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Quark-Hadron Duality

Wally Melnitchouk

Jefferson Lab

Outline

1. Introduction
2. Bloom-Gilman duality
   - *resonances & scaling*
3. Duality in QCD
   - *moments & higher twists*
4. Global & local duality
   - *dynamical quark models*
5. DIS at low Q^2
   - *target mass corrections*

1.
Introduction

Quark-hadron duality

Complementarity between *quark* and *hadron* descriptions of observables

$$\sum_{hadrons} = \sum_{quarks}$$

Can use either set of complete basis states to describe all physical phenomena

Duality in Nature

- Duality between quarks *(high energy)* and hadrons *(low energy)* manifests itself in many processes

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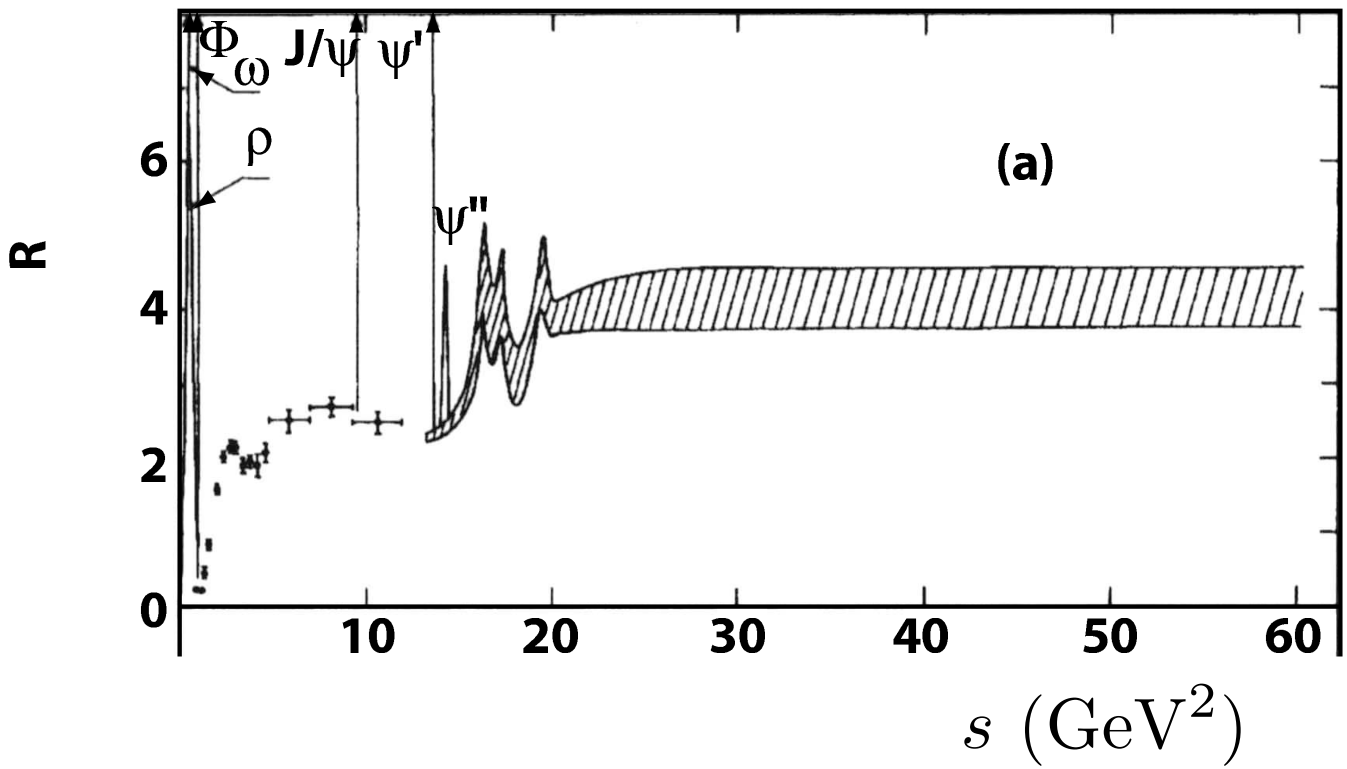
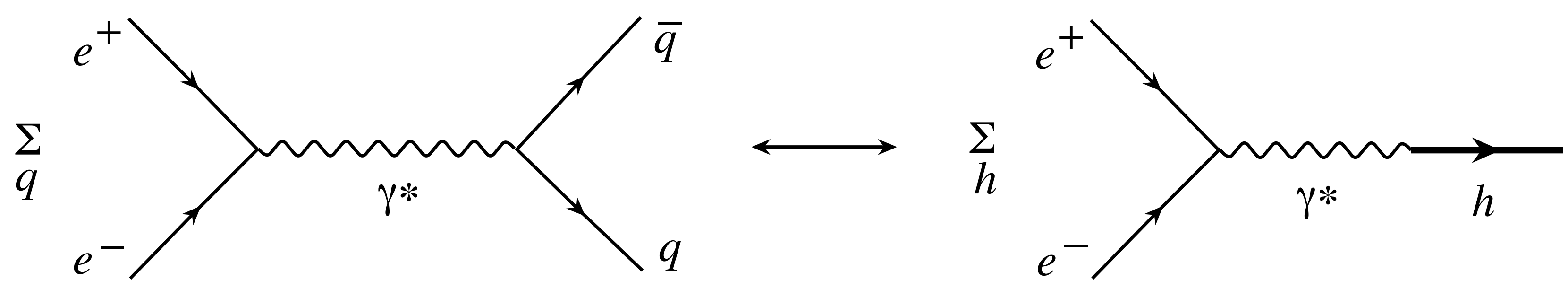
- *e*+ *e*- annihilation

  - *total hadronic cross section at high energy averages resonance cross section*

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$





Duality in Nature

- Duality between quarks *(high energy)* and hadrons *(low energy)* manifests itself in many processes

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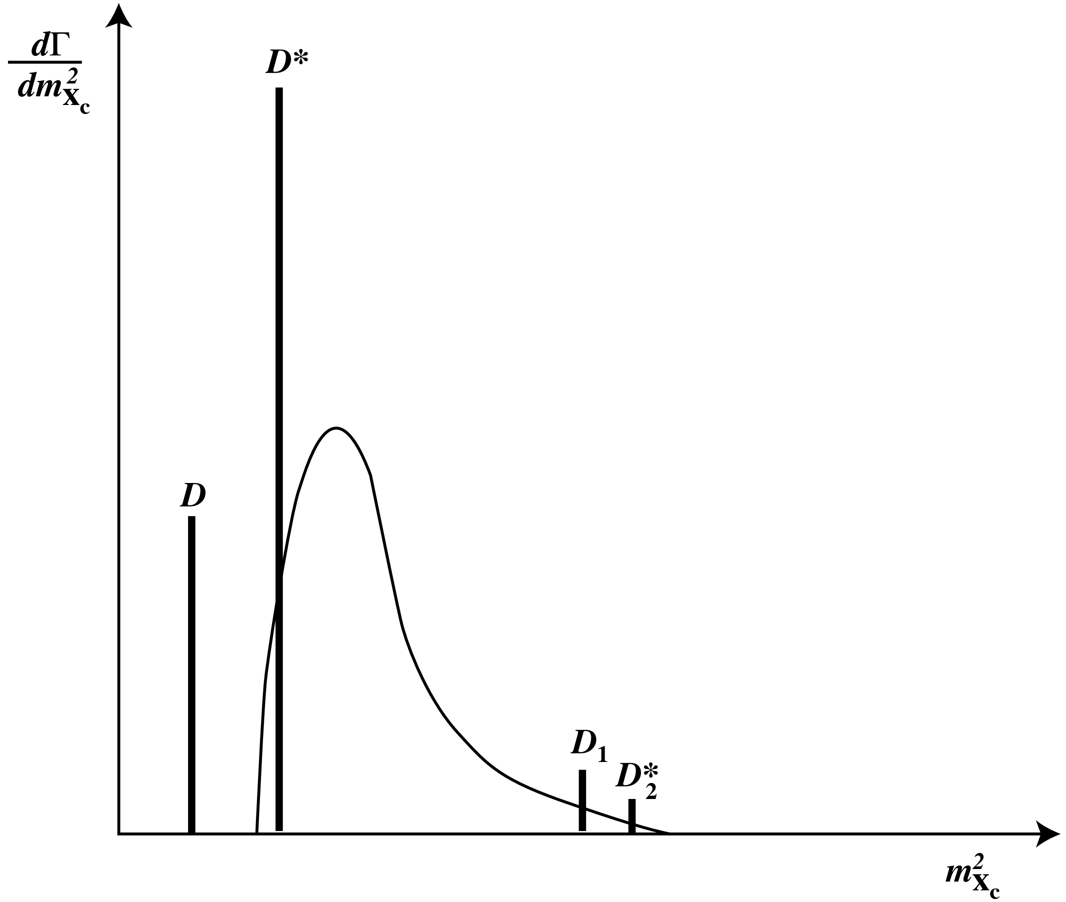
- Heavy meson decays

  - *duality between hadronic & quark descriptions of decays in* $m_Q \to \infty$ *limit*



$$B \to D\, l\, \nu$$

$$\Updownarrow$$

$$b \to c\, l\, \nu$$

Voloshin, Shifman, Sov. J. Nucl. Phys. 41 (1985) 120
Isgur, Phys. Lett. B448 (1999) 111

Duality in Nature

- Duality between quarks *(high energy)* and hadrons *(low energy)* manifests itself in many processes

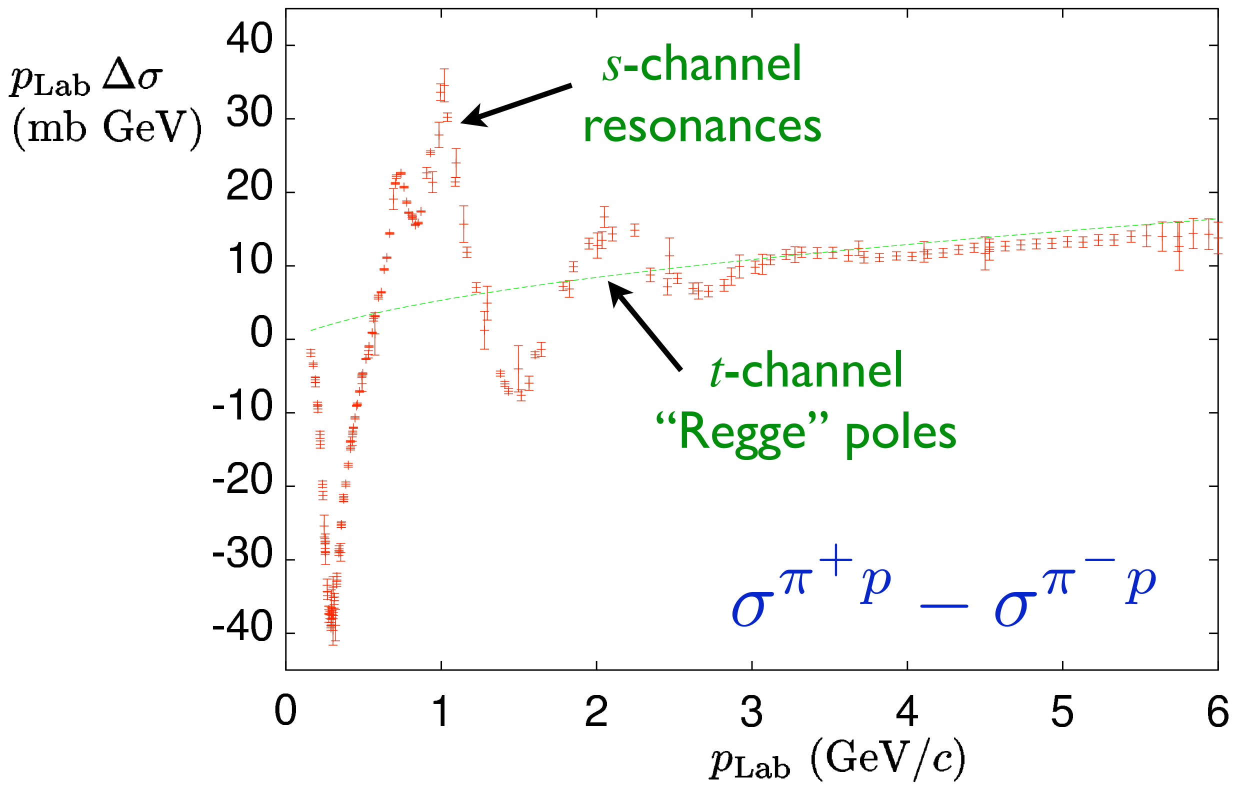
- $e+$ $e-$ annihilation

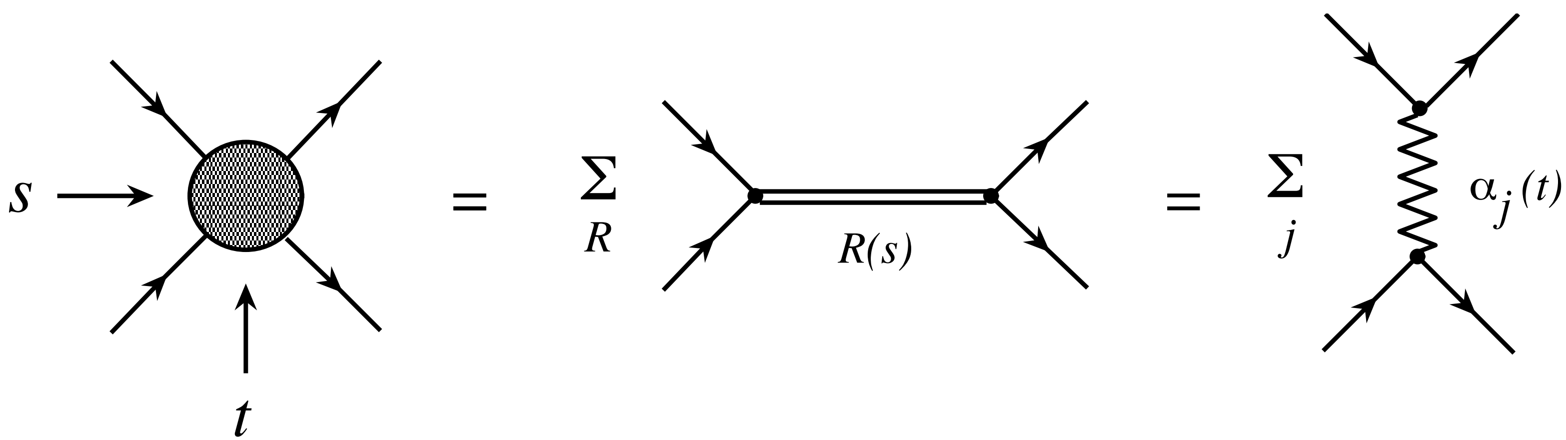
  - *total hadronic cross section at high energy averages resonance cross section*

- Heavy meson decays

  - *duality between hadronic & quark descriptions of decays in* $m_Q \to \infty$ *limit*

- Duality between s-channel resonances and t-channel (Regge) poles in hadronic reactions

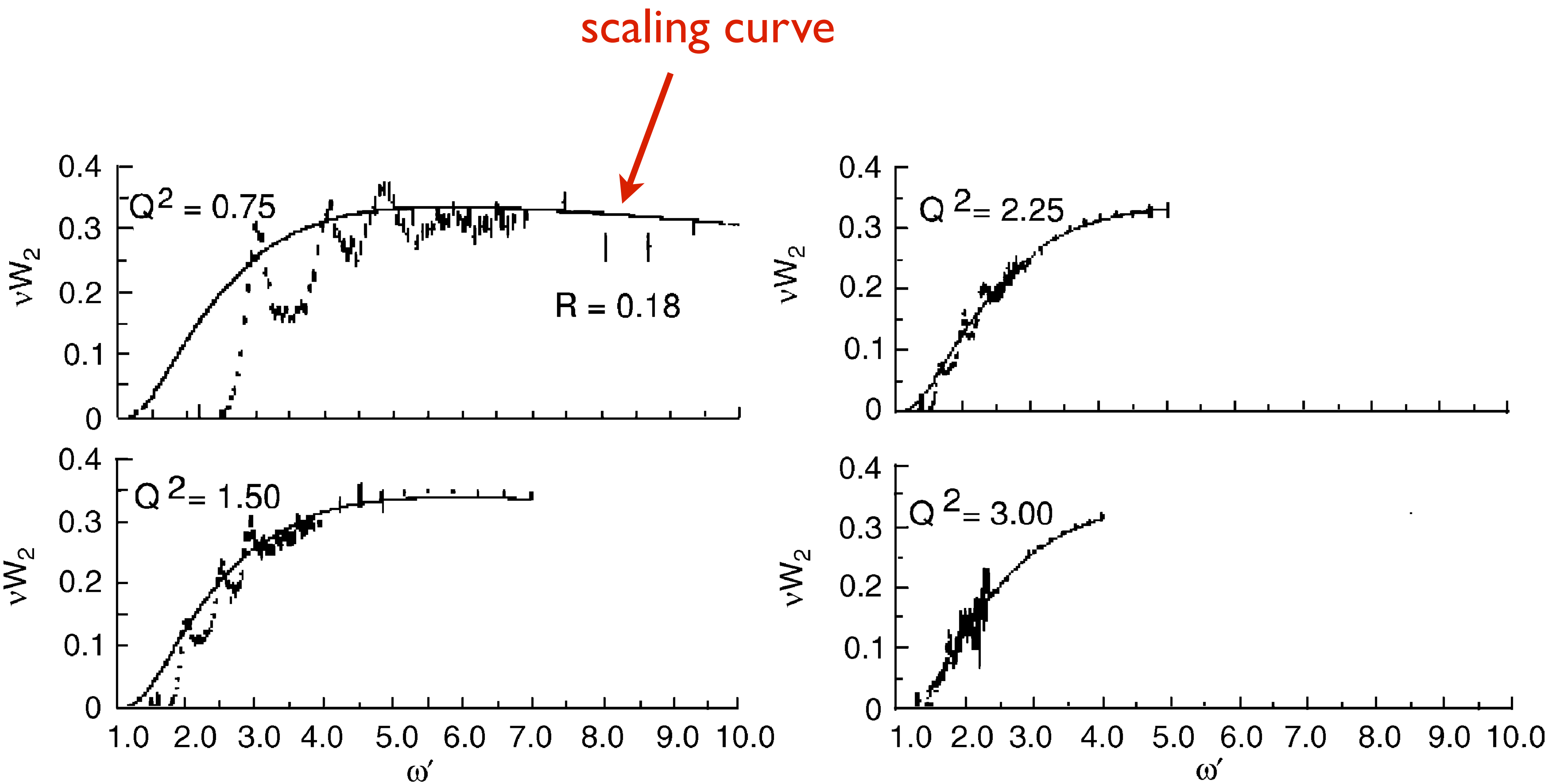




"Finite energy sum rules"

Igi (1962), Dolen, Horn, Schmidt (1968)

2.
Bloom-Gilman duality

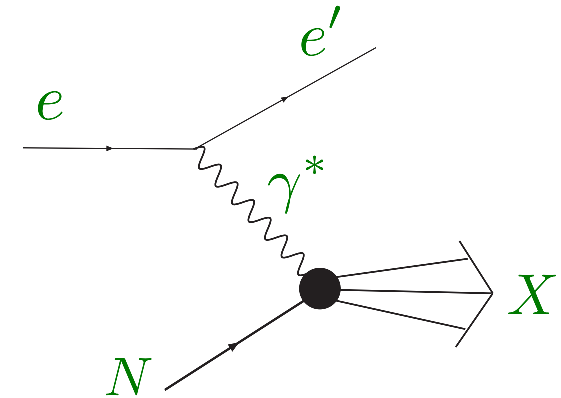


Bloom, Gilman, Phys. Rev. Lett. 85 (1970) 1185

resonance – scaling duality in proton $\nu W_2 = F_2$ structure function

Electron scattering

Inclusive cross section for $eN \to eX$



$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2 \cos^2\frac{\theta}{2}}{Q^4}\left(2\tan^2\frac{\theta}{2}\frac{F_1}{M} + \frac{F_2}{\nu}\right)$$

$$\left.\begin{aligned} \nu &= E - E' \\ Q^2 &= \vec{q}^{\,2} - \nu^2 = 4EE'\sin^2\frac{\theta}{2} \end{aligned}\right\} \quad x = \frac{Q^2}{2M\nu}$$

Bjorken scaling variable

F_1 , F_2 “structure functions”

$\longrightarrow$ contain all information about structure of nucleon

$\longrightarrow$ functions of $x,\ Q^2$ in general

Bloom-Gilman duality

Average over (strongly Q^2 dependent) resonances $\approx Q^2$ independent scaling function

