Quark-Hadron Duality

Here: Emphasize Duality in Pion Electroproduction

Rolf Ent

Factorization proof →

Core quark-gluon calculation + Universal Function

Examples of Universal Functions

Inclusive Scattering	Parton Distribution Functions
Semi-Inclusive Scattering	Fragmentation Functions
Exclusive Scattering	Generalized Parton Distributions

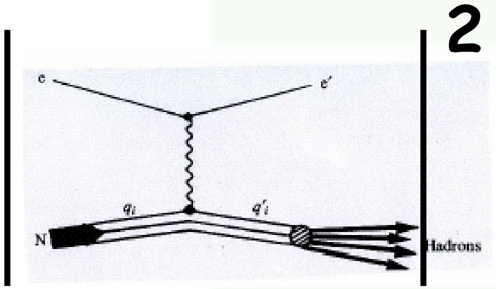
Cross sections scale if they follow the energy-momentum dependence of the core quark-gluon calculation

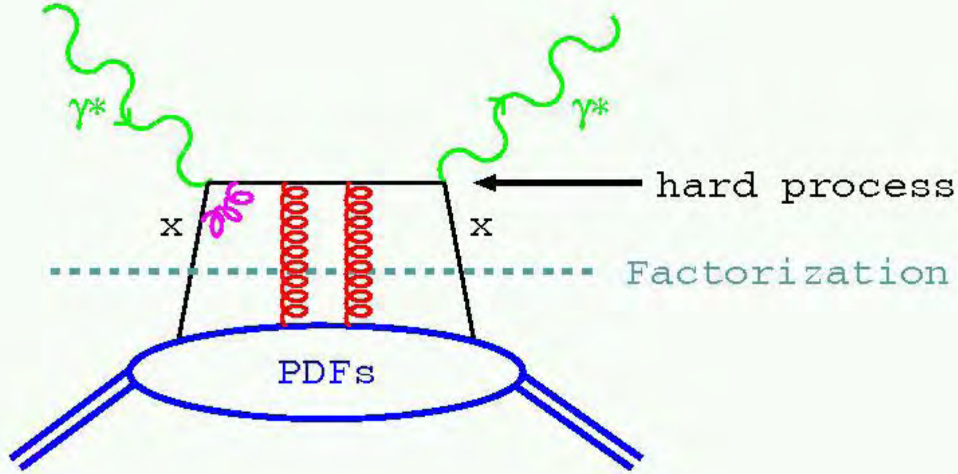
Question:

When do cross sections scale, and how do we experimentally "prove" factorization

Things may not be as they seem… (Quark-Hadron Duality!!!)

Deep Inelastic Scattering





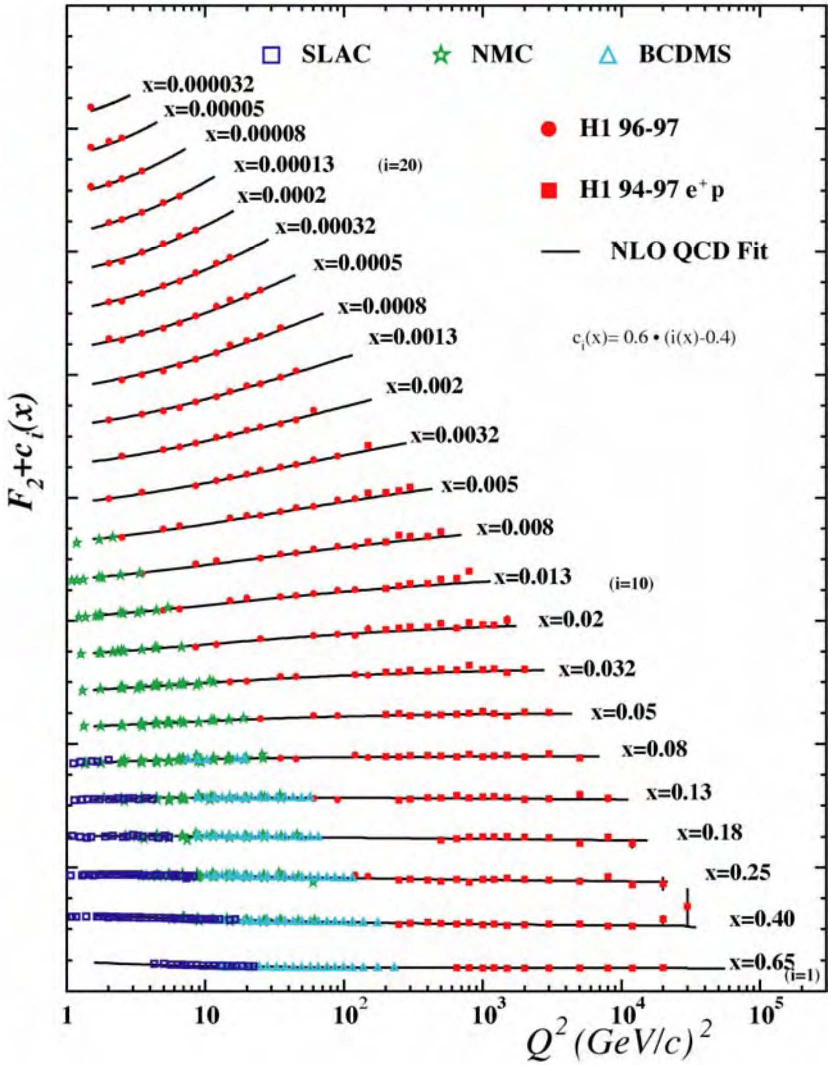
$$\frac{d\sigma}{d\Omega dE'} \sim \sigma_{Mott} \left(\sum_i e_i^2 x \left[q_i(x, Q^2) + \bar{q}_i(x, Q^2) \right] \right)$$

Bjorken Limit: $Q^2 \to \infty, \nu \to \infty$

Empirically DIS region is where **logarithmic scaling** is observed

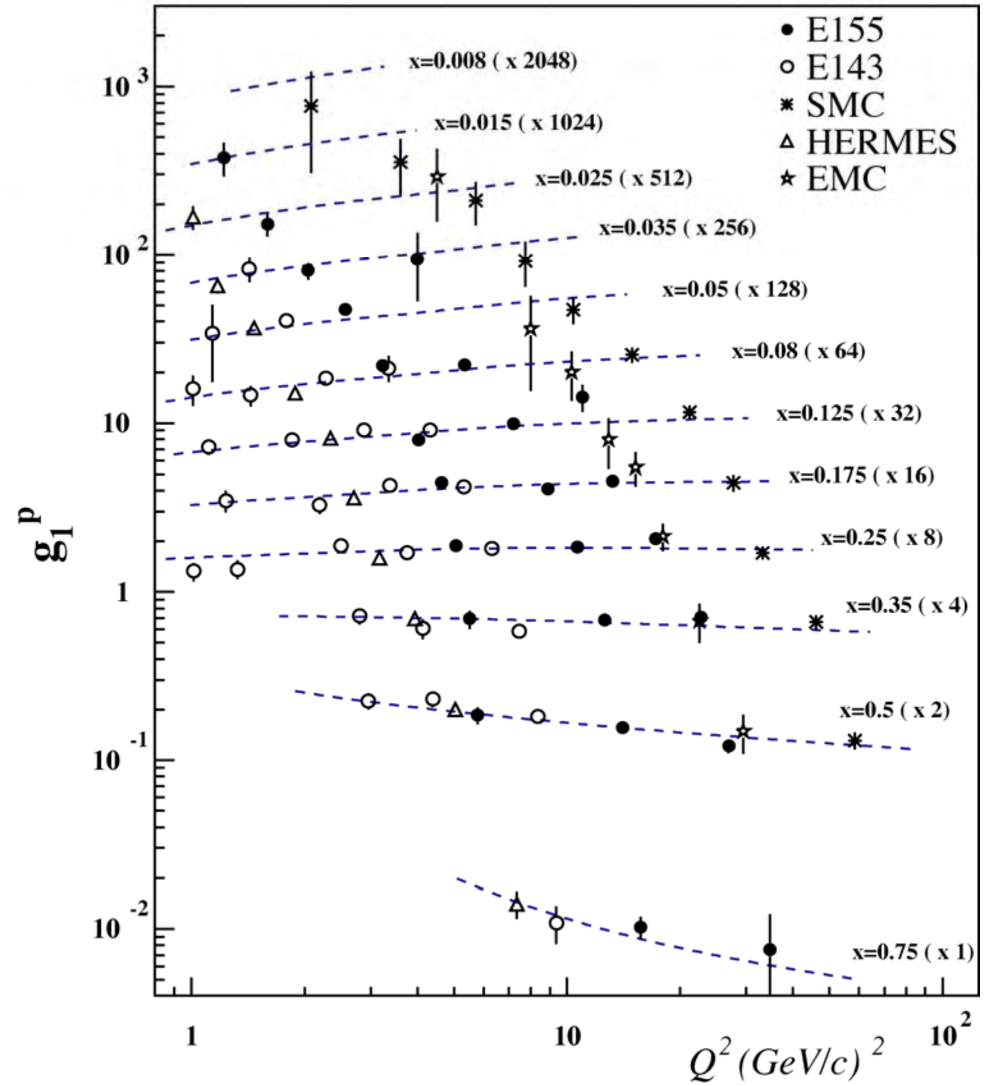
$Q^2 > 1$ (GeV/c)^2, $W^2 > 4$ GeV^2

World data on F_2^p



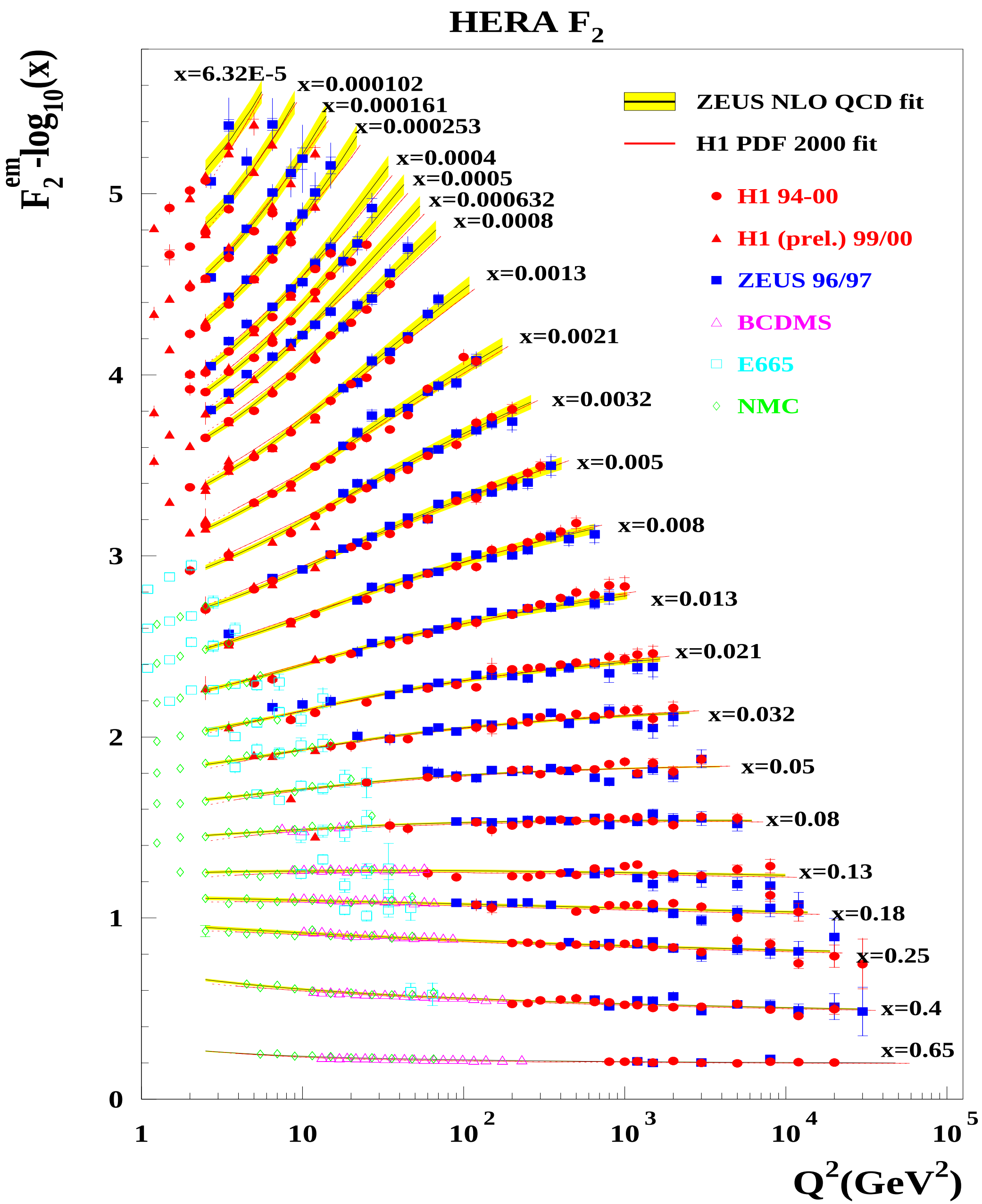
→ 50% of momentum carried by gluons

World data on g_1^p

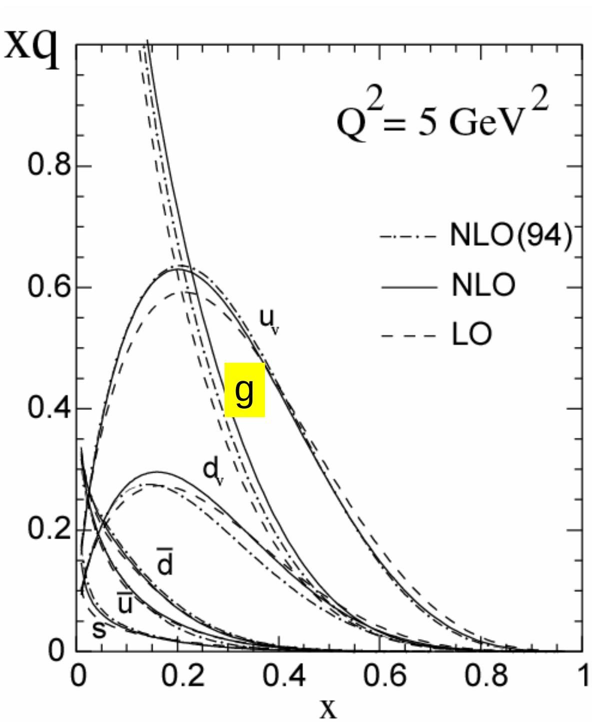


→ 20% of proton spin carried by quark spin

World Data on F_2^p Structure Function

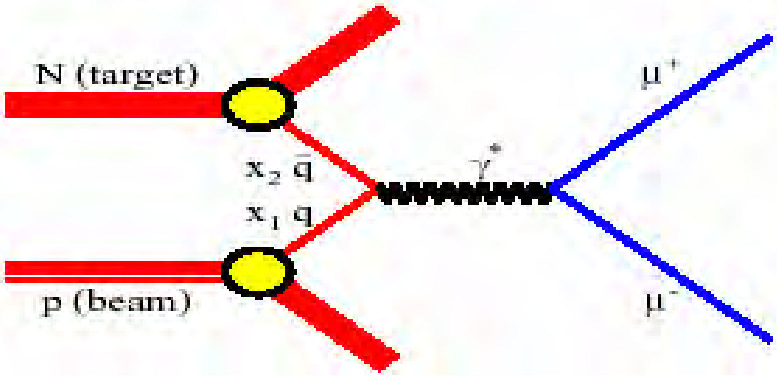


In general, Next-to-Leading-Order (NLO) perturbative QCD (DGLAP) fits do a good job of reproducing the data over the full measurement range.



The Drell-Yan Process : $pN \rightarrow \mu^+ \mu^- X$

Drell-Yan Pocess



Electromagnetic process

Time-like photon

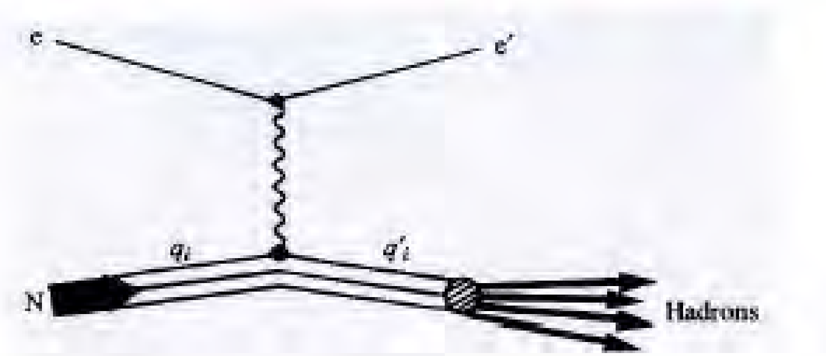
Two hadrons are involved

Two leptons are detected

Measure invariant mass (M)

and Feynman-x of the di-lepton

Deep-Inelastic Scattering



Electromagnetic process

Space-like photon

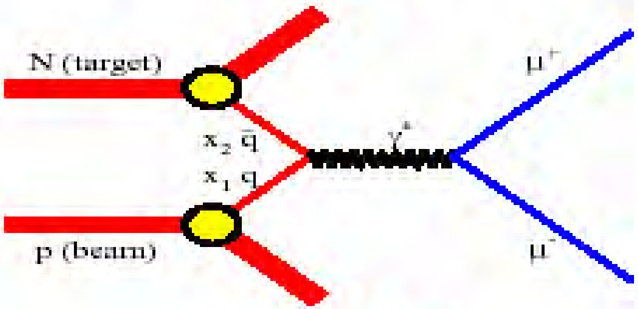
One hadron is involved

One lepton is detected

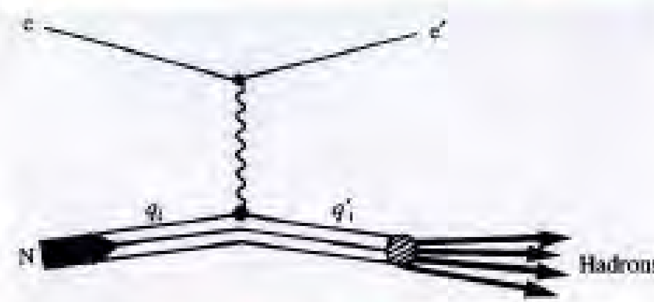
Measure Q^2 and Bjorken-x

The Drell-Yan Process: $pN \rightarrow \mu^+\mu^- X$

Drell-Yan Pocess



Deep-Inelastic Scattering



$x = Q^2 / 2m_N\nu \qquad y = \nu / E \qquad \nu = E - E'$

DIS:

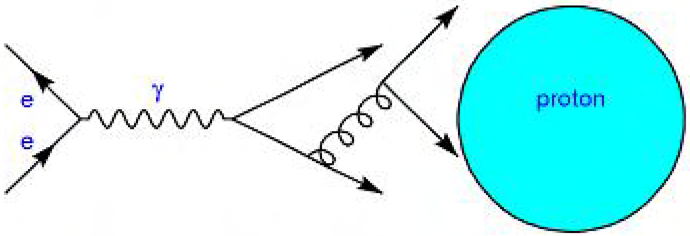
$$\frac{d\sigma}{dxdy} = \frac{2\pi\alpha^2}{Q^4} s\left[1+(1-y)^2\right]\sum_i e_i^2 x[q_i(x)+\overline{q}_i(x)]$$

$M^2 = x_1 x_2 s \qquad x_F = x_1 - x_2$

Drell-Yan:

$$\frac{d\sigma}{dM^2 dx_F} = \frac{4\pi\alpha^2}{9M^2 s}\frac{1}{x_1+x_2}\sum_i e_i^2\left[q_i(x_1)\overline{q}_i(x_2)+\overline{q}_i(x_1)q_i(x_2)\right]$$

In Proton Rest Frame



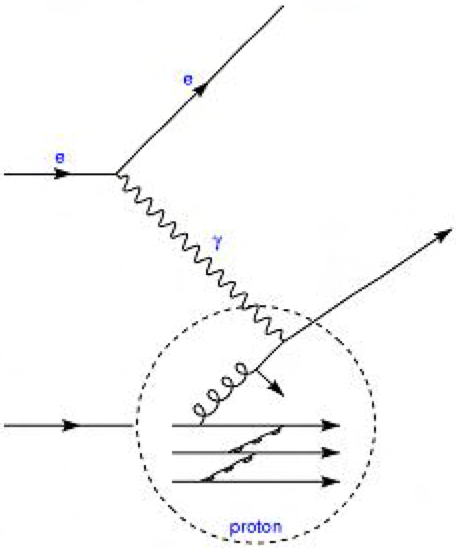
$$d^2\sigma/dW\,dQ^2 = \Gamma\,(\sigma_T + \varepsilon\,\sigma_L) = \Gamma\sigma_T\,(1 + \varepsilon R)$$

Γ is flux of photons
$\sigma_{T,L}$ are cross sections for transversely, and longitudinally polarized photons to scatter from proton, with R their ratio.
ε is the relative flux

$$F_2 = Q^2/4\pi^2\alpha\ \ (\sigma_T + \varepsilon\,\sigma_L)$$

In Infinite Momentum Frame

(Bjorken Limit: $Q^2,\nu \rightarrow$ infinity, with $x = Q^2/2M\nu$ fixed)



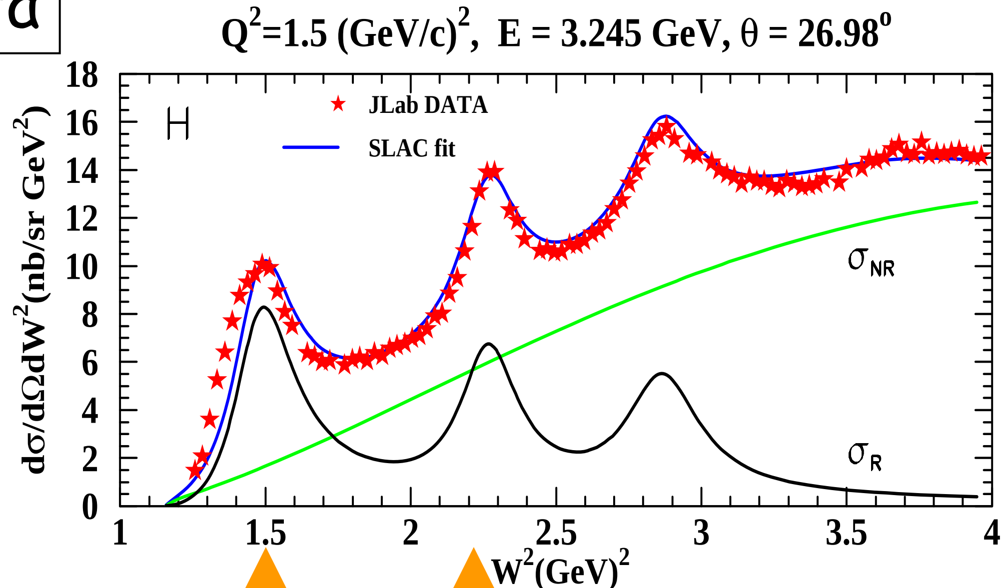
$$d^2\sigma/dxdQ^2 = 2\pi\alpha^2/xQ^4[(1+(1-y)^2)F_2 - y^2F_L]$$

$F_2 = \Sigma_f\, e^2_f\, x\,\{q(x,Q^2) + \overline{q}(x,Q^2)\}$, where e_f is the parton charge and $q(x,Q^2)$ the parton distribution

$F_L = 0$ in Leading Order (LO) in the Parton Model, but can be non-zero after gluon radiation.

