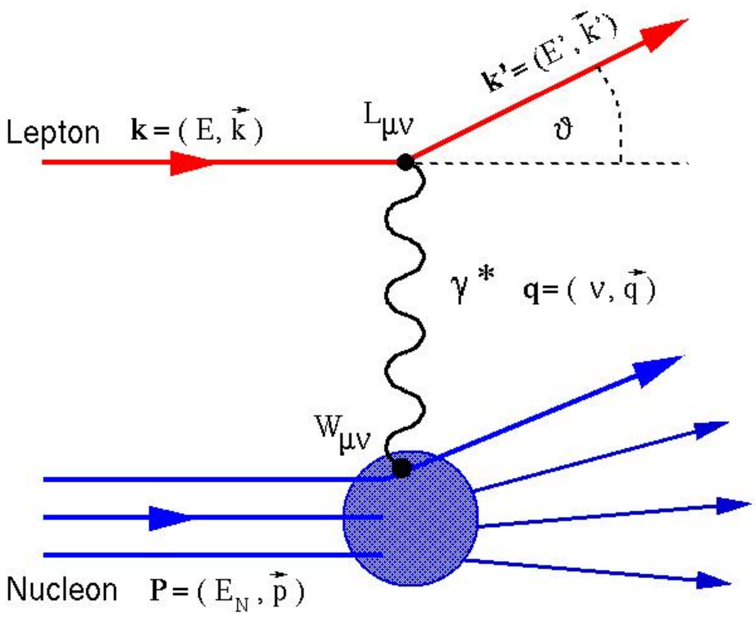
Some Electron Scattering Basics

- kinematics:



$L_{\mu\nu}$: lepton tensor

$W_{\mu\nu}$: hadron tensor

- Four-momentum transfer:

$$q^2 = (E - E')^2 - (\vec{k} - \vec{k}') \cdot (\vec{k} - \vec{k}') =$$

$$= m_e^2 + m_{e'}^2 - 2(EE' - |\vec{k}||\vec{k}'|\cos\theta) =$$

$$\approx \boxed{-4EE'\sin^2\tfrac{\theta}{2} \equiv -Q^2}$$

- Mott Cross Section ($c=1$):

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \frac{4\alpha^2 E'^2}{Q^4}\cos^2\frac{\theta}{2} \cdot \frac{E'}{E}$$

$$= \frac{4\alpha^2 E'^2}{16E^2E'^2\sin^4\frac{\theta}{2}}\cos^2\frac{\theta}{2} \cdot \frac{1}{1+\frac{E}{M}(1-\cos\theta)}$$

$$= \frac{\alpha^2\cos^2\frac{\theta}{2}}{4E^2\sin^4\frac{\theta}{2}} \cdot \frac{1}{1+\frac{E}{M}(2\sin^2\frac{\theta}{2})}$$

Electron scattering of a spinless point particle

Excited states of the nucleon

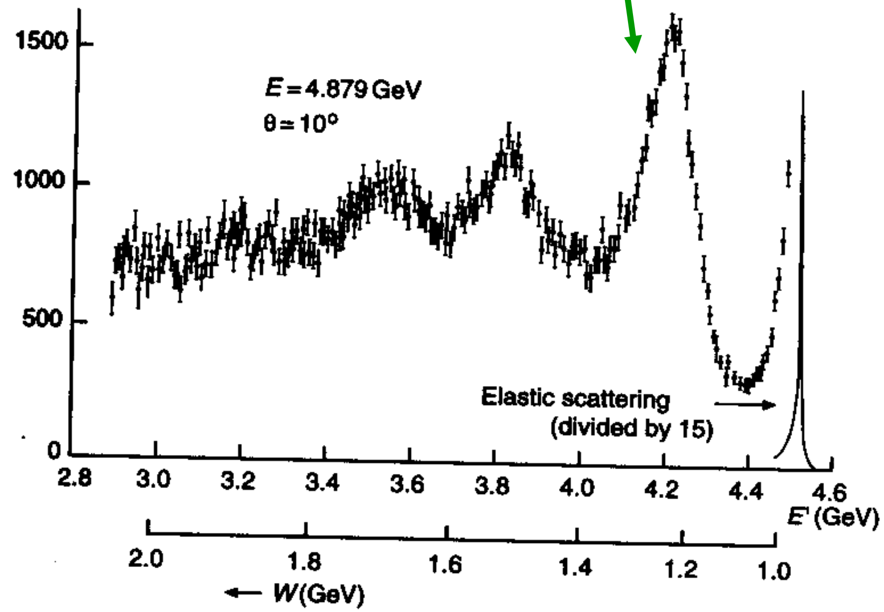
- Scatter 4.9 GeV electrons from a hydrogen target:
- Why use invariant energy?
  - A: $E_x \sim 10$ MeV$<< m_A$: small E_{rec}
  - p: $E_x \sim 500$ MeV$\sim m_p$: large E_{rec}
- Evaluate invariant energy of virtual-photon proton system

$$W^2 = (P_p + q)^2 = P^2 + 2Pq + q^2$$

- In the lab-frame: $P = (m_p, 0) \rightarrow$

$$\boxed{W^2 = m_p^2 + 2m_p\nu - Q^2}$$

$\Delta(1232)$



- Observation excited states:

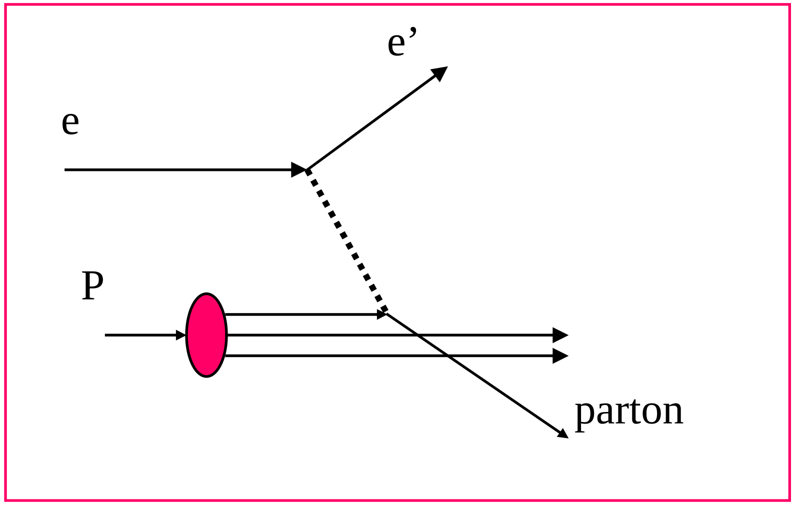
Nucleons are composite

→ What do we see in the data for $W > 2$ GeV ?

