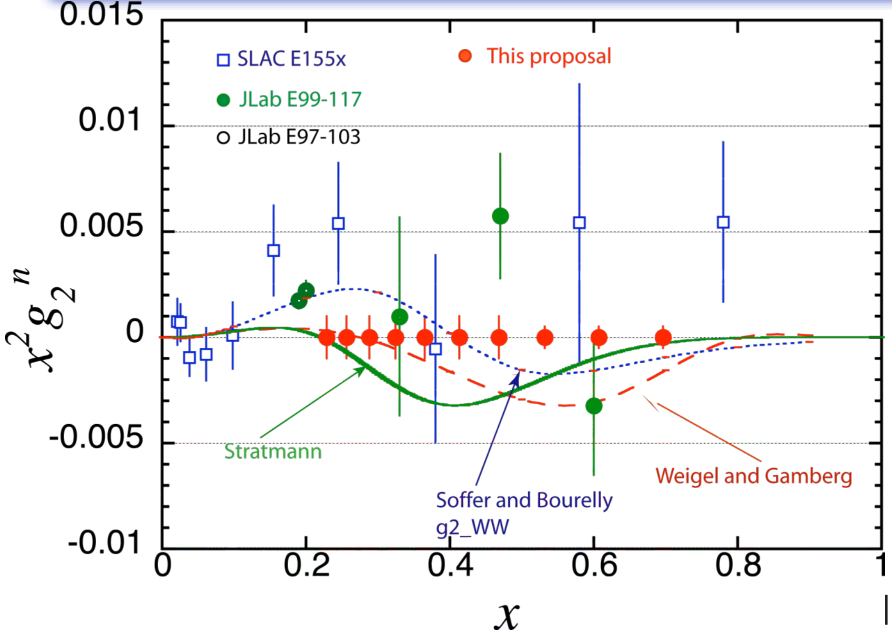
Kinematics of the proposed measurements

Two beam energies
4.6 and 5.7 GeV
(4 pass, 5 pass)

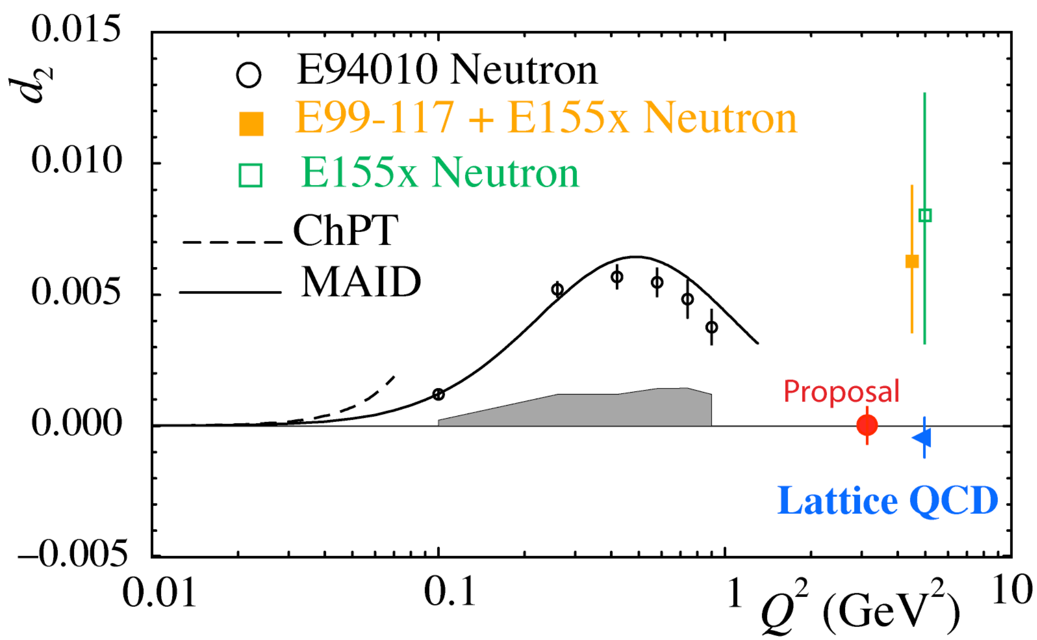
BigBite fixed at single scattering angle (θ=45°)
(data divided into 10 bins during analysis)



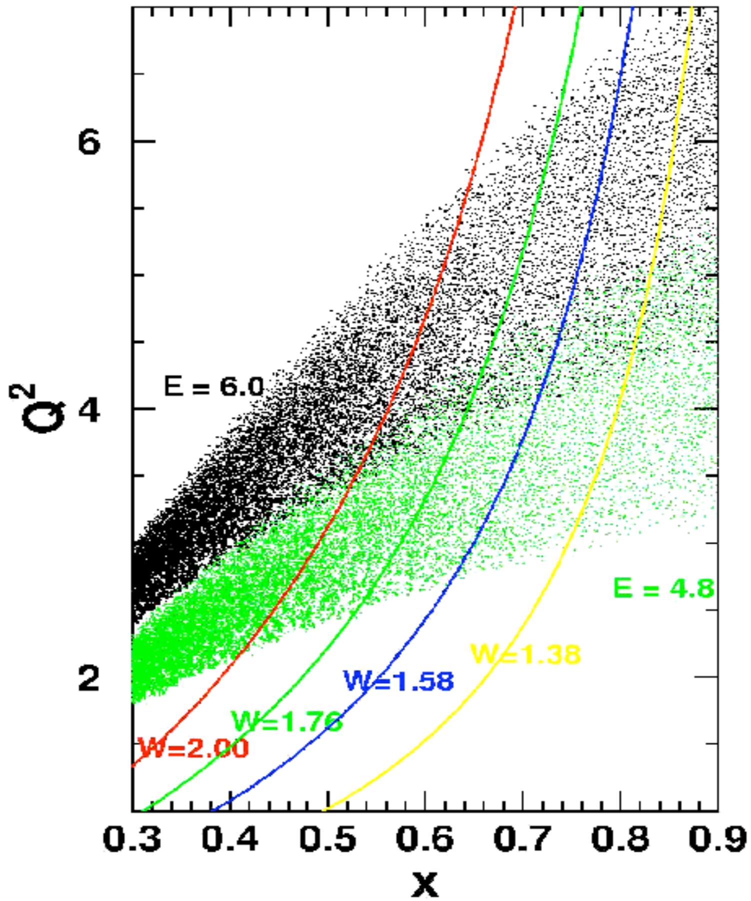
Expected precision in Experiment E06-114