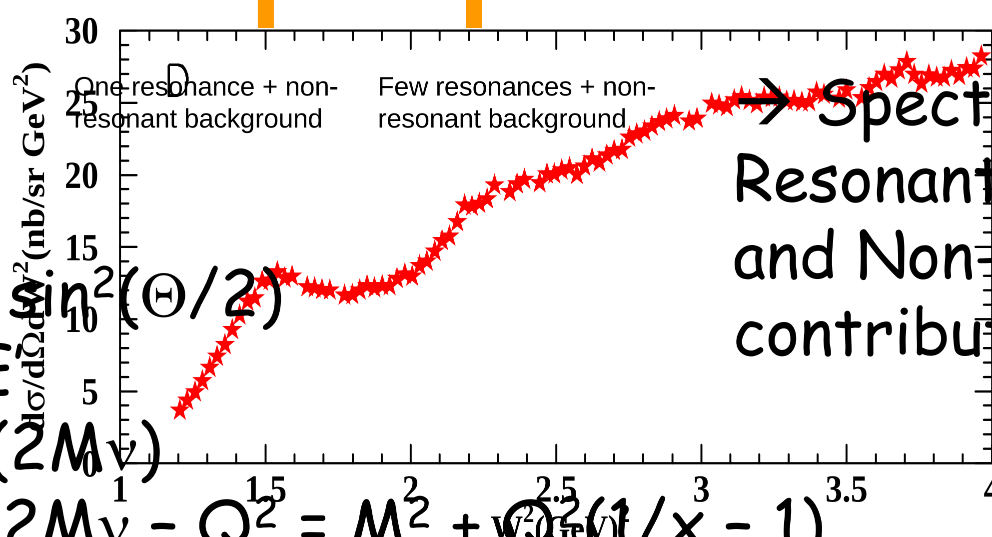
Inclusive Nucleon Resonance Electroproduction

$^{1}H(e,e')$ data



Non-resonant contribution

Resonant contribution



→ Spectrum consists of Resonant Contributions and Non-resonant contributions

$$Q^2 = 4EE' \sin^2(\Theta/2)$$

$$\nu = E - E'$$

$$x = Q^2/(2M\nu)$$

$$W^2 = M^2 + 2M\nu - Q^2 = M^2 + Q^2(1/x - 1)$$

Quark-Hadron Duality

complementarity between quark and hadron descriptions of observables

At high enough energy:

Hadronic Cross Sections

averaged over appropriate energy range

Perturbative

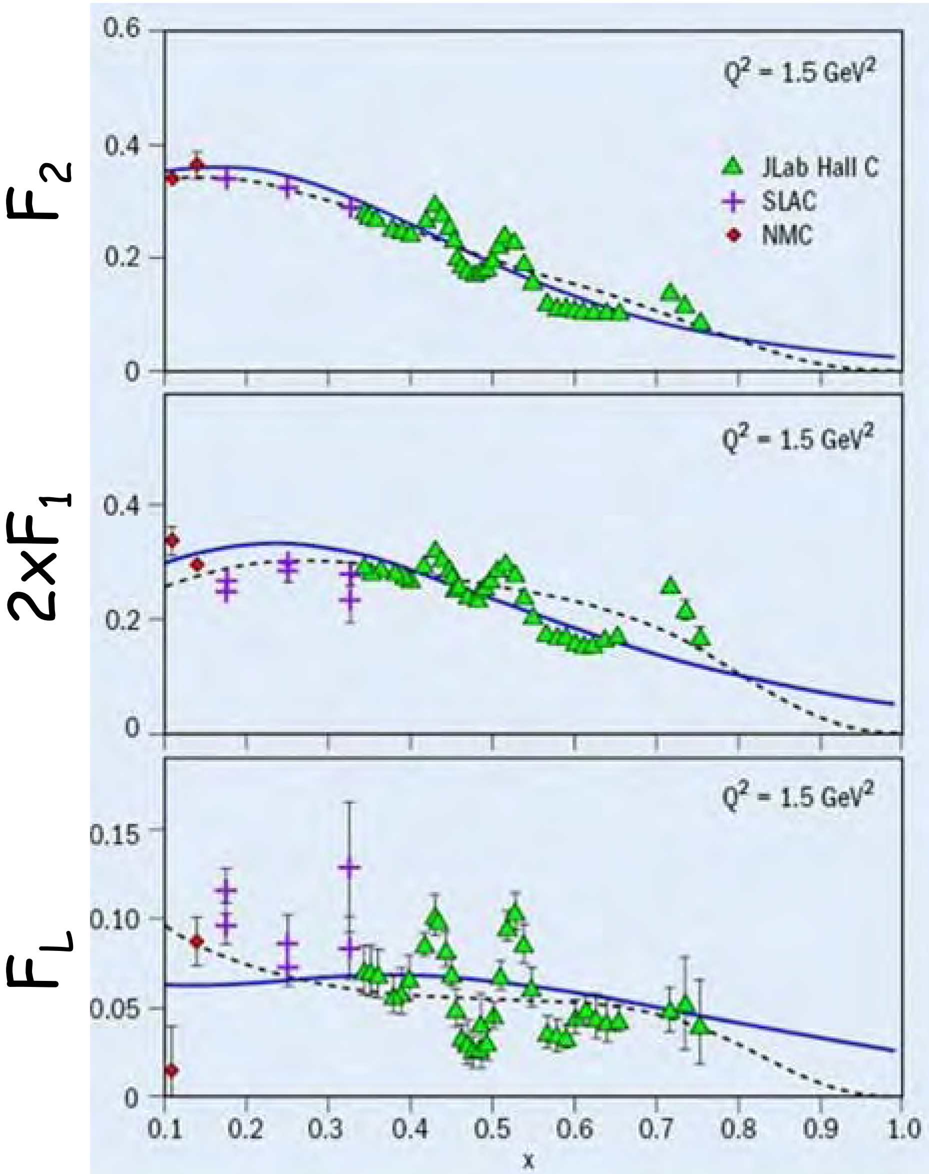
Quark-Gluon Theory

$$\sum_{hadrons} = \sum_{quarks+gluons}$$

Can use either set of complete basis states to describe physical phenomena

But why also in limited local energy ranges?

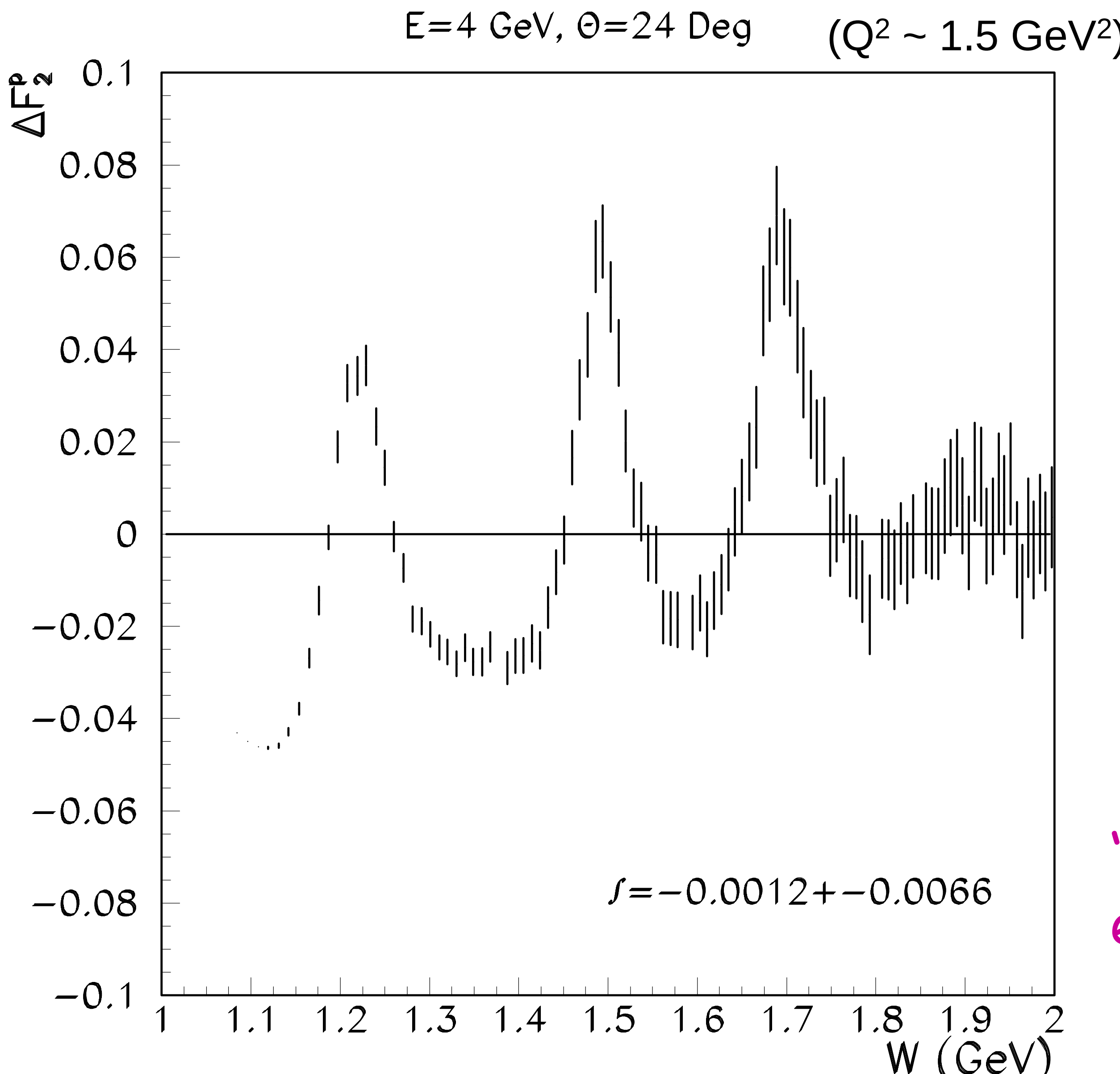
E94-110 : Separated Structure Functions
Duality works well for F_2, $2xF_1$ (F_T), and F_L



- The resonance region is, on average, well described by NNLO QCD fits.
- This implies that Higher-Twist (FSI) contributions cancel, and are on average small. "Quark-Hadron Duality"
- The result is a smooth transition from Quark Model Excitations to a Parton Model description, or a smooth quark-hadron transition.
- This explains the success of the parton model at relatively low W^2 (=4 GeV^2) and Q^2 (=1 GeV^2).

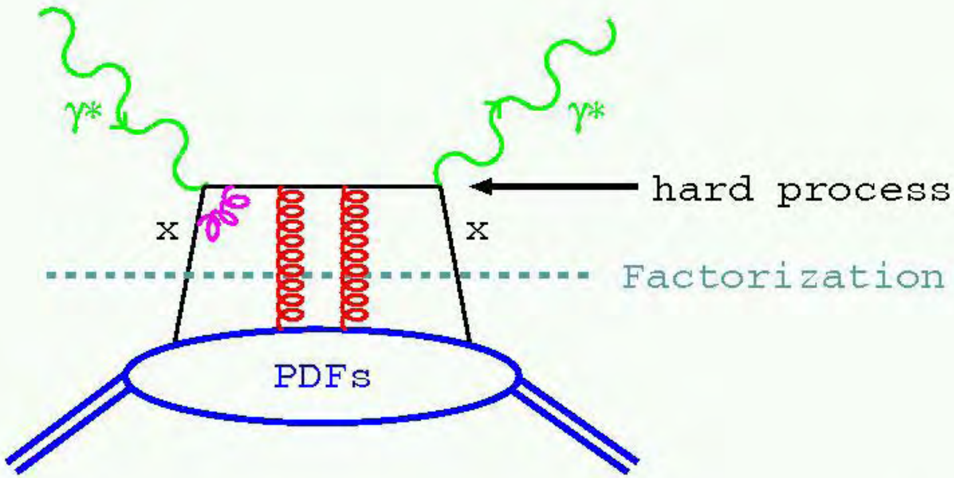
"The successful application of duality to extract known quantities suggests that it should also be possible to use it to extract quantities that are otherwise kinematically inaccessible."
(CERN Courier, December 2004)

Quantification: Resonance Region F_2
w.r.t. Alekhin NNLO Scaling Curve



"typical" example

Deep Inelastic Scattering



$$\frac{d\sigma}{d\Omega dE'} \sim \sigma_{Mott} \left(\sum_i e_i^2 x \left[q_i(x, Q^2) + \bar{q}_i(x, Q^2) \right] \right)$$

Bjorken Limit: $Q^2 \rightarrow \infty, \nu \rightarrow \infty$

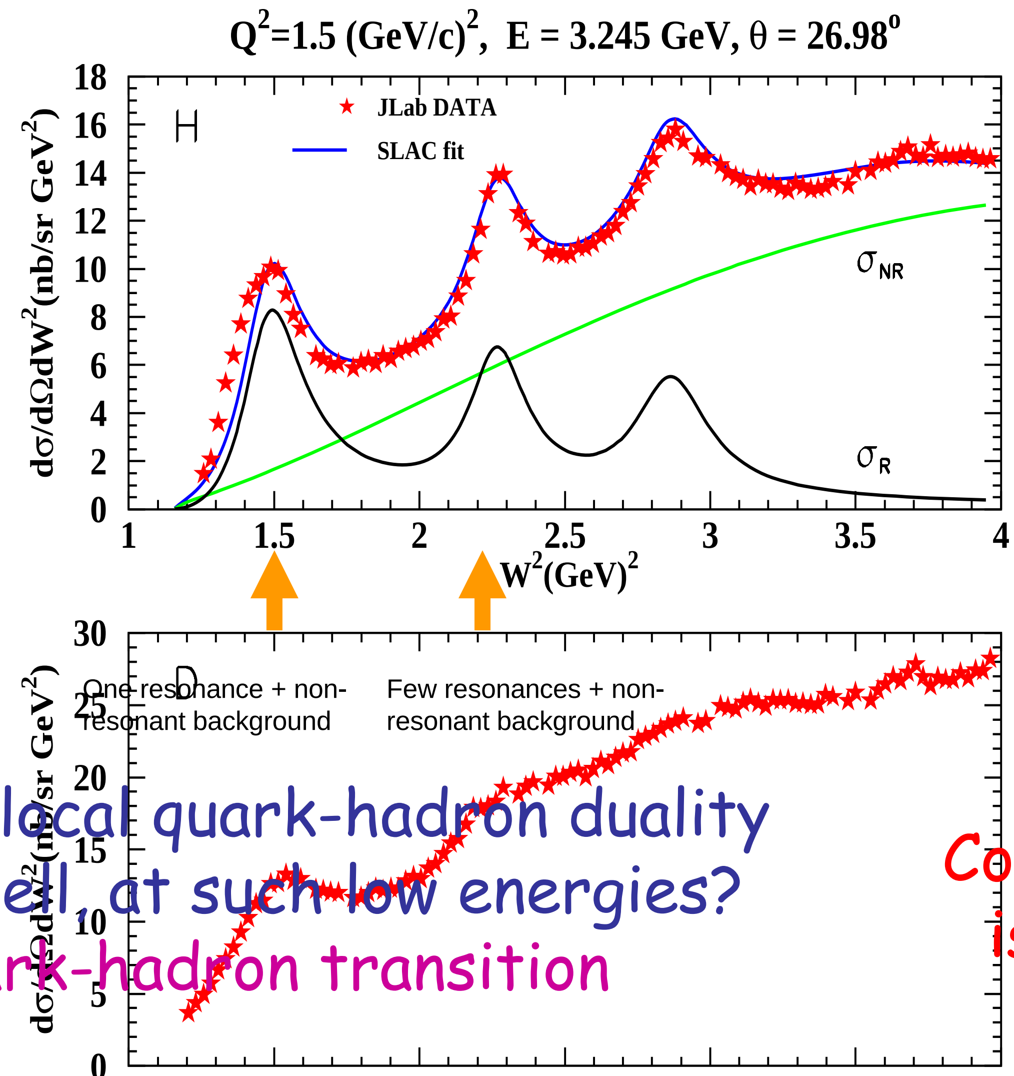
Empirically DIS region is where **logarithmic scaling** is observed

$Q^2 > 1$ (GeV/c)^2, $W^2 > 4$ GeV^2

Even worse : **Averaged** over W^2 region "works" also for $W^2 < 4$ GeV^2

Quark-Hadron Duality

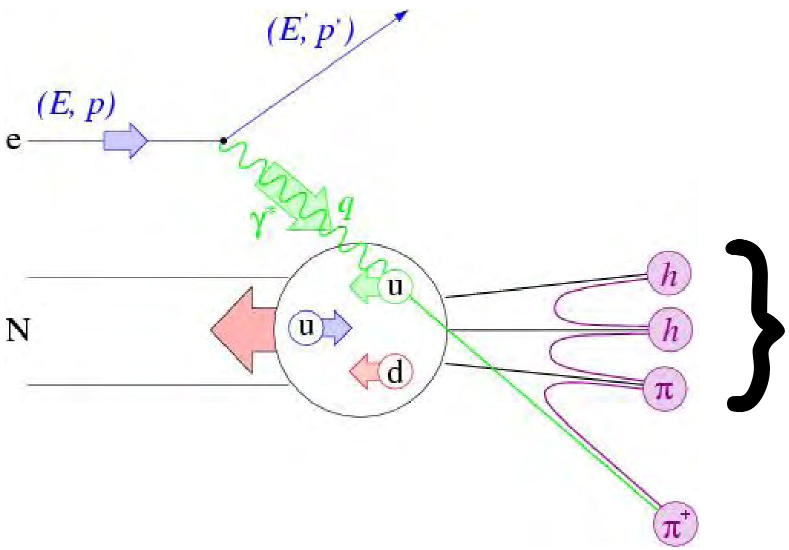
If one integrates over all resonant and non-resonant states, quark-hadron duality should be shown by any model. This is simply unitarity.
However, quark-hadron duality works also, for $Q^2 > 0.5$ (1.0) GeV^2, to better than 10 (5) % for the F_2 structure function in both the N-Δ and N-S_{11} region!
(Obviously, duality does not hold on top of a peak! -- One needs an appropriate energy range)



Why does local quark-hadron duality work so well, at such low energies?
~ quark-hadron transition

Confinement is local ….