The Quark-Parton Model

- Assumptions:
  - Neglect masses and p_T's
  - Proton constituent = Parton
  - Impulse Approximation:

Quasi-elastic scattering off partons



- Lets assume: $p_{quark} = xP_{proton}$

$$(xP+q)^2 = p'^2_{quark} = m^2_{quark} \approx 0$$

  - Since $xP^2 \leq M^2 << Q^2$ it follows:

$$2xP \cdot q + q^2 \approx 0 \Rightarrow x = \frac{Q^2}{2Pq} = \frac{Q^2}{2M\nu}$$

Definition Bjorken scaling variable

- Check limiting case:

$$W^2 = M_p^2 + 2M_p\nu - Q^2 \xrightarrow{x \to 1} M_p^2$$

- Therefore:

  $x = 1$: elastic scattering

  and $0 < x < 1$

Structure Functions F_1, F_2

- Introduce dimensionless structure functions:

$$F_1 = MW_1 \text{ and } F_2 = \nu W_2 \Rightarrow \frac{d\sigma}{dE'd\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_M \frac{1}{\nu}\left[F_2(x) + \frac{2\nu}{M} F_1(x)\tan^2(\theta/2)\right]$$

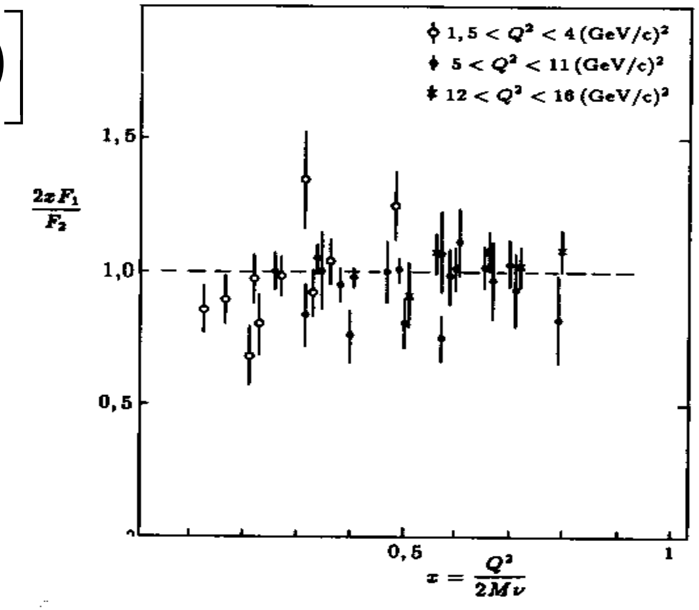
- Rewrite this in terms of : $\tau = Q^2 / 4m^2_{quark}$ (elastic e-q scatt.: $2m_q\nu = Q^2$)

$$\frac{d\sigma}{dE'd\Omega} \Big/ \left(\frac{d\sigma}{d\Omega}\right)_M = \frac{1}{\nu}\left[F_2(x) + 2\frac{Q^2}{4m_q^2}\frac{4m_q^2}{Q^2}\frac{\nu}{M}F_1(x)\tan^2(\theta/2)\right] =$$

$$= \frac{1}{\nu}\left[F_2(x) + 2\tau \cdot 2xF_1(x)\tan^2(\theta/2)\right]$$

$$\boxed{\text{if } F_2(x) = 2xF_1(x)}$$

$$= \frac{1}{\nu}F_2(x)\left[1 + 2\tau\tan^2(\theta/2)\right]$$



- Experimental data for $2xF_1(x) / F_2(x)$

  → *quarks have spin 1/2*

  (if bosons: no spin-flip $\Rightarrow F_1(x) = 0$)

Interpretation of $F_1(x)$ and $F_2(x)$

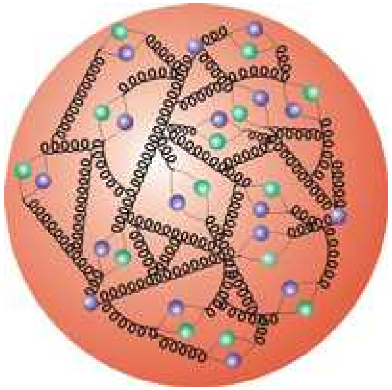
- In the quark-parton model:

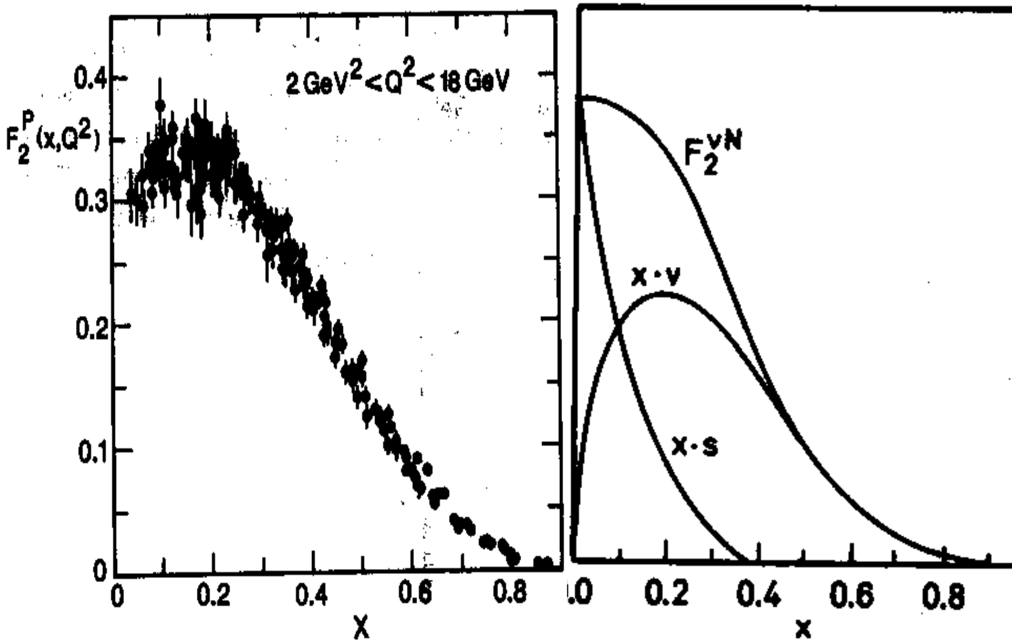
  $$F_1(x) = \sum_f \tfrac{1}{2} z_f^2 [q_f(x) + \bar{q}_f(x)]$$

  [and $F_2 = 2xF_1$ analogously]

  Quark momentum distribution

- Heisenberg requires:
  - Gluon emission: presence of virtual $q\bar{q}$-pairs





- Distinguish
  - Valence quarks (N-prop.)
  - Sea quarks
  - derived from: $F_3^\nu = \sum (q_f(x) - \bar{q}_f(x))$

$$\sigma_{DIS}^{\nu,\bar{\nu}} \propto (1-y)\,F_2^\nu + y^2 x F_1^\nu \pm y(1-\tfrac{y}{2}) x F_3^\nu$$

The quark structure of nucleons

- Quark quantum numbers:
  - Spin: ½ $\Rightarrow S_{p,n}$ = (↑↑↓) = ½
  - Isopin: ½ $\Rightarrow I_{p,n}$ = (↑↑↓) = ½
- Why fractional charges?
  - Extreme baryons: $Z = (-1,+2)$

$$\boxed{-1 \le 3z_q \le +2 \;\Rightarrow\; -\tfrac{1}{3} \le z_q \le +\tfrac{2}{3}}$$

  - Assign: $z_{up} = +\,2/3,\;\; z_{down} = -\,1/3$
- Three families:

$$\begin{pmatrix} u \\ d \end{pmatrix}\begin{pmatrix} c \\ s \end{pmatrix}\begin{pmatrix} t \\ b \end{pmatrix} \qquad \begin{array}{l} \Rightarrow z = +\tfrac{2}{3};\; m_u << m_{c\,(\approx 1.5\,\mathrm{GeV})} << m_t \\ \Rightarrow z = -\tfrac{1}{3};\; m_d << m_{s\,(\approx 0.3\,\mathrm{GeV})} << m_b \end{array}$$

  - $m_{c,b,t} >> m_{u,d,s}$: no role in p,n

- Structure functions:

$$F_2^p = x[\tfrac{1}{9}(d_v^p + d_s^p + \bar{d}_s^p) + \tfrac{4}{9}(u_v^p + u_s^p + \bar{u}_s^p) + \tfrac{1}{9}(s_s + \bar{s}_s)]$$
$$F_2^n = x[\tfrac{1}{9}(d_v^n + d_s^n + \bar{d}_s^n) + \tfrac{4}{9}(u_v^n + u_s^n + \bar{u}_s^n) + \tfrac{1}{9}(s_s + \bar{s}_s)]$$

  - Isospin symmetry:

$$u_v^n = d_v^p,\; d_v^n = u_v^p,\; \bar{u}_s^n = \bar{d}_s^n = \bar{u}_s^p = \bar{d}_s^p$$

  - 'Average' nucleon $F_2(x)$ with $q(x) = q_v(x) + q_s(x)$ etc.

$$F_2^N = \tfrac{1}{2}(F_2^p + F_2^n)$$
$$= \tfrac{5}{18} x \cdot \sum_{u,d} (q(x) + \bar{q}(x)) + \tfrac{1}{9} x \cdot [s_s(x) + \bar{s}_s(x)]$$

- Neutrinos:

$$F_2^\nu = x[(d_v + d_s + \bar{d}_s) + (u_v + u_s + \bar{u}_s) + (s_s + \bar{s}_s)]$$
$$= x[(d + u + s) + (\bar{d}_s + \bar{u}_s + \bar{s}_s)] = \boxed{x \sum_{u,d,s} (q(x) + \bar{q}(x))}$$

Fractional quark charges

- Neglect strange quarks $\Rightarrow$

$$\frac{F_2^{e,N}}{F_2^{\nu,N}} \approx \frac{5}{18}$$

  – Data confirm factor 5/18:

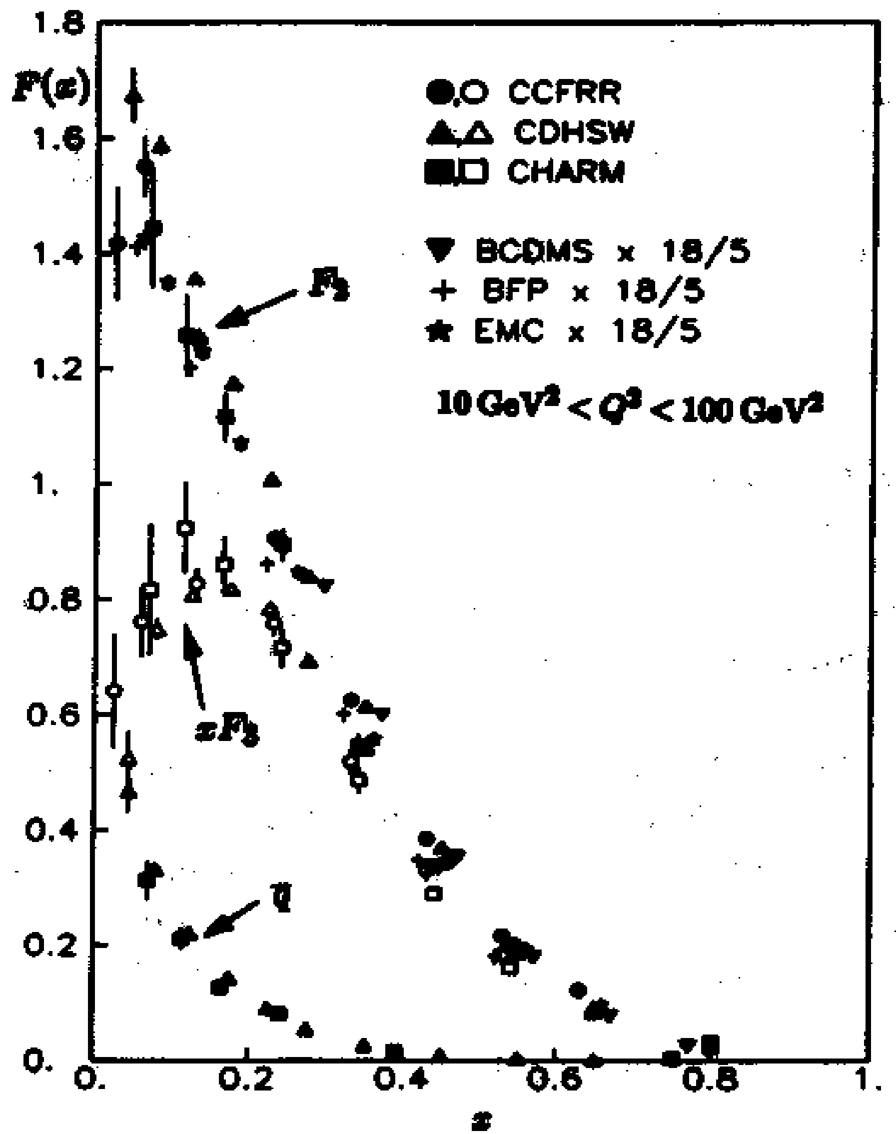
Evidence for fractional charges

- Fraction of proton momentum carried by quarks:

$$\int_0^1 F_2^{\nu,N}(x)dx = \tfrac{18}{5}\int_0^1 F_2^{e,N}(x)dx \approx 0.5$$

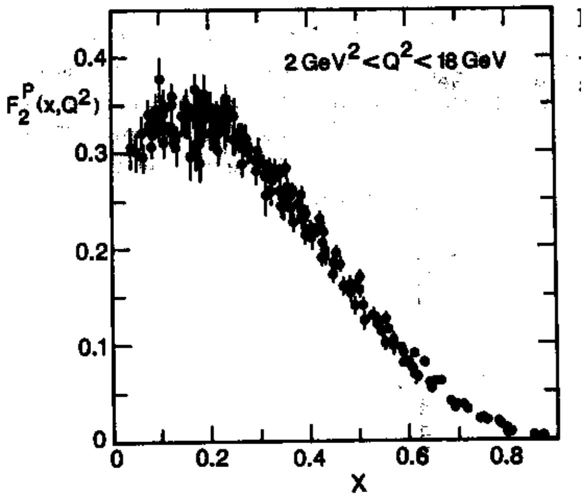
  – 50% of momentum due to non-electro-weak particles:

Evidence for gluons



Modern data

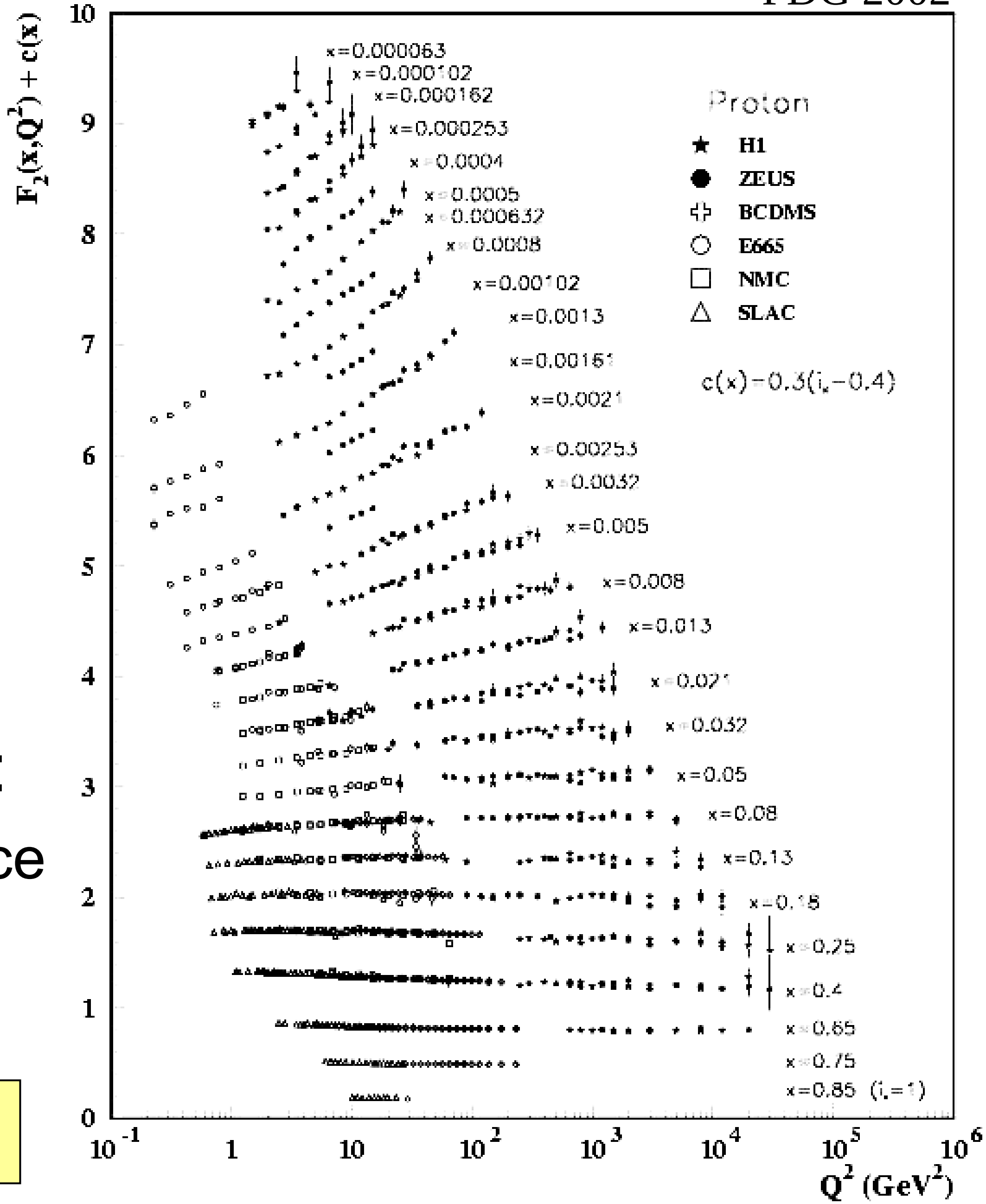
- First data (1980):