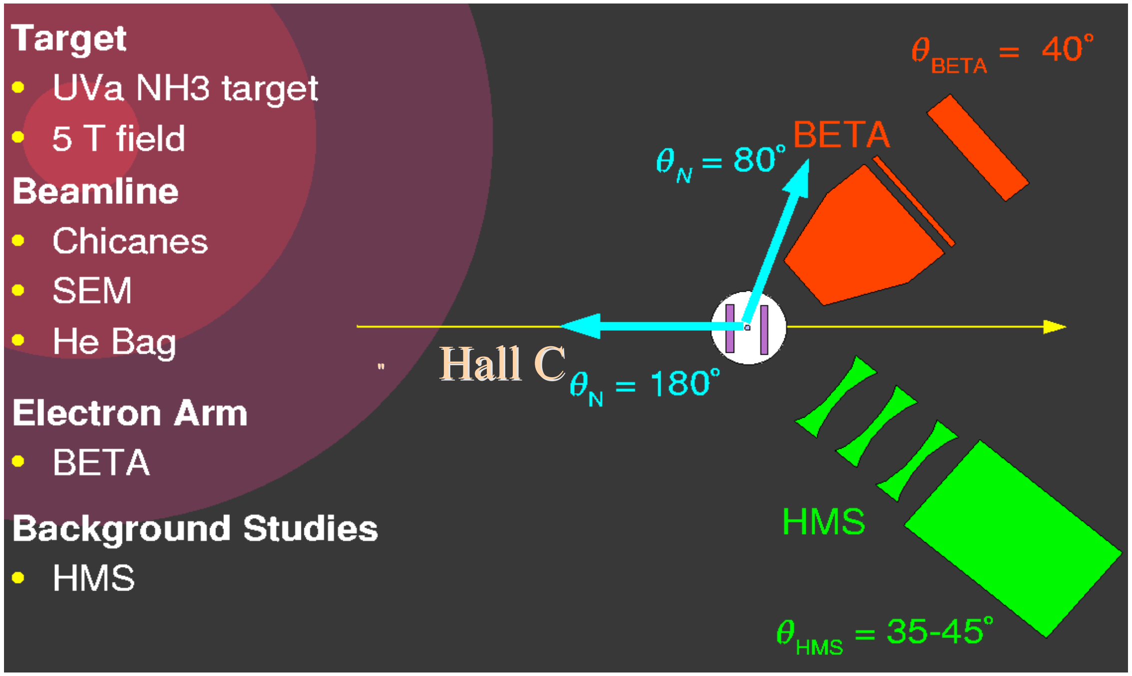


- At large Q^2, d_2 coincides with the reduced twist-3 matrix element of gluon and quark operators
- At low Q^2, d_2 is related to the spin polarizabilities