Flavor Decomposition through semi-inclusive DIS



DIS probes only the sum of quarks and anti-quarks → requires assumptions on the role of sea quarks $\sum e_q^2 (q + \bar{q})$

Solution: Detect a final state hadron in addition to scattered electron

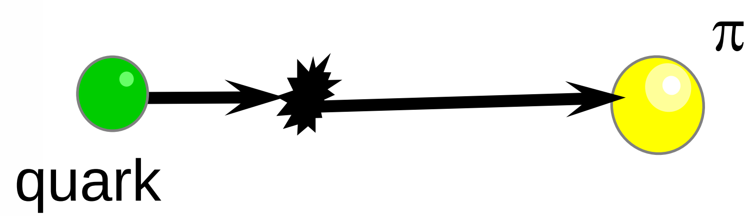
→ Can 'tag' the flavor of the struck quark by measuring the hadrons produced: 'flavor tagging'

$$\frac{1}{\sigma}\frac{d\sigma}{dz}(ep \to hX) = \frac{\sum_q e_q^2 f_q(x) D_q^h(z)}{\sum_q e_q^2 f_q(x)}$$

$f_q(x)$: parton distribution function

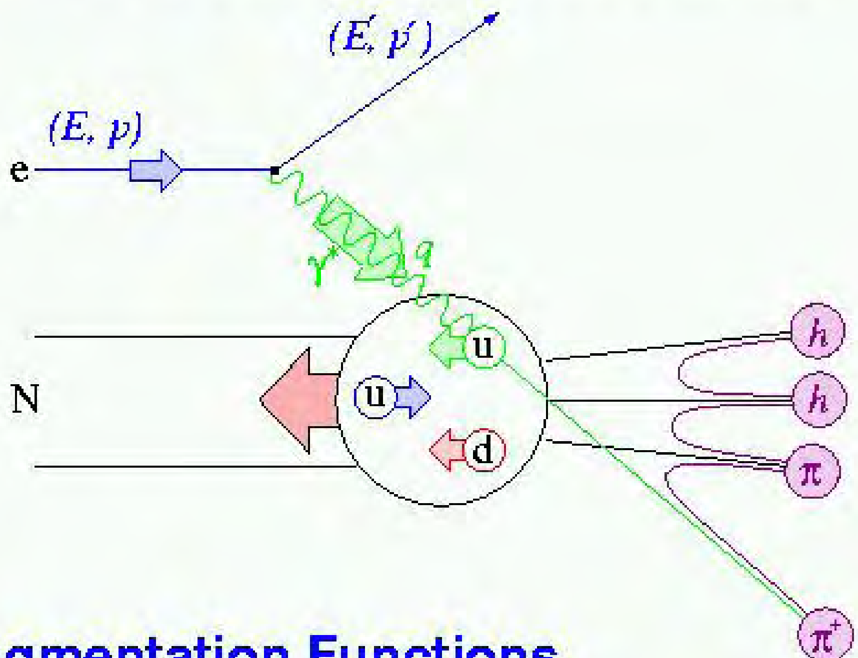
$D_q^h(z)$: fragmentation function

Fragmentation Function



Hard Matrix Elem → Parton Shower → **Hadronization**

Perturbative QCD calculations — Non-perturbative models



$$z = E_h/E_q = E_h/\nu$$

Fragmentation Functions

Universal fragmentation function $D(z)$ describes the transition ($q \rightarrow hadron$) ... "distribution of hadrons within a quark". cf. other universal function = PDF $q(x)$ ... "distribution of quarks within a hadron".

Universal fragmentation function $D(z)$ describes the transition $(q \to hadron)$ ... "distribution of hadrons within a quark". cf. other universal function = PDF $q(x)$ ... "distribution of quarks within a hadron".

$$\frac{d\sigma}{dz}(e^+e^- \to hX) = \sum_q \sigma(e^+e^- \to q\bar{q})[D_q^h(z) + D_{\bar{q}}^h(z)]$$

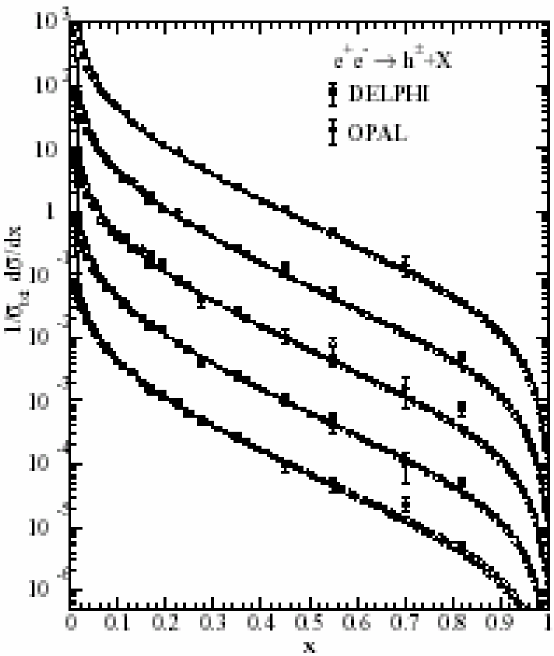
$$\frac{1}{\sigma}\frac{d\sigma}{dz}(ep \to hX) = \frac{\sum_q e_q^2 q(x) D_q^h(z)}{\sum_q e_q^2 q(x)}$$

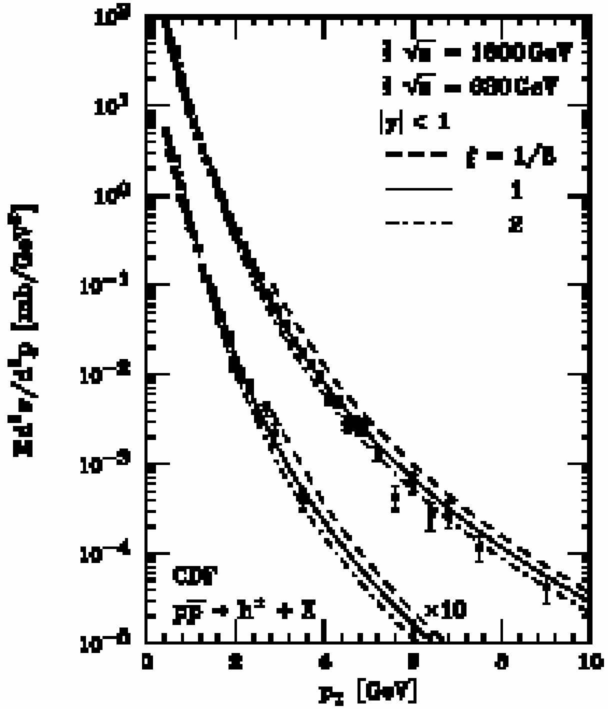
Relations

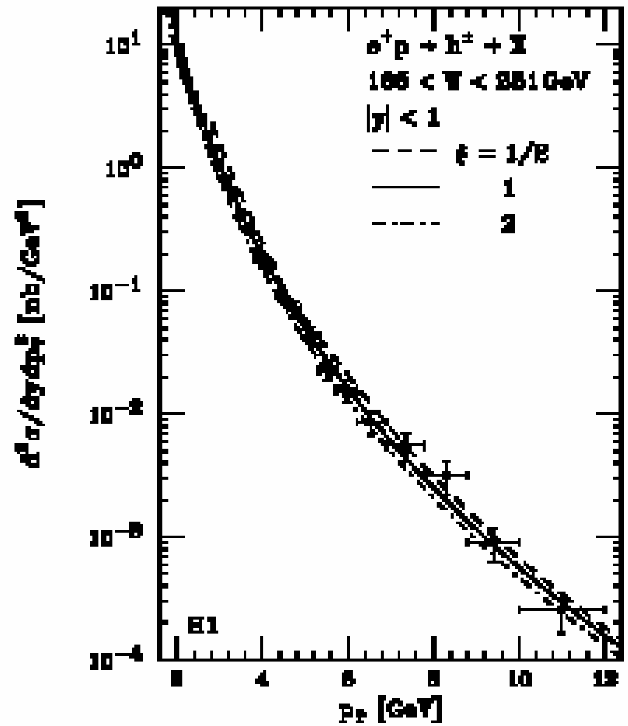
- Momentum conservation: $\sum_h \int_0^1 D_q^h(z)dz = 1$
- Probability conservation: $\sum_q \int_{z_{\min}} [D_q^h(z) + D_{\bar{q}}^h(z)]dz = n_h$
  (n_h = average multiplicity)

Universality of fragmentation functions

$e^+e^- \to h^\pm + X$ $\qquad$ $p\bar{p} \to h^\pm + X$ $\qquad$ $e^+p \to e^+ + h^\pm + X$







Fragmentation functions parametrize our lack of understanding of the **non-perturbative** **hadron formation** process.

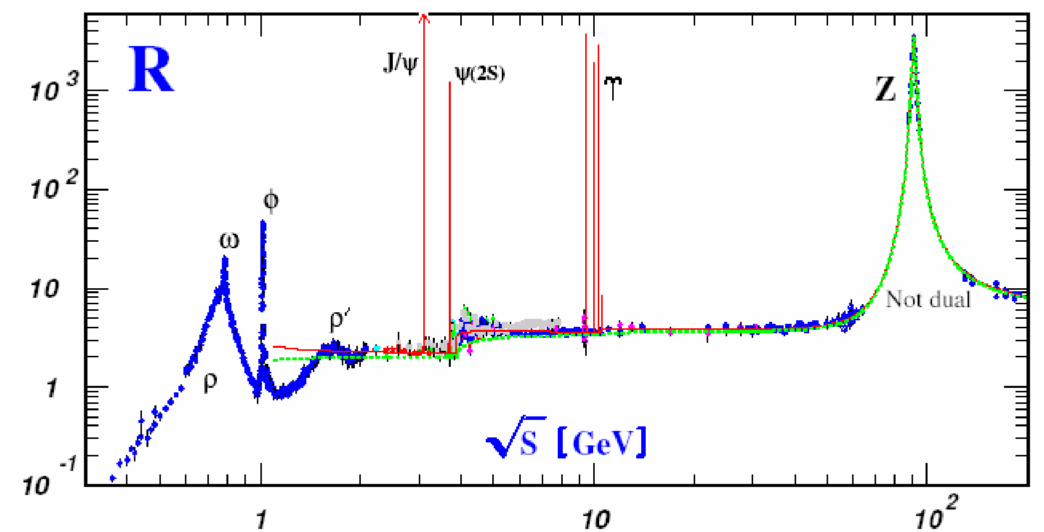
$$\frac{1}{\sigma}\frac{d\sigma}{dz}(ep \rightarrow hX) = \frac{\sum_q e_q^2 q(x) D_q^h(z)}{\sum_q e_q^2 q(x)} \quad \text{where } z \equiv E_h/E_q = E_h/\nu$$

We should write: $D(z, p_t, Q^2)$ ... have observed these features:

- **z-scaling** : Fragmentation functions depend almost entirely on energy fraction $z = E_h/E_q$, with only a **weak logarithmic Q^2-dependence** (scaling violations) $\rightarrow$ z-scaling independent of process and energy.

$$Q^2\frac{\delta}{\delta Q^2}D_i^h(x, Q^2) = \sum_j \int_x^1 \frac{dz}{z} P_{ji}(z, \alpha(Q^2)) D_j^h(x/z, Q^2)$$

$$R = \frac{\sigma(e^+e^- \to X)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



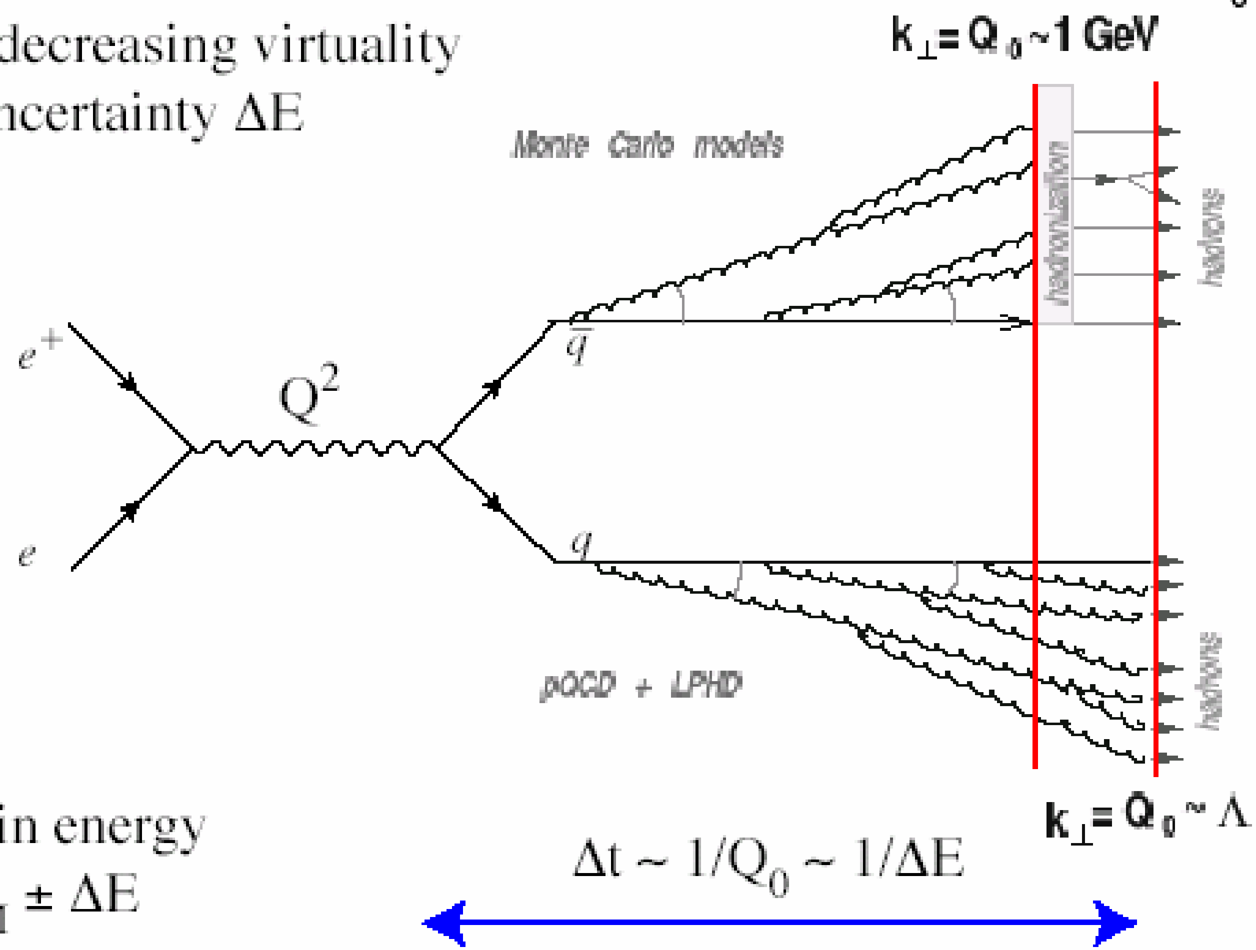
$$\lim_{E\to\infty} \frac{\sigma(e^+e^- \to X)}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum_q e_q^2$$

Only evidence of hadrons produced is narrow states oscillating around step function

"Resonances build the parton subprocess cross section because of a separation of scales between hard and soft processes."
Confinement is Local

From: Paul Hoyer, Summary Talk Workshop on Duality, Frascati 2005

Final state evolves with decreasing virtuality and decreasing energy uncertainty ΔE

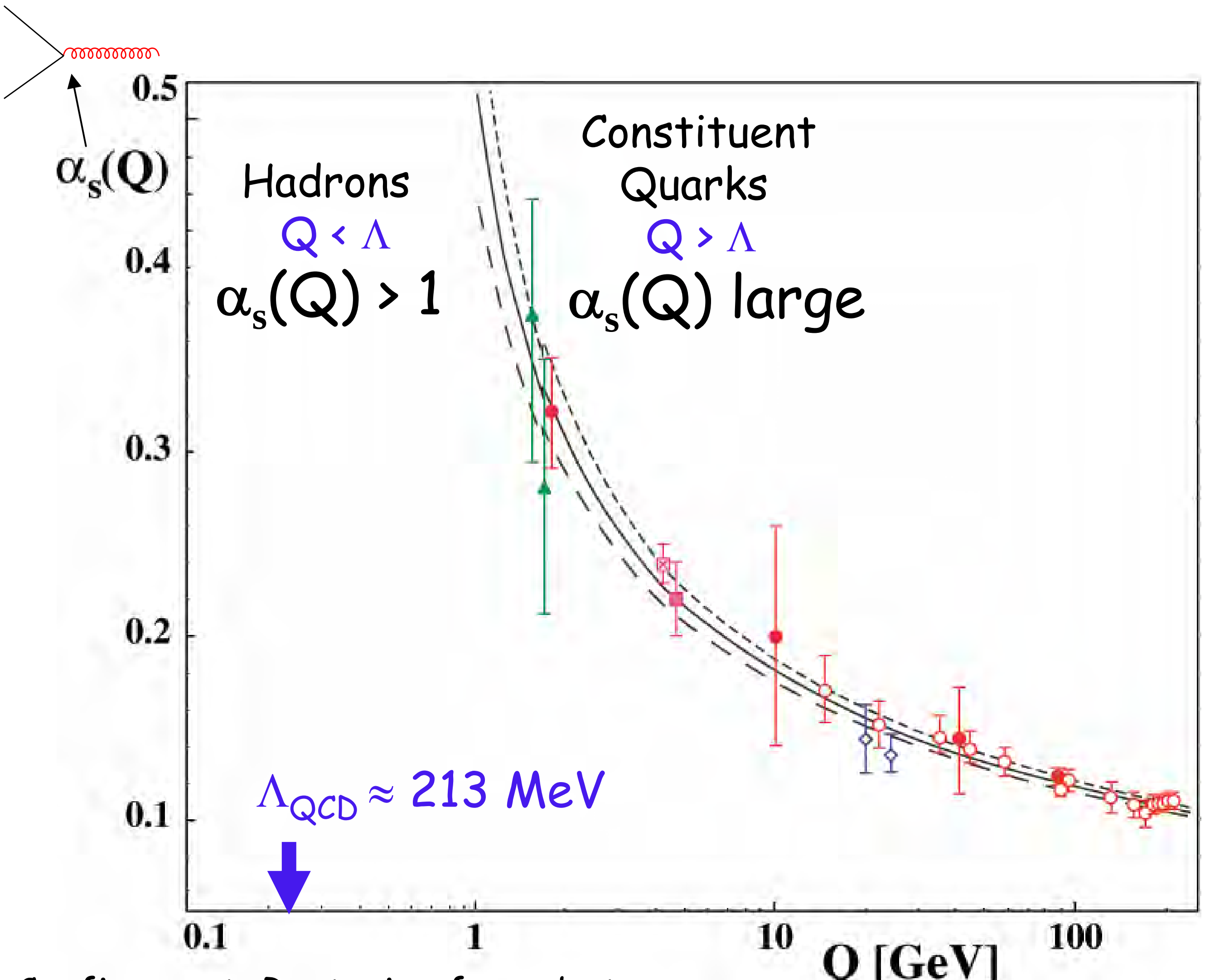


Evolution beyond virtuality $Q_0^2 \sim 2E\Delta E$ involves $k_\perp \sim Q_0$ and is unitary:

Measured cross section in energy uncertainty interval $E_{CM} \pm \Delta E$ must average to pQCD cross section at resolution Q_0^2 .

"The softer is confinement physics, the smaller Q_0 can be chosen, and the more local is duality → duality suggests principal features in confinement region, but our field theory tools are limited in this region."

QCD and the Parton-Hadron Transition

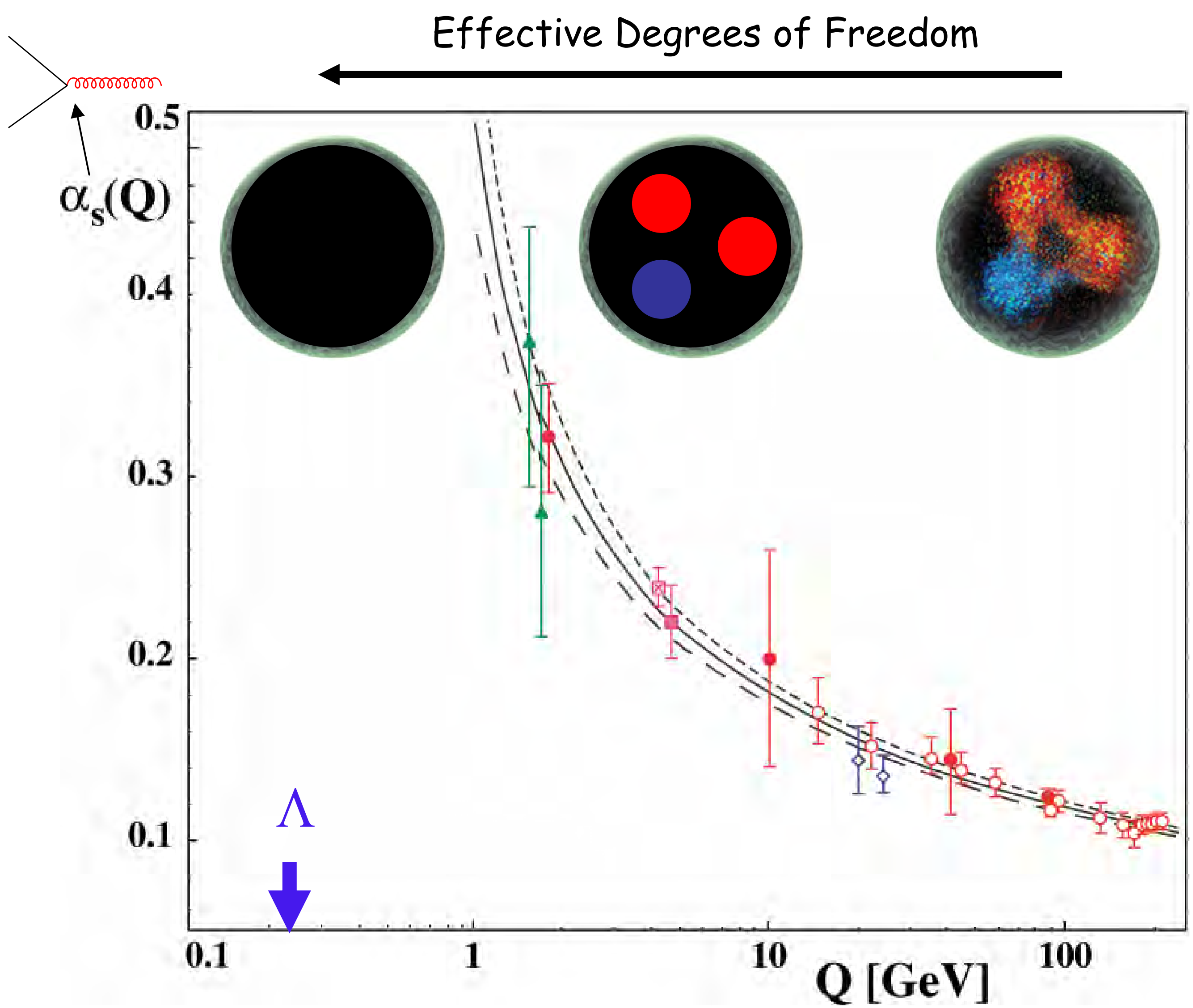


Asymptotically the photon couples to quarks, yet *confinement* ensures that only hadronic final states are observed.