- "Scaling violations":
  - weak Q^2 dependence
  - rise at low x
  - what physics??

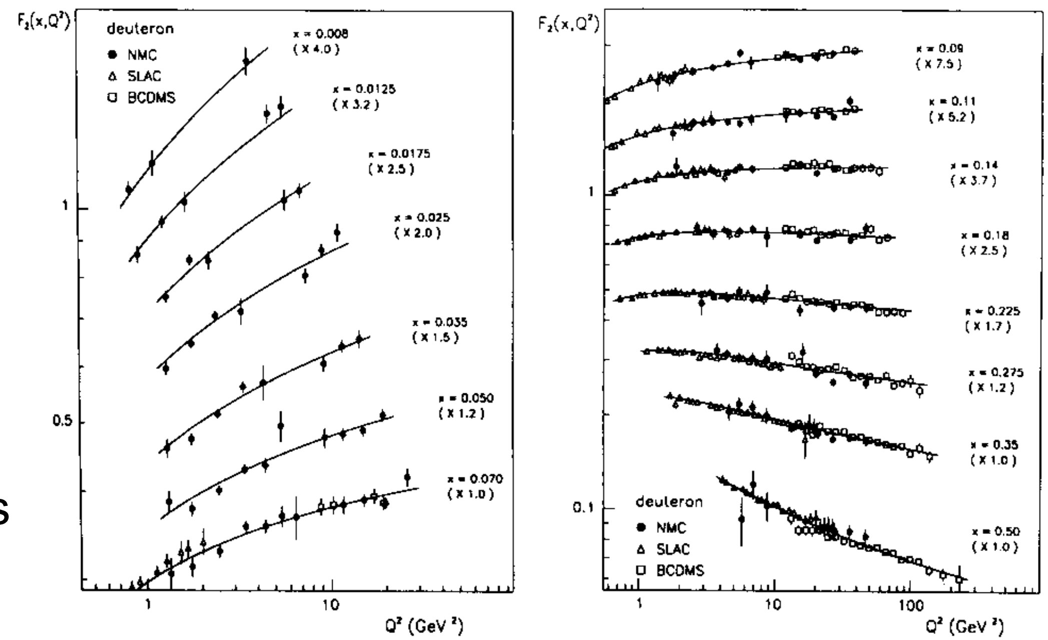
….. QCD

PDG 2002



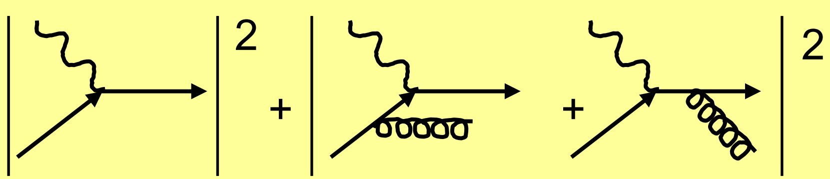
QCD predictions: scaling violations

- Originally: $F_2 = F_2(x)$
  - but also Q^2-dependence
- Why scaling violations?
  - if Q^2 increases:
    - $\Rightarrow$ more resolution ($\sim 1/Q^2$)
    - $\Rightarrow$ more sea quarks +gluons



- QCD improved QPM:

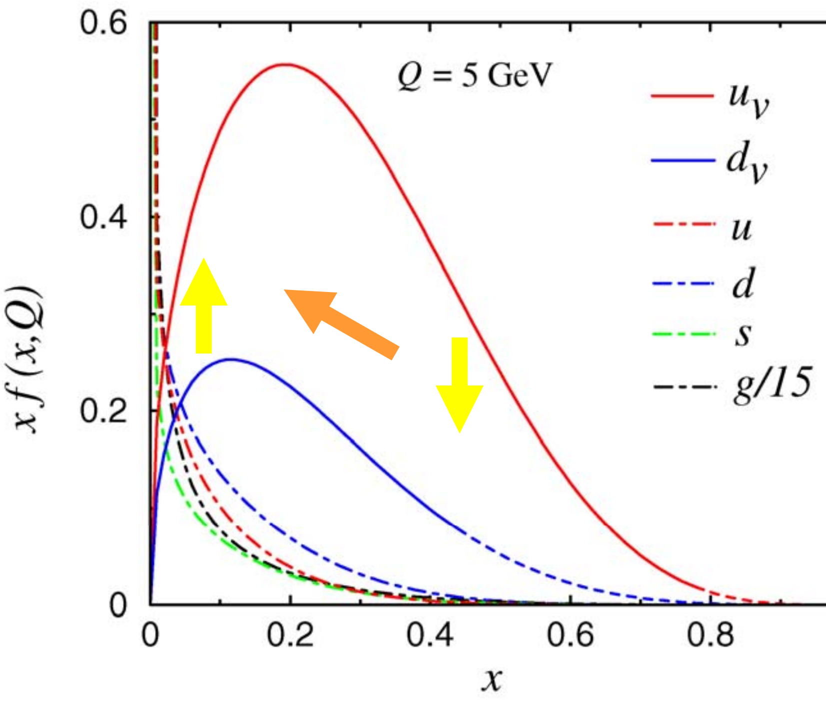
$$\frac{F_2(x,Q^2)}{x} =$$

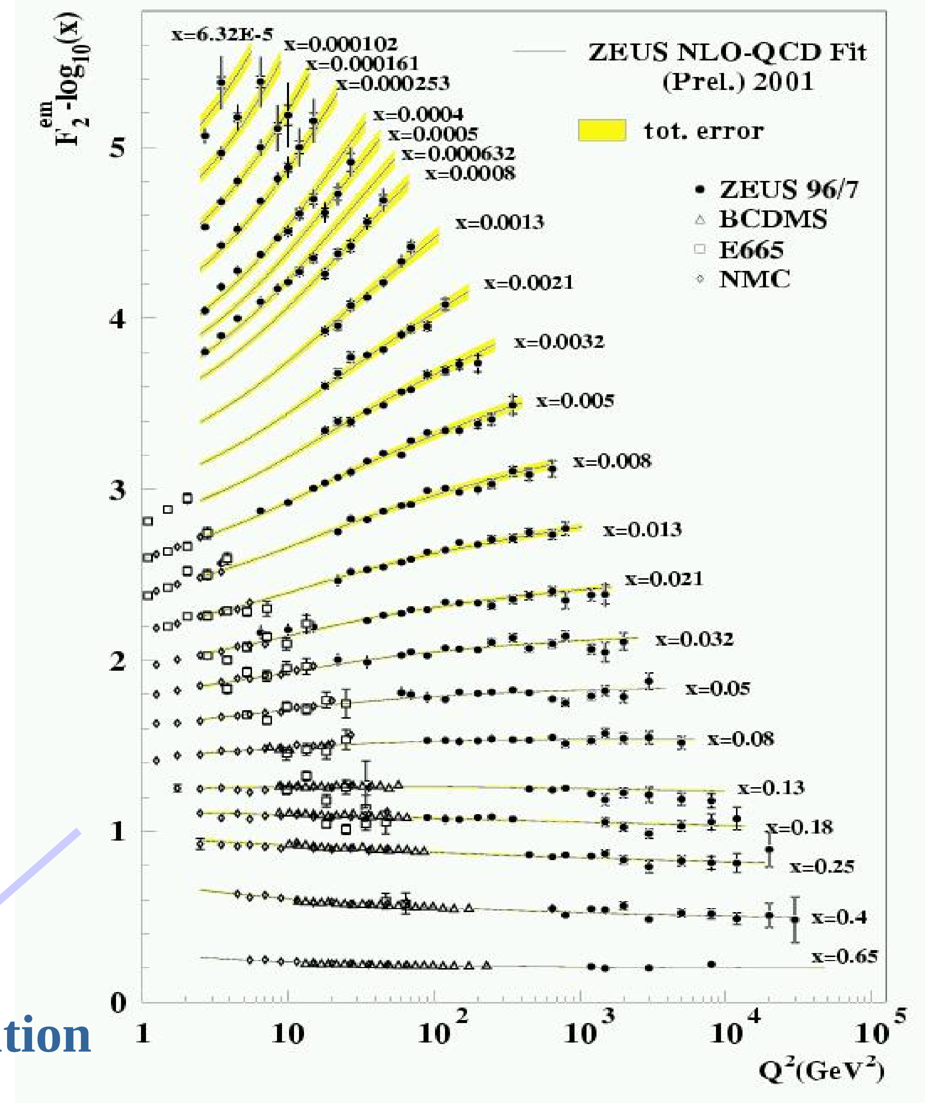


- Officially known as: Altarelli-Parisi Equations (“DGLAP”)

Q^2 Evolution of the Proton F_2 Structure Function

F_2 Structure Function measured over impressive range of x and Q^2





One can extract Parton Distribution Functions f(x,Q) and do the Q^2 evolution

QCD fits of $F_2(x,Q^2)$ data

- Free parameters:
  - coupling constant:

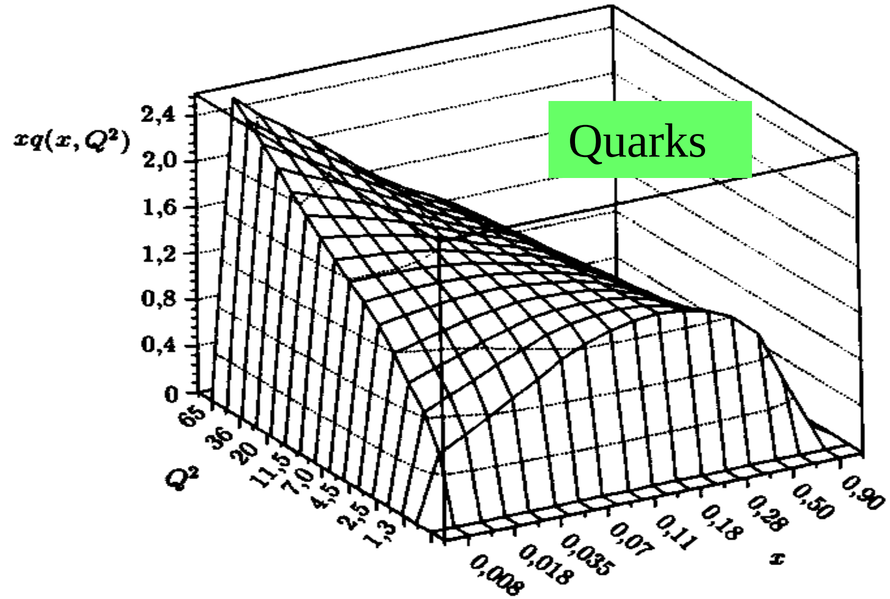
$$\alpha_s = \frac{12\pi}{(33-n_f)\ln(Q^2/\Lambda^2)} \approx 0.16$$