SANE experiment in Hall C



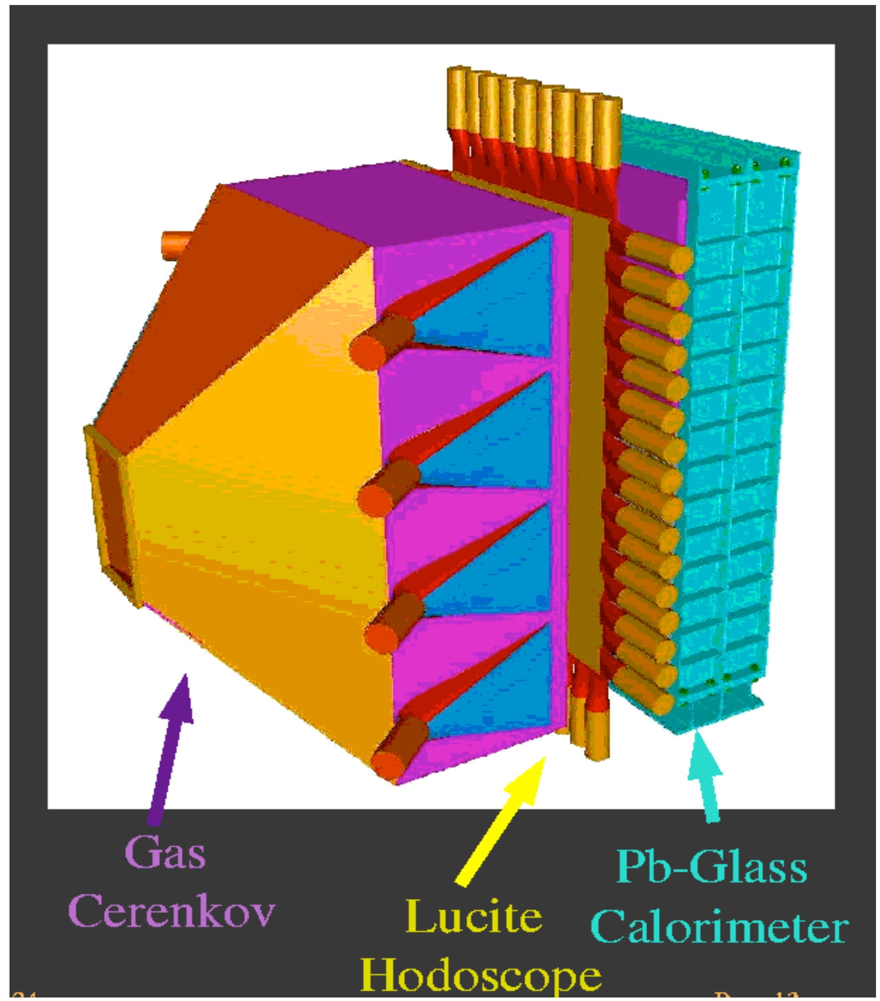


- Two beam energies:
  - **6 GeV (black)**
  - **4.8 GeV (green)**

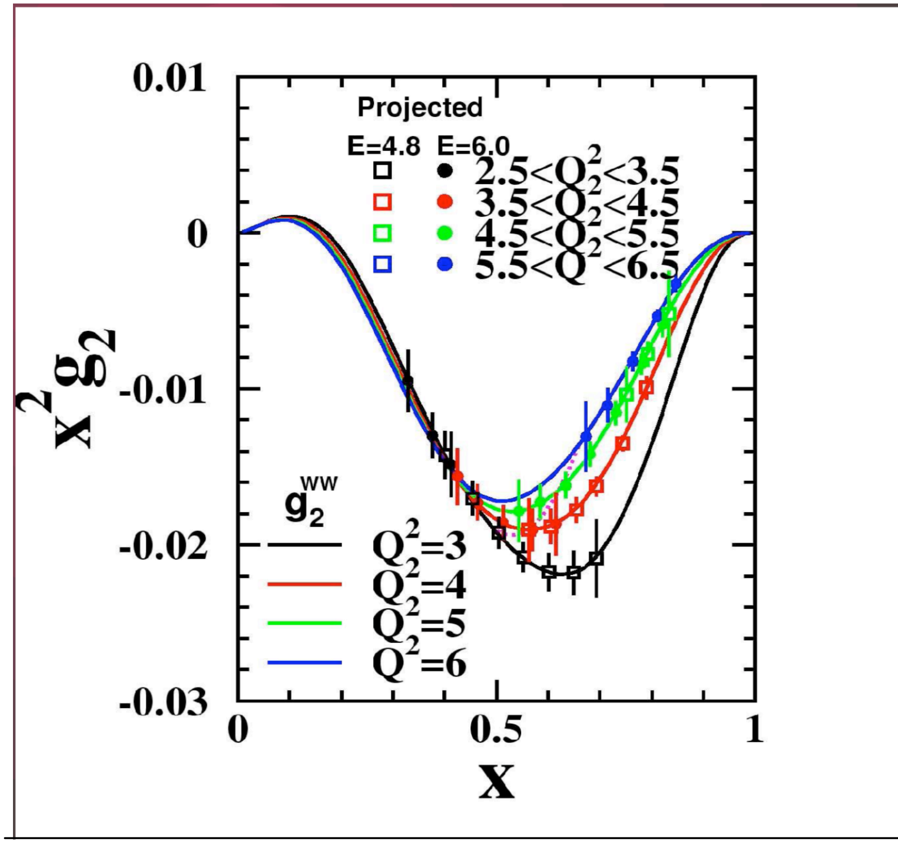
CEBAF polarized beam
- 85 nA
- 75% beam polarization