Finite energy sum rule for eN scattering

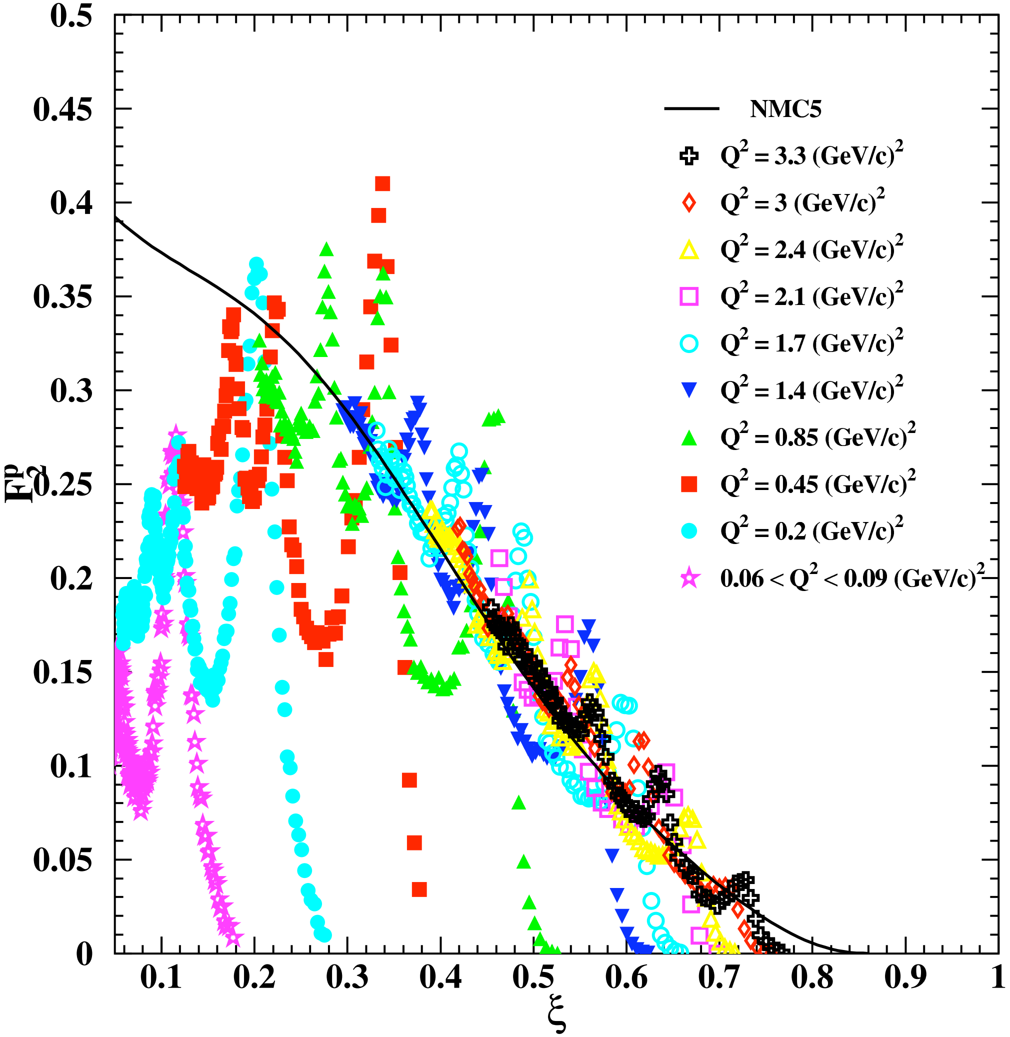
$$\frac{2M}{Q^2}\int_0^{\nu_m} d\nu\ \nu W_2(\nu, Q^2) = \int_1^{\omega'_m} d\omega'\ \nu W_2(\omega')$$

measured structure function
(function of ν and Q^2)

scaling function
(function of ω' only)

$$\omega' = \frac{1}{x} + \frac{M^2}{Q^2}$$

Bloom-Gilman duality



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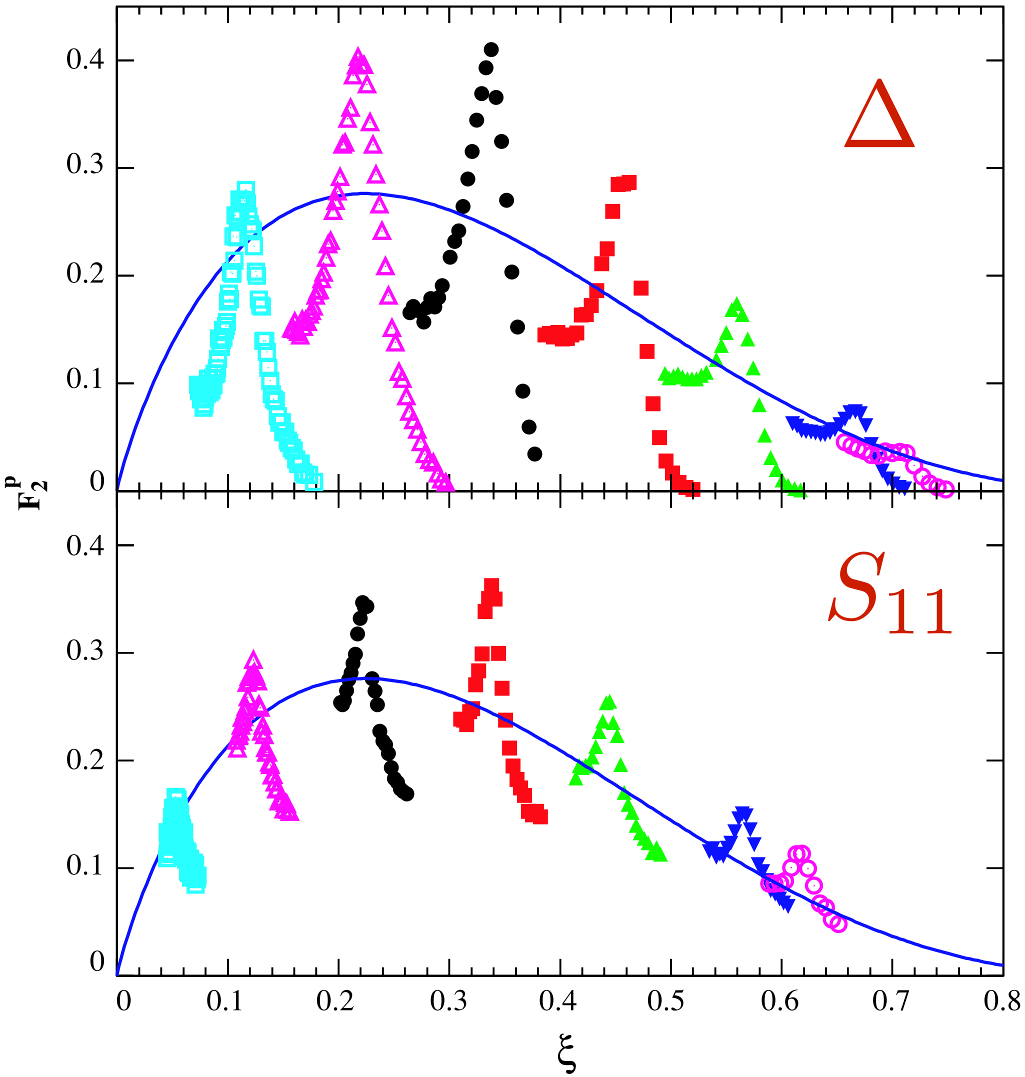
Niculescu et al., Phys. Rev. Lett. 85 (2000) 1182

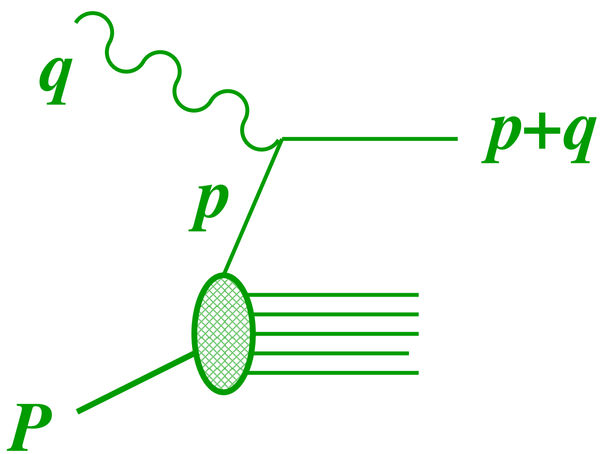
Average over (strongly Q^2 dependent) resonances $\approx$ Q^2 independent scaling function

Local Bloom-Gilman duality



$$\xi = \frac{2x}{1+\sqrt{1+4M^2x^2/Q^2}}$$ Nachtmann scaling variable

Scaling variables



$$(p+q)^2 = m_q^2 \quad \begin{cases} m_q = 0 \\ p_T = 0 \end{cases}$$

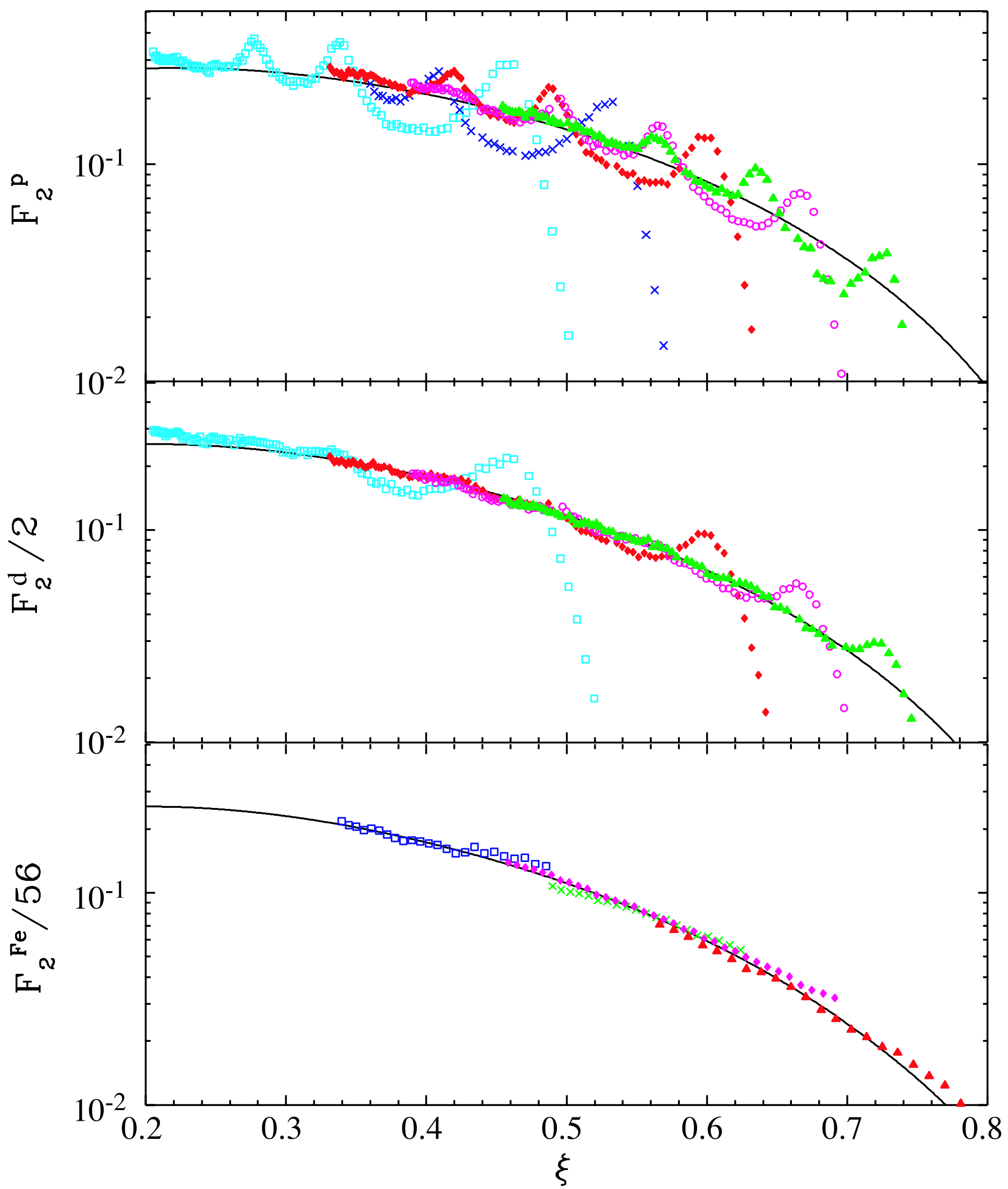
light-cone fraction of target's momentum carried by parton

$$\xi = \frac{p^+}{P^+} = \frac{p^0 + p^z}{M}$$

$$\Longrightarrow \quad \xi = \frac{2x}{1+\sqrt{1+4x^2M^2/Q^2}} \quad \rightarrow \quad x \quad \text{as} \quad Q^2 \rightarrow \infty$$

Nachtmann scaling variable

Nuclear structure functions



for larger nuclei, Fermi motion does resonance averaging automatically !

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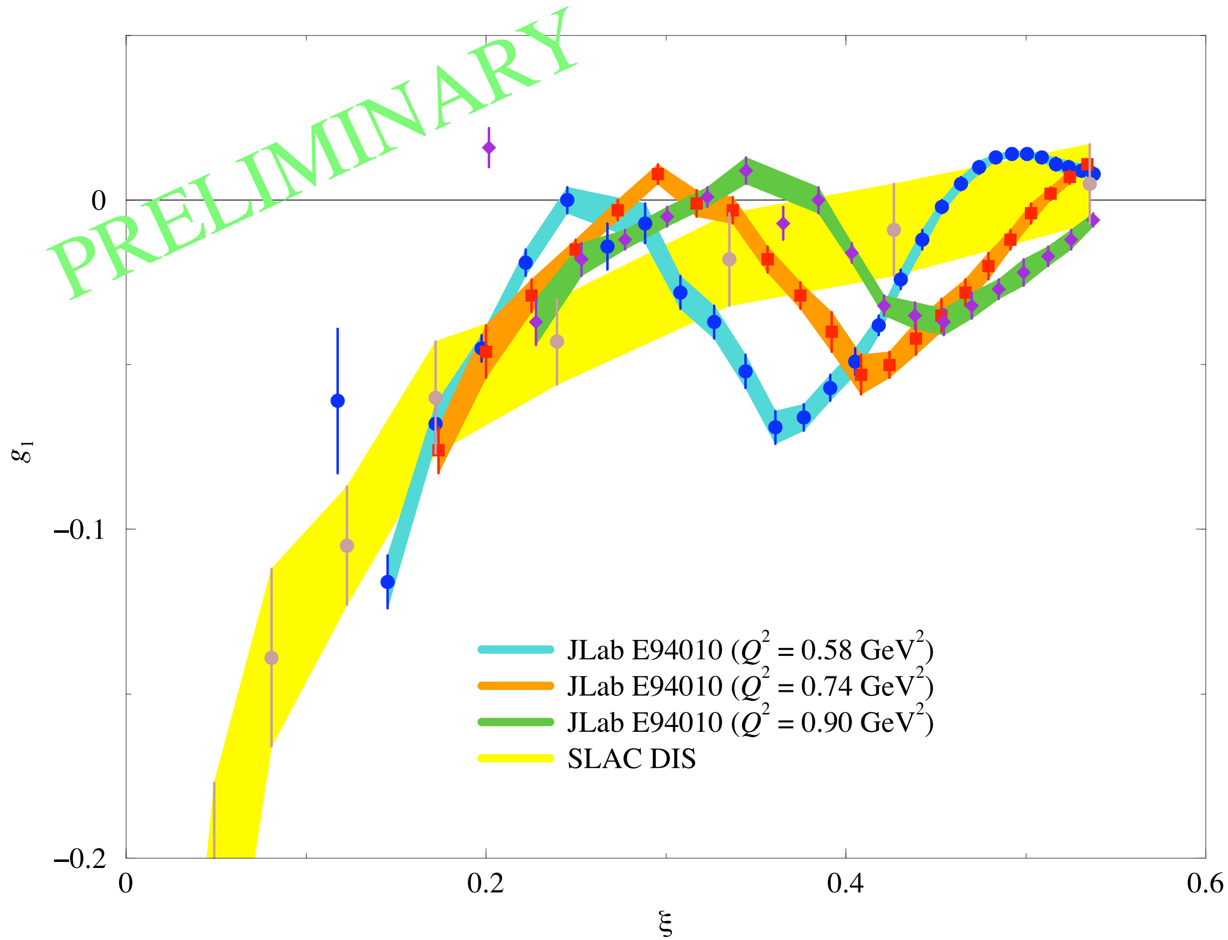
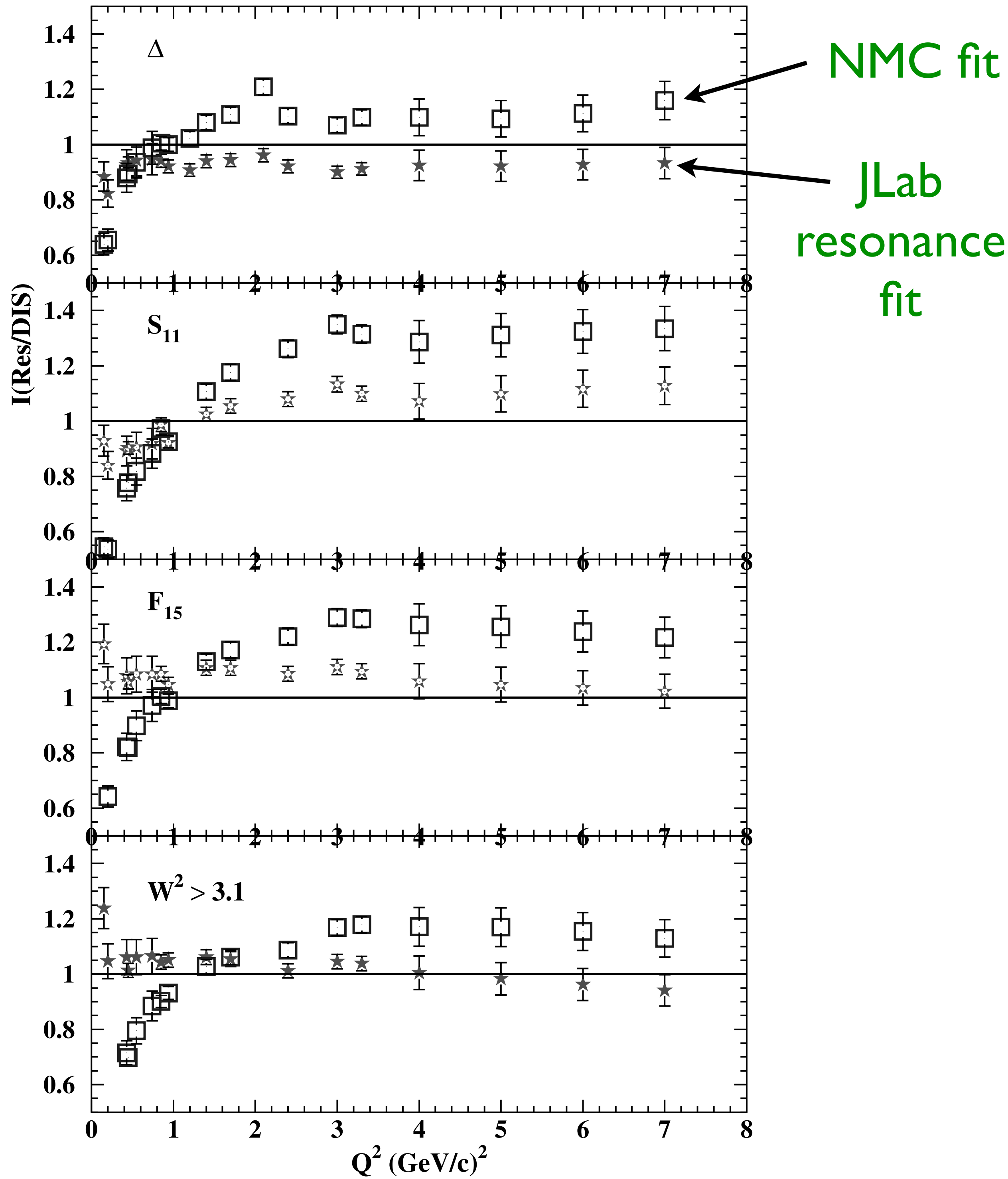
Neutron (^{3}He) g_1 structure function



Liyanage et al. (JLab Hall A)