Asymptotically Free Quarks
$Q \gg \Lambda$
$\alpha_s(Q)$ small

Confinement: Restoring force between quarks at large distances equivalent to 10 tons, no matter how far apart!

QCD and the Parton-Hadron Transition



One parameter, Λ_{QCD},
~ Mass Scale or
Inverse Distance Scale
where $a_s(Q)$ = infinity

"Separates" Confinement
and Perturbative Regions

Mass and Radius of the
Proton are (almost)
completely governed by

$\Lambda_{QCD} \approx 213$ MeV

Quarks that appear in
QCD Lagrangian "drag"
some of the local QCD
fields around them →
Protons are balls of
energy

Nuclear Physics in terms of protons, neutrons and pion exchange is a very good effective model.

Momentum transfer Q is negligible

Protons and Neutrons in terms of constituent (valence) quarks is a very decent effective model:

the Constituent Quark Model works surprisingly well.

Momentum transfer Q is small

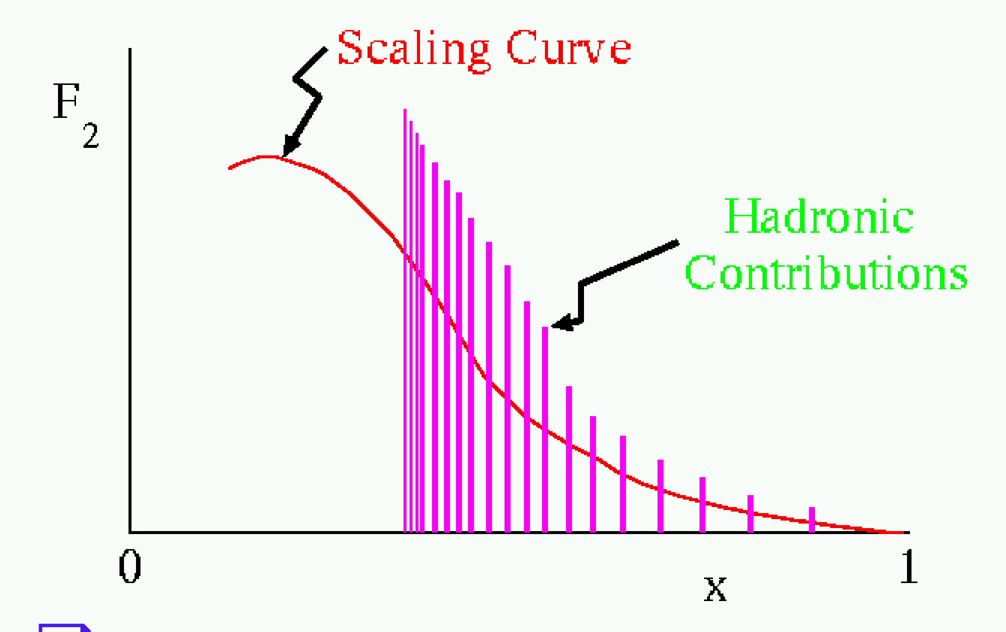
Looking deep inside protons and neutrons, they are really balls of energy, with lots of gluons and quark-antiquark pairs popping in and out of existence.

Momentum transfer Q is “large”

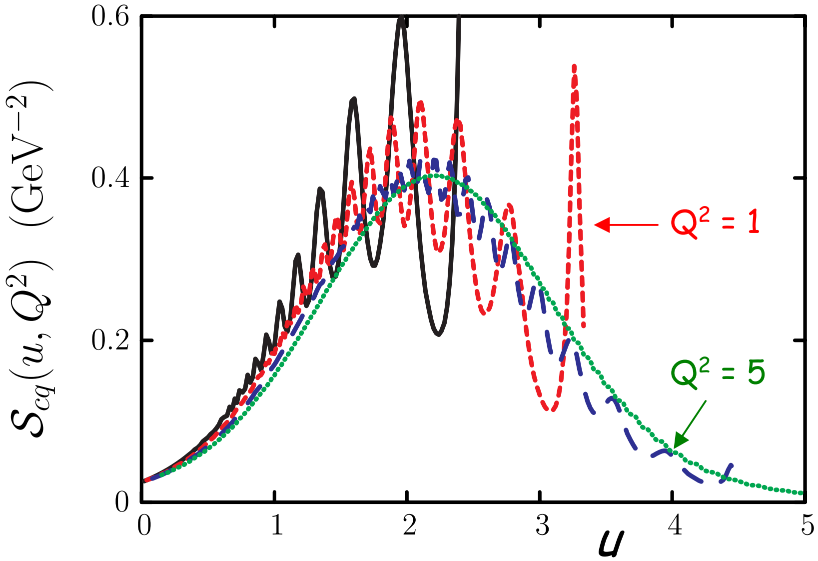
Quark-Hadron Duality – Theoretical Efforts

N. Isgur et al : $N_c \to \infty$

qq infinitely narrow resonances
qqq only resonances



One heavy quark, Relativistic HO



Scaling occurs rapidly!

- Distinction between Resonance and Scaling regions is spurious
- Bloom-Gilman Duality must be invoked even in the Bjorken Scaling region
  → Bjorken Duality

F. Close et al : SU(6) Quark Model

How many resonances does one need to average over to obtain a complete set of states to mimic a parton model?
→56 and 70 states o.k. for closure
→Similar arguments for e.g. DVCS and semi-inclusive reactions

How does the square of the sum become the sum of the squares?

Close and Isgur, Phys. Lett. B509, 81 (2001)

$F_1 \sim \sigma_{1/2} + \sigma_{3/2}$ $g_1 \sim \sigma_{1/2} - \sigma_{3/2}$

56 + 70 → *Closure*

Relative Photoproduction Strengths of **56,0$^+$** and **70,1$^-$** Multiplets

$SU(6)$:	$[56,0^+]^2 8$	$[56,0^+]^4 10$	$[70,1^-]^2 8$	$[70,1^-]^4 8$	$[70,1^-]^2 10$
F_1^p	9	8	9	0	1
F_1^n	4	8	1	4	1
g_1^p	9	-4	9	0	1
g_1^n	4	-4	1	-2	1
W			~ 1.53	~ 1.7	~ 1.7

Relative Longitudinal Production Strengths

$SU(6)$:	$[56,0^+]^2 8$	$[56,0^+]^4 10$	$[70,1^-]^2 8$	$[70,1^-]^4 8$	$[70,1^-]^2 10$
F_L^p	1	0	1	0	1
F_L^n	0	0	1	0	1

Conclusion: destructive interference between hadronic states of different symmetries critical feature of duality

Relative Photoproduction Strengths of **56,0$^+$** and **70,1$^-$** Multiplets

$SU(6)$:	[56,0$^+$]^{2}8	[56,0$^+$]^{4}10	[70,1$^-$]^{2}8	[70,1$^-$]^{4}8	[70,1$^-$]^{2}10
F_1^p	9	8	9	0	1
F_1^n	4	8	1	4	1
g_1^p	9	-4	9	0	1
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Relative Longitudinal Production Strengths

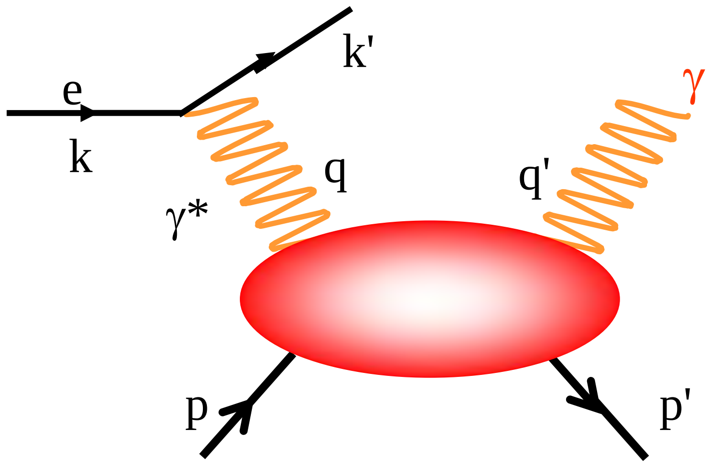
$SU(6)$:	[56,0$^+$]^{2}8	[56,0$^+$]^{4}10	[70,1$^-$]^{2}8	[70,1$^-$]^{4}8	[70,1$^-$]^{2}10
F_L^p	1	0	1	0	1
F_L^n	0	0	1	0	1

Predictions

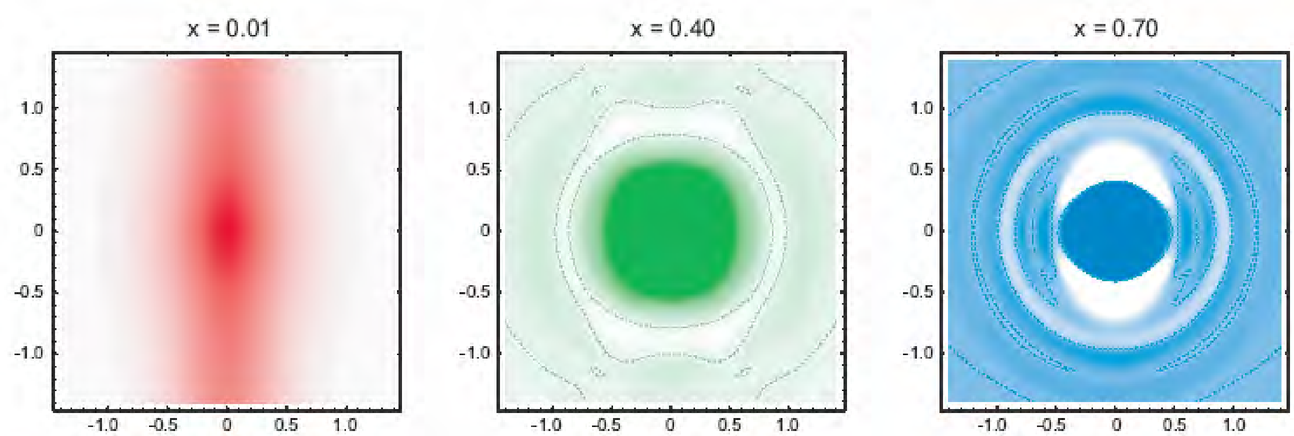
- **Second Resonance Region has too much strength for proton**
- **Third Resonance Region has more impact for neutron**
- **Breakdown in Duality for $Q^2 \leq$ 0.5: both electric and magnetic contribute comparably**
- **Duality in magnetic strength down to Q^2 = 0**

Generalized Parton Distributions and Nucleon Tomography

A Major new direction in Hadron Physics aimed at the 3-D mapping of the quark structure of the nucleon.

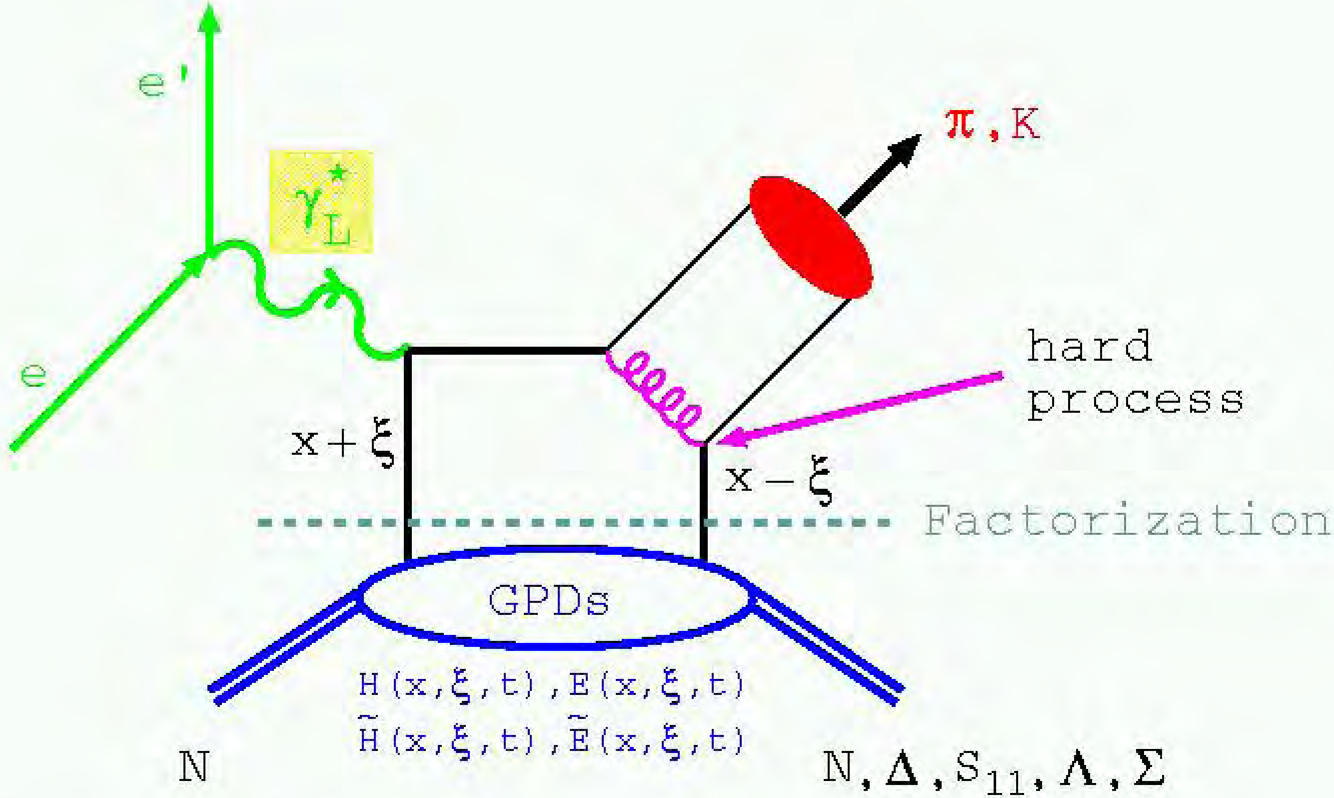


Simplest process:
Deep-Virtual Compton Scattering



Model calculation of u-quark distributions in protons at different momentum fractions.

Generalized Parton Distributions and Pion Electroproduction



For factorization to be strictly valid

$$\sigma_L \sim Q^{-6}, \sigma_T \sim Q^{-8}$$

and

Onset of Color Transparency

For factorization to be strictly valid

$$\sigma_L \sim Q^{-6}, \sigma_T \sim Q^{-8}$$

and

Onset of Color Transparency

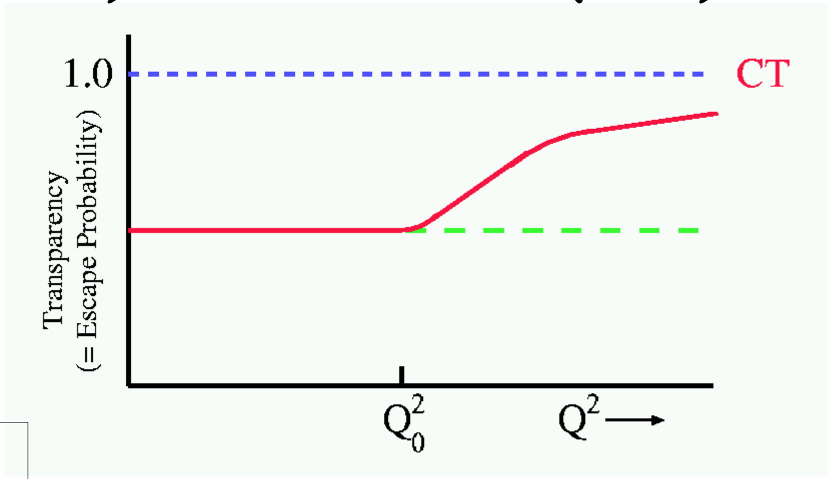
Support and Problems

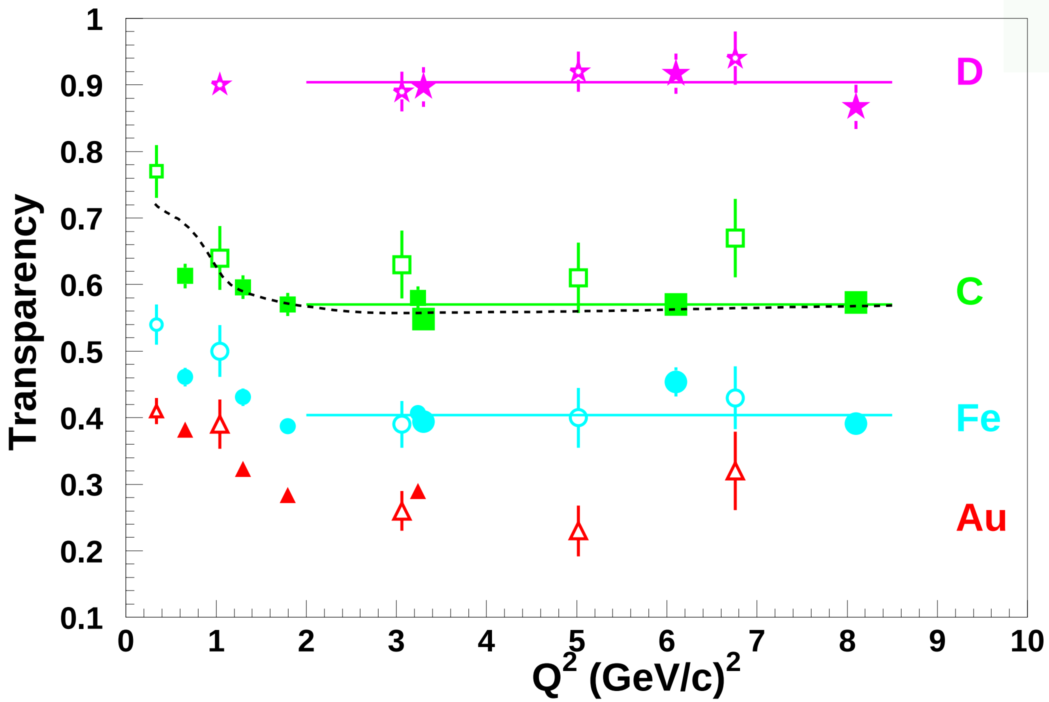
- $\sigma_L/\sigma_T \sim Q^2$ for (e,e$'\pi^+$) and $(\mu, \mu'\rho^\circ)$ at $Q^2 \sim$ few (GeV/c)^2
- Pion Form Factor predicted to be soft to $Q^2 \sim 10$ (GeV/c)^2
- FNAL A$(\mu, \mu'\rho^\circ)$ data show some hint of CT at $Q^2 \geq 3$ (GeV/c)^2
- FNAL Di-jet data seem to see full CT at $Q^2 \simeq 10$ (GeV/c)^2

Search for Color Transparency in Quasi-free A(e,e'p) Scattering

*E94-139, Spokespersons: **Rolf Ent** (JLab) + Richard Milner (MIT)*

From fundamental considerations (quantum mechanics, relativity, nature of the strong interaction) it is predicted (Brodsky, Mueller) that fast protons scattered from the nucleus will have decreased final state interactions





Constant value line fits give good description χ^2/df = 1

Conventional Nuclear Physics Calculation by Pandharipande *et al.* (dashed) also gives good description

→No sign of CT yet

Search for Color Transparency in A(e,e'π) Reactions

E01-107, Spokespersons: Dipangkar Dutta (Duke/TUNL) + Rolf Ent (JLab)

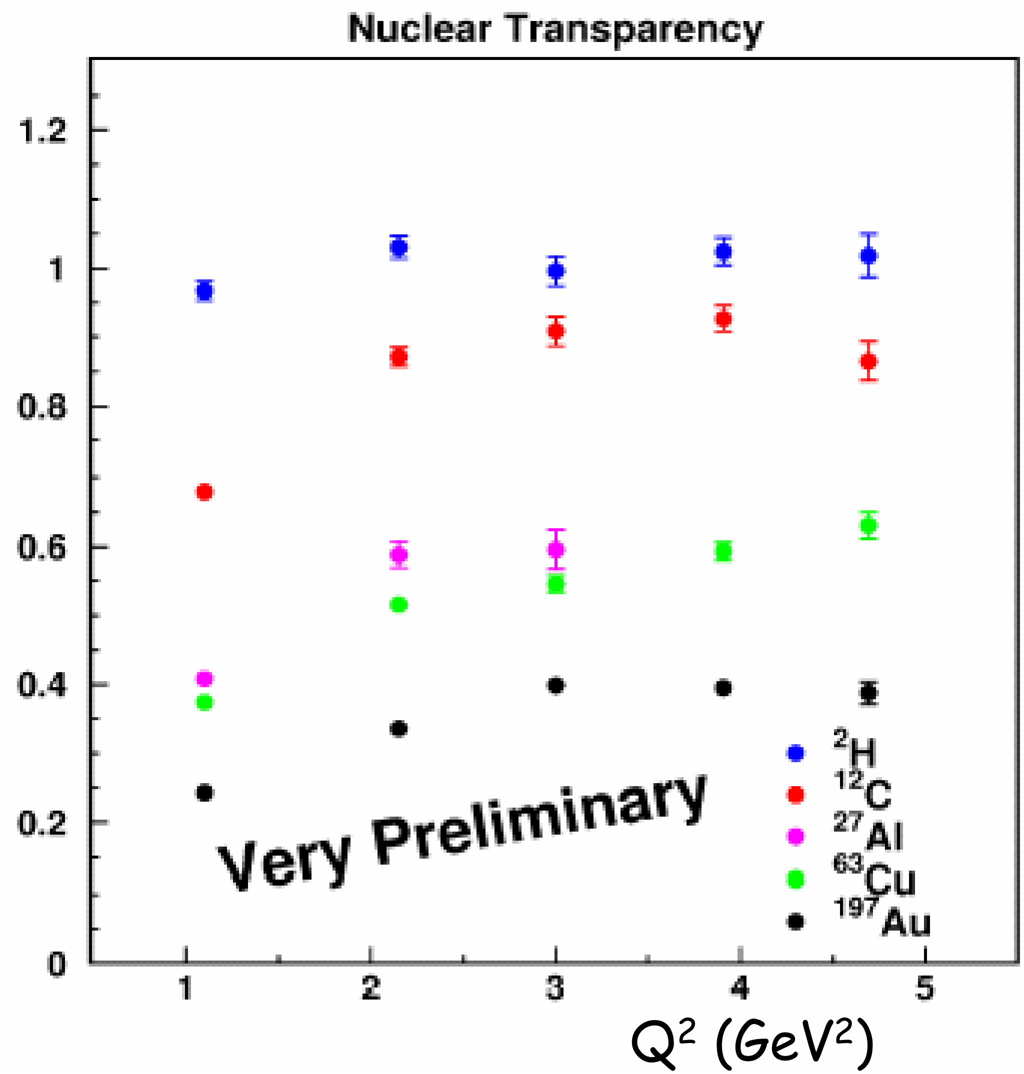
If FNAL A(π,dijet) data interpretation is correct, we should see onset of CT in A(e,e'π) at Q^2 ~ 5 GeV^2

$$T = \frac{(Y_{DATA}/Y_{MC})_A}{(Y_{DATA}/Y_{MC})_{Hydrogen}}$$

Analysis by Ben Clasie (MIT)

Model dependency of ratios still +/- 10%

No smoking gun for color transparency.