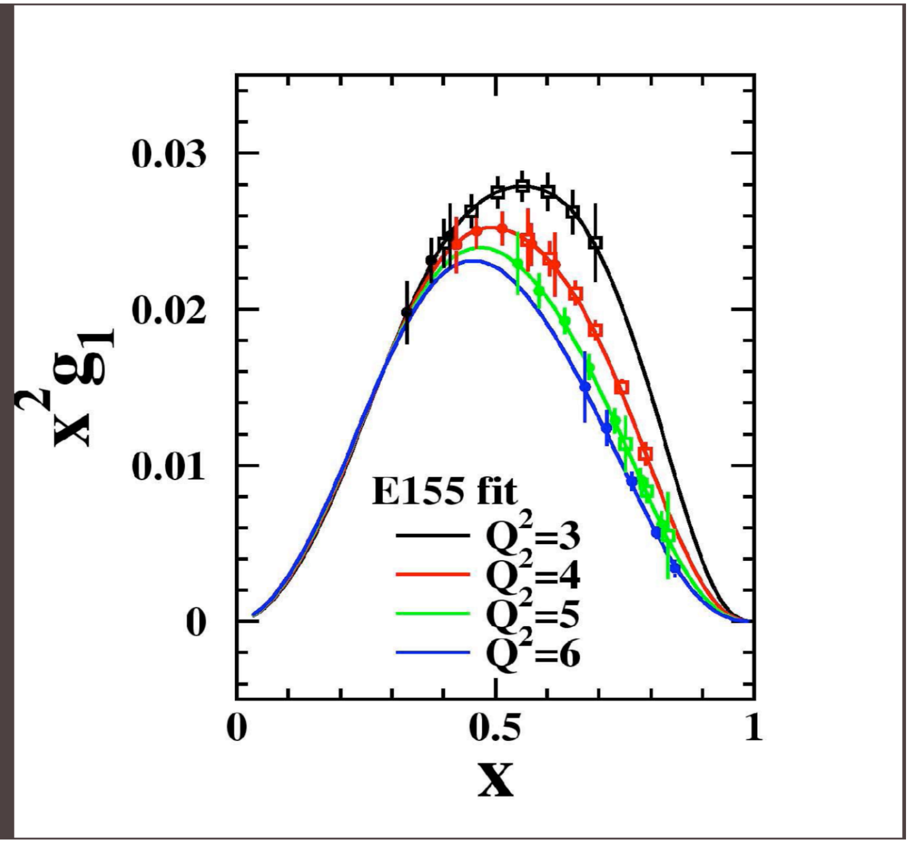
BETA detector

- Three subsystems:
  - Lead glass calorimeter BigCal: main detector
  - Gas Cherenkov (N): additional pion rejection
  - Lucite hodoscope: tracking
- Target field sweeps low *E* background
- Characteristics
  - Effective solid angle (with cuts) = 0.194 sr
  - Energy resolution 5%/√ *E(GeV)*
  - *angular resolution = 2°*
  - *1000:1 pion rejection*

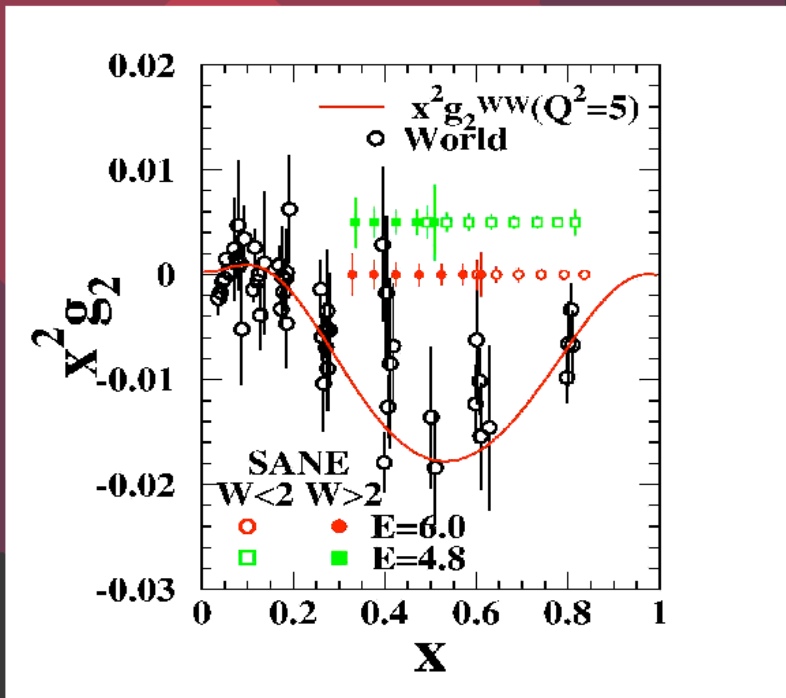


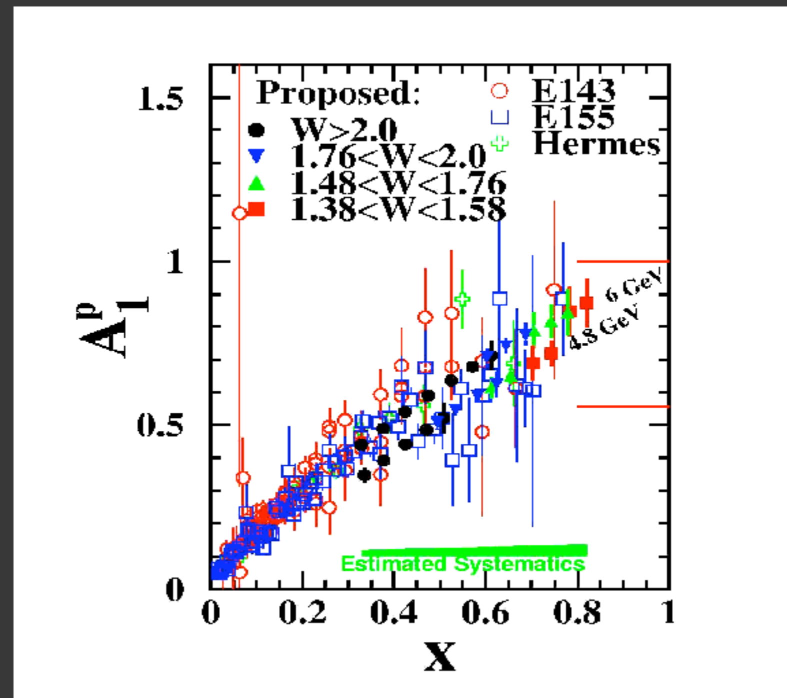
SANE experiment g_2, g_1 projected errors