Integrated strength

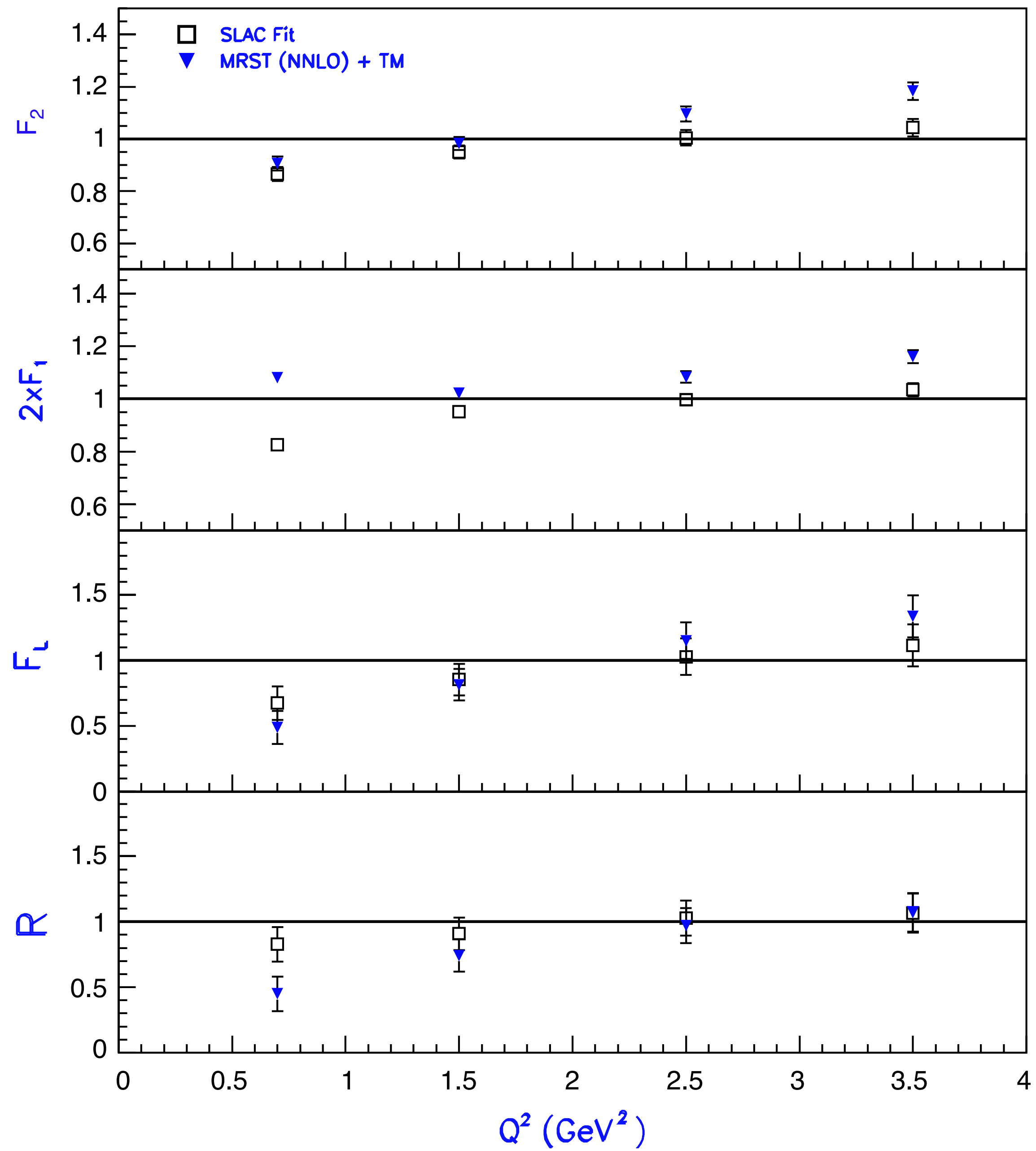


~10% agreement for $Q^2 > 1$ GeV^2

Niculescu et al., Phys. Rev. Lett. 85 (2000) 1186

Moments

data from longitudinal-transverse separation !



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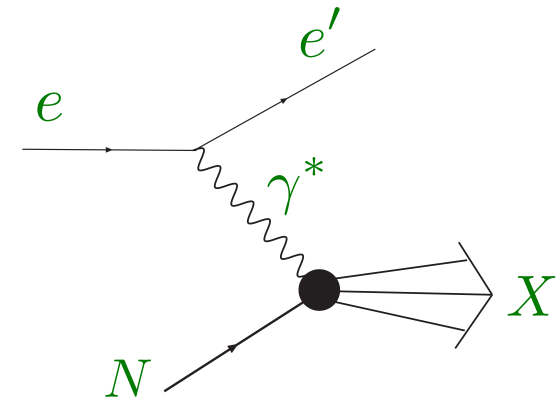
3.
Duality in QCD

Electron scattering

Inclusive cross section for $eN \to eX$



leptonic tensor

$$\frac{d^2\sigma}{d\Omega dE'} \sim L^{\mu\nu}\, W_{\mu\nu}$$

Hadronic tensor

$$\begin{aligned} W_{\mu\nu} &= \sum_X \langle X|J_\mu(z)|N\rangle\, \langle N|J_\nu(0)|X\rangle\, \delta^4(p+q-p_X) \\ &= \int d^4z\; e^{iq\cdot z}\, \langle N|J_\mu(z)J_\nu(0)|N\rangle \end{aligned}$$

using completeness (sum over ALL states X)

$$\sum_X |X\rangle\langle X| = 1$$

"duality"

in general, $N \to X$ transition matrix element very complicated

at large Q^2 and large ν ("Bjorken limit") things simplify ...

- Wilson Operator Product Expansion

  Expand product of currents $J(z)J(0)$ in a series of (nonperturbative) local operators $\widehat{\mathcal{O}}$ and (perturbative) coefficient functions C_n

$$J(z)J(0) \;\sim\; \sum_n C_n(z^2)\; z^{\mu_1} z^{\mu_2} \ldots z^{\mu_n}\; \widehat{\mathcal{O}}_{\mu_1\mu_2\cdots\mu_n}$$

- Matrix elements of $\widehat{\mathcal{O}}_{\mu_1\mu_2\cdots\mu_n}$

M^2/Q^2 corrections

$$\langle N|\widehat{\mathcal{O}}_{\mu_1\mu_2\cdots\mu_n}|N\rangle \;=\; \mathcal{A}_n(\mu^2)\; p_{\mu_1} p_{\mu_2} \cdots p_{\mu_n} \;-\; \text{traces}$$

- Moments of structure function F_2

$$M_n(Q^2) \equiv \int_0^1 dx\ x^{n-2}\ F_2(x, Q^2)$$
$$= \sum_i \widetilde{C}_n^i(Q^2)\ \mathcal{A}_n^i(Q^2/\mu^2)$$

where $\widetilde{C}_n(Q^2)$ is Fourier transform of $C_n(z^2)$

- Reconstruct structure function from moments via inverse Mellin transform

- **Parton model:** $F_2(x, Q^2) = x \sum_q e_q^2\ q(x, Q^2)$

probability to find quark type "q" in nucleon, carrying (light-cone) momentum fraction x

Duality and the OPE

Operator product expansion

⟹ expand moments of structure functions in powers of $1/Q^2$

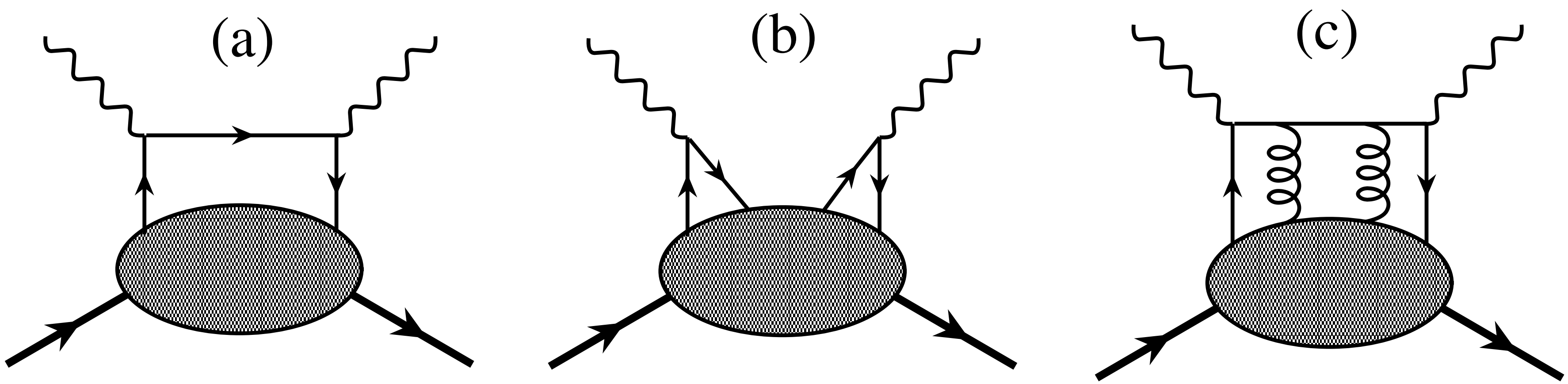
$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x, Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

matrix elements of operators with specific "twist" τ

$$\tau = \mathrm{dimension} - \mathrm{spin}$$

Higher twists



$\tau = 2$

$\tau > 2$

single quark scattering

qq and qg correlations

$e.g.\quad \bar{\psi}\ \gamma_\mu\ \psi$

$e.g.\ \ \bar{\psi}\ \gamma_\mu\ \psi\ \bar{\psi}\ \gamma_\nu\ \psi$

or $\quad \bar{\psi}\ \tilde{G}_{\mu\nu}\gamma^\nu\ \psi$

Duality and the OPE

Operator product expansion

⟹ expand moments of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x,Q^2)$$
$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

If moment $\approx$ independent of Q^2

⟹ higher twist terms $A_n^{(\tau>2)}$ small

Duality and the OPE

Operator product expansion

⟹ expand moments of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx \; x^{n-2} \; F_2(x, Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

Duality $\Longleftrightarrow$ suppression of higher twists

de Rujula, Georgi, Politzer,
Ann. Phys. 103 (1975) 315

Applications of duality

If higher twists are small (duality "works")

$\Rightarrow$ can use single-parton approximation to describe structure functions

$\Rightarrow$ extract *leading twist* parton distributions

If duality is violated, and if violations are small

$\Rightarrow$ can use duality violations to extract *higher twist* matrix elements

$\Rightarrow$ learn about nonperturbative qq or qg correlations

Example:

Lowest moment of g_1

$$\Gamma_1(Q^2) = \int_0^1 dx\ g_1(x, Q^2)$$
$$= \mu_2 + \frac{\mu_4}{Q^2} + \frac{\mu_6}{Q^4} + \cdots$$

Twist 2

$$\mu_2 = \frac{1}{2} \sum_q e_q^2\ \Delta q\ = \frac{1}{2}\left(\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right)$$

$$\Delta q = \int dx\ (q^{\uparrow} - q^{\downarrow})$$

Example:

Lowest moment of g_1

$$\Gamma_1(Q^2) = \int_0^1 dx \; g_1(x, Q^2)$$
$$= \mu_2 + \frac{\mu_4}{Q^2} + \frac{\mu_6}{Q^4} + \cdots$$

Twist 2

$$\mu_2^{p(n)} = \left(\pm \frac{1}{12} g_A + \frac{1}{36} a_8 \right) C_{ns}(Q^2) + \frac{1}{9} \Delta\Sigma \; C_s(Q^2)$$

triplet *octet* *RGI singlet axial charge*

Higher twist terms

$1/Q^2$ correction to g_1 moment

$$\mu_4 = \frac{1}{9} M^2 \left(a_2 + 4d_2 + 4f_2 \right)$$

target mass correction

quark-gluon correlations

Higher twist terms

$1/Q^2$ correction to g_1 moment

$$\mu_4 = \frac{1}{9} M^2 \left(a_2 + 4d_2 + 4f_2\right)$$

$$d_2 \;\rightarrow\; \langle N|\; \bar{\psi}\; \widetilde{G}^{\mu\{\nu}\gamma^{\alpha\}}\; \psi\; |N\rangle$$

twist 3

$$f_2 \;\rightarrow\; \langle N|\; \bar{\psi}\; \widetilde{G}^{\mu\nu}\gamma_{\nu}\; \psi\; |N\rangle$$

twist 4

Color polarizabilities

$1/Q^2$ correction to g_1 moment

$$\mu_4 = \frac{1}{9} M^2 \left(a_2 + 4 d_2 + 4 f_2 \right)$$

color *electric* polarizability

$$\chi_E \;=\; \frac{1}{3}(4d_2 + 2f_2) \;\sim\; \langle \vec{j}_a \times \vec{E}_a \rangle_z$$

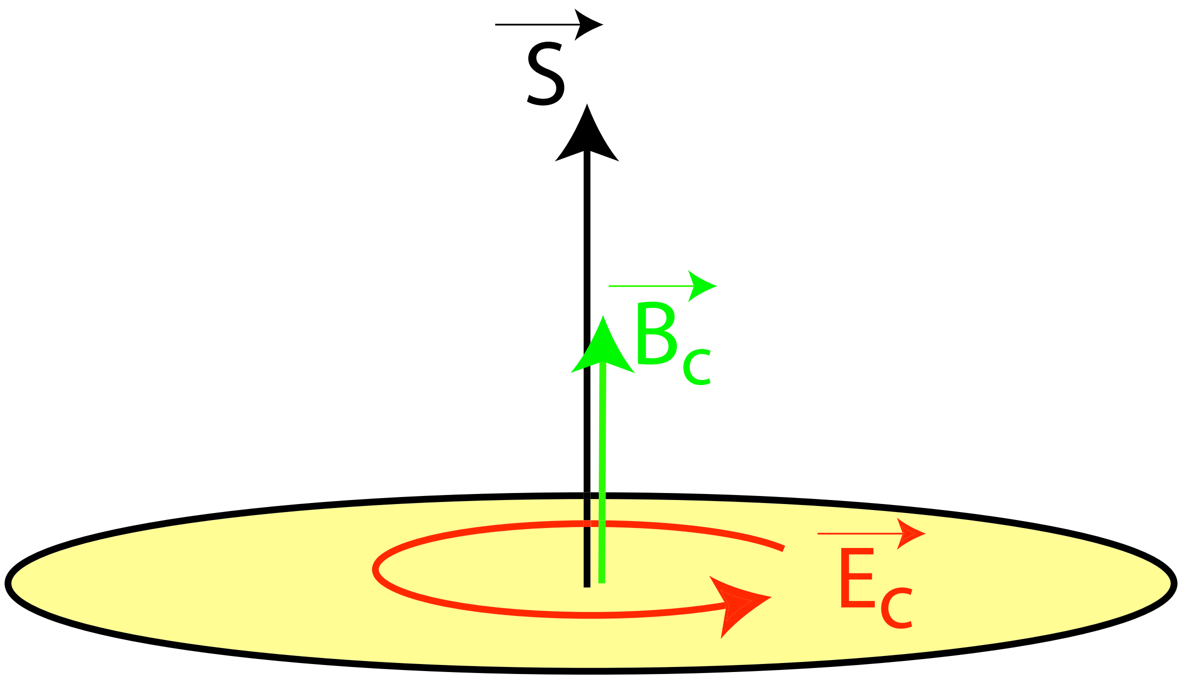
$$j^\mu_a = g_s \psi \gamma^\mu \mathbf{t}_a \psi$$

color *magnetic* polarizability

$$\chi_B \;=\; \frac{1}{3}(4d_2 - f_2) \;\sim\; \langle j^0_a \; \vec{B}_a \rangle_z$$

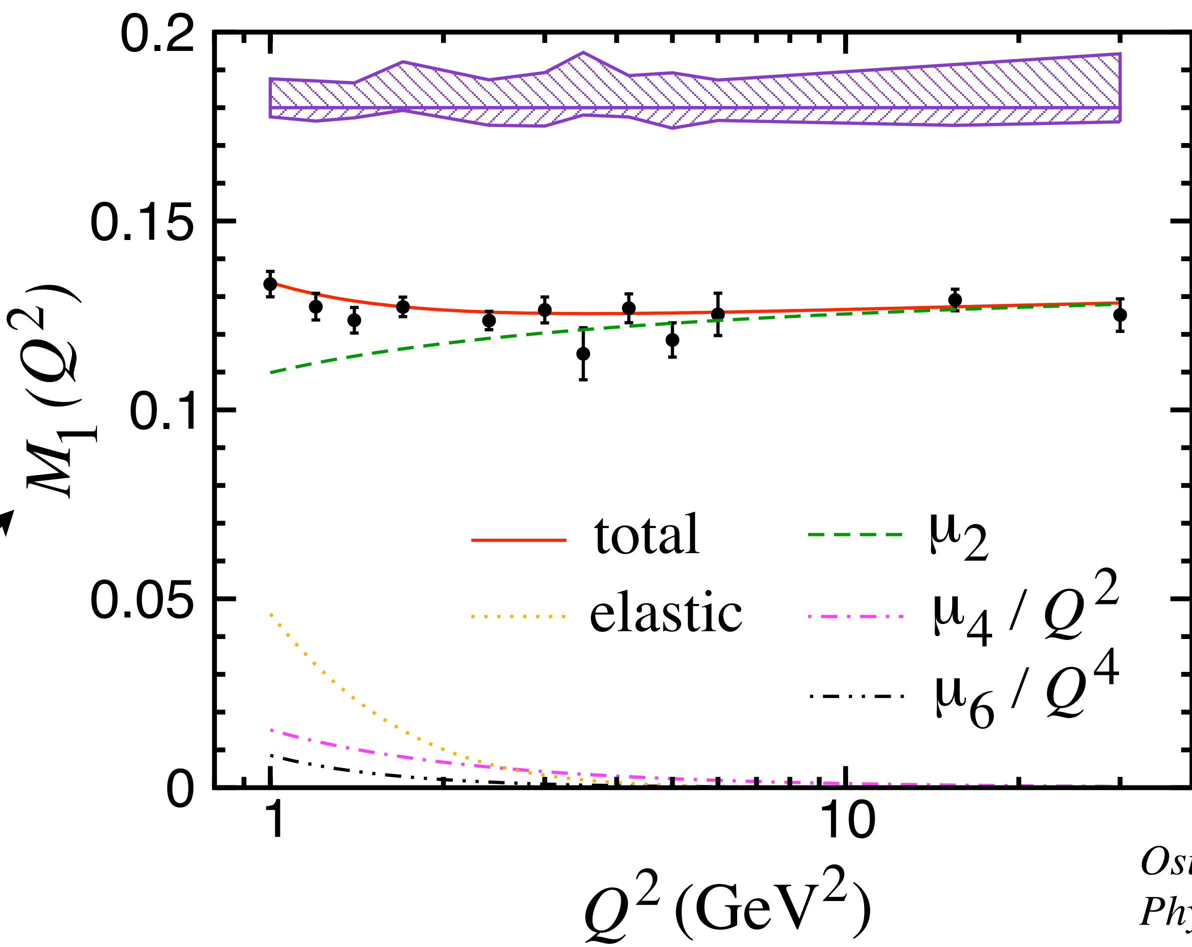
Ji (1995), Schafer, Mankiewicz, ... (1995)

Color polarizabilities



response of collective color electric and magnetic fields
to spin of nucleon

Proton g_1 moment



Nachtmann moment

Osipenko, WM et al, Phys. Lett. B 609 (2005) 259

$$M_1 = \int_0^1 dx \frac{\xi^2}{x^2}\left[g_1\left(\frac{x}{\xi} - \frac{M^2 x\xi}{9Q^2}\right) - g_2 \frac{4M^2x^2}{3Q^2}\right] \;=\; \mu_2 + \frac{4M^2}{9Q^2} f_2 + \cdots$$

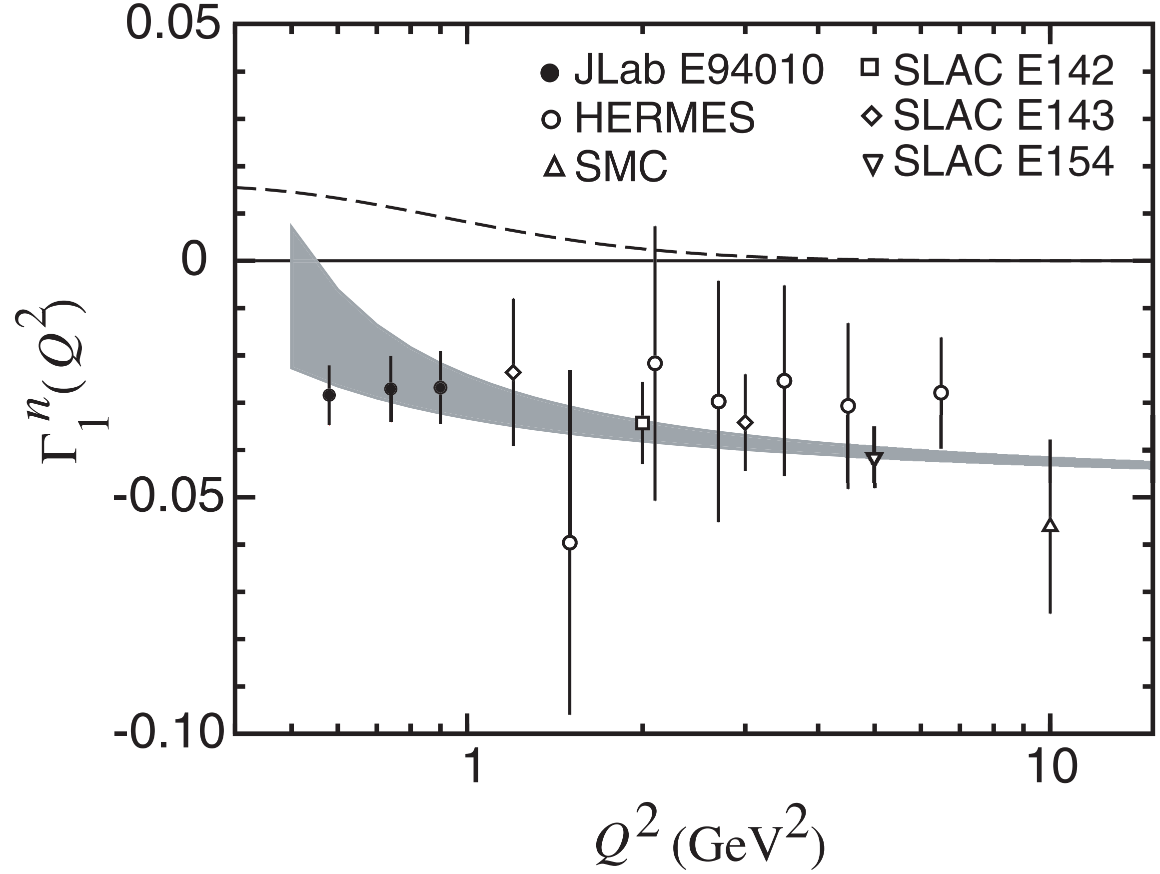
$$\chi_E^p = 0.026 \pm 0.015 \text{ (stat)} \pm 0.021 \text{ (sys)}$$

$$\chi_B^p = -0.013 \pm 0.007 \text{ (stat)} \pm 0.011 \text{ (sys)}$$

Compare with theoretical calculations:

	χ_E^p	χ_B^p
QCD sum rules	-0.04	0.01
MIT bag	0.05	0.02
Instanton	-0.03	0.02
Lattice	?	?