Factorization Proofs guide our universal description of processes, giving access to the long-range dynamics of confinement

Many experimental approaches to factorization possible

- Verify that the cross sections **scale** , i.e. follow the energy-momentum dependence of the core quark-gluon calculation – This should render us **universal functions**
- Verify the factorization by indepently verifying the kinematic dependence of the factorized terms – e.g. x **and** z dependence
- Look for observables that should occur in the region of factorization – e.g. **L/T ratios** , **Color Transparency**

In the end, none of these are **fool proof** , as evidenced by **quark-hadron duality** , and the fact that these approaches independently are **necessary** , but **not sufficient** , proof for the factorization to be rigid.

Low-Energy Factorization

Or … Duality in Pion Electroproduction …

Rolf Ent

Will try to convince you why factorization appears to work well at low energies, and how this is related to Quark-Hadron Duality

Quark-Hadron Duality in
- $e^+ + e^- \rightarrow$ hadrons
- $e + p \rightarrow e' + X$
- Pion Electroproduction

Low-Energy Factorization

Conclusions

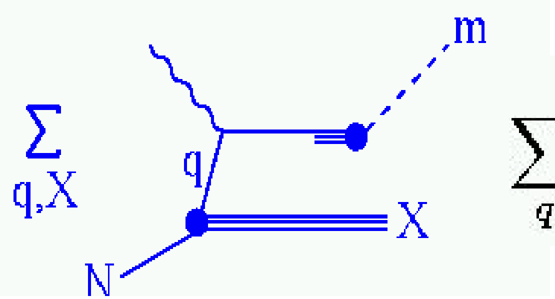
Duality in (Semi-)Exclusive Processes

Inclusive-Exclusive connection: Bjorken and Kogut impose "correspondence principle": demanding continuity of the dynamics from one region of kinematics to the other → relates exclusive cross sections at low energy to inclusive production at high energies

Momentum Spectrum of produced hadrons in an inclusive reaction
$\gamma^*N \to MX$

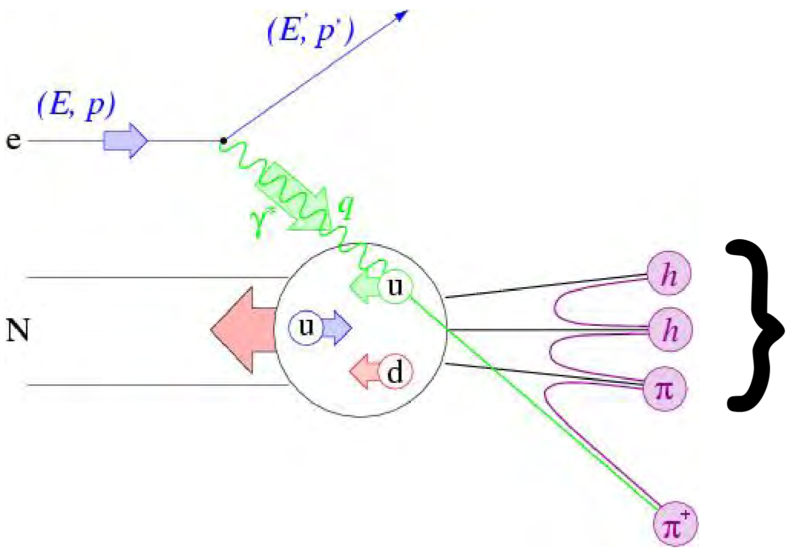
Used as argument that there should be a region where quark-hadron duality holds in exclusive reactions and semi-inclusive reactions.

→ Quark-hadron duality predicted to also appear in semi-inclusive scattering processes (Carlson (1998), Afanasev, Carlson, Wahlquist, Phys. Rev. D 62, 074011 (2000))



$$\sum_q e_q^2 \, q(x) \, D_{q\to M}(z)$$

Flavor Decomposition through semi-inclusive DIS

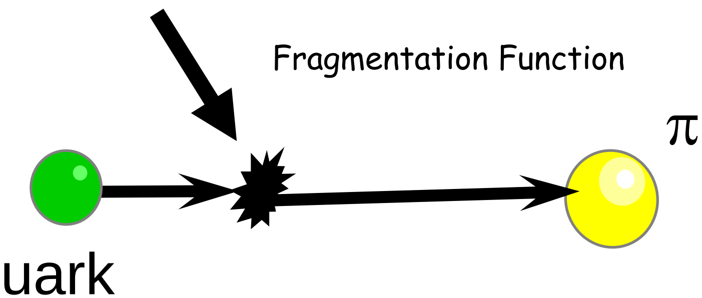


DIS probes only the sum of quarks and anti-quarks → requires assumptions on the role of sea quarks $\sum e_q^2(q+\bar{q})$

Solution: Detect a final state hadron in addition to scattered electron

→ Can 'tag' the flavor of the struck quark by measuring the hadrons produced: 'flavor tagging'

$$\sum e_q^2\, q(x)\, D_{q\to M}(z)$$



(e,e') $W^2 = M^2 + Q^2\,(1/x - 1)$

For M_m small, $\vec{p}_m$ collinear with $\vec{\gamma}$, and $Q^2/\nu^2 \ll 1$

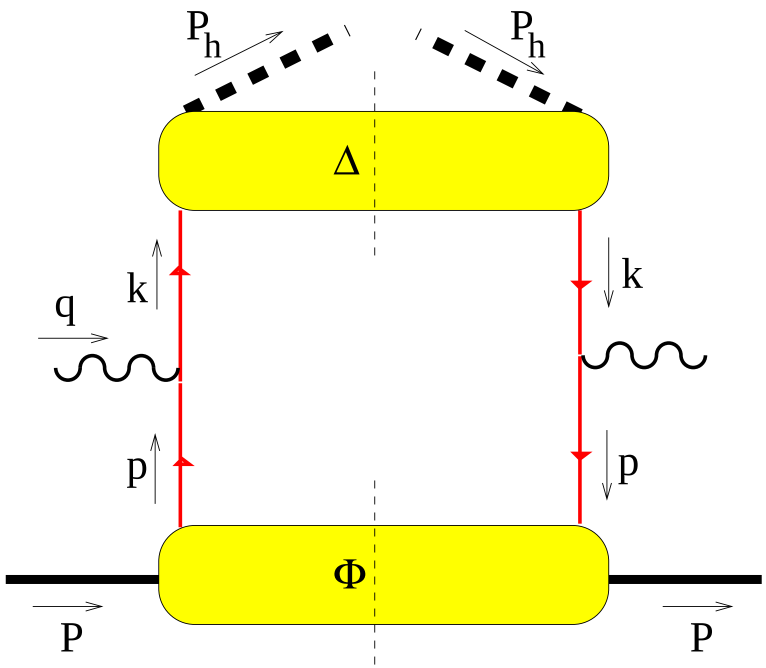
(e,e'm) $W'^2 = M^2 + Q^2\,(1/x - 1)(1 - z)$

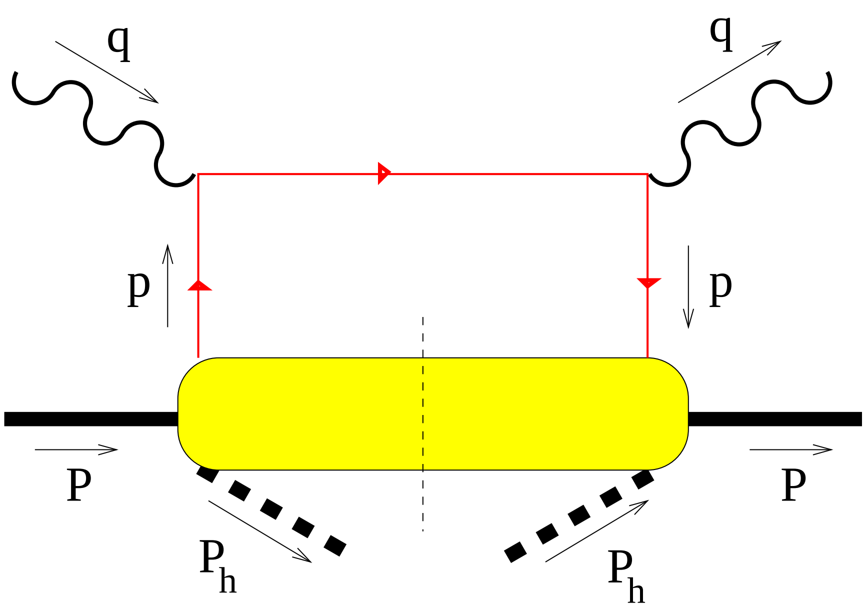
$z = E_m/\nu$

Factorization

$$\sum_q e_q^2\, q(x)\, D_{q\to M}(z)$$

Both Current and Target Fragmentation Processes Possible. At Leading Order:





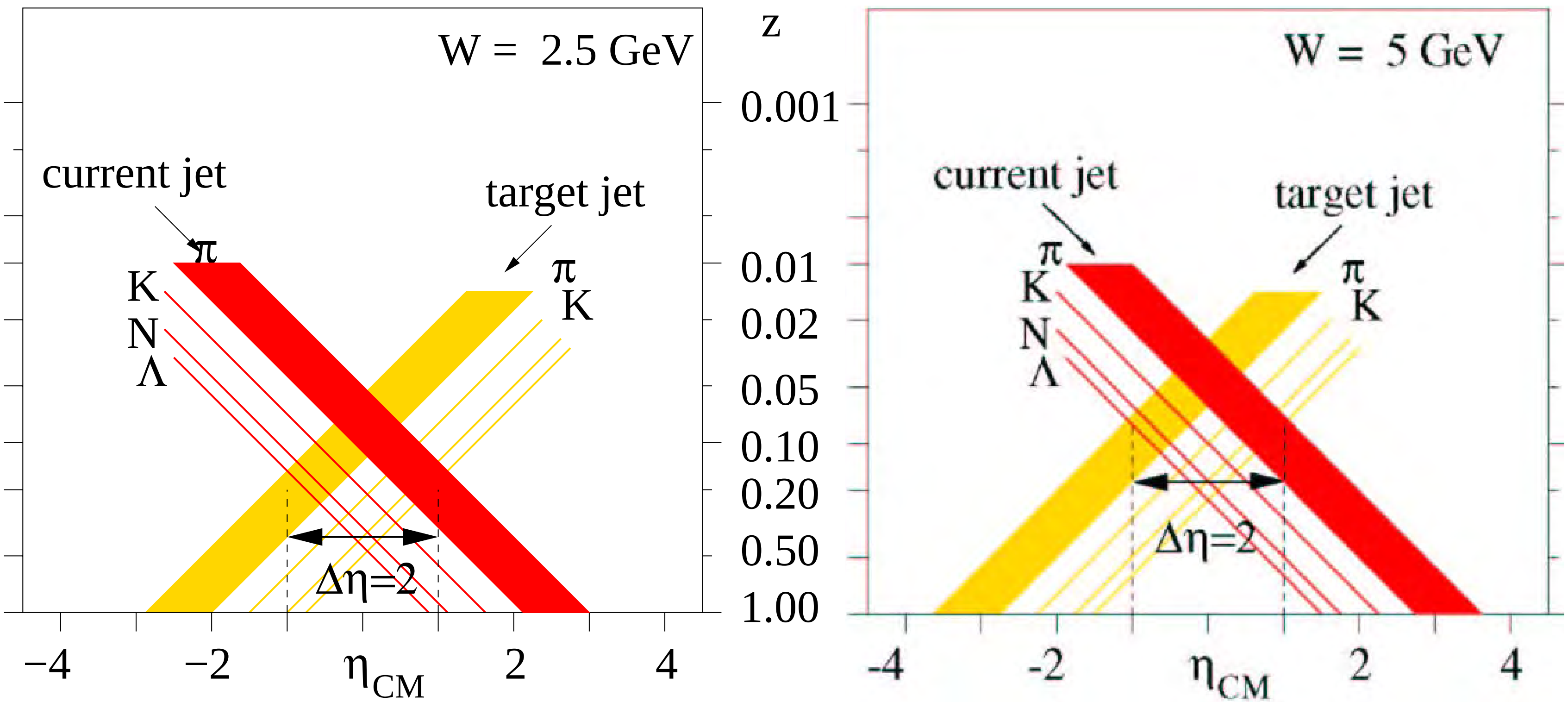
Berger Criterion $\Delta\eta > 2$ Rapidity gap for factorization

Separates Current and Target Fragmentation Region in Rapidity

Factorization

$$\sum_q e_q^2 \, q(x) \, D_{q \to M}(z)$$

P.J. Mulders, hep-ph/0010199 (EPIC Workshop, MIT, 2000)



At large z-values easier to separate current and target fragmentation region
→ for fast hadrons factorization (Berger Criterion) "works" at lower energies

At W = 2.5 GeV: z > 0.4
(Typical JLab)

At W = 5 GeV: z > 0.2
(Typical HERMES)

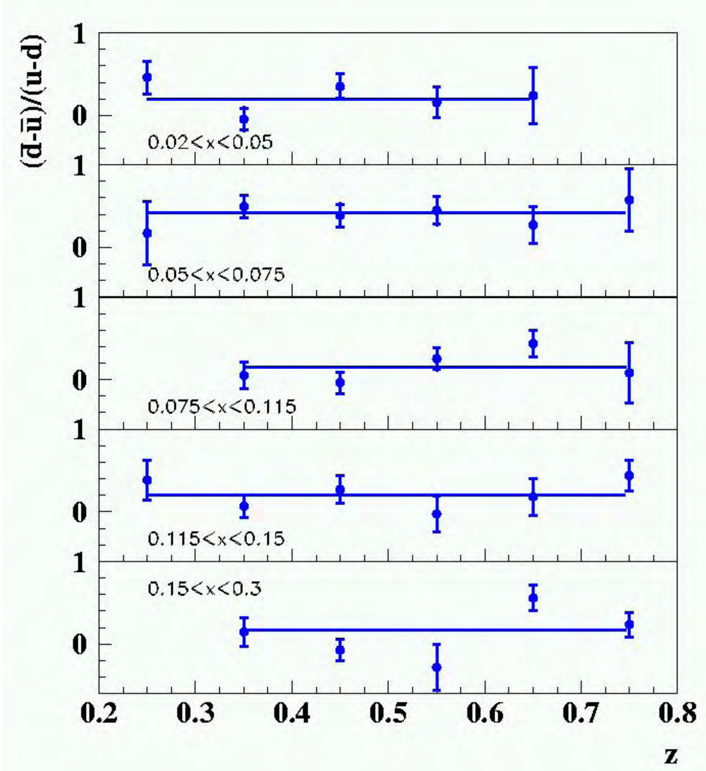
Low-Energy Factorization?

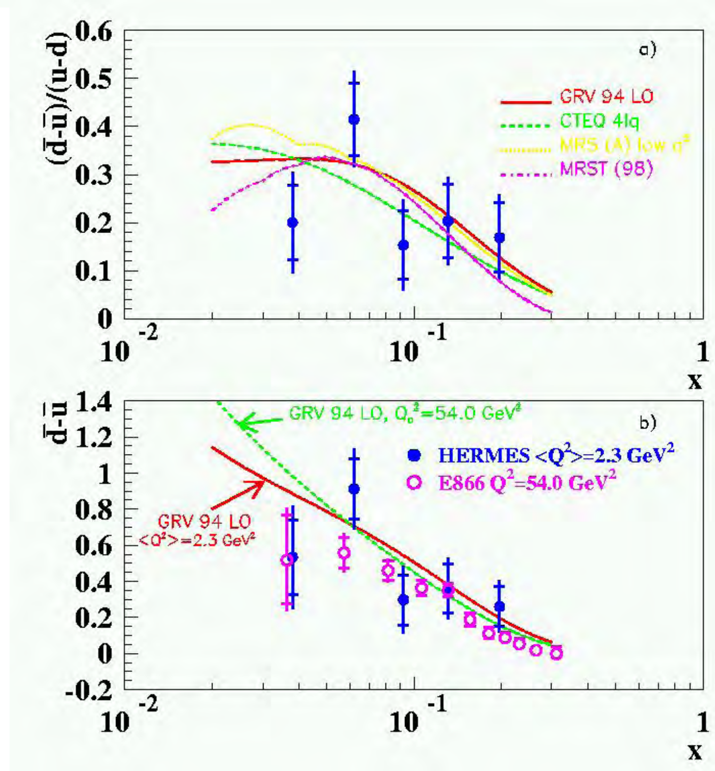
HERMES Collaboration, PRL 81, 5519 (1998)

Flavor Asymmetry of the Light Quark Sea

Not just balls of energy….

$$\frac{\bar{d}(x)-\bar{u}(x)}{u(x)-d(x)} \sim f(D^+(z)/D^-(z))\,\frac{N_p^{\pi^-}(x,z)-N_n^{\pi^-}(x,z)}{N_p^{\pi^+}(x,z)-N_n^{\pi^+}(x,z)}$$



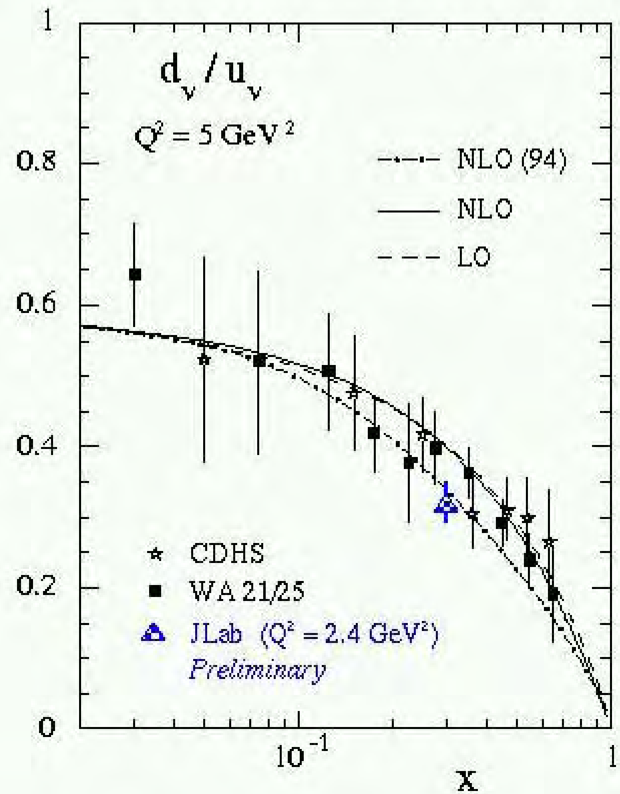


- Data do, within statistics, confirm independence of x and z dependence, for z > 0.25
- $x \sim 0.1$, $Q^2 \sim 2.3\ \mathrm{GeV}^2 \rightarrow$ W ~ 5 GeV
- HERMES data at $Q^2 \sim 2.3\ \mathrm{GeV}^2$ agree well with FNAL Drell-Yan data at $Q^2 = 54\ \mathrm{GeV}^2$

Low-Energy Factorization at JLab?