Proton g_2 and A_1

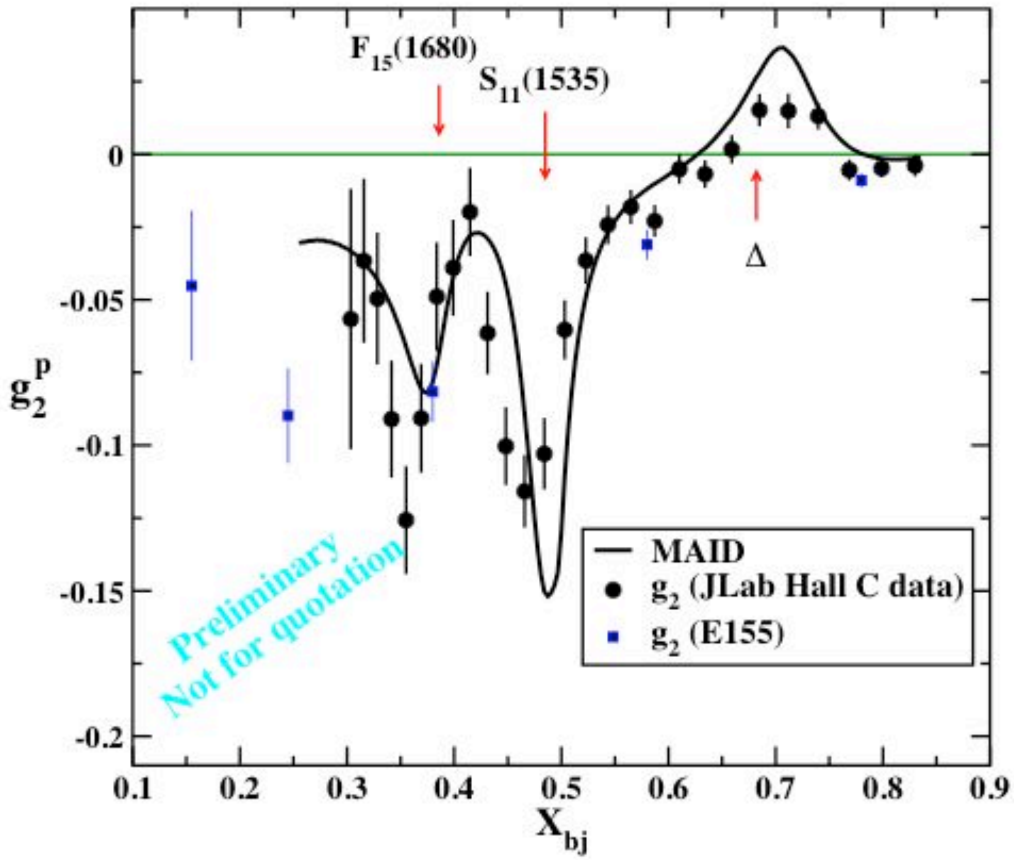




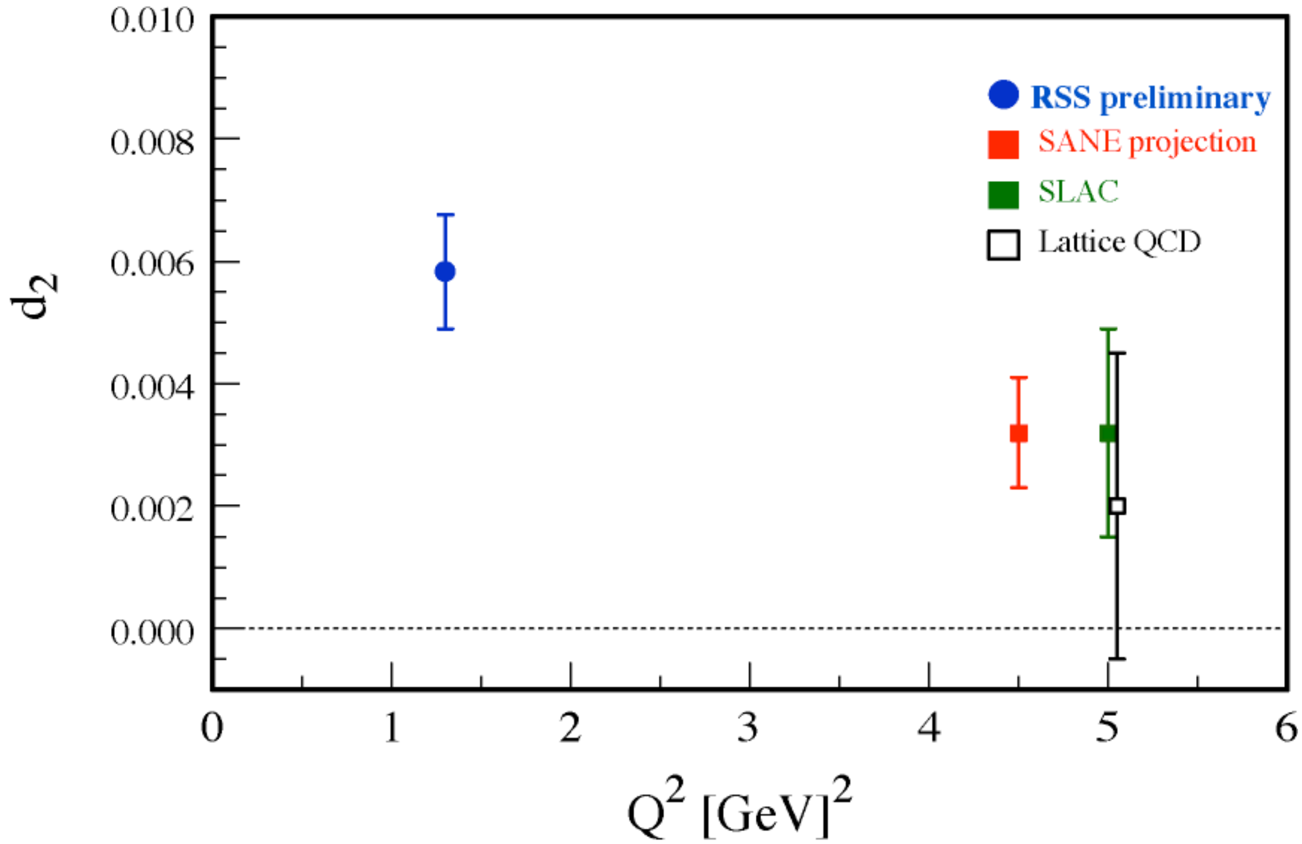
- DIS data up to $x = 0.6$; Resonances measured down to $W = 1.38$ GeV
- g_2 measured in region of most sensitivity for d_2