Neutron g_1 moment



Meziani, WM et al,
Phys. Lett. B613 (2005) 148

Γ_1^n extracted from $\Gamma_1^{^3\mathrm{He}}$ data correcting for nuclear effects

$$\chi_E^n = +0.033 \pm 0.029$$

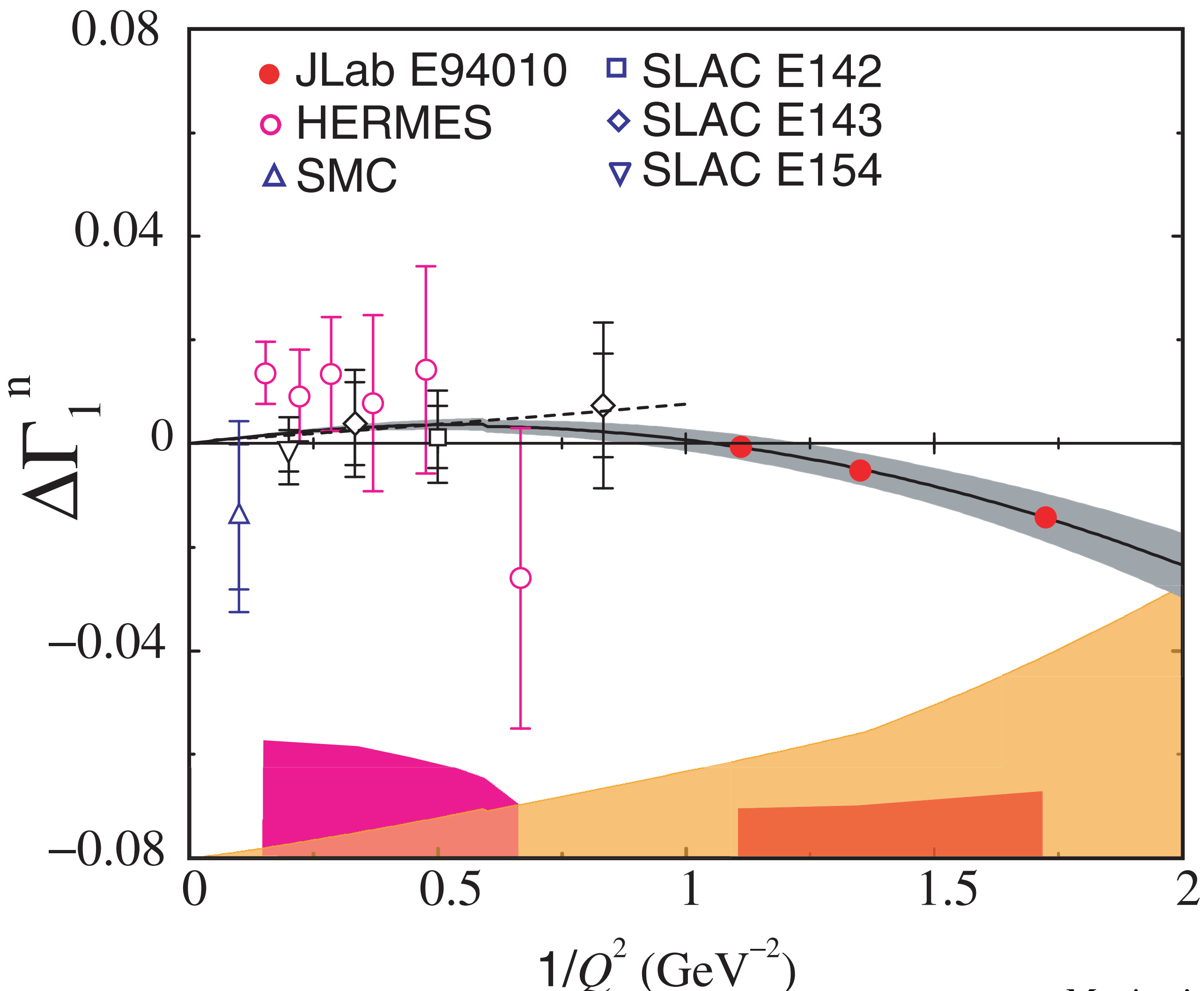
$$\chi_B^n = -0.001 \pm 0.016$$

Compare with theoretical calculations:

	χ_E^n	χ_B^n
QCD sum rules	-0.04	-0.02
MIT bag	0.00	0.00
Instanton	0.03	-0.01
Lattice	?	?

Neutron g_1 moment

$\longrightarrow$ higher twist contribution



Meziani, WM et al.,
Phys. Lett. B613 (2005) 148

Total higher twist $\sim zero$ at $Q^2 \sim 1-2\ \mathrm{GeV}^2$

- nonperturbative interactions between quarks and gluons not dominant at these scales

- suggests *strong cancellations* between resonances, resulting in dominance of *leading twist*

- OPE does not tell us *why* higher twists are small !

Can we understand this behavior dynamically?

How do cancellations between *coherent* resonances produce *incoherent* scaling function?

4.
Local duality

Coherence vs. incoherence

Exclusive form factors

⟹ *coherent* scattering from quarks

$$d\sigma \sim \left(\sum_i e_i \right)^2$$

Inclusive structure functions

⟹ *incoherent* scattering from quarks

$$d\sigma \sim \sum_i e_i^2$$

⟹ How can *square of a sum* $\approx$ *sum of squares* ?

Pedagogical model

Two quarks bound in a harmonic oscillator potential

$\Longrightarrow$ exactly solvable spectrum

Structure function given by sum of squares of transition form factors

$$F(\nu, \mathbf{q}^2) \sim \sum_n \left|G_{0,n}(\mathbf{q}^2)\right|^2 \delta(E_n - E_0 - \nu)$$

Charge operator $\Sigma_i \, e_i \exp(i\mathbf{q} \cdot \mathbf{r}_i)$ excites

even partial waves with strength $\propto (e_1 + e_2)^2$

odd partial waves with strength $\propto (e_1 - e_2)^2$

Pedagogical model

Resulting structure function

$$F(\nu, \mathbf{q}^2) \ \sim \ \sum_n \left\{ (e_1 + e_2)^2 \ G^2_{0,2n} + (e_1 - e_2)^2 \ G^2_{0,2n+1} \right\}$$

If states degenerate, cross terms $(\sim e_1 e_2)$ cancel when averaged over nearby even and odd parity states

Minimum condition for duality:

⟹ *at least one complete set of <u>even</u> and <u>odd</u> parity resonances must be summed over*

Close, Isgur, Phys. Lett. B509 (2001) 81

Quark model

Even and odd parity states generalize to $\mathbf{56}^+$ $(L{=}0)$ and $\mathbf{70}^-$ $(L{=}1)$ multiplets of spin-flavor SU(6)

$\Longrightarrow$ scaling occurs if contributions from $\mathbf{56}^+$ and $\mathbf{70}^-$ have equal overall strengths

Simplified case: magnetic coupling of γ^* to quark

$\Longrightarrow$ expect dominance over electric at large Q^2

Quark model

Even and odd parity states generalize to $\mathbf{56}^+$ ($L=0$) and $\mathbf{70}^-$ ($L=1$) multiplets of spin-flavor SU(6)

$\Longrightarrow$ scaling occurs if contributions from $\mathbf{56}^+$ and $\mathbf{70}^-$ have equal overall strengths

representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
F_1^p	$9\rho^2$	$8\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2+9\lambda^2$
F_1^n	$(3\rho+\lambda)^2/4$	$8\lambda^2$	$(3\rho-\lambda)^2/4$	$4\lambda^2$	λ^2	$(9\rho^2+27\lambda^2)/2$
g_1^p	$9\rho^2$	$-4\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2-3\lambda^2$
g_1^n	$(3\rho+\lambda)^2/4$	$-4\lambda^2$	$(3\rho-\lambda)^2/4$	$-2\lambda^2$	λ^2	$(9\rho^2-9\lambda^2)/2$

$\lambda\ (\rho) =$ (anti) symmetric component of ground state wfn.

Quark model

SU(6) limit $\Longrightarrow$ $\lambda = \rho$

$SU(6):$	$[\mathbf{56},\mathbf{0}^+]^\mathbf{2}\mathbf{8}$	$[\mathbf{56},\mathbf{0}^+]^\mathbf{4}\mathbf{10}$	$[\mathbf{70},\mathbf{1}^-]^\mathbf{2}\mathbf{8}$	$[\mathbf{70},\mathbf{1}^-]^\mathbf{4}\mathbf{8}$	$[\mathbf{70},\mathbf{1}^-]^\mathbf{2}\mathbf{10}$	*total*
F_1^p	9	8	9	0	1	27
F_1^n	4	8	1	4	1	18
g_1^p	9	-4	9	0	1	15
g_1^n	4	-4	1	-2	1	0

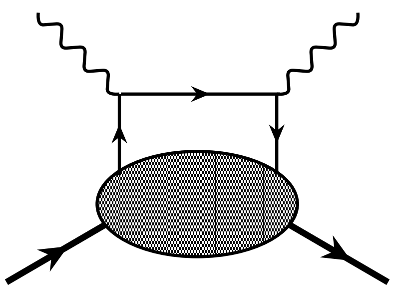
Summing over all resonances in $\mathbf{56}^+$ and $\mathbf{70}^-$ multiplets

$$\Longrightarrow \quad R^{np} = \frac{F_1^n}{F_1^p} = \frac{2}{3} \qquad A_1^p = \frac{g_1^p}{F_1^p} = \frac{5}{9} \qquad A_1^n = \frac{g_1^n}{F_1^n} = 0$$

Quark model

- proton wave function

$$p^{\uparrow} = -\frac{1}{3} d^{\uparrow} (uu)_1 - \frac{\sqrt{2}}{3} d^{\downarrow} (uu)_1 + \frac{\sqrt{2}}{6} u^{\uparrow} (ud)_1 - \frac{1}{3} u^{\downarrow} (ud)_1 + \frac{1}{\sqrt{2}} u^{\uparrow} (ud)_0$$



interacting quark

spectator diquark

diquark spin

$\longrightarrow$ $u(x) = 2\, d(x)$ for all x $\longrightarrow$ $\dfrac{F_2^n}{F_2^p} = \dfrac{4u + d}{u + 4d} = \dfrac{2}{3}$

Quark model

SU(6) limit $\Longrightarrow$ $\lambda = \rho$

$SU(6):$	$[\mathbf{56},\mathbf{0}^+]^\mathbf{2}\mathbf{8}$	$[\mathbf{56},\mathbf{0}^+]^\mathbf{4}\mathbf{10}$	$[\mathbf{70},\mathbf{1}^-]^\mathbf{2}\mathbf{8}$	$[\mathbf{70},\mathbf{1}^-]^\mathbf{4}\mathbf{8}$	$[\mathbf{70},\mathbf{1}^-]^\mathbf{2}\mathbf{10}$	*total*
F_1^p	9	8	9	0	1	27
F_1^n	4	8	1	4	1	18
g_1^p	9	-4	9	0	1	15
g_1^n	4	-4	1	-2	1	0

Summing over all resonances in $\mathbf{56}^+$ and $\mathbf{70}^-$ multiplets

$\Longrightarrow$ $R^{np} = \frac{F_1^n}{F_1^p} = \frac{2}{3}$ $\quad A_1^p = \frac{g_1^p}{F_1^p} = \frac{5}{9}$ $\quad A_1^n = \frac{g_1^n}{F_1^n} = 0$

$\Longrightarrow$ as in quark-parton model !

Quark model

SU(6) limit $\Longrightarrow$ $\lambda = \rho$

$SU(6):$	$[\mathbf{56}, \mathbf{0}^+]^\mathbf{2}\mathbf{8}$	$[\mathbf{56}, \mathbf{0}^+]^\mathbf{4}\mathbf{10}$	$[\mathbf{70}, \mathbf{1}^-]^\mathbf{2}\mathbf{8}$	$[\mathbf{70}, \mathbf{1}^-]^\mathbf{4}\mathbf{8}$	$[\mathbf{70}, \mathbf{1}^-]^\mathbf{2}\mathbf{10}$	*total*
F_1^p	9	8	9	0	1	27
F_1^n	4	8	1	4	1	18
g_1^p	9	-4	9	0	1	15
g_1^n	4	-4	1	-2	1	0

$\Longrightarrow$ expect duality to appear earlier for F_1^p than F_1^n

$\Longrightarrow$ earlier onset for g_1^n than g_1^p

$\Longrightarrow$ cancellations *within* multiplets for g_1^n

Quark model

SU(6) may be $\approx$ valid at $x \sim 1/3$

But significant deviations at large x

$\Longrightarrow$ which combinations of resonances reproduce behavior of structure functions at large x?

Model	SU(6)	No $^4\mathbf{10}$	No $^2\mathbf{10}$, $^4\mathbf{10}$	No $S_{3/2}$	No $\sigma_{3/2}$	No ψ_λ
R^{np}	2/3	10/19	1/2	6/19	3/7	1/4
A_1^p	5/9	1	1	1	1	1
A_1^n	0	2/5	1/3	1	1	1

gives $\Delta u/u > 1$ $\Longrightarrow$ *inconsistent with duality*

Quark model

SU(6) may be $\approx$ valid at $x \sim 1/3$

But significant deviations at large x

$\Rightarrow$ which combinations of resonances reproduce behavior of structure functions at large x?

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R^{np}	2/3	10/19	1/2	6/19	3/7	1/4
A_1^p	5/9	1	1	1	1	1
A_1^n	0	2/5	1/3	1	1	1

$^4\mathbf{10}\ [\mathbf{56}^+]$ and $^4\mathbf{8}\ [\mathbf{70}^-]$ suppressed

Quark model

SU(6) may be $\approx$ valid at $x \sim 1/3$

But significant deviations at large x

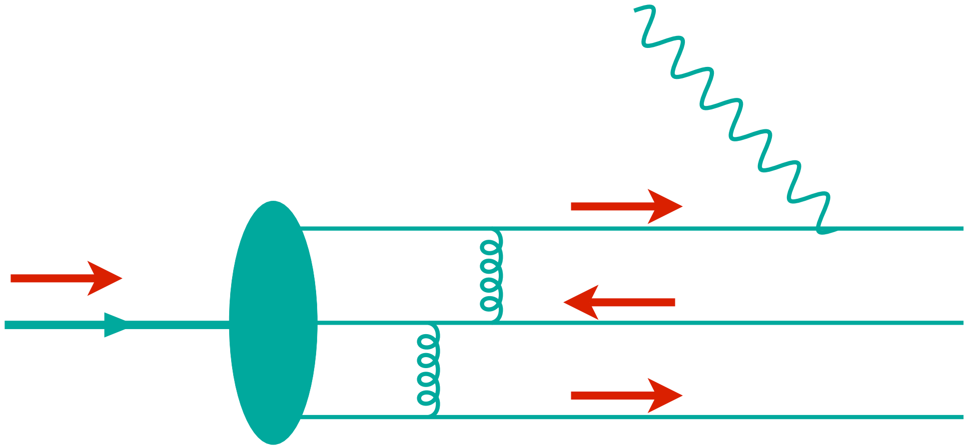
→ which combinations of resonances reproduce behavior of structure functions at large x?

Model	SU(6)	No $^4\mathbf{10}$	No $^2\mathbf{10}$, $^4\mathbf{10}$	No $S_{3/2}$	No $\sigma_{3/2}$	No ψ_λ
R^{np}	2/3	10/19	1/2	6/19	3/7	1/4
A_1^p	5/9	1	1	1	1	1
A_1^n	0	2/5	1/3	1	1	1

helicity 3/2 suppression

hard gluon exchange

at large x, helicity of struck quark = helicity of hadron



$q^{\uparrow} \gg q^{\downarrow}$

$\Longrightarrow$ helicity-zero diquark dominant in $x \to 1$ limit

$\longrightarrow$ $$\frac{d}{u} \to \frac{1}{5}$$

$\longrightarrow$ $$\frac{F_2^n}{F_2^p} \to \frac{3}{7}$$

Farrar, Jackson 1975

$N \to N^*$ transitions for helicity-1/2 dominance

SU(6) representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
$F_1^p = g_1^p$	9	2	9	0	1	21
$F_1^n = g_1^n$	4	2	1	1	1	9

polarization asymmetries $A_1^N \to 1$

→ cf. pQCD "counting rules"

→ hard gluon exchange between quarks

neutron to proton ratio $F_2^n / F_2^p \to 3/7$

→ cf. "helicity retention" model

Farrar, Jackson, Phys. Rev. Lett. 35 (1975) 1416

Quark model

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But significant deviations at large x

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R^{np}	2/3	10/19	1/2	6/19	3/7	1/4
A_1^p	5/9	1	1	1	1	1
A_1^n	0	2/5	1/3	1	1	1

e.g. through $\vec{S}_i \cdot \vec{S}_j$ interaction between quarks $\Longleftarrow$ suppression of symmetric part of spin-flavor wfn.