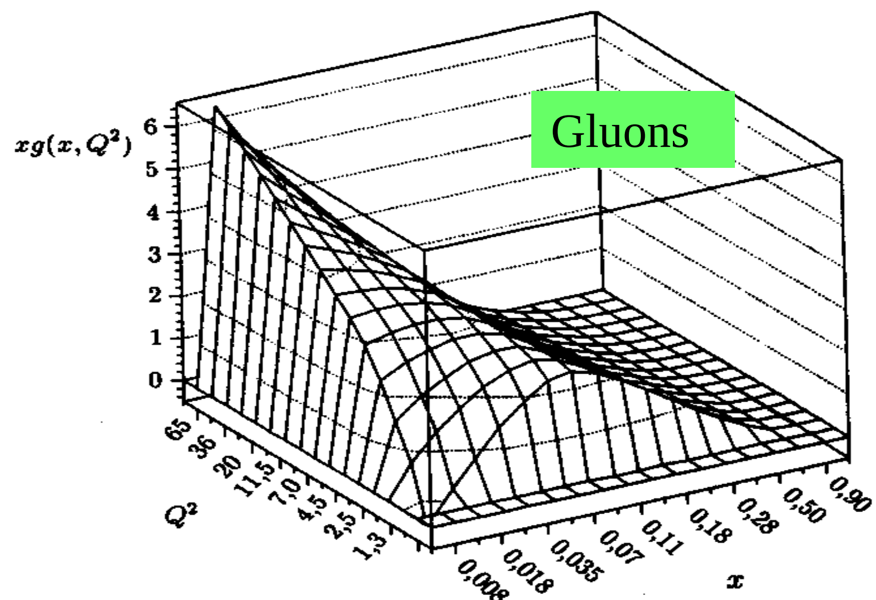
  - quark distribution $q(x,Q^2)$
  - gluon distribution $g(x,Q^2)$
- Successful fit:

Corner stone of QCD

- Nucleon structure:

Unique self-replicating structure





QCD predictions: the running of α_s

- pQCD valid if $\alpha_s << 1$:

  $\Rightarrow \; Q^2 > 1.0\ (\text{GeV/c})^2$

- pQCD calculation:

$$\boxed{\alpha_s(Q^2) = \frac{12\pi}{(33-2n_f)\cdot\ln(Q^2/\Lambda^2)}}$$

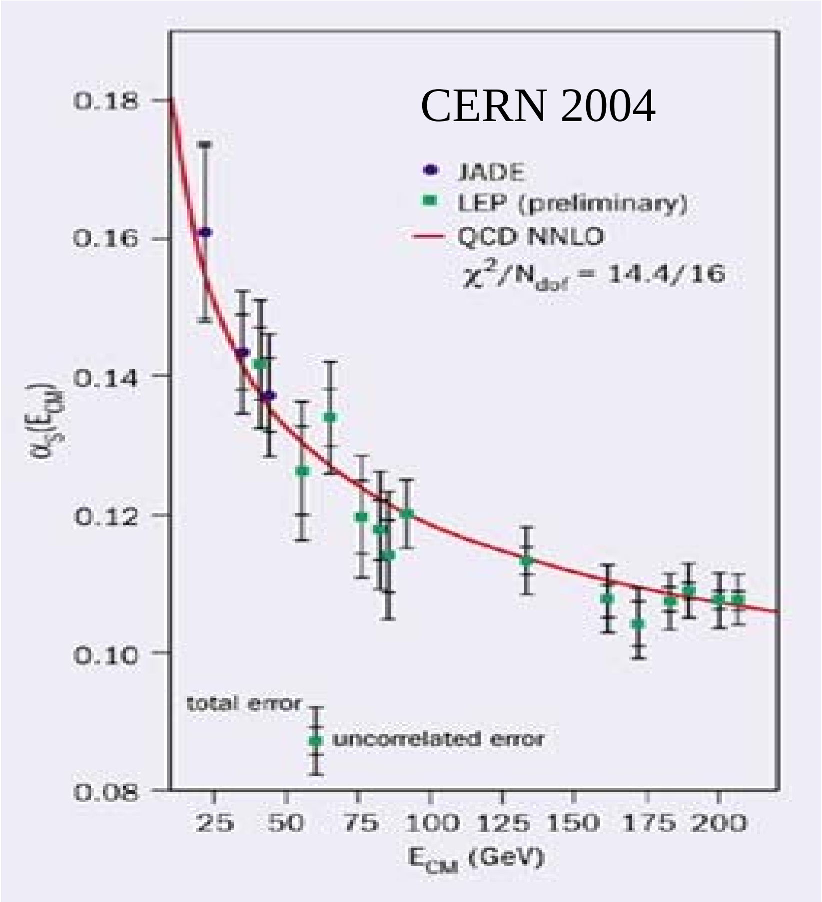
  – with Λ_{exp} = 250 MeV/c:

$$\boxed{Q^2 \to \infty \quad \Rightarrow \quad \alpha_s \to 0}$$

$\Rightarrow$ asymptotic freedom

$$\boxed{Q^2 \to 0 \quad \Rightarrow \quad \alpha_s \to \infty}$$

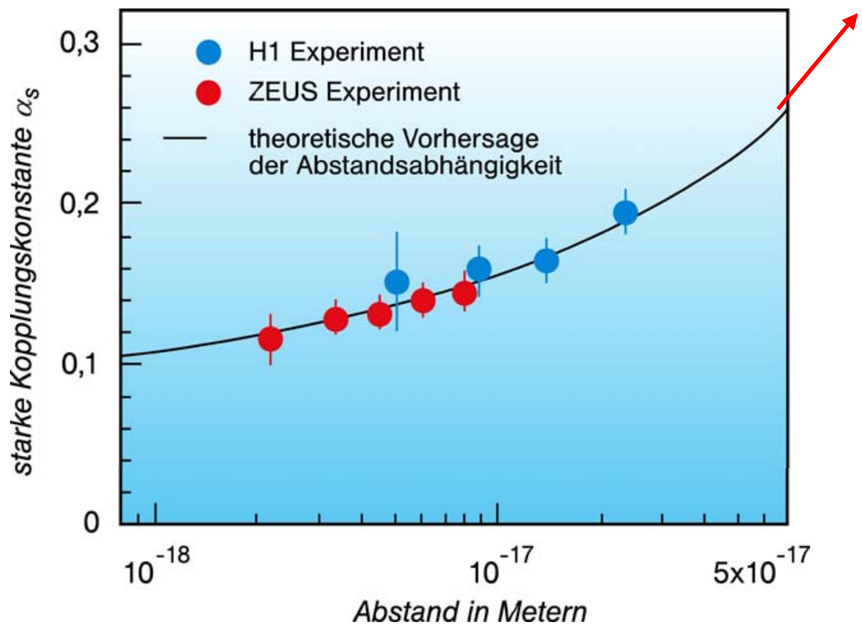
$\Rightarrow$ confinement



Running coupling constant is best quantitative test of QCD.

The problem of QCD

- Extrapolate α_s to the size of the proton, 10^{-15} m:



$$l \to r_{proton} \quad \Rightarrow \quad \alpha_s > 1$$

- If $\alpha_s > 1$ perturbative expansions fail…

$\rightarrow$ *Non-perturbative QCD:*

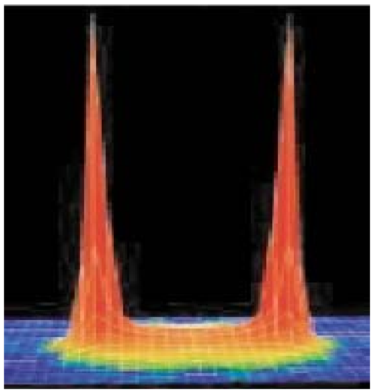
– Proton structure & spin

– Confinement

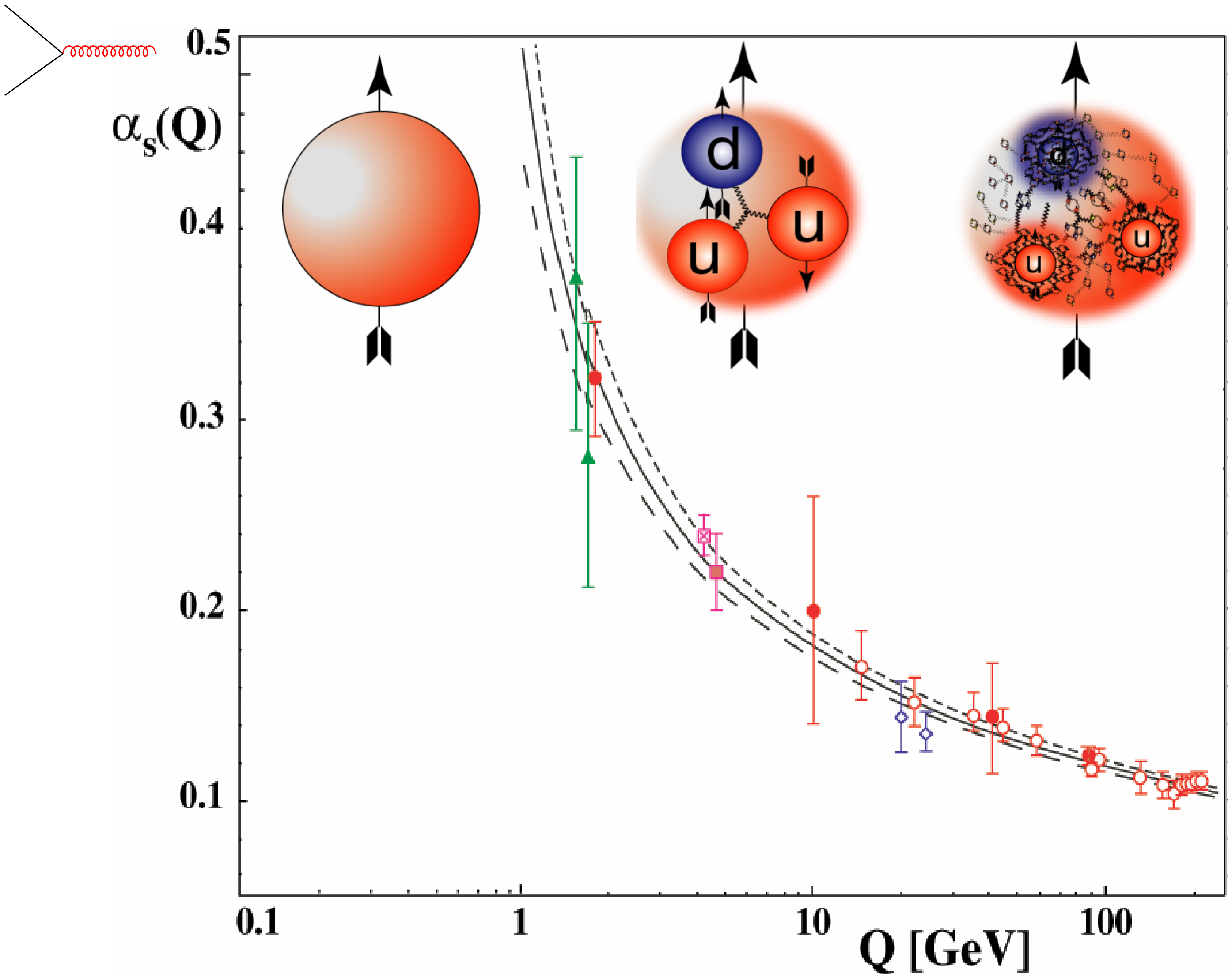
– Nucleon-Nucleon forces

– Hadron spectroscopy…..

Lattice QCD simulations…



QCD and the Parton-Hadron Transition



Hadrons

Nucleons

Quarks and Gluons