Hall C Resonance Spin Structure g_2 results

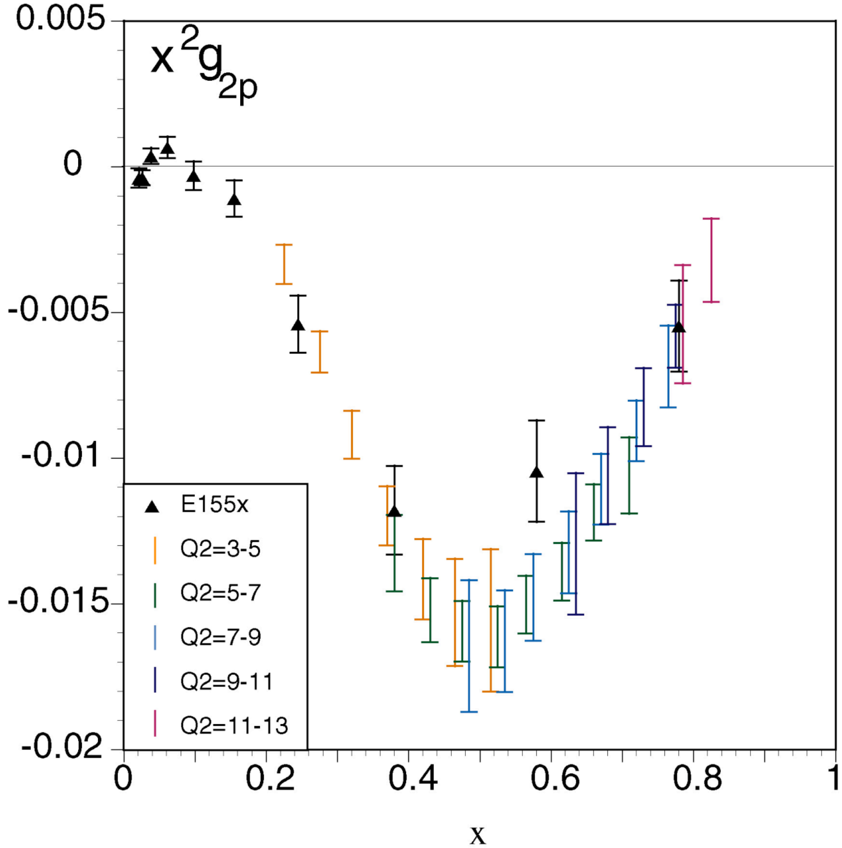
Jlab E01-006 Collaboration, O. Rondon, M. Jones spokespersons

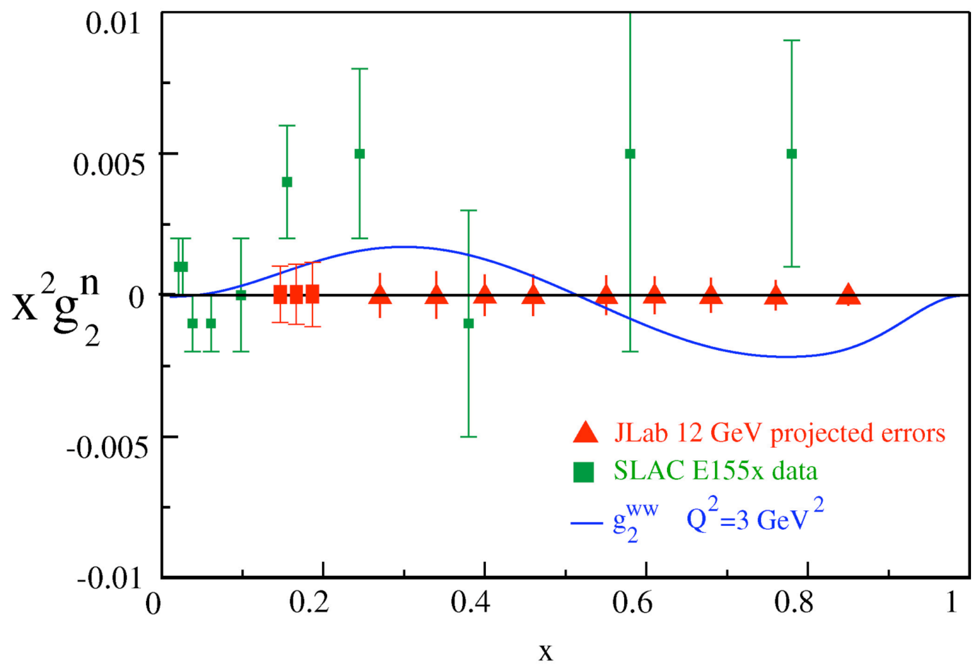


SANE d_2^p projection



g_2 at JLab with 11 GeV





Generalized Spin Polarizabilities of the Neutron

$$
\begin{aligned}
T(\nu, Q^2) &= \varepsilon'^* \cdot \varepsilon f_T(\nu, Q^2) + f_L(\nu, Q^2) \\
&\quad + i\sigma \cdot (\varepsilon'^* \times \varepsilon) g_{TT}(\nu, Q^2) - i\sigma \cdot [(\varepsilon'^* - \varepsilon) \times \hat{q}] g_{LT}(\nu, Q^2)
\end{aligned}
$$