Valence quarks

- scalar diquark dominance

$M_\Delta > M_N \quad \Longrightarrow \quad (qq)_1$ has larger energy than $(qq)_0$

$\Longrightarrow$ scalar diquark dominant in $x \to 1$ limit

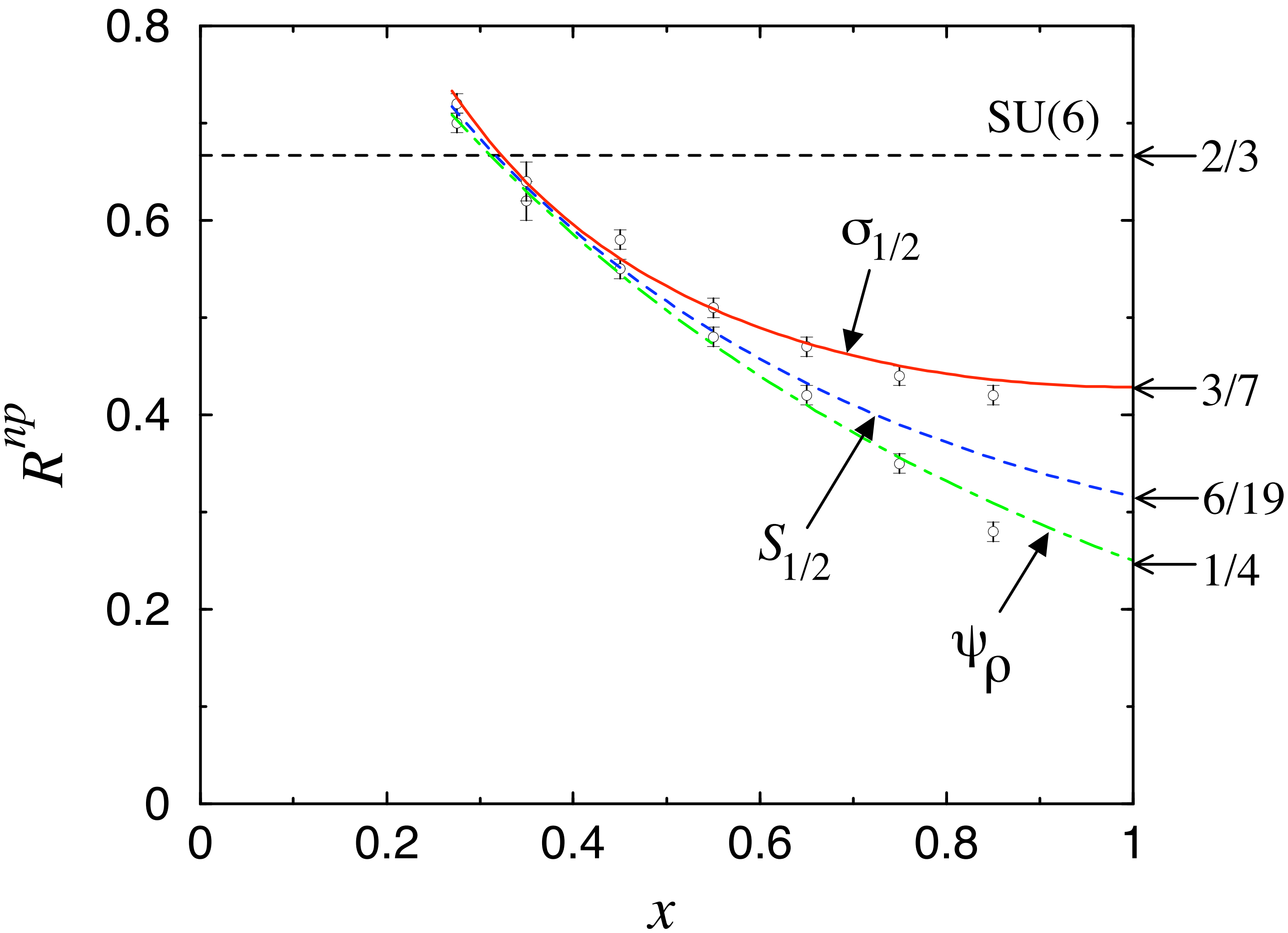
since only u quarks couple to scalar diquarks

$\longrightarrow \quad \dfrac{d}{u} \to 0$

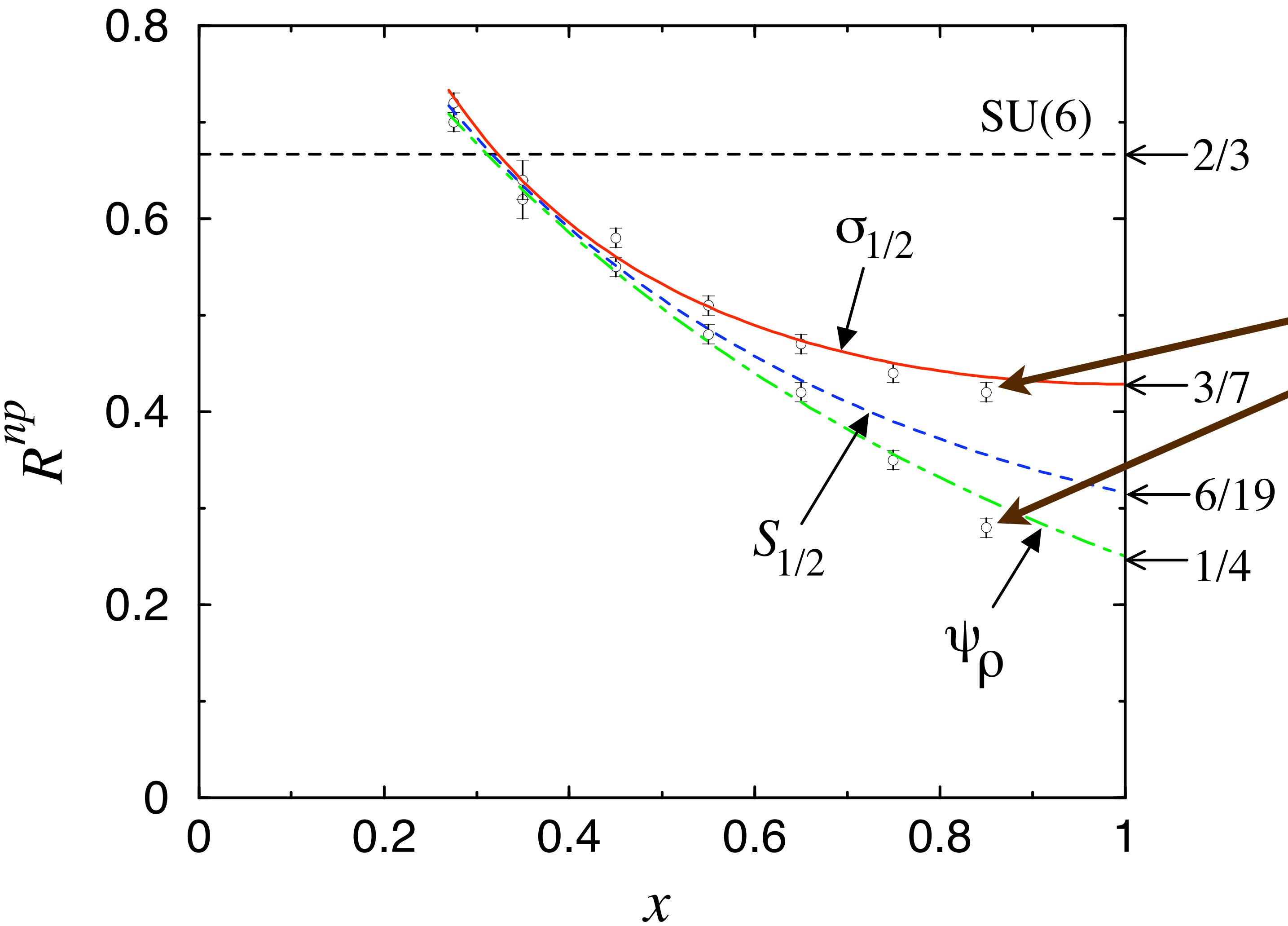
$\longrightarrow \quad \dfrac{F_2^n}{F_2^p} \to \dfrac{1}{4}$

Feynman 1972, Close 1973, Close/Thomas 1988

Fit to $\begin{cases} \text{SU(6) symmetry at } x \sim 1/3 \\ \text{SU(6) breaking at } x \sim 1 \end{cases}$

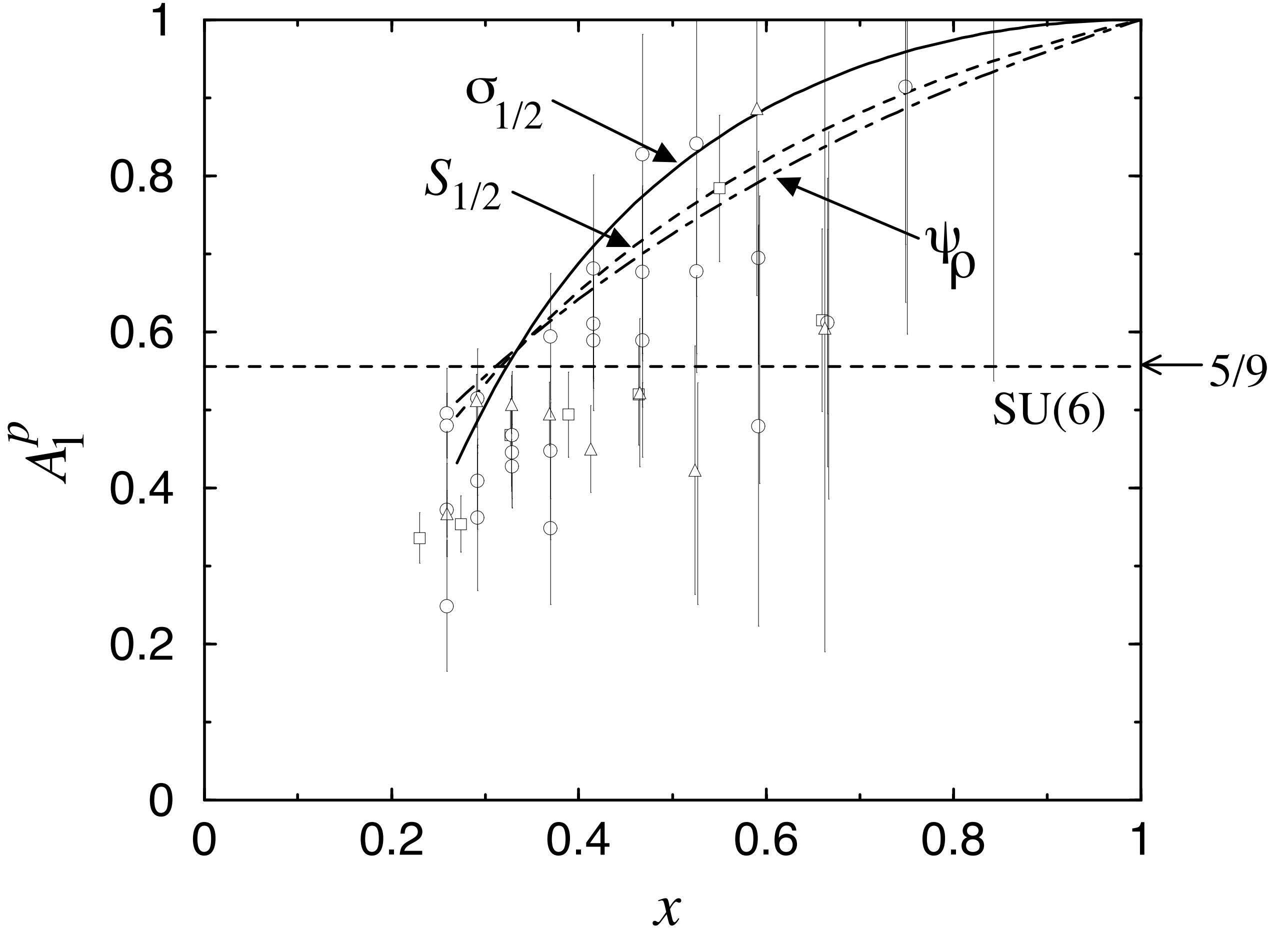


Fit to $\begin{cases} \text{SU(6) symmetry at } x \sim 1/3 \\ \text{SU(6) breaking at } x \sim 1 \end{cases}$

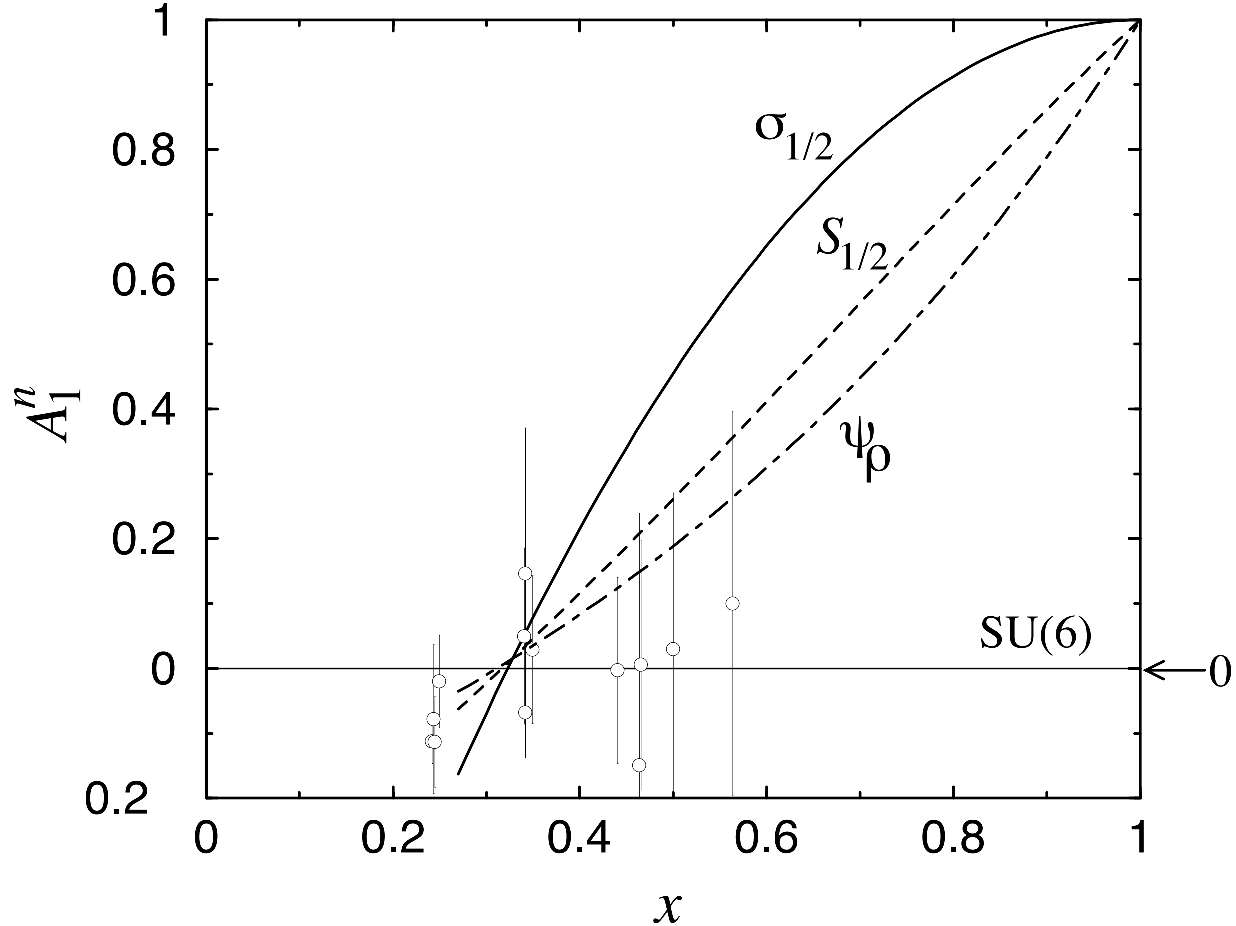


uncertainty in F_2^n due to nuclear effects in deuteron

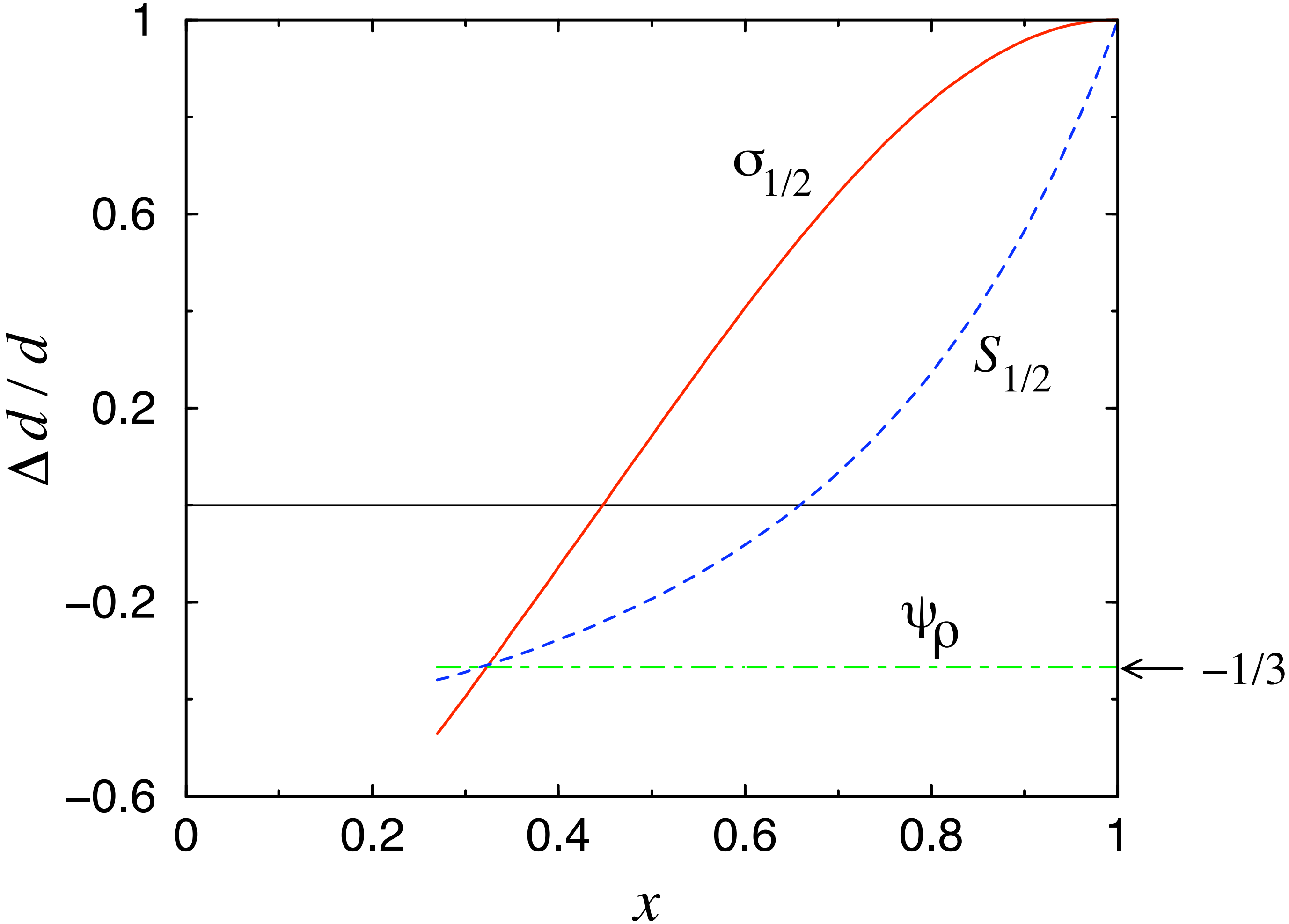
Polarization asymmetry A_1^p



Polarization asymmetry A_1^n



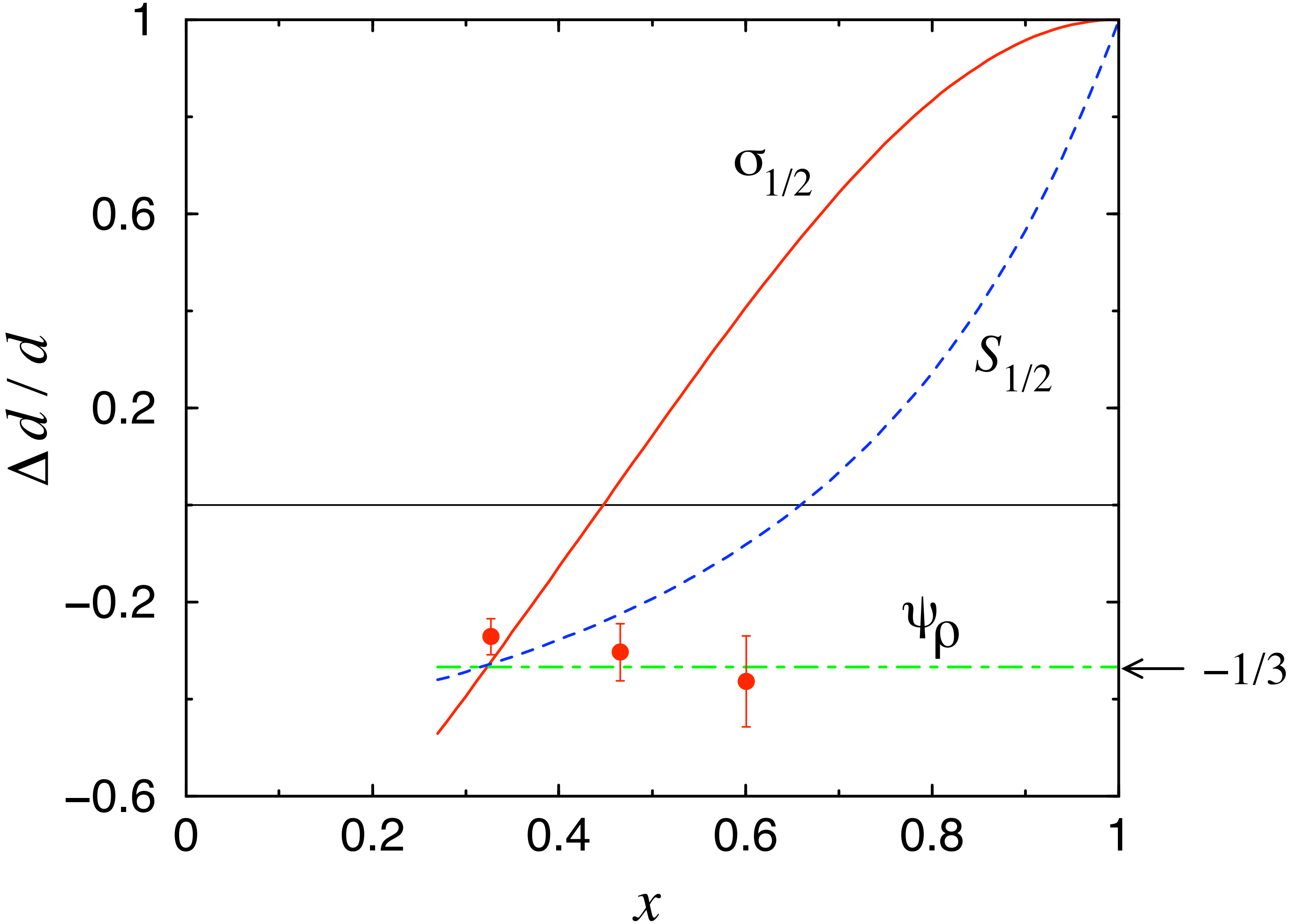
$$\frac{\Delta d}{d} = \frac{4}{15} A_1^n \left(4 + \frac{u}{d}\right) - \frac{1}{15} A_1^p \left(1 + 4\frac{u}{d}\right)$$



$$\frac{u}{d} = \frac{4 - R^{np}}{4R^{np} - 1}$$

Close, WM
PRC68 (2003) 035210

$$\frac{\Delta d}{d} = \frac{4}{15} A_1^n \left(4 + \frac{u}{d}\right) - \frac{1}{15} A_1^p \left(1 + 4\frac{u}{d}\right)$$



$$\frac{u}{d} = \frac{4 - R^{np}}{4R^{np} - 1}$$

Zheng et al. (JLab Hall A)
PRL (2004) 012004

➡ nonperturbative physics still dominant at $x \sim 0.6$!