$\nu \approx 4$ GeV, $Q^2 = 2.3$ GeV^2, $W^2 = 5$ GeV^2, $z > 0.5$

- Extracted fragmentation functions agree reasonably well with NLO fragmentation function fit to high-energy e^+e^- data

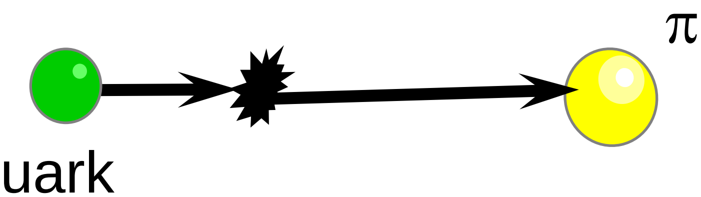


- Extraction of d_v/u_v agrees well with high energy data

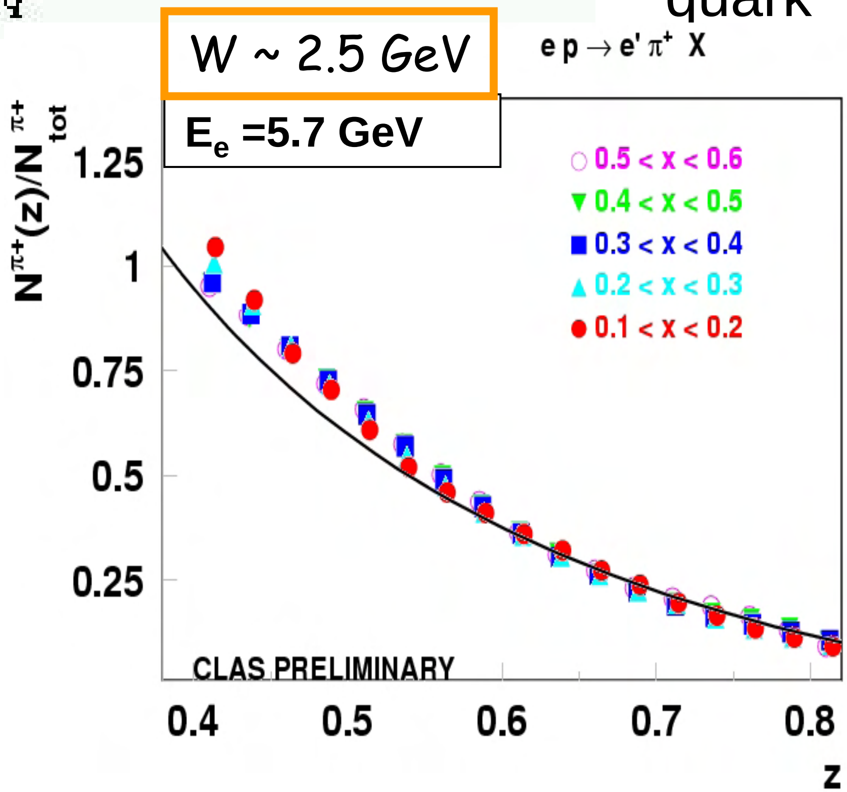
Low-Energy Factorization?

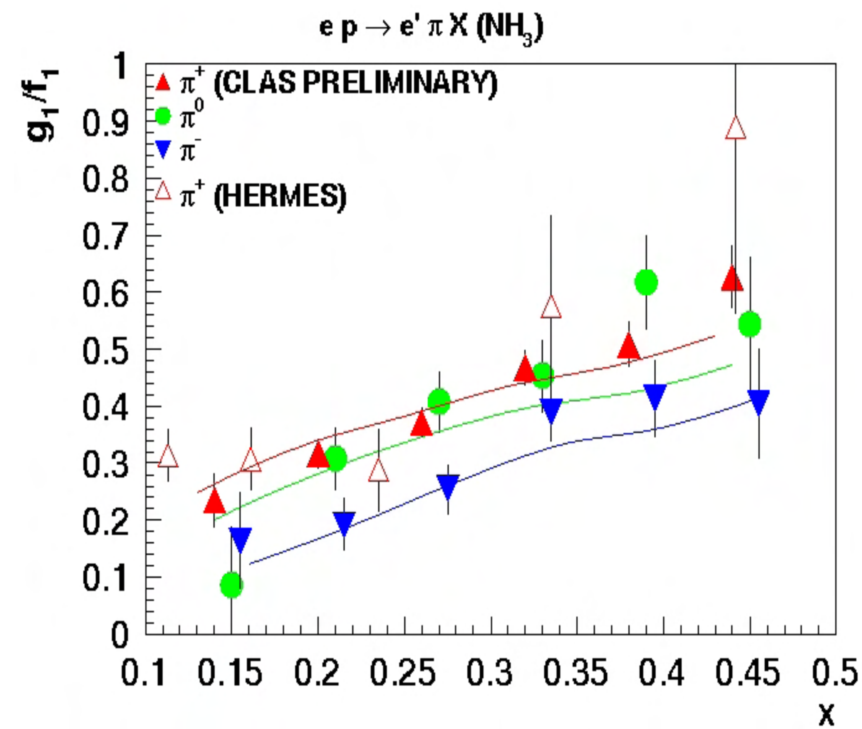
CLAS Collaboration,
H. Avakian et al.

$$\sum_q e_q^2 \, q(x) \, D_{q\to M}(z)$$



Collinear Fragmentation



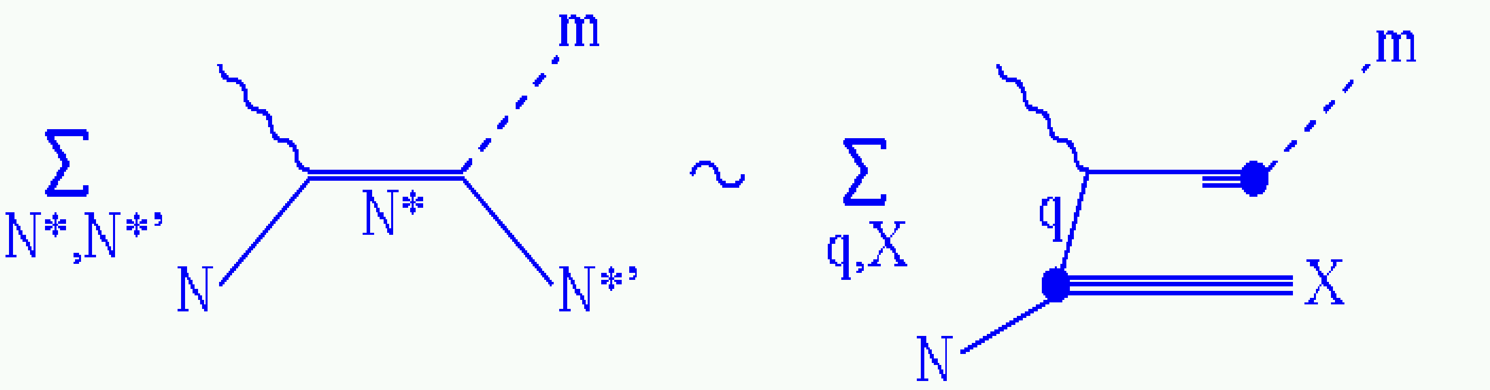


No significant variation observed in z distributions of π^+ for different **x** ranges (0.4 < z < 0.7, M_X > 1.5 GeV)

x dependence of CLAS A_1^p (A_2=0) consistent with HERMES data (at x3 higher Q^2) and with PEPSI (LUND)MC.

Duality in Meson Electroproduction

hadronic description quark-gluon description



$$\sum_{N'^*} \left| \sum_{N^*} F_{\gamma^* N \to N^*}(Q^2, W^2)\, \mathcal{D}_{N^* \to N'^* M}(W^2, W'^2) \right|^2 \qquad \sum_q e_q^2\, q(x)\, D_{q \to M}(z)$$

Transition Form Factor — Decay Amplitude — Fragmentation Function

Requires non-trivial cancellations of decay angular distributions
If duality is not observed, factorization is questionable

Duality and factorization possible for $Q^2, W^2 \leq 3$ GeV^2
(Close and Isgur, Phys. Lett. B509, 81 (2001))

E00-108 data

$$\sum e_q^2\, q(x)\, D_{q\to M}(z)$$

A little bit more complicated….

P_t, ϕ

$$\frac{\frac{d\sigma}{d\Omega_e dE_e dx dp_\perp^2 d\phi}}{\frac{d\sigma}{d\Omega_e dE_e}} = \frac{dN}{dz}\, b \exp(-bp_\perp^2)\, \frac{1 + A\cos(\phi) + B\cos(2\phi)}{2\pi}$$

$q, \bar{q}$

$$dN/dz \cong \Sigma_i e_i^2[q_i(x,Q^2)D_{qi}^m(z,Q^2) + \bar{q}_i(x,Q^2)D_{\bar{q}i}^m(z,Q^2)]$$

Add fragmentation process to general
Hall C MC Simulation Package ("SIMC")

Input parameters:

Pdf's (q_i, q_i)	: CTEQ5M
FF's (D_{qi})	: Binnewies et al., given as ($D^+ + D^-$)
D^-/D^+	: from HERMES
P_t (b)	: from our data (b = 4.37)
ϕ	: assume no ϕ dep.

E00-108 data

$$\sum e_q^2\, q(x)\; D_{q\to M}(z)$$

A little bit more complicated….

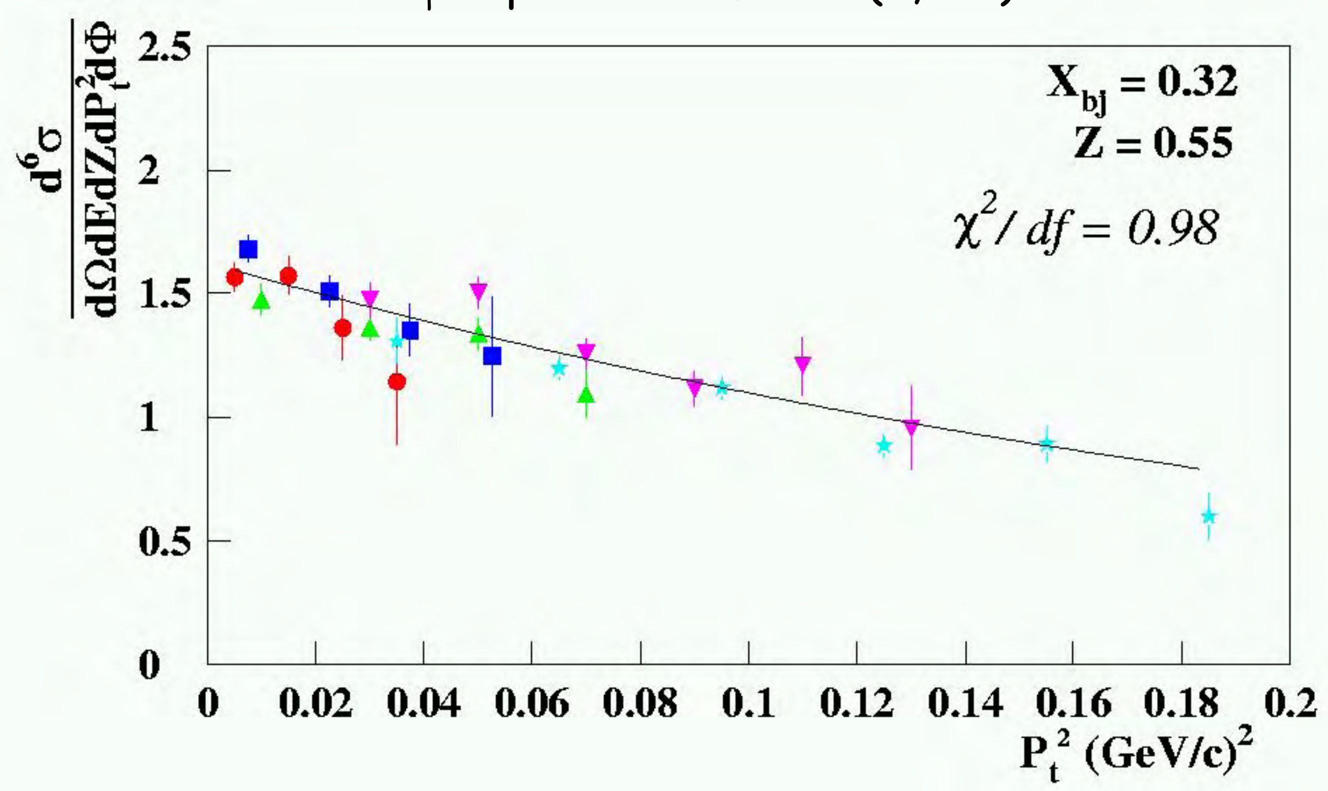
P_t, ϕ

$$\frac{\frac{d\sigma}{d\Omega_e\, dE_e\, dx\, dp_\perp^2\, d\phi}}{\frac{d\sigma}{d\Omega_e\, dE_e}} = \frac{dN}{dz}\, b\exp(-bp_\perp^2)\, \frac{1 + A\cos(\phi) + B\cos(2\phi)}{2\pi}$$

q, $\bar{q}$

$$dN/dz \cong \Sigma_i e_i^2 [q_i(x,Q^2) D_{qi}^m(z,Q^2) + \bar{q}_i(x,Q^2) D_{\bar{q}i}^m(z,Q^2)]$$

P_t dependence for $^1H(e,e'\pi^-)X$



(W' ~ 1.8 GeV)

E00-108 data

$$\sum e_q^2\, q(x)\; D_{q\to M}(z)$$

A little bit more complicated....

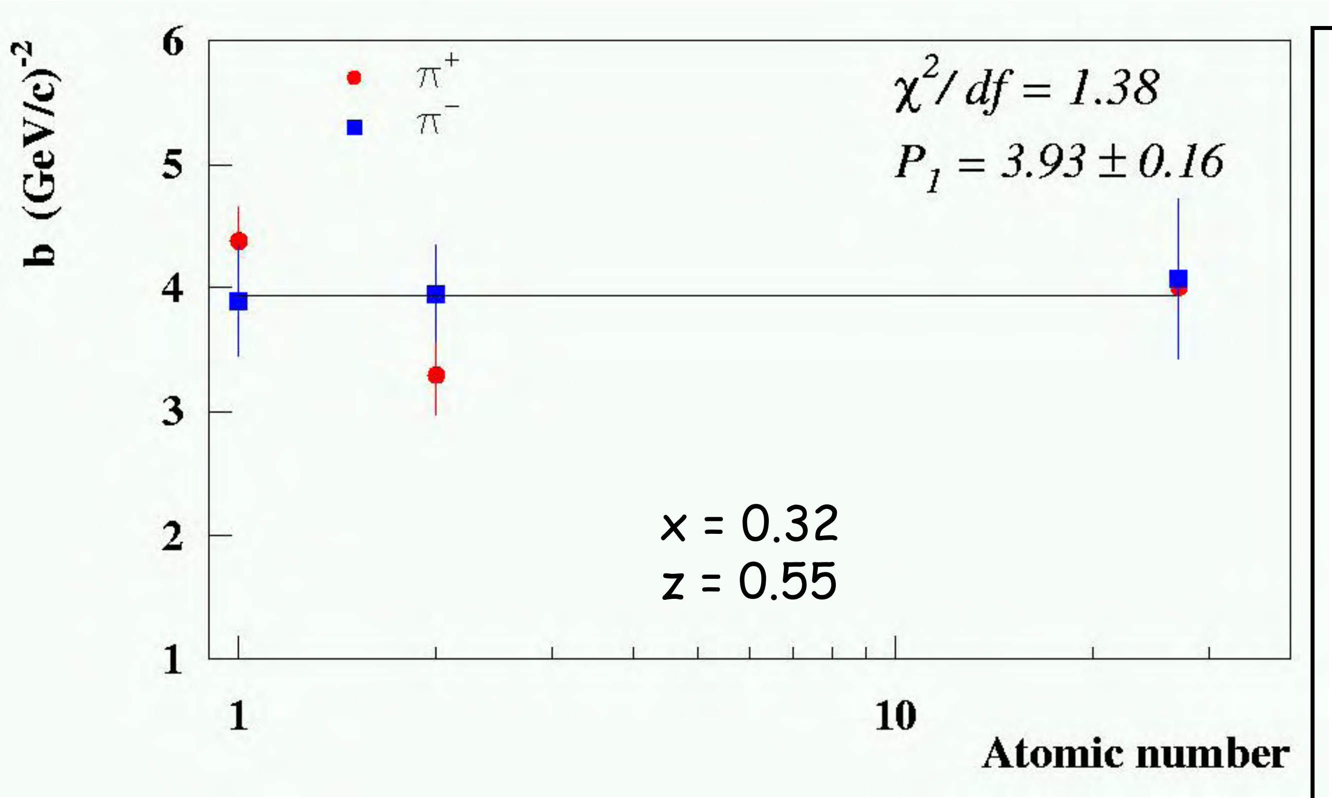
P_t, ϕ

$$\frac{\frac{d\sigma}{d\Omega_e dE_e dx dp_\perp^2 d\phi}}{\frac{d\sigma}{d\Omega_e dE_e}} = \frac{dN}{dz} b \exp(-bp_\perp^2) \frac{1 + A\cos(\phi) + B\cos(2\phi)}{2\pi}$$

q, $\bar{q}$

$$dN/dz \cong \Sigma_i e_i^2 [q_i(x,Q^2) D_{qi}^m(z,Q^2) + \bar{q}_i(x,Q^2) D_{\bar{q}i}^m(z,Q^2)]$$

P_t dependence for $^1H(e,e'\pi^{+/-})X$, $^2H(e,e'\pi^{+/-})X$, $^{27}Al(e,e'\pi^{+/-})X$



[Scaling in Electropion Production earlier shown with Cornell data by Calogeracos, Dombey and West, who also show that the p_t distribution is, within uncertainties, independent of kinematics, with much of the data at W' < 2 GeV: Phys. Rev. D 51, 6075 (1995).]

Scaling in Electropion Production

Calogeracos, Dombey, and West, Phys. Rev. D 51, 6075 (1995)

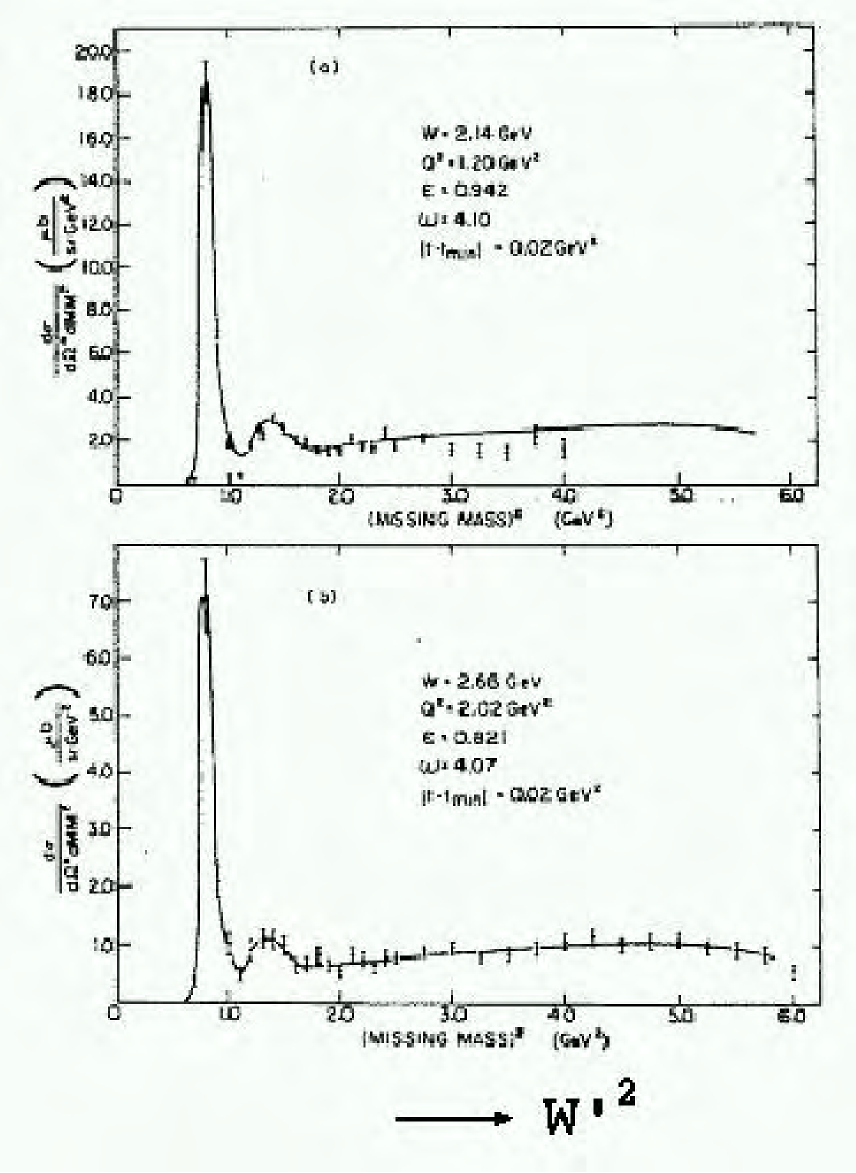
$$s^2 \frac{d^2\sigma}{dt\,dW'^2} = F(x,t,W'^2)$$

Cornell : $x = 0.24$
$t = 0.02$
(end of '70's)

$\sqrt{s}$ = 2.66 vs. 3.14

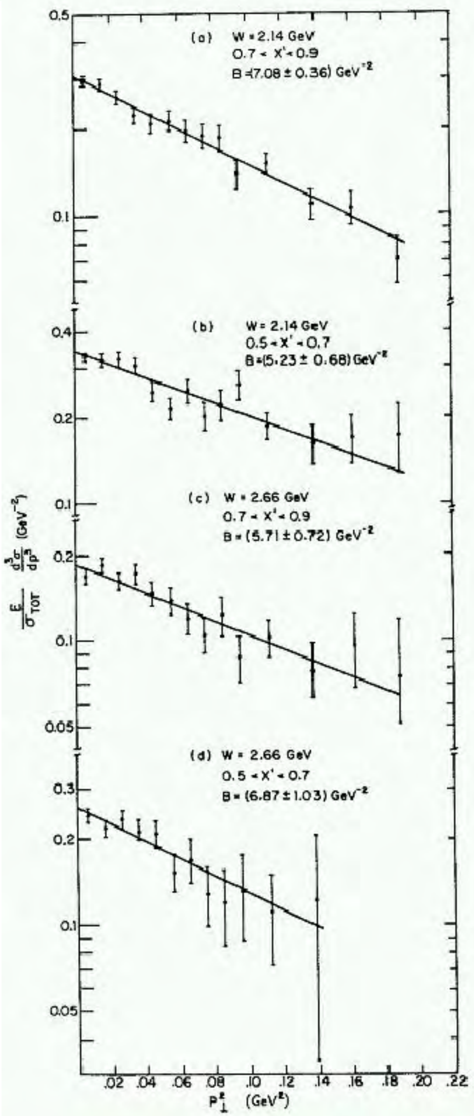
(i.e. Q^2 and W^2 different)