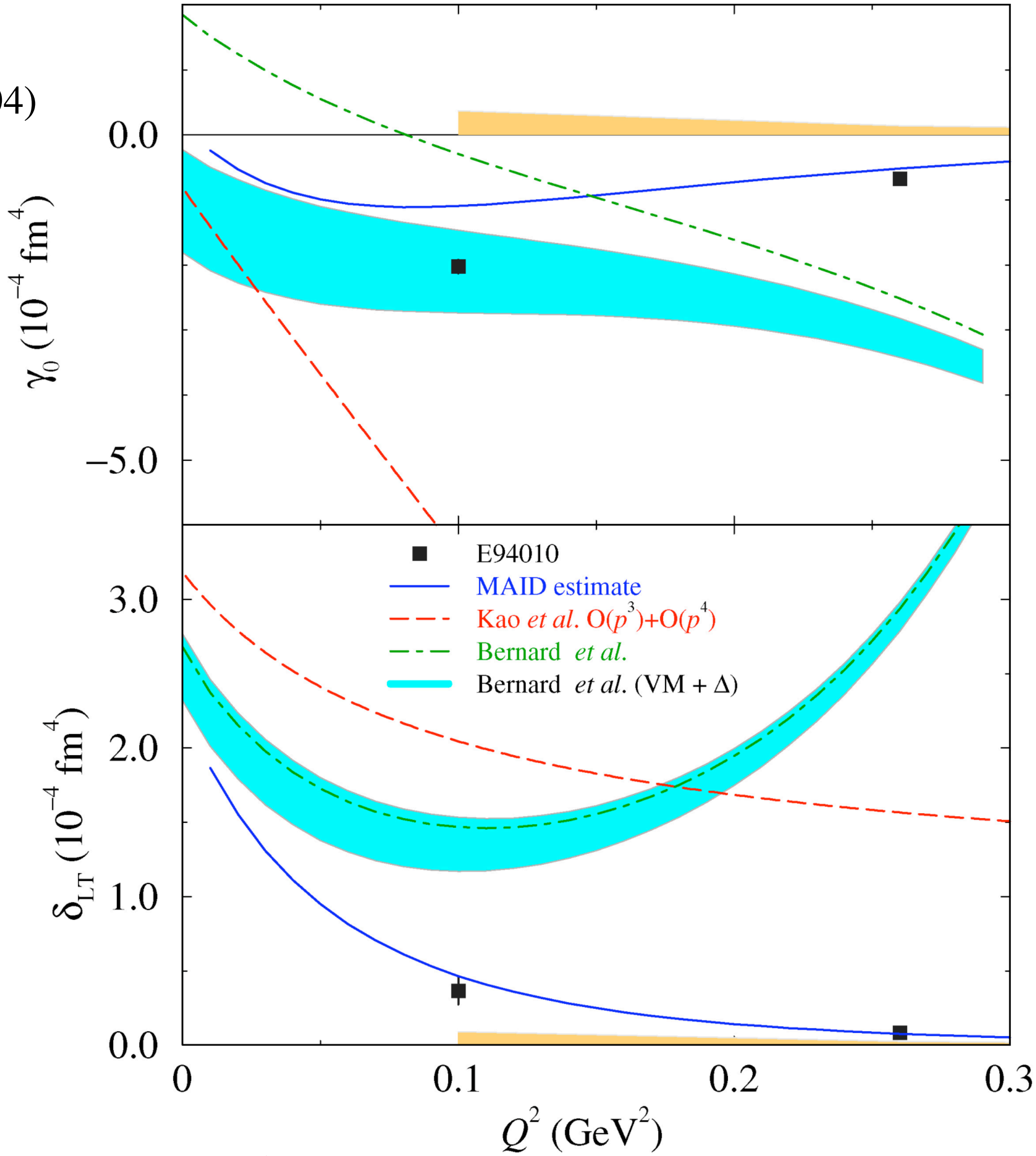
$$
\begin{aligned}
\mathrm{Re}\, g_{TT}{}^{\mathrm{nonpole}}(\nu, Q^2) &= \frac{2\alpha_{\mathrm{em}}}{M^2} I_A(Q^2)\nu + \gamma_0(Q^2)\nu^3 + \mathcal{O}(\nu^5) \\
\mathrm{Re}\, g_{LT}{}^{\mathrm{nonpole}}(\nu, Q^2) &= \frac{2\alpha_{\mathrm{em}}}{M^2} Q I_3(Q^2) + Q\delta_{LT}(Q^2)\nu^2 + \mathcal{O}(\nu^4)
\end{aligned}
$$

$$
\begin{aligned}
\gamma_0(Q^2) &= \frac{16 M^2 \alpha_{\mathrm{em}}}{Q^6} \int_0^{x_0} x^2 \left\{ g_1(x, Q^2) - \frac{Q^2}{\nu^2} g_2(x, Q^2) \right\} dx \\
\delta_{LT}(Q^2) &= \frac{16 M^2 \alpha_{\mathrm{em}}}{Q^6} \int_0^{x_0} x^2 \left\{ g_1(x, Q^2) + g_2(x, Q^2) \right\} dx
\end{aligned}
$$