λ suppression model $\Rightarrow$ identical production rates in $\mathbf{56}^+$ and $\mathbf{70}^-$ channels

representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
F_1^p	$9\rho^2$	$8\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2+9\lambda^2$
F_1^n	$(3\rho+\lambda)^2/4$	$8\lambda^2$	$(3\rho-\lambda)^2/4$	$4\lambda^2$	λ^2	$(9\rho^2+27\lambda^2)/2$
g_1^p	$9\rho^2$	$-4\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2-3\lambda^2$
g_1^n	$(3\rho+\lambda)^2/4$	$-4\lambda^2$	$(3\rho-\lambda)^2/4$	$-2\lambda^2$	λ^2	$(9\rho^2-9\lambda^2)/2$

representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
$F_1^{\nu p}$	0	$24\lambda^2$	0	0	$3\lambda^2$	$27\lambda^2$
$F_1^{\nu n}$	$(9\rho+\lambda)^2/4$	$8\lambda^2$	$(9\rho-\lambda)^2/4$	$4\lambda^2$	λ^2	$(81\rho^2+27\lambda^2)/2$
$g_1^{\nu p}$	0	$-12\lambda^2$	0	0	$3\lambda^2$	$-9\lambda^2$
$g_1^{\nu n}$	$(9\rho+\lambda)^2/4$	$-4\lambda^2$	$(9\rho-\lambda)^2/4$	$-2\lambda^2$	λ^2	$(81\rho^2-9\lambda^2)/2$

λ suppression model ⟹ identical production rates in $\mathbf{56}^+$ and $\mathbf{70}^-$ channels

representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
F_1^p	$9\rho^2$	$8\cancel{\lambda}^2$	$9\rho^2$	0	$\cancel{\lambda}^2$	$18\rho^2+9\lambda^2$
F_1^n	$(3\rho+\cancel{\lambda})^2/4$	$8\cancel{\lambda}^2$	$(3\rho-\cancel{\lambda})^2/4$	$4\cancel{\lambda}^2$	$\cancel{\lambda}^2$	$(9\rho^2+27\lambda^2)/2$
g_1^p	$9\rho^2$	$-4\cancel{\lambda}^2$	$9\rho^2$	0	$\cancel{\lambda}^2$	$18\rho^2-3\lambda^2$
g_1^n	$(3\rho+\cancel{\lambda})^2/4$	$-4\cancel{\lambda}^2$	$(3\rho-\cancel{\lambda})^2/4$	$-2\cancel{\lambda}^2$	$\cancel{\lambda}^2$	$(9\rho^2-9\lambda^2)/2$

representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
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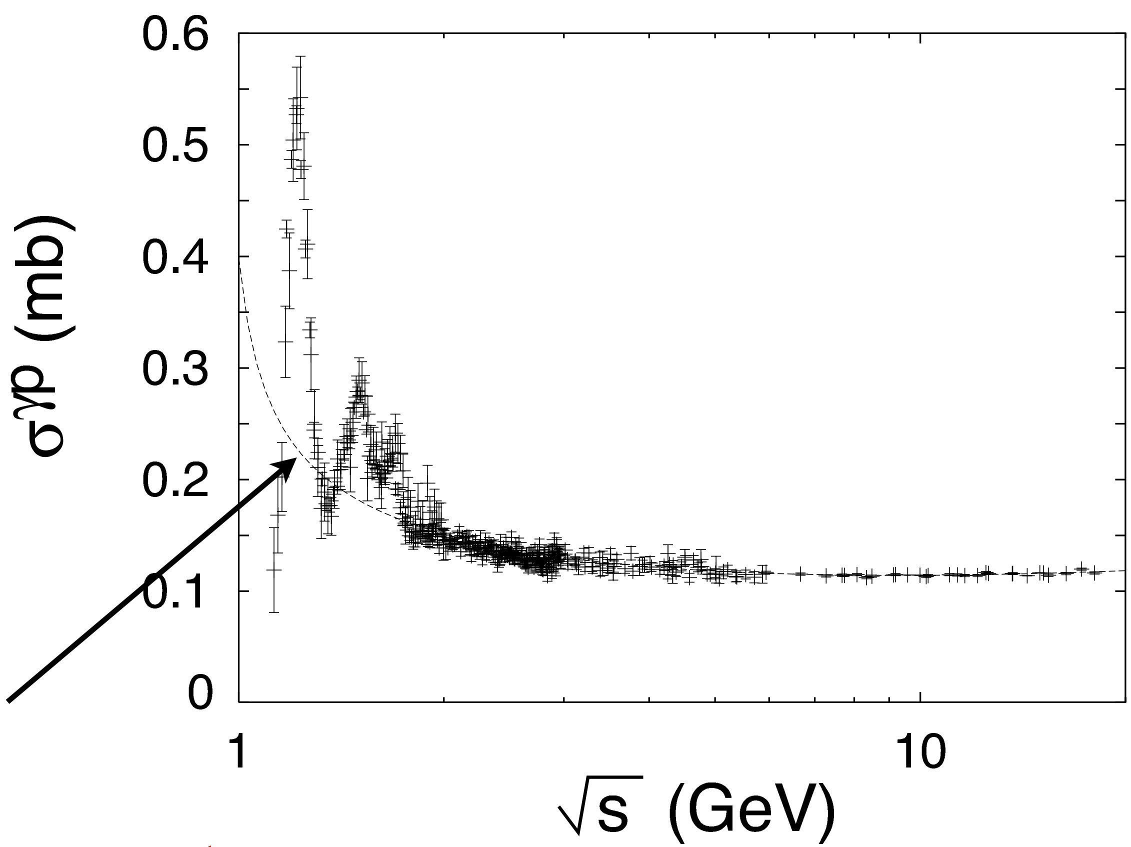
$g_1^{\nu p}$	0	$-12\cancel{\lambda}^2$	0	0	$3\cancel{\lambda}^2$	$-9\lambda^2$
$g_1^{\nu n}$	$(9\rho+\cancel{\lambda})^2/4$	$-4\cancel{\lambda}^2$	$(9\rho-\cancel{\lambda})^2/4$	$-2\cancel{\lambda}^2$	$\cancel{\lambda}^2$	$(81\rho^2-9\lambda^2)/2$

⟹ important test for future experiments

5.
DIS at low Q^2

- as Q^2 decreases, pQCD description (twist expansion) breaks down
  - → near real photon point expand in Q^2 rather than $1/Q^2$
  - → intriguing indications of duality even at $Q^2 = 0$



high-energy Regge fit

$$\sigma^{\gamma p} = X(2M\nu)^{\alpha_P - 1} + Y(2M\nu)^{\alpha_R - 1}$$

Donnachie, Landshoff (1992)

- low Q^2 behavior constrained by (electromagnetic) gauge invariance

$$\left.\begin{aligned} F_2(x,Q^2) &\to Q^2 \\ F_L(x,Q^2) &\to Q^4 \end{aligned}\right\} \quad as \;\; Q^2 \to 0$$

- cf. ν scattering - axial current partially conserved

$$F_2^{\nu}(x,Q^2) \to f_{\pi}^2\, \sigma^{\pi N} \;\; as\; Q^2 \to 0$$

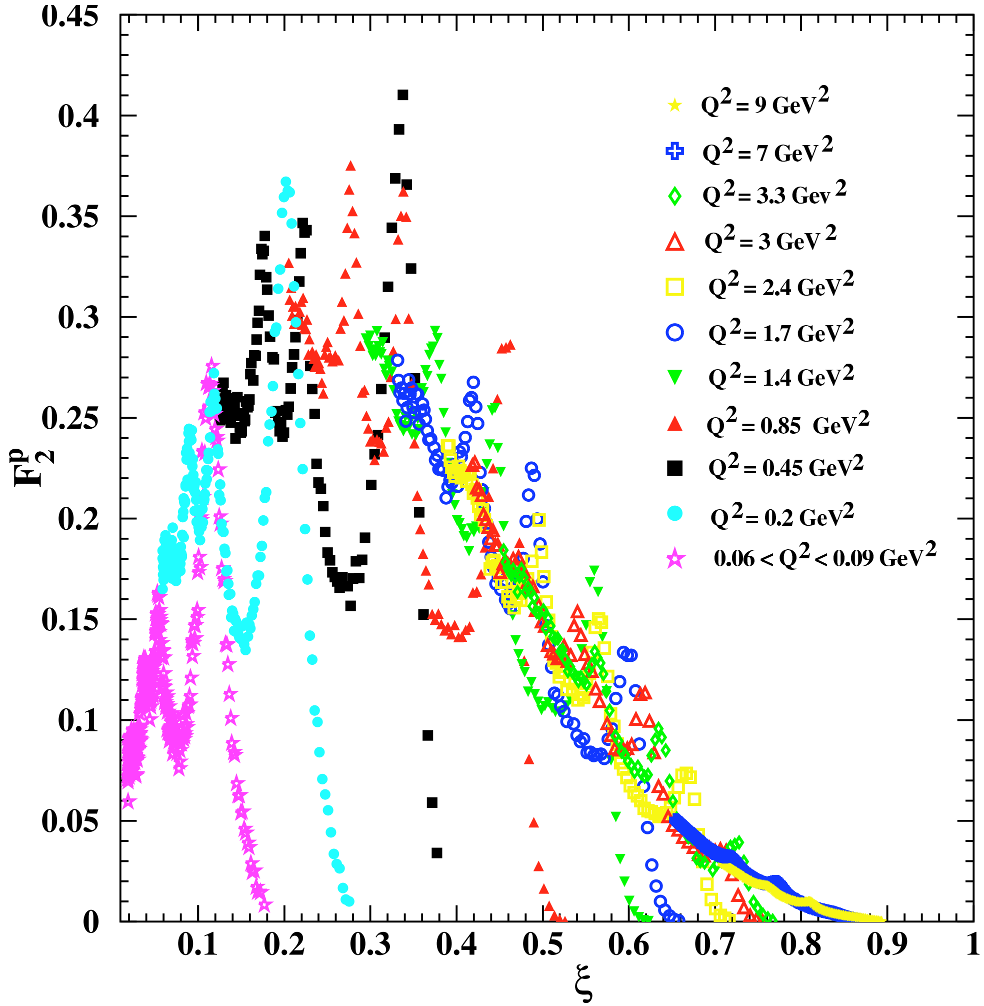
→ model for F_2^{ν} at low Q^2

$$F_2^{\nu} = Q^2 \underbrace{\left(\frac{f_{\rho}}{1+Q^2/m_{\rho}^2}\right)^2 \sigma^{\rho N}}_{\text{VMD}} + f_{\pi}^2 \underbrace{\left(\frac{1}{1+Q^2/m_{A_1}^2}\right)^2 \sigma^{\pi N}}_{\text{PCAC}}$$

gauge invariance or dynamics?

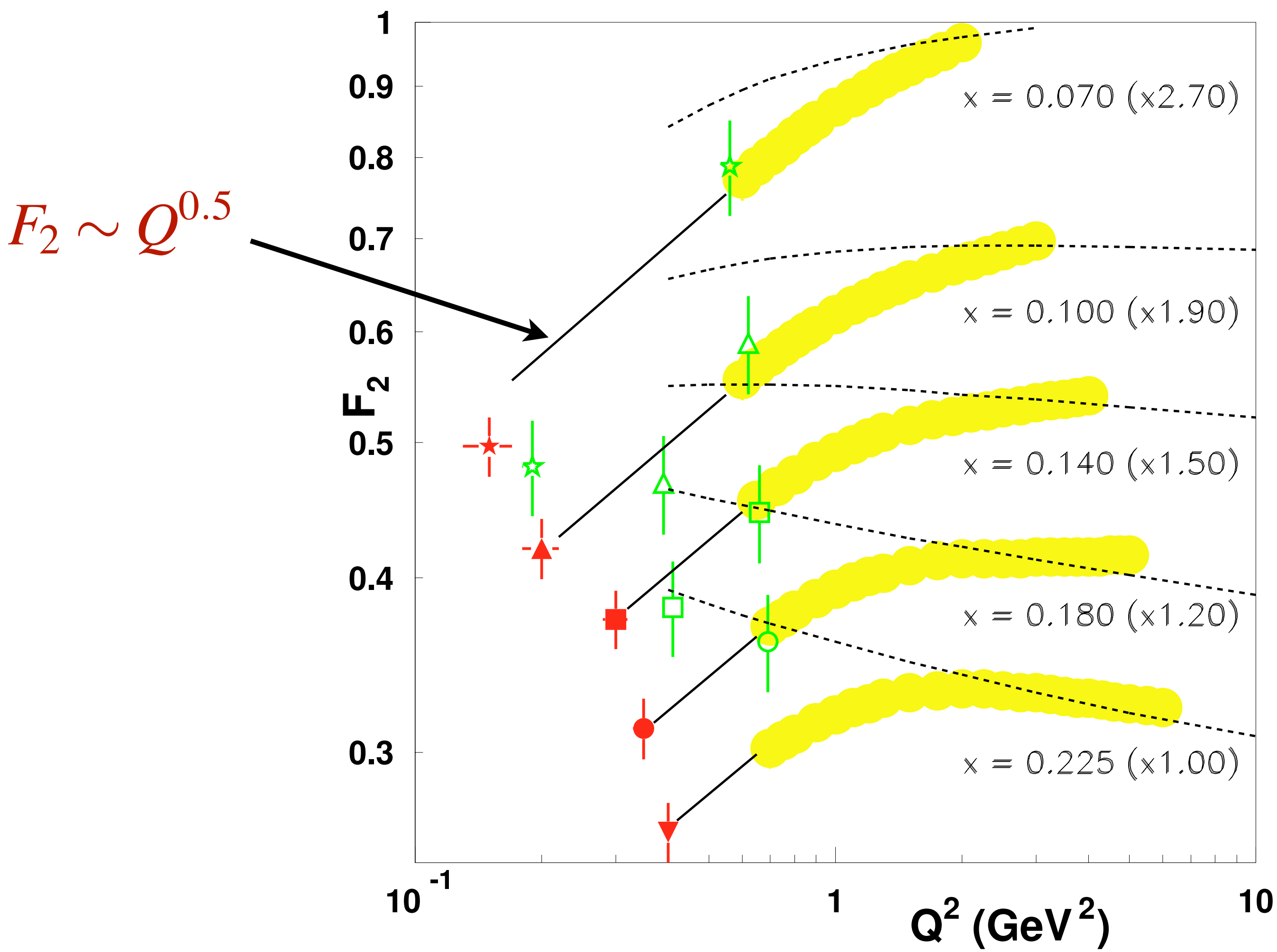
F_2 valence-like at low Q^2 ?

→ cf. xF_3



Niculescu et al., Phys. Rev. Lett. 85 (2000) 1182

gauge invariance or dynamics?



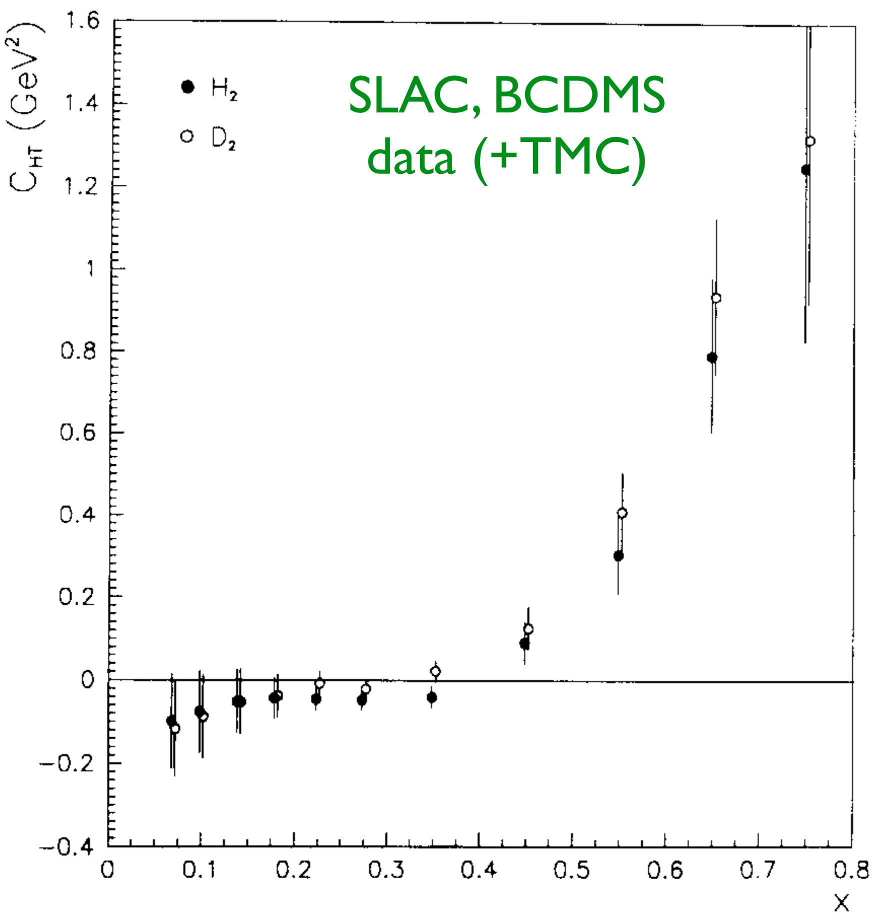
Niculescu et al., Phys. Rev. Lett. 85 (2000) 1182

→ need lower Q^2 before behavior driven by gauge inv.