→ Scaling factor of 2.4

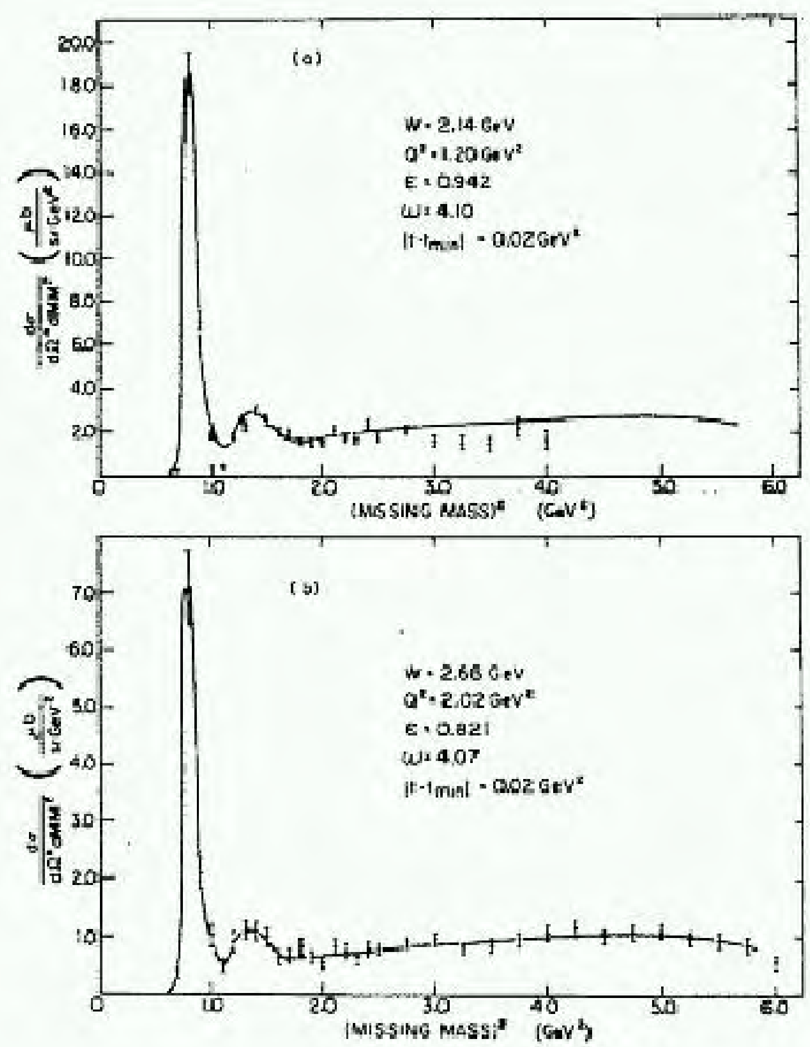


Scaling in Electropion Production

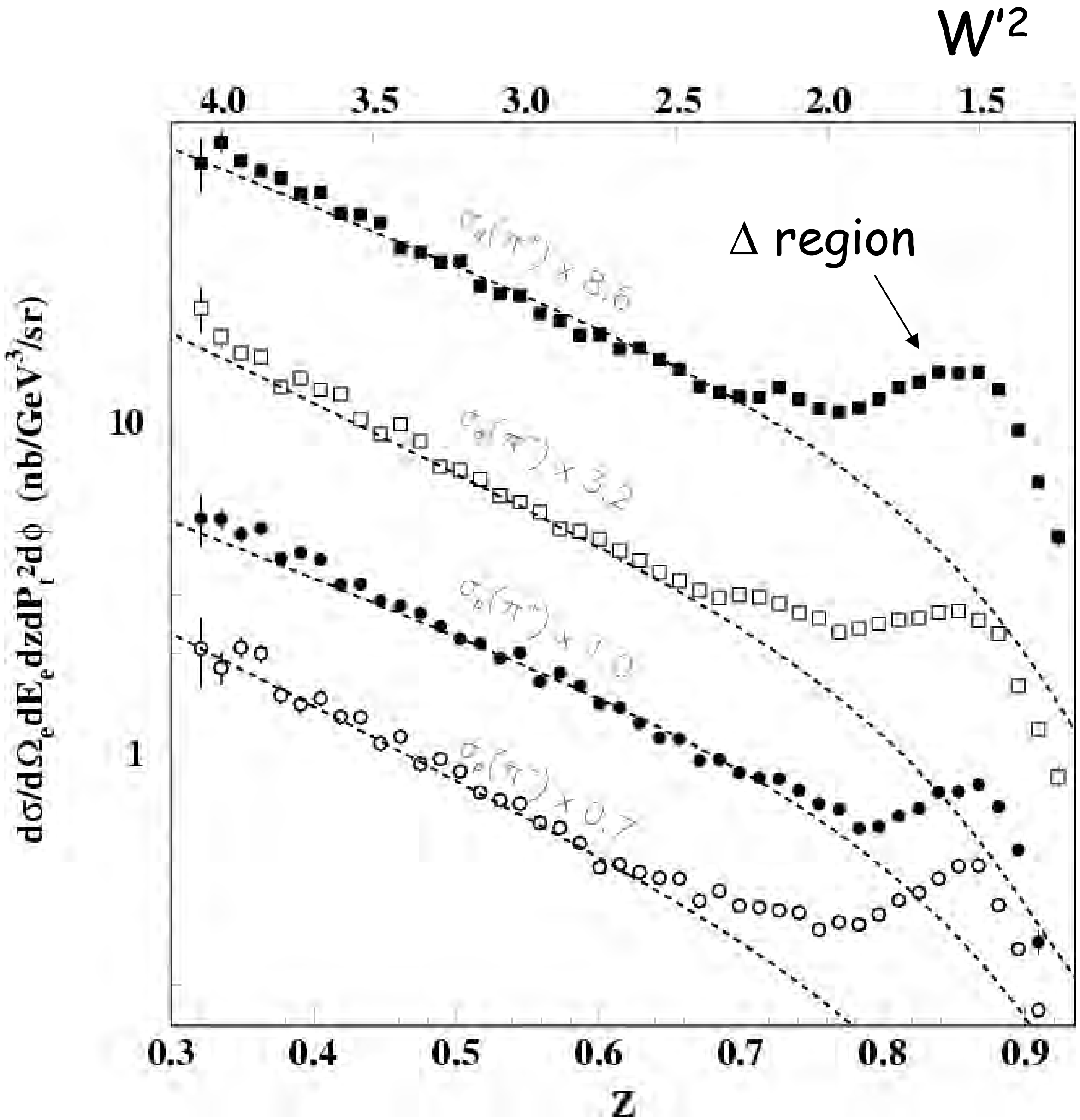
Calogeracos, Dombey, and West, Phys. Rev. D 51, 6075 (1995)



Same P_T dependence



→ W'^2



Dashed curves: SIMC

$$\sum e_q^2 \, q(x) \, D_{q \to M}(z)$$

Surprising similarity between low-energy factorization ansatz and JLab $^1H(e,e'\pi^+)X$, $^1H(e,e'\pi^-)X$, $^2H(e,e'\pi^+)X$, and $^2H(e,e'\pi^-)X$ data.

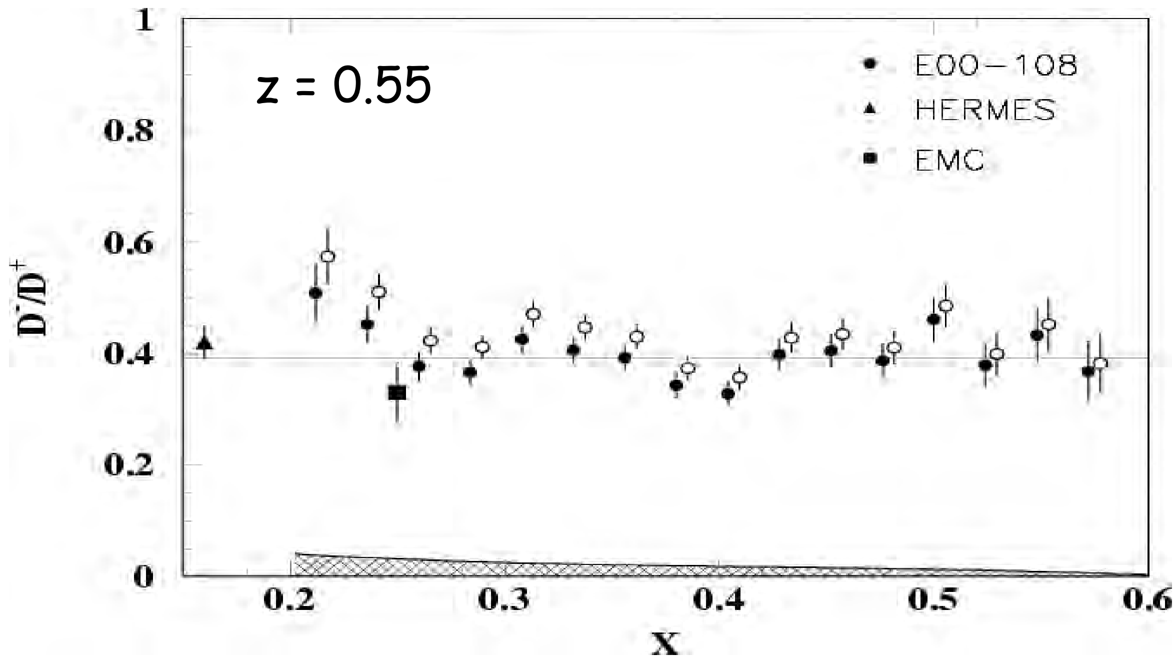
Berger Criterion "works"

From deuterium data: $D^-/D^+ = (4 - N_\pi^+/N_\pi^-)/(4N_\pi^+/N_\pi^- - 1)$

$$\sum_q e_q^2\, q(x)\, D_{q\rightarrow M}(z)$$

→ D^-/D^+ ratio should be *independent* of x

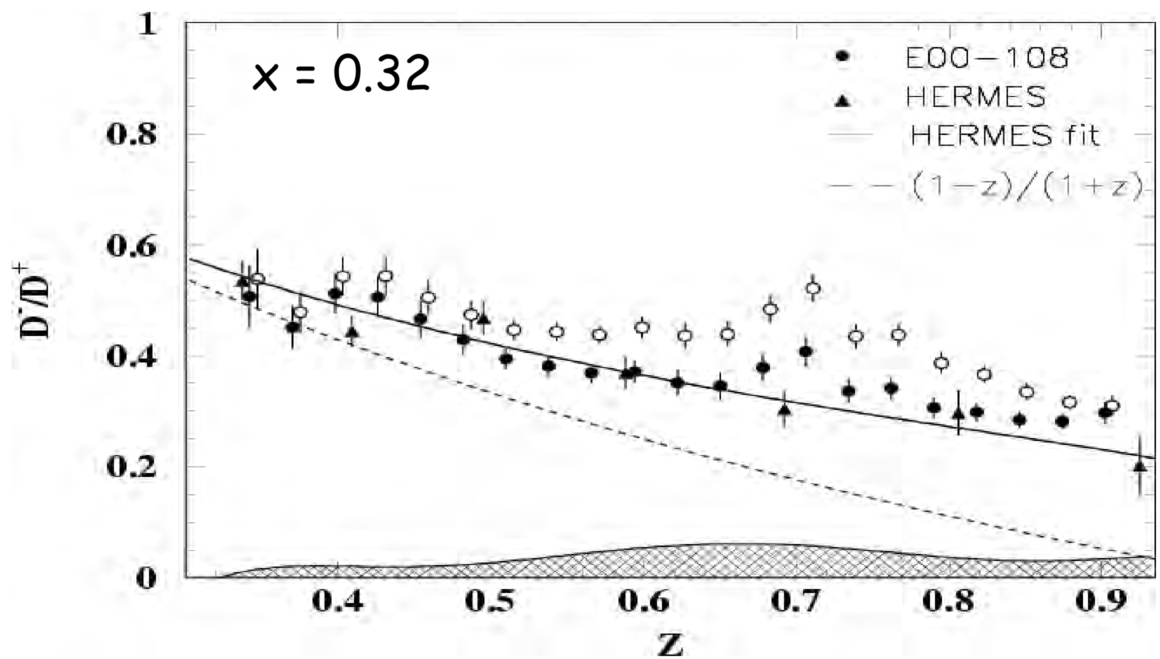
(D⁻ is the "favored", and D⁺ is the "unfavored" fragmentation function)



but *should* depend on z

→ Find similar slope versus z as HERMES, but slightly larger values for D^-/D^+, and definitely not the values expected from Feynman-Field independent fragmentation at large z!

(Note: JLab Data *all* in the nucleon resonance region!)



W′ = 2 GeV ~ z = 0.35 → data predominantly in "resonance region"
What happened to the resonances?

From deuterium data: $D^-/D^+ = (4 - N_\pi^+/N_\pi^-)/(4N_\pi^+/N_\pi^- - 1)$

The Origins of Quark-Hadron Duality – Semi-Inclusive Hadroproduction

F. Close et al : SU(6) Quark Model

How many resonances does one need to average over to obtain a complete set of states to mimic a parton model? →56 and 70 states o.k. for closure

Destructive interference leads to factorization and duality

$$F(\gamma N \to \pi X)(x,z) = \sum_{N^*,N^{*\prime}} F_{\gamma^* N \to N^*}(Q^2, W^2)\, \mathcal{D}_{N^* \to N^{*\prime}\pi}(W^2, W'^2)$$

$$\sim \sum_q e_q^2\, q(x)\, D_{q\to\pi}(z)$$

$SU(6)$ and $SU(3) \times SU(2)$ Multiplet Contributions to $\pi^{\pm}$ Photoproduction

W'	$p(\gamma,\pi^+)W'$	$p(\gamma,\pi^-)W'$	$n(\gamma,\pi^+)W'$	$n(\gamma,\pi^-)W'$
56;8	100	0	0	25
56;10	32	24	96	8
70;^{2}8	64	0	0	16
70;^{4}8	16	0	0	4
70;^{2}10	4	3	12	1
Total	216	27	108	54

Predictions: Duality obtained by end of second resonance region

Factorization and approximate duality for $Q^2, W^2 < 3$ GeV^2

Duality in Pion Electroproduction

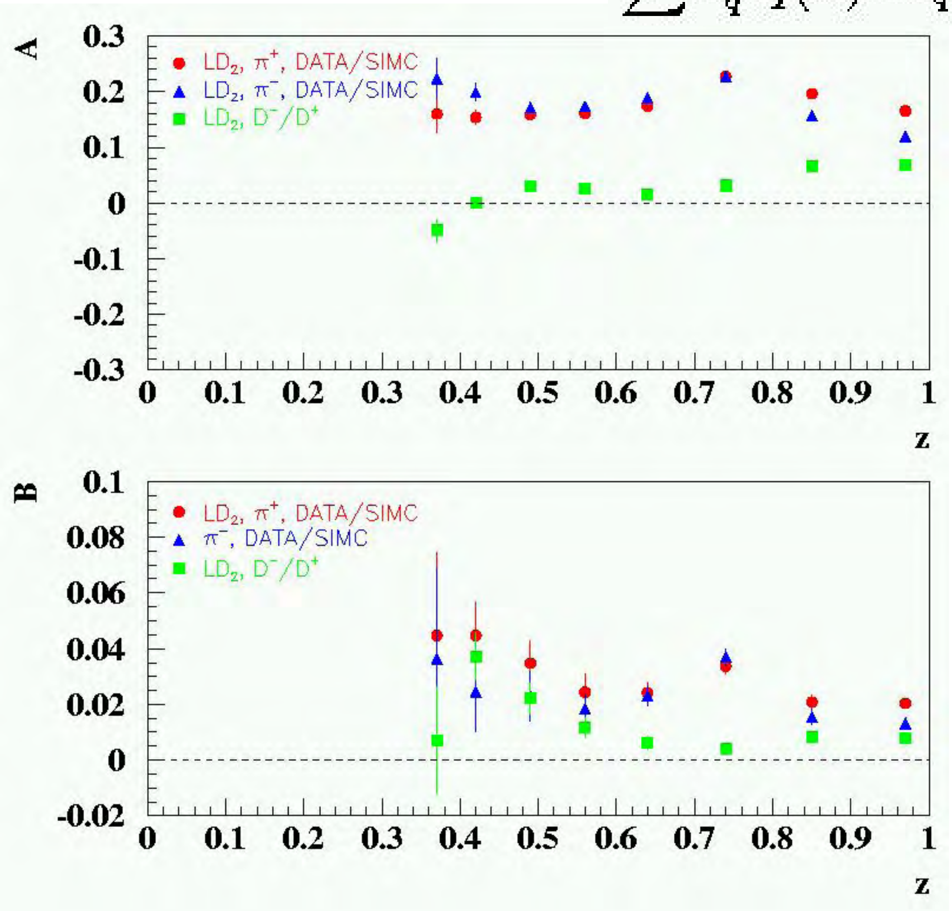
... or Why does factorization appear to work well at low energies ...

E00-108, Spokespersons: Hamlet Mkrtchyan (Yerevan), Gabriel Niculescu (JMU), Rolf Ent (JLab)

$$\frac{\frac{d\sigma}{d\Omega_e dE_e dx dp_\perp^2 d\phi}}{\frac{d\sigma}{d\Omega_e dE_e}} = \frac{dN}{dz} b \exp(-bp_\perp^2) \frac{1 + A\cos(\phi) + B\cos(2\phi)}{2\pi}$$

$$\frac{dN}{dz} \rightarrow \sum e_q^2\, q(x)\, D_{q\to M}(z)$$

$A \sim \sigma_{LT}$, $B \sim \sigma_{TT}$



The pion electroproduction cross section itself has a noticable ϕ dependence, but this effect nearly cancels in the ratio D^-/D^+.

How Can We Verify Factorization?

Neglect sea quarks and assume no p_t dependence to parton distribution functions

→ Fragmentation function dependence drops out in Leading Order

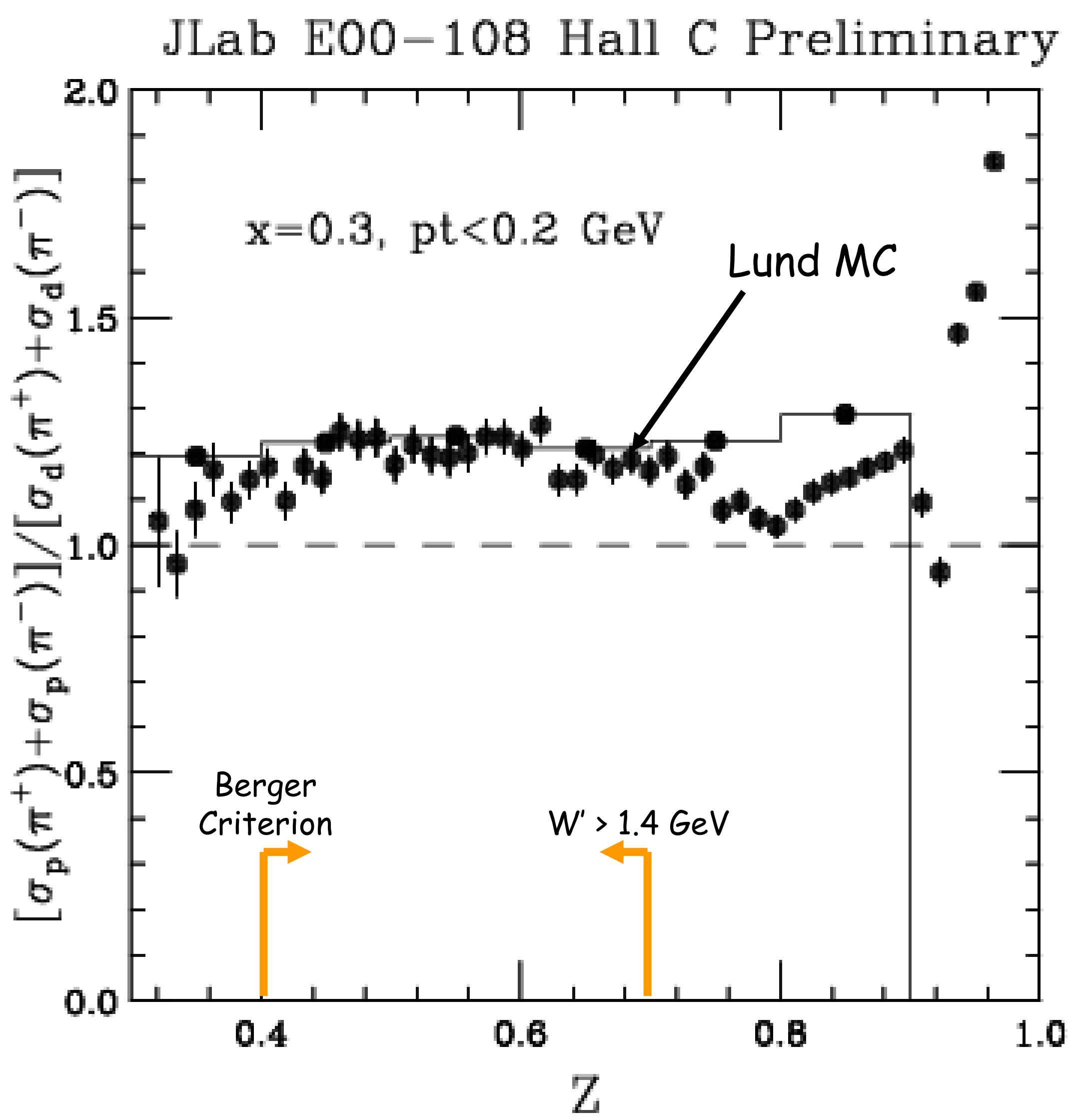
$$\rightarrow [\sigma_p(\pi^+) + \sigma_p(\pi^-)]/[\sigma_d(\pi^+) + \sigma_d(\pi^-)]$$