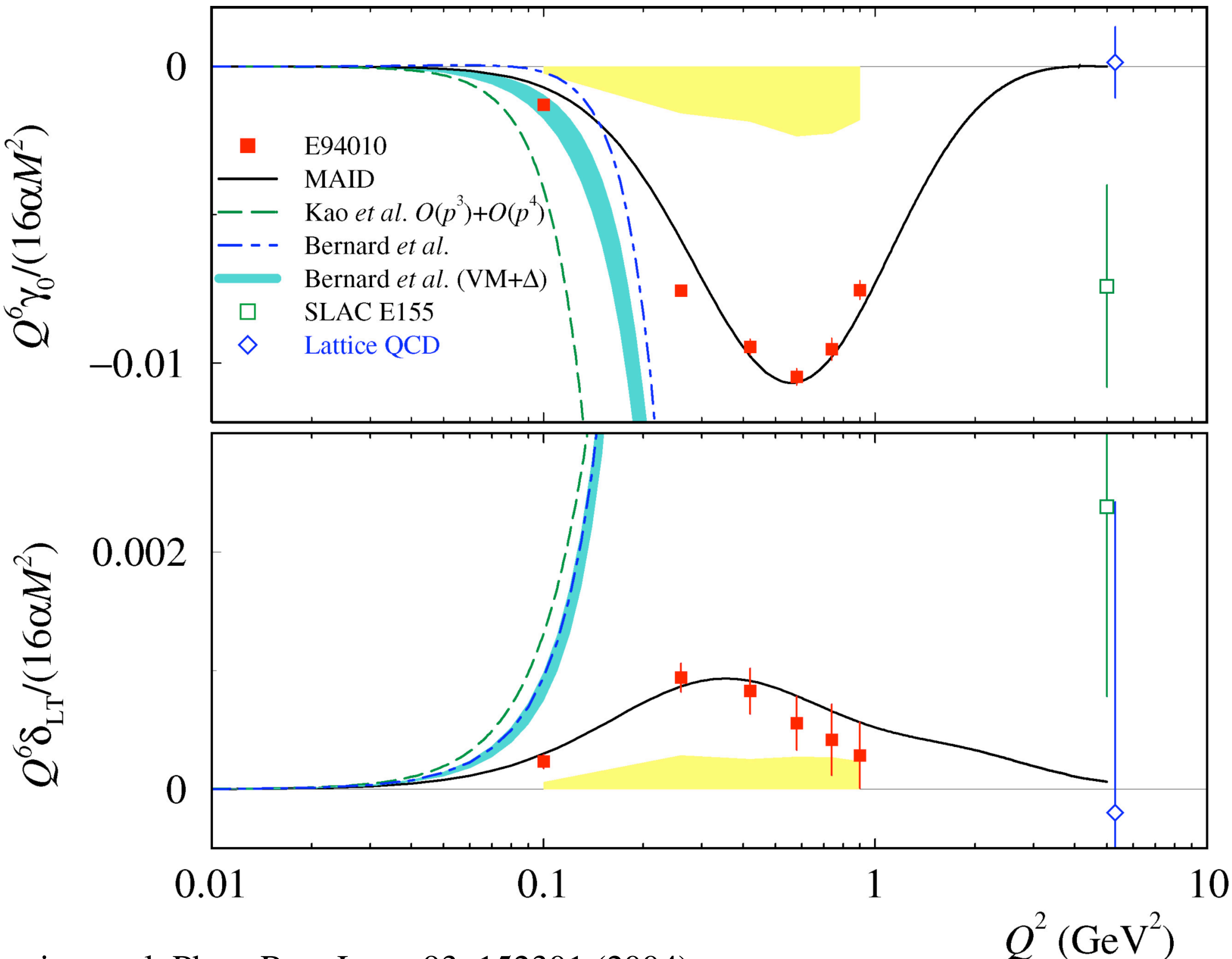
$$
\delta_{LT}(Q^2) \to \frac{1}{3}\gamma_0(Q^2), \quad Q^2 \to \infty
$$

Spin polarizabilities at low Q^2

M. Amarian et al.,
Phys. Rev. Lett. 93, 152301 (2004)



Q^2 evolution of the Spin Polarizabilities



M. Amarian et al. Phys. Rev. Lett. 93, 152301 (2004)

Summary

- Helicity independent and dependent distributions will be determined at large x with precision. Nucleon models will be tested and moments of these distributions evaluated and tested against Lattice QCD.

- The Burkhardt-Cottingham sum rule seems "verified" in ^{3}He and the neutron within errors for $Q^2 < 1$ GeV^2; a region dominated by the elastic and Delta resonance contributions which approximately cancel each other.

- Precision measurements of g_1 and g_2 in the range $1 < Q^2 < 4$ GeV^2 are needed for reducing the error on the extraction of the color polarizabilities. This will be pursued at JLab 11 GeV.

- In the next few years will have SANE, E06-14, the neutron measurement may turn out to be surprising. This program will be pursued at 12 GeV.

- More investigation is needed to understand the discrepancy between chiral perturbation calculations and the data for the spin polarizabilities δ_{LT} at low Q^2. Proton data are needed too