Phenomenological higher twists

- usually parameterized as

$$F_2(x,Q^2) = F_2^{LT}(x,Q^2)\left(1+\frac{C(x)}{Q^2}\right)$$



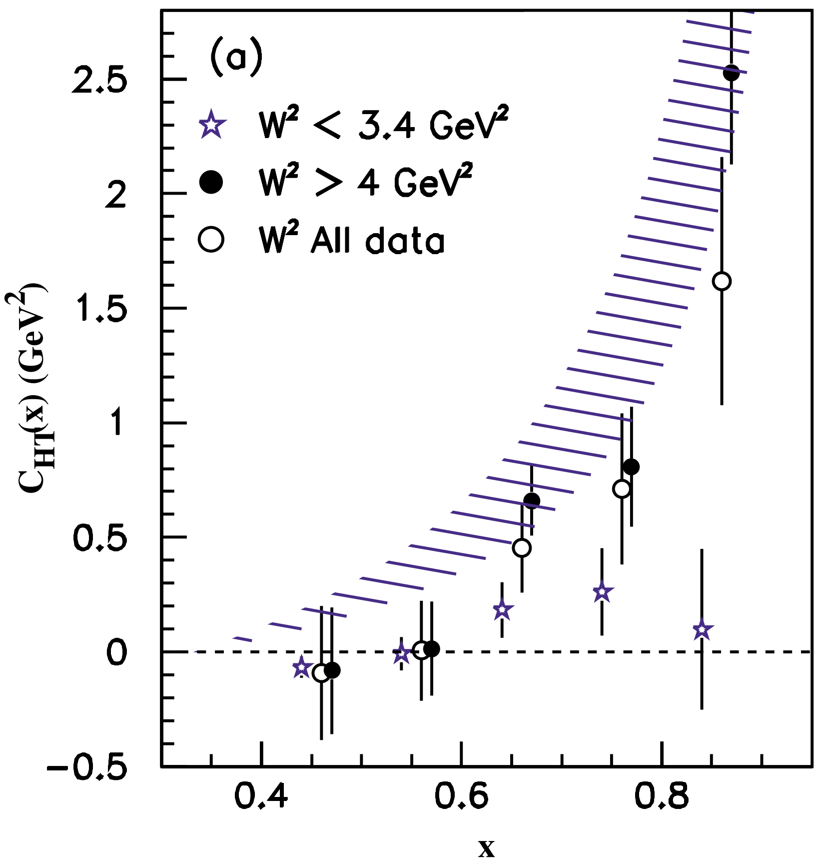
Virchaux, Milsztajn,
Phys. Lett. B274 (1992) 221

Phenomenological higher twists

- more recent JLab data analysis

$$F_2(x,Q^2) = F_2^{LT}(x,Q^2)\left(1 + \frac{C_{HT}(x)}{Q^2} + \Delta H(x,Q^2)\right)$$

+ TMC
+ large-x resummation



large-x resummation reduces C_{HT}

lower-W data require negative $1/Q^4$ term

Liuti, Ent, Keppel, Niculescu, Phys. Rev. Lett. 89 (2002) 162001

Phenomenological higher twists

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$$F_2(x,Q^2) = F_2^{LT}(x,Q^2)\left(1 + \frac{C_{HT}(x)}{Q^2} + \Delta H(x,Q^2)\right)$$

+ TMC
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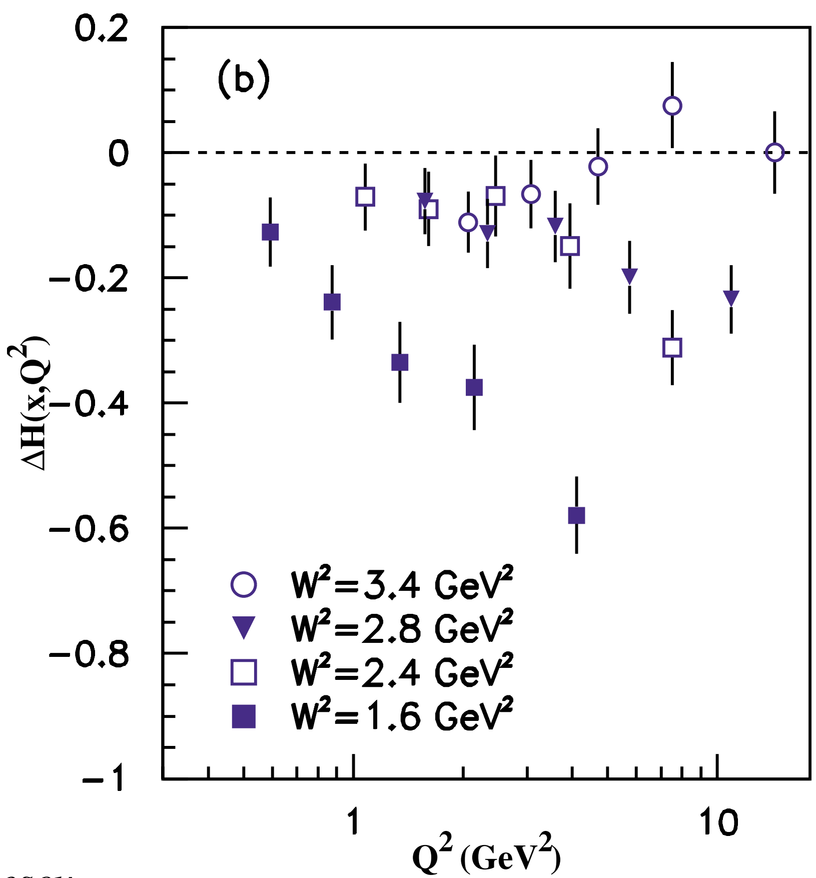


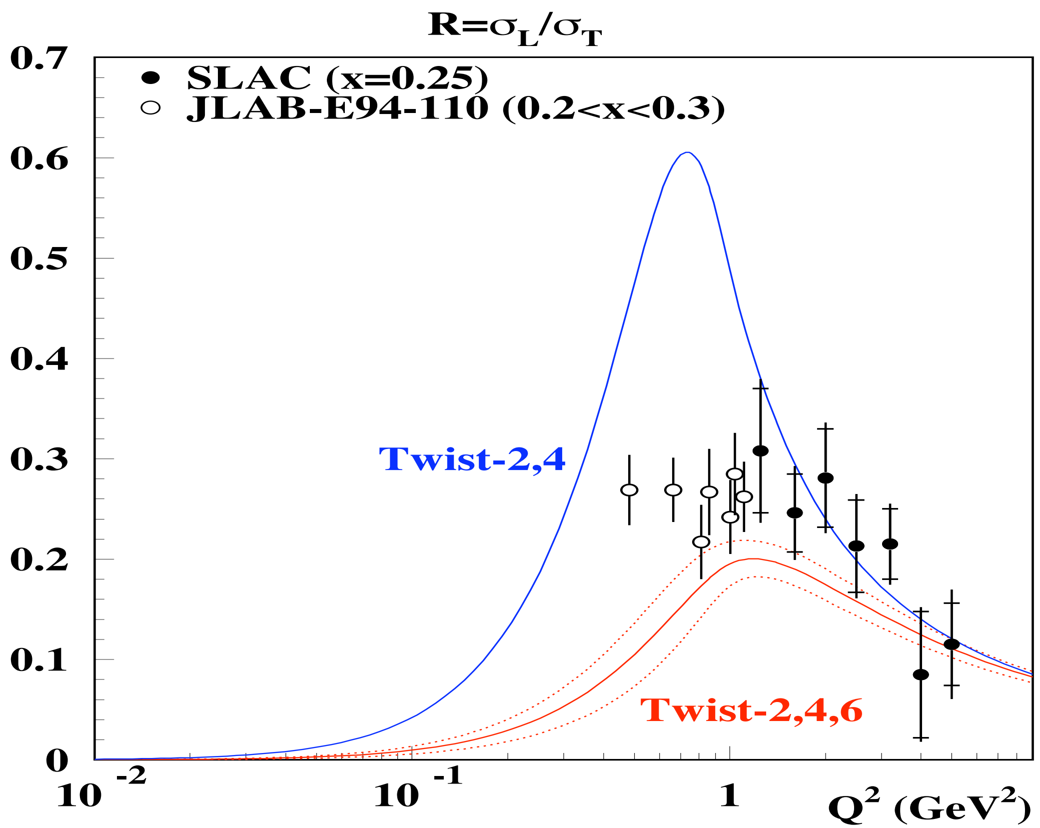
→ large-x resummation reduces C_{HT}

→ lower-W data require negative $1/Q^4$ term

Liuti, Ent, Keppel, Niculescu,
Phys. Rev. Lett. 89 (2002) 162001

Phenomenological higher twists

- extrapolation to low Q^2
  (Alekhin, Kulagin, Petti 2005)



large twist 6!
convergence?

NB: $R^{\nu} \not\to 0 \quad as \quad Q^2 \to 0$

5.
DIS at low Q^2:
target mass corrections

Operator Product Expansion

Georgi, Politzer (1976)

$$\int d^4x \; e^{iq\cdot x} \langle N|T(J^\mu(x)J^\nu(0))|N\rangle$$

$$= \sum_k \left(-g^{\mu\nu}q^{\mu_1}q^{\mu_2} + g^{\mu\mu_1}q^\nu q^{\mu_2} + q^\mu q^{\mu_1}g^{\nu\mu_2} + g^{\mu\mu_1}g^{\nu\mu_2}Q^2\right)$$

$$\times q^{\mu_3}\cdots q^{\mu_{2k}} \frac{2^{2k}}{Q^{4k}} \underbrace{A_{2k}\Pi_{\mu_1\cdots\mu_{2k}}}_{\langle N|\mathcal{O}_{\mu_1\cdots\mu_{2k}}|N\rangle}$$

$$\Pi_{\mu_1\cdots\mu_{2k}} = p_{\mu_1}\cdots p_{\mu_{2k}} - (g_{\mu_i\mu_j} \text{ terms})$$

$$= \sum_{j=0}^{k} (-1)^j \frac{(2k-j)!}{2^j(2k)^j} g\cdots g\; p\cdots p$$

traceless, symmetric rank-$2k$ tensor

n-th moment of F_2 structure function

$$M_2^n(Q^2) = \int dx\ x^{n-2}\ F_2(x, Q^2)$$

$$= \sum_{j=0}^{\infty} \left(\frac{M^2}{Q^2}\right)^j \frac{(n+j)!}{j!(n-2)!} \frac{A_{n+2j}}{(n+2j)(n+2j-1)}$$

$$\Rightarrow \quad A_n = \int_0^1 dy\ y^n\ F(y)$$

"quark distribution function"

$$F(y) = \frac{F_2(y)}{y^2}$$

inverse Mellin transform (+ tedious manipulations)

$$F_2^{\rm GP}(x,Q^2) = \frac{x^2}{r^3} F(\xi) \; + \; 6 \frac{M^2}{Q^2} \frac{x^3}{r^4} \int_\xi^1 d\xi' \; F(\xi')$$

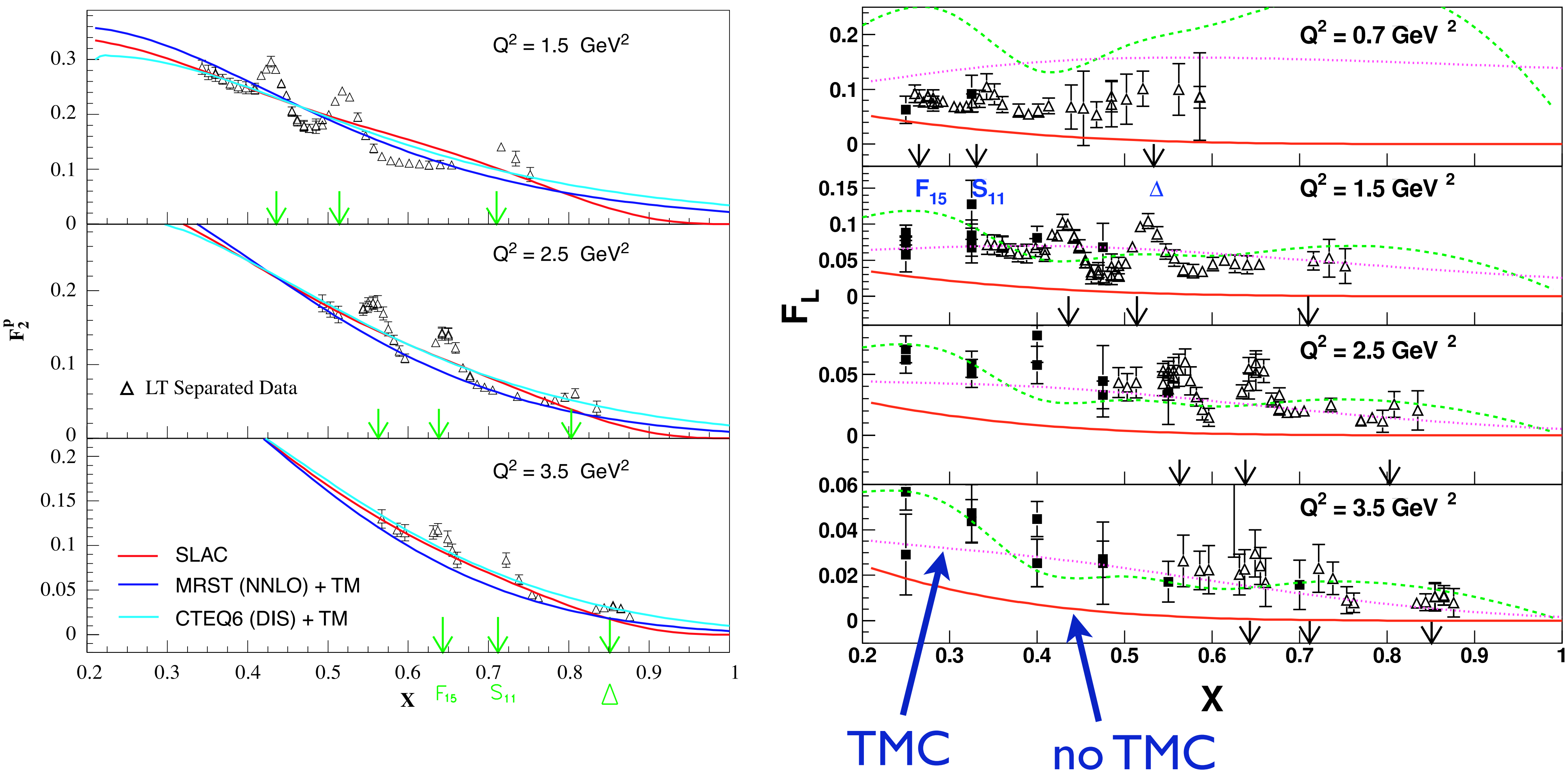
$$+ \; 12 \frac{M^4}{Q^4} \frac{x^4}{r^5} \int_\xi^1 d\xi' \; \int_{\xi'}^1 d\xi'' \; F(\xi'')$$

$$\xi = \frac{2x}{1+r} \qquad\qquad r = \sqrt{1+4x^2M^2/Q^2}$$

... similarly for other structure functions F_1, F_L

duality in F_2 and F_L structure functions

Christy et al. (2005)



➡ TMCs significant at large x^2/Q^2, especially for F_L

Threshold problem

- if $F(y) \sim (1-y)^{\beta}$ at large y

  then since $\xi_0 \equiv \xi(x=1) < 1$

  ⇨ $F(\xi_0) > 0$

  ⇨ $F_i^{\rm TMC}(x=1, Q^2) > 0$

  is this physical?

  ⇨ problem with GP formulation?

Possible solutions

- Johnson/Tung - modified threshold factor

Nachtmann moment

$$\mu_2^n(Q^2) = \int_0^1 dx \frac{\xi^{n+1}}{x^3} \left(\frac{3 + 3(n+1)r + n(n+2)r^2}{(n+2)(n+3)} \right) F_2(x, Q^2)$$

→ supposed to remove TMCs explicitly from SF moment

Possible solutions

- Johnson/Tung - modified threshold factor

Nachtmann moment

$$\mu_2^n(Q^2) = \int_0^1 dx \frac{\xi^{n+1}}{x^3} \left(\frac{3 + 3(n+1)r + n(n+2)r^2}{(n+2)(n+3)} \right) F_2(x, Q^2)$$

➡ n fixed, $Q^2 \to \infty$

$$\mu_2^n(Q^2) \to (\ln Q^2/\Lambda^2)^{-\lambda_n} \; A_n$$

➡ $n \to \infty$, Q^2 fixed

$$\mu_2^n(Q^2) \to \xi_0^n(Q^2) \; \widetilde{\mu}_2^n(Q^2)$$

"regularized" amplitudes
(threshold-independent)

Possible solutions

- Johnson/Tung - modified threshold factor

Nachtmann moment

$$\mu_2^n(Q^2) = \int_0^1 dx \frac{\xi^{n+1}}{x^3} \left(\frac{3 + 3(n+1)r + n(n+2)r^2}{(n+2)(n+3)} \right) F_2(x, Q^2)$$

ansatz $\mu_2^n(Q^2) = \xi_0^n(Q^2)\ (\ln Q^2/\Lambda^2)^{-\lambda_n}\ A_n$

➡ consistent with asymptotic pQCD behavior

➡ not unique!

Bitar, Johnson, Tung
PLB 83B (1979) 114

Possible solutions

- Johnson/Tung - modified threshold factor

moreover, if identify A_n with $M_2^n = \int_0^1 dx\ x^{n-2}\ F_2(x)$

$$\mu_2^n(Q^2) = \xi_0^n(Q^2)\ M_2^n(Q^2)$$

$\Longrightarrow$ $$M_2^n(Q^2) = \mu_2^n(Q^2) + \frac{nM^2}{Q^2} M_2^n + \cdots$$

cf. exact expression

$$M_2^n(Q^2) = \mu_2^n(Q^2) + \frac{n(n-1)}{n+2} \frac{M^2}{Q^2} M_2^{n+2} + \cdots$$

$\Longrightarrow$ inconsistency at low Q^2 ?

Alternative solution

- work with ξ_0 dependent PDFs

→ n-th moment A_n of distribution function

$$A_n = \int_0^{\xi_{\max}} d\xi \; \xi^n \; F(\xi)$$

→ what is $\xi_{\max}$?

- GP use $\xi_{\max} = 1, \;\; \xi_0 < \xi < 1$ unphysical
- strictly, should use $\xi_{\max} = \xi_0$

Steffens, WM
PRC 73 (2006) 055202

Alternative solution

- what is effect on phenomenology?

  $\longrightarrow$ try several "toy distributions"

standard TMC ("sTMC")

$$q(\xi) = \mathcal{N}\, \xi^{-1/2}\, (1-\xi)^3\,, \qquad \xi_{\max} = 1$$

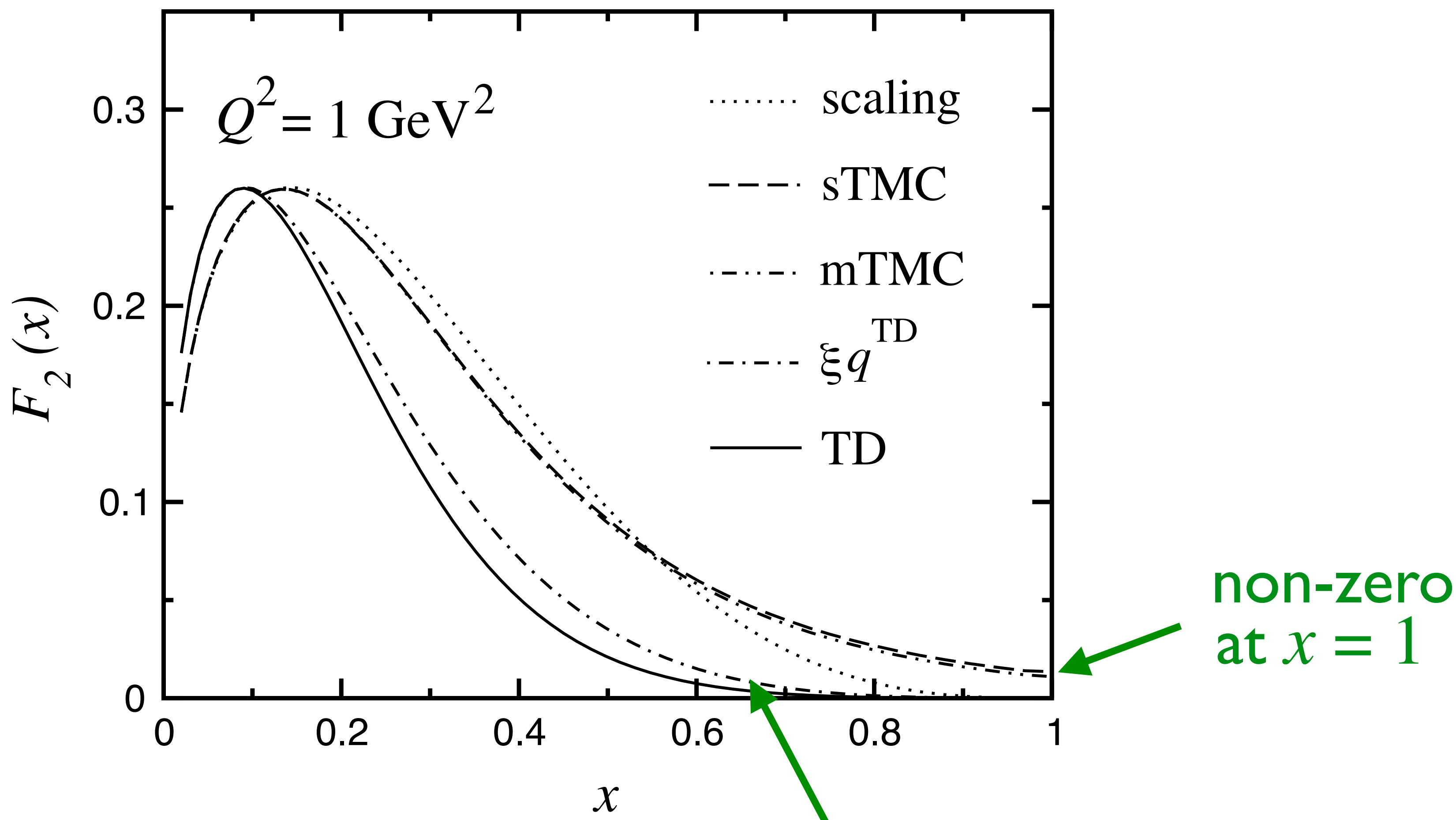
modified TMC ("mTMC")

$$q(\xi) = \mathcal{N}\, \xi^{-1/2}\, (1-\xi)^3\, \Theta(\xi - \xi_0), \qquad \xi_{\max} = \xi_0$$

threshold dependent ("TD")

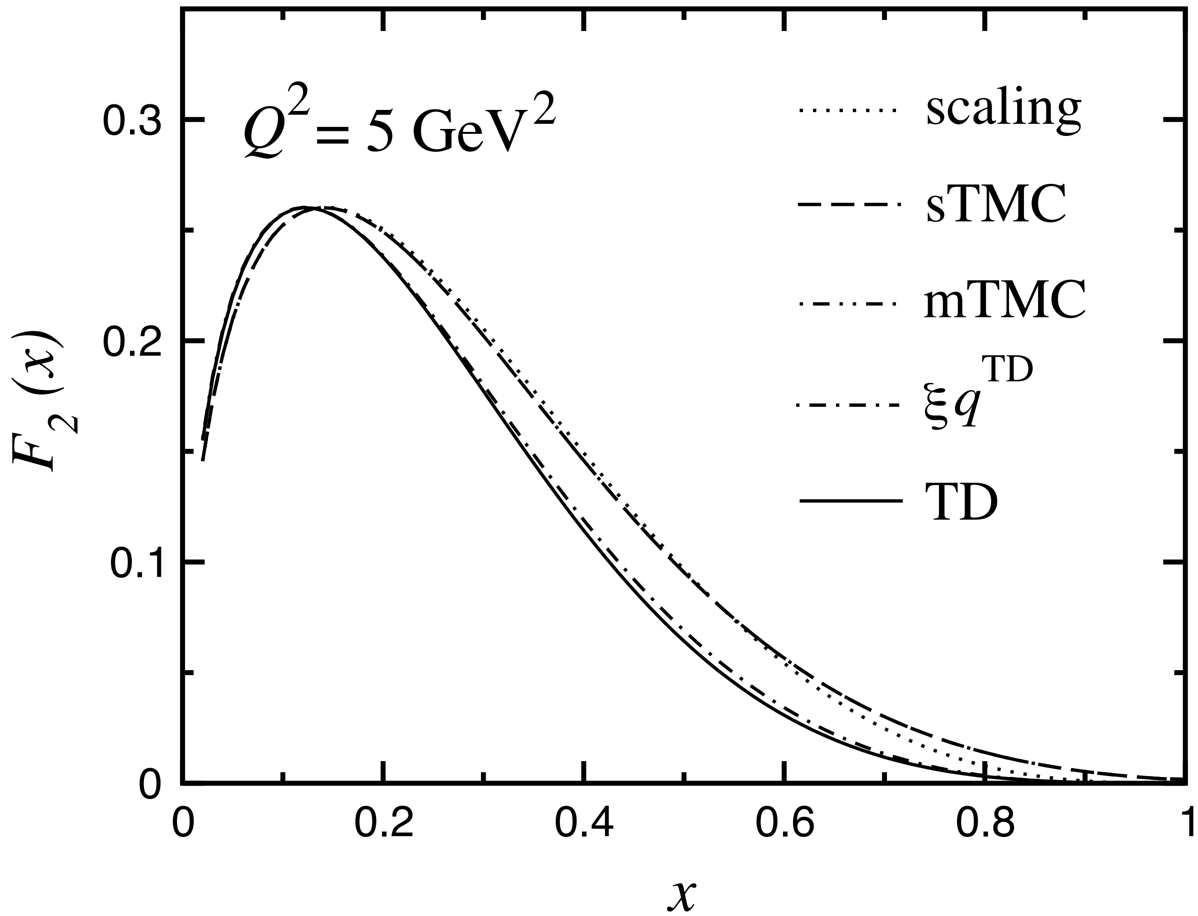
$$q^{\mathrm{TD}}(\xi) = \mathcal{N}\, \xi^{-1/2}\, (\xi_0 - \xi)^3\,, \qquad \xi_{\max} = \xi_0$$

TMCs in F_2



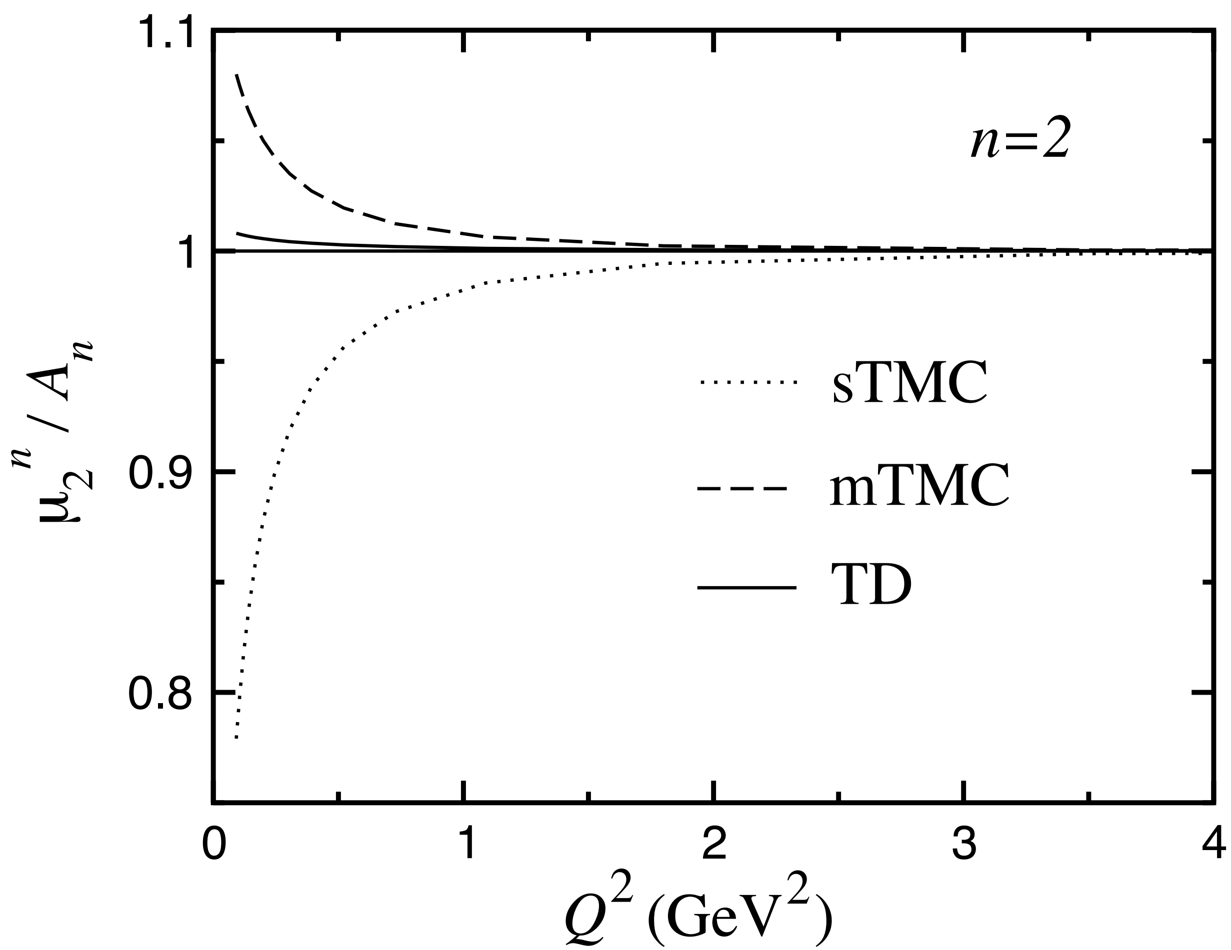
→ correct threshold behavior for "TD" correction

TMCs in F_2



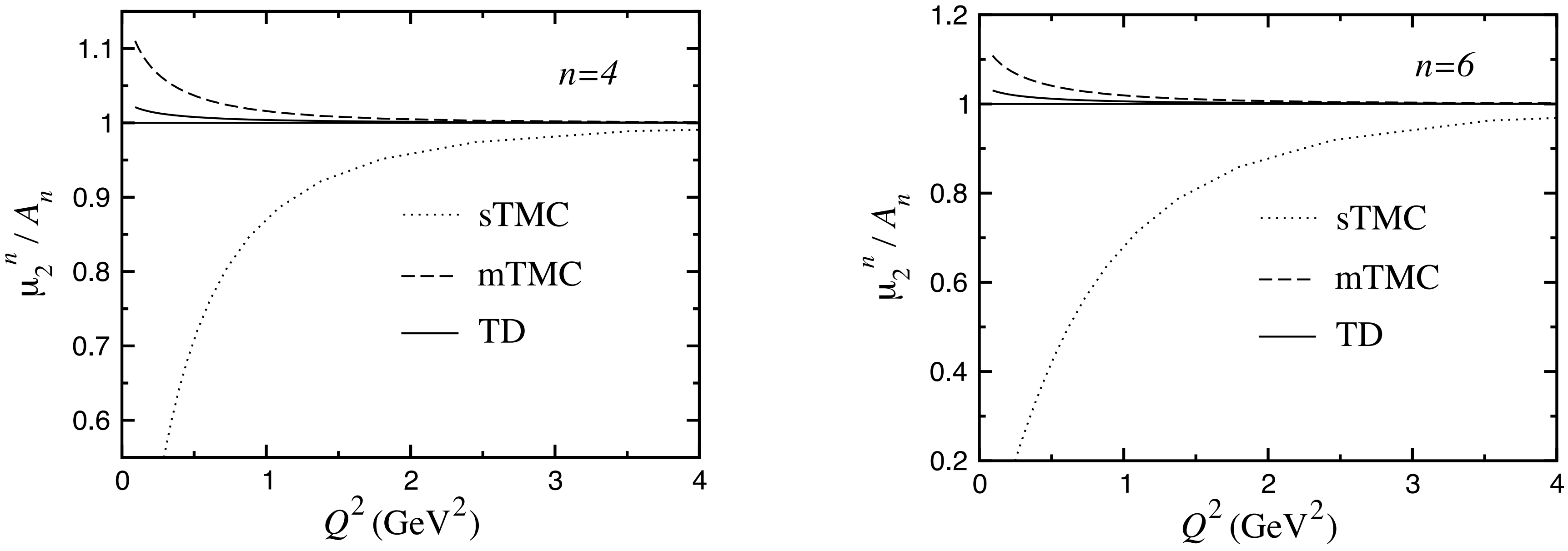
→ effect small at higher Q^2

Nachtmann F_2 moments



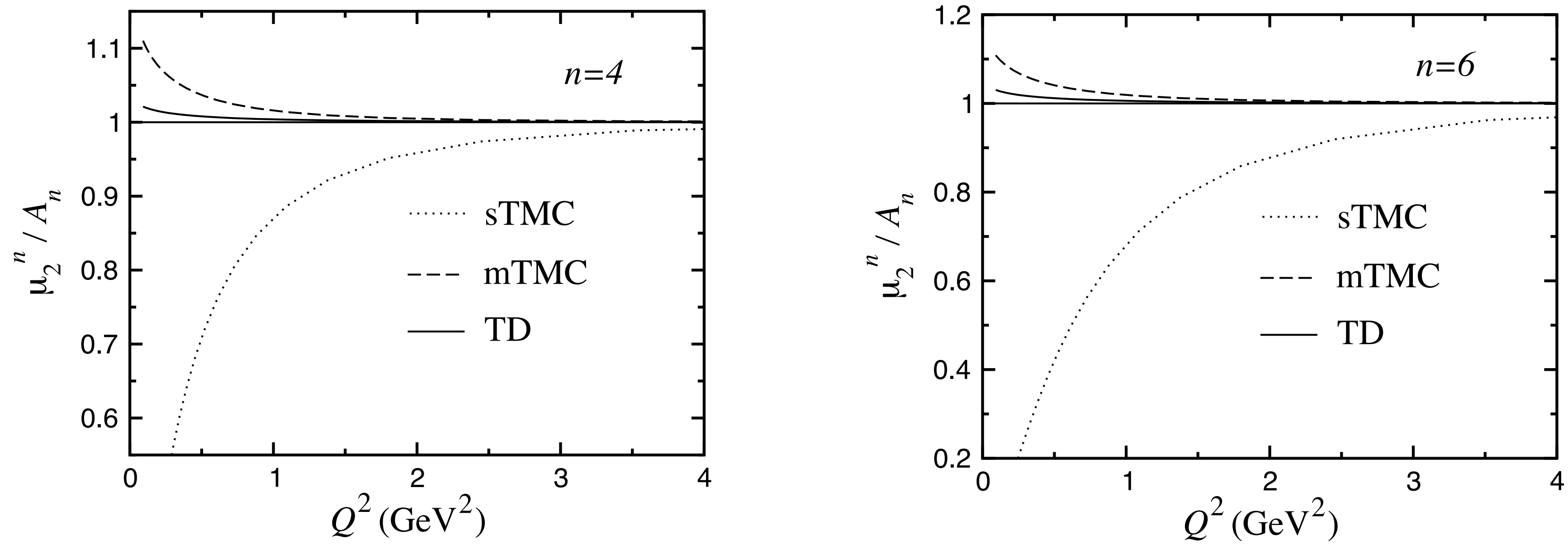
→ moment of structure function agrees with moment of PDF to 1% down to very low Q^2

Nachtmann F_2 moments



$\longrightarrow$ higher moments show much weaker Q^2 dependence than sTMC & mTMC prescriptions

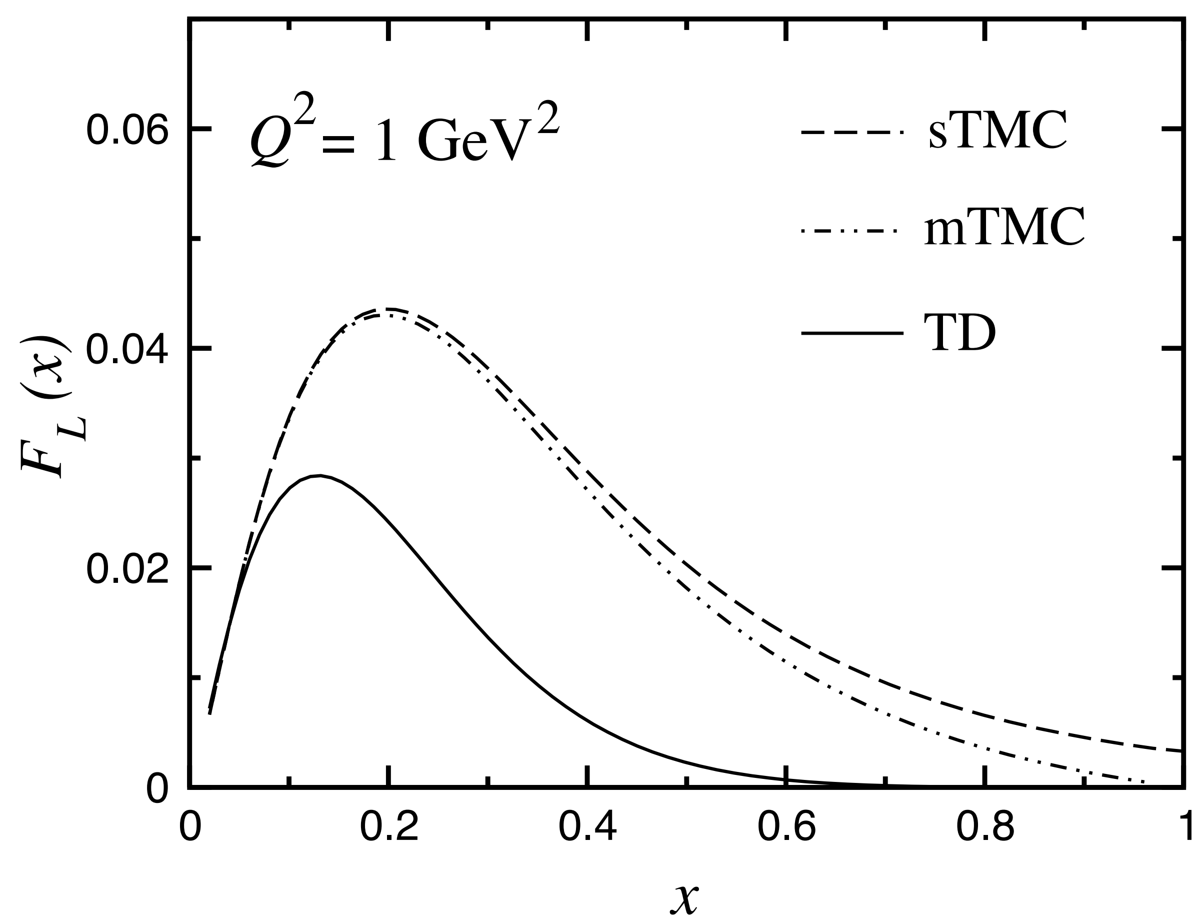
Nachtmann F_2 moments



$$\rightarrow \quad \frac{\mu_2^n(\text{finite } Q^2)}{A_n(\text{finite } Q^2)} = \frac{\mu_2^n(Q^2 \to \infty)}{A_n(Q^2 \to \infty)}$$

→ extract PDFs from structure function data at lower Q^2

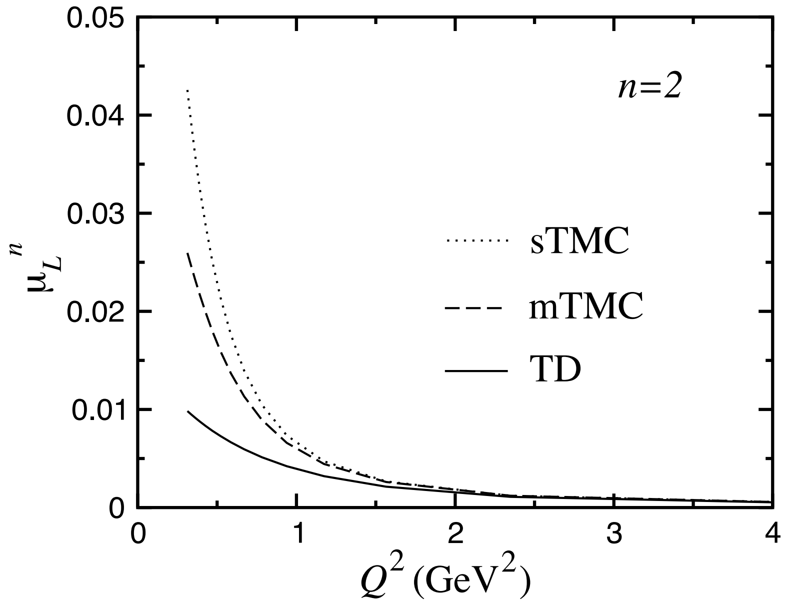
TMCs in F_L

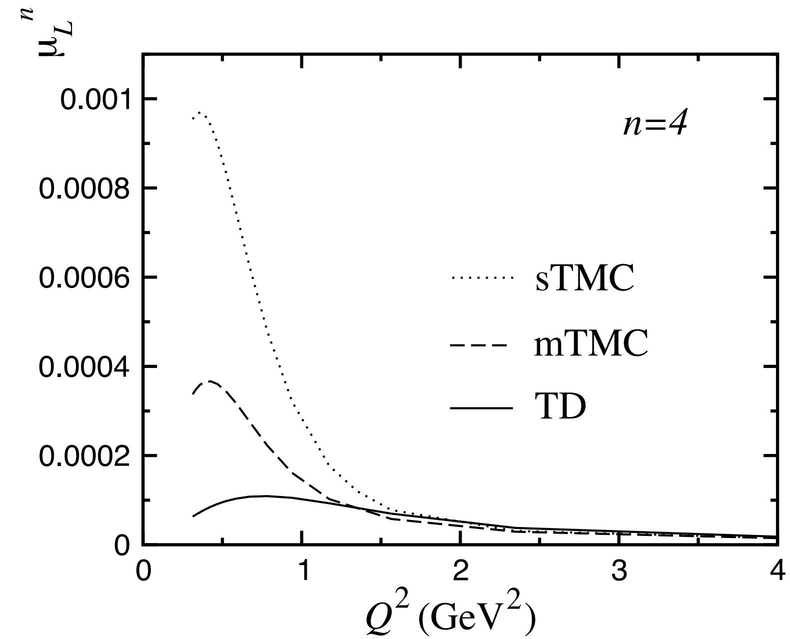


→ correct threshold behavior for "TD" correction

→ reduced TMC effect *cf.* sTMC and mTMC

Nachtmann F_L moments





→ weaker Q^2 dependence for TD prescription

Summary

- Remarkable confirmation of quark-hadron duality in structure functions
  → higher twists "small" down to low Q^2 $(\sim 1\ \text{GeV}^2)$

- Use duality violations to extract higher twist matrix elements
  → color polarizabilities

- OPE "organizes" duality violations in terms of higher twists
  but need quark models to understand origin of resonance cancellations

- Intriguing low-Q^2 behavior
  → importance of TMC's at large x^2M^2/Q^2

Summary

- References: WM, Ent, Keppel, Phys. Rept. 406 (2005) 127

- Fall 2006 graduate course: *Hadronic Physics: from Quarks to Nuclei*

  Monday, Wednesday 11:00 a.m. - 12:30 p.m., from Sep. 6
  (CEBAF Center Auditorium)

The End