$$= [4u(x) + d(x)]/[5(u(x) + d(x))]$$

$$\sim \sigma_p/\sigma_d$$ *independent of z and p_t*

Dependence on z

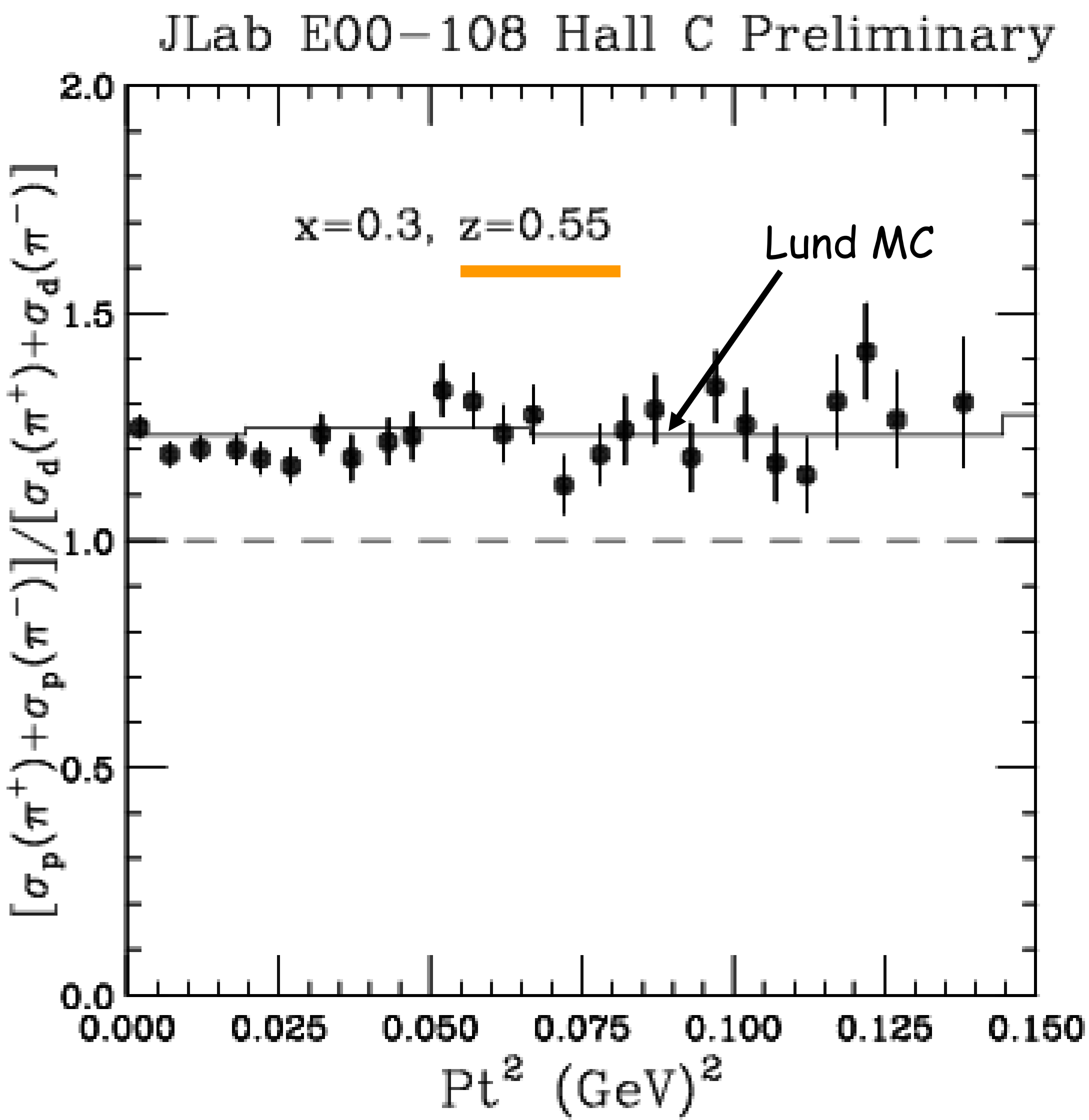
No Radiative Corrections applied yet!



Still to quantify (preliminary data only) …

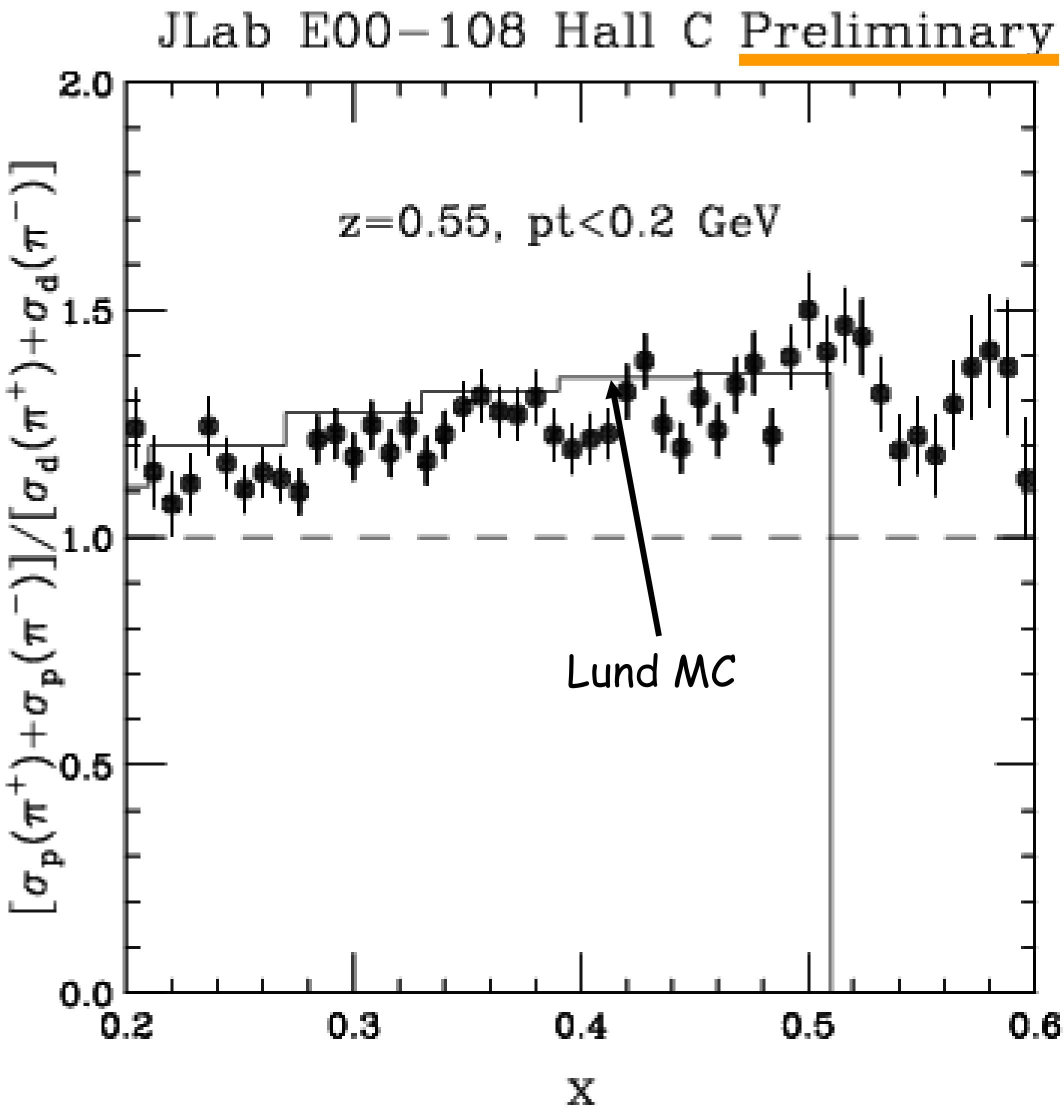
Dependence on p_t

No Radiative Corrections applied yet!



… but data not inconsistent with factorization

Dependence on x



No Radiative Corrections applied yet!

… trend of expected x-dependence, but slightly low?

How Can We Verify Factorization?

Neglect sea quarks and assume no p_t dependence to parton distribution functions

→ Fragmentation function dependence drops out in Leading Order

$$\rightarrow [\sigma_p(\pi^+) + \sigma_p(\pi^-)]/[\sigma_d(\pi^+) + \sigma_d(\pi^-)]$$

$$= [4u(x) + d(x)]/[5(u(x) + d(x))]$$

$$\sim \sigma_p/\sigma_d$$ *independent of z and p_t*

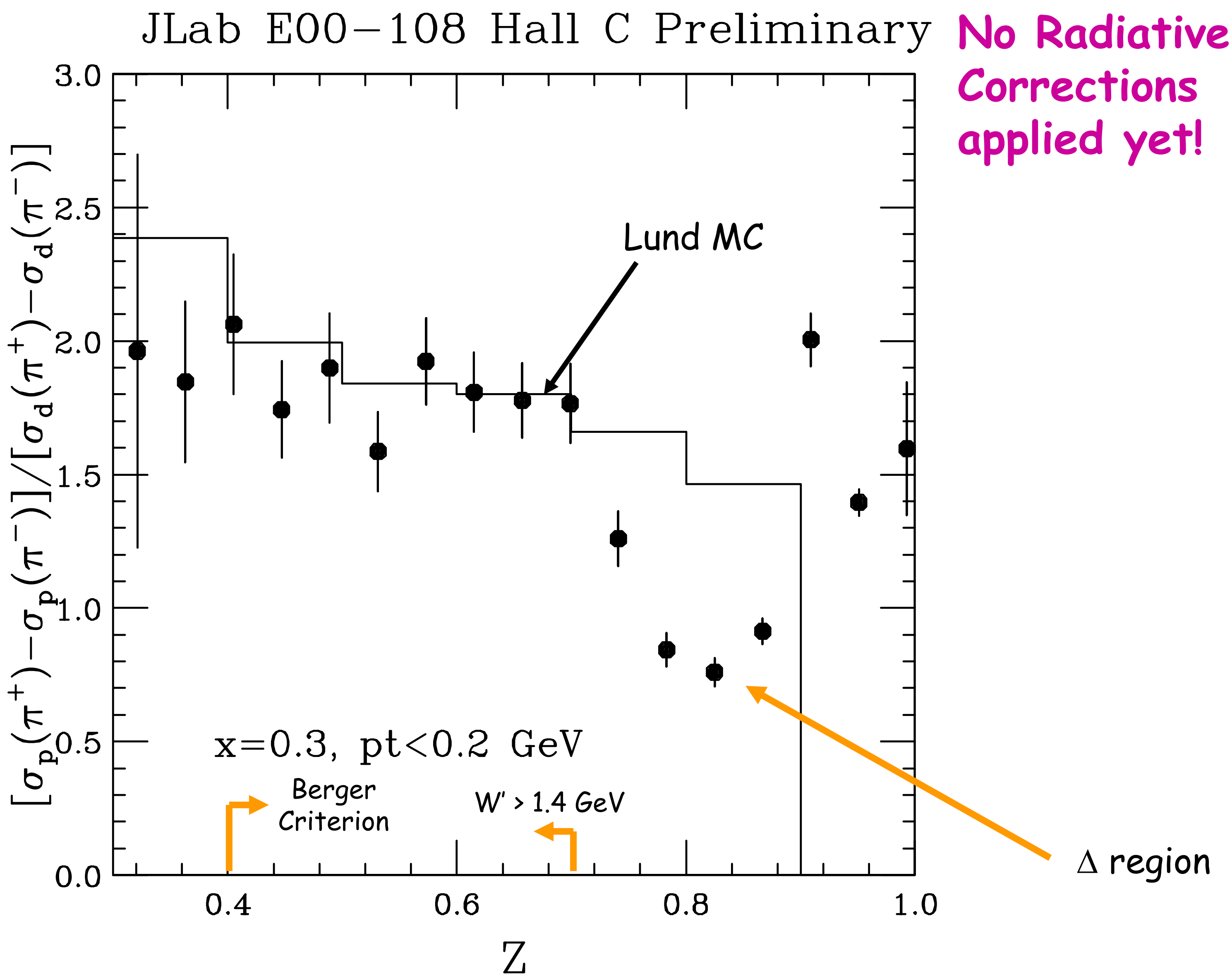
$$\rightarrow [\sigma_p(\pi^+) - \sigma_p(\pi^-)]/[\sigma_d(\pi^+) - \sigma_d(\pi^-)]$$

$$= [4u(x) - d(x)]/[3(u(x) + d(x))]$$

independent of z and p_t,
but more sensitive to assumptions

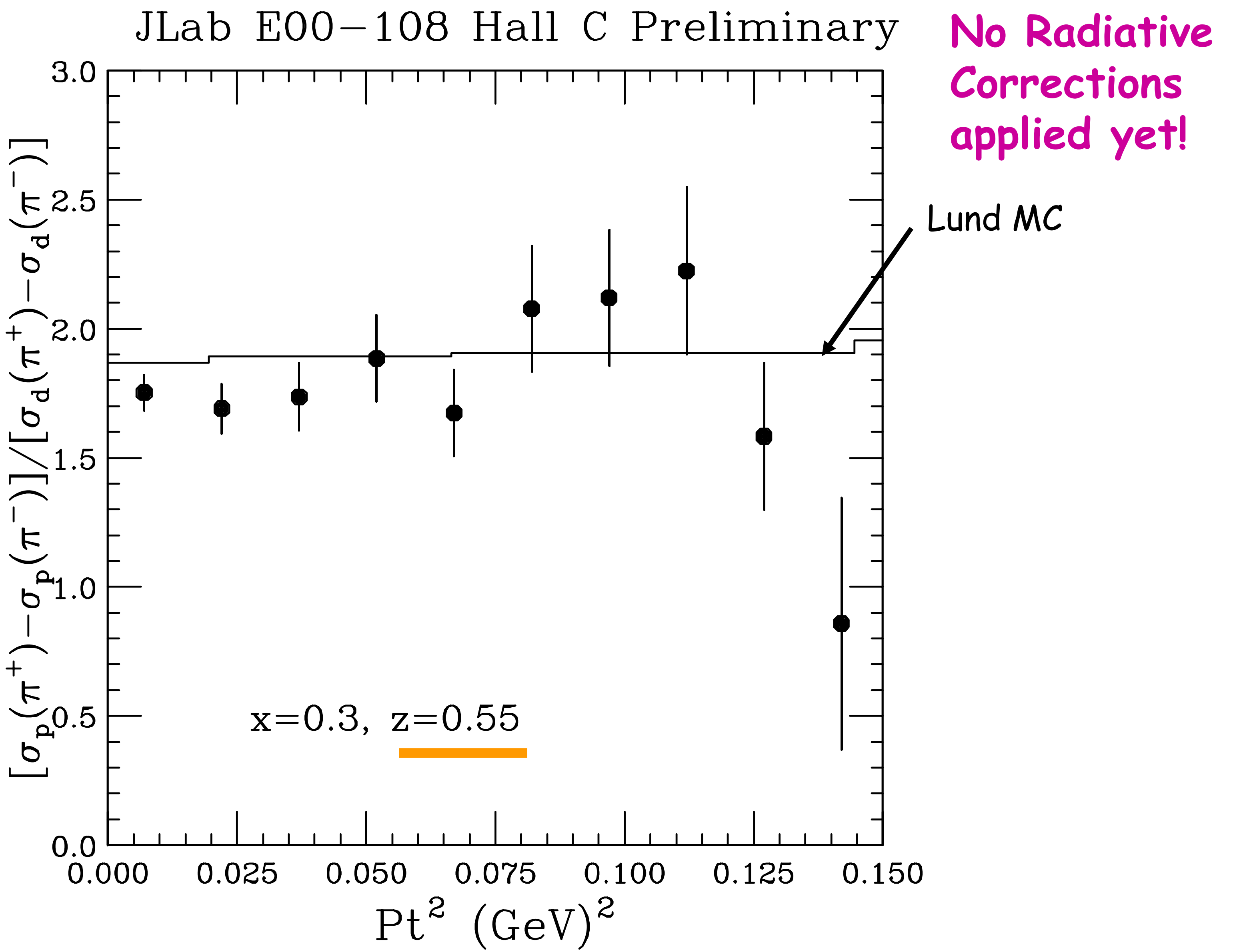
Dependence on z

No Radiative Corrections applied yet!



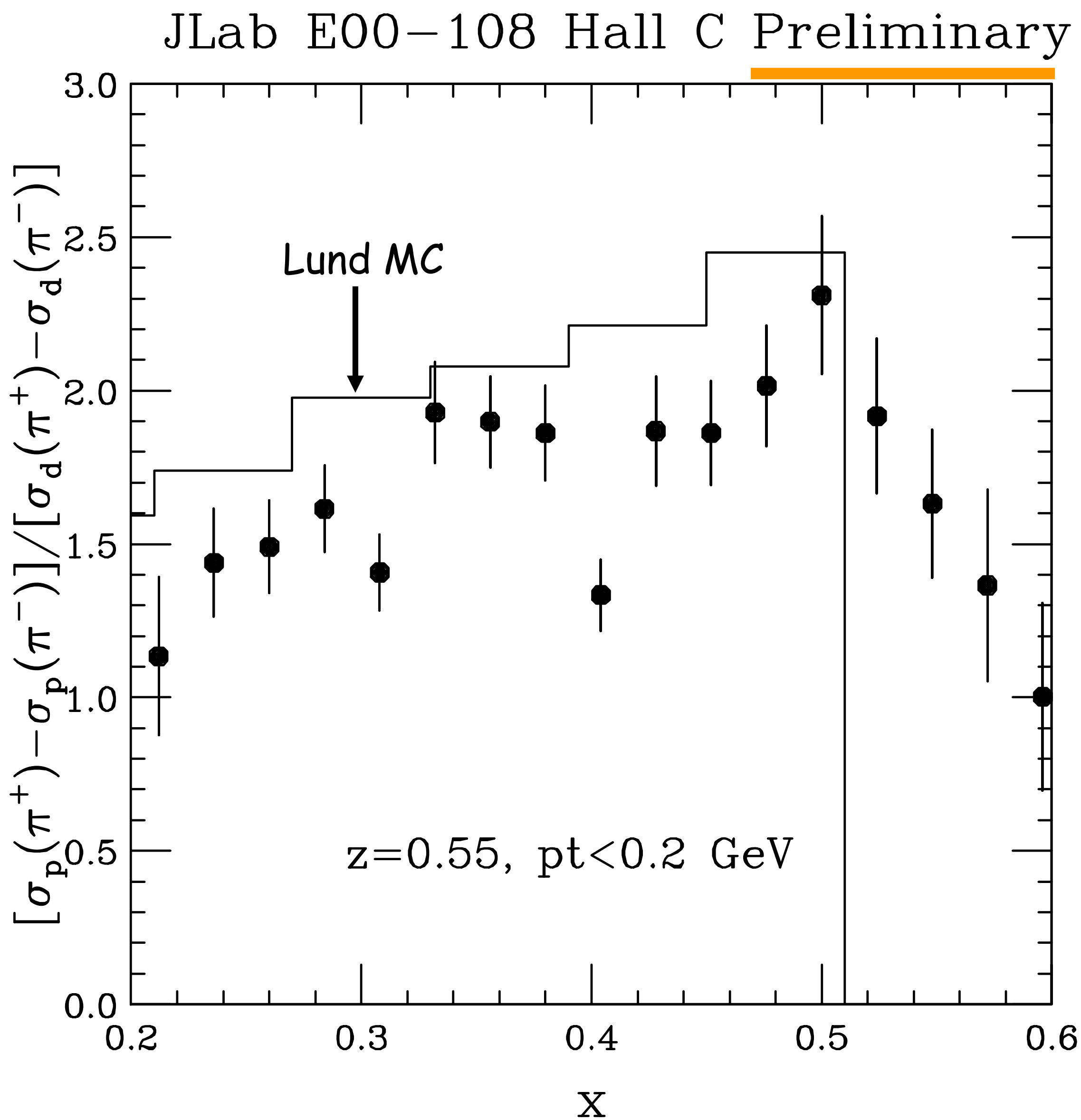
Difference clearly more sensitive … but 0.4 < z < 0.7 seems to work

Dependence on p_t



… but data again not inconsistent with factorization

Dependence on x



No Radiative Corrections applied yet!

... and again slightly low compared to expected x-dependence.

Duality in Pion Electroproduction

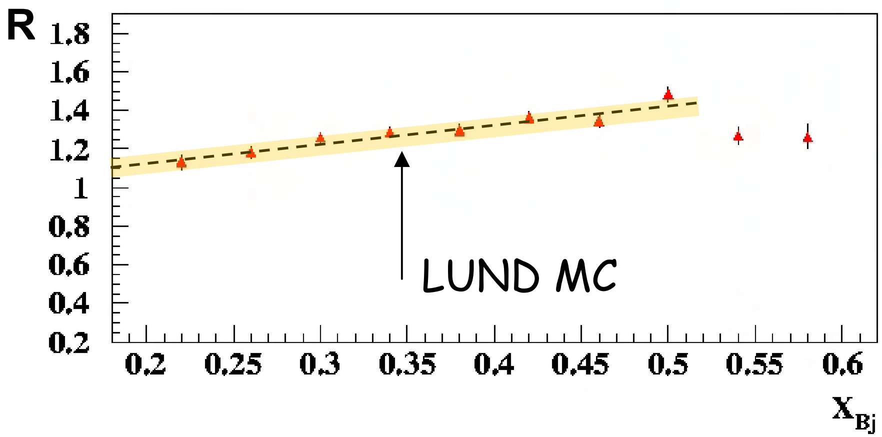
... or Why does factorization appear to work well at low energies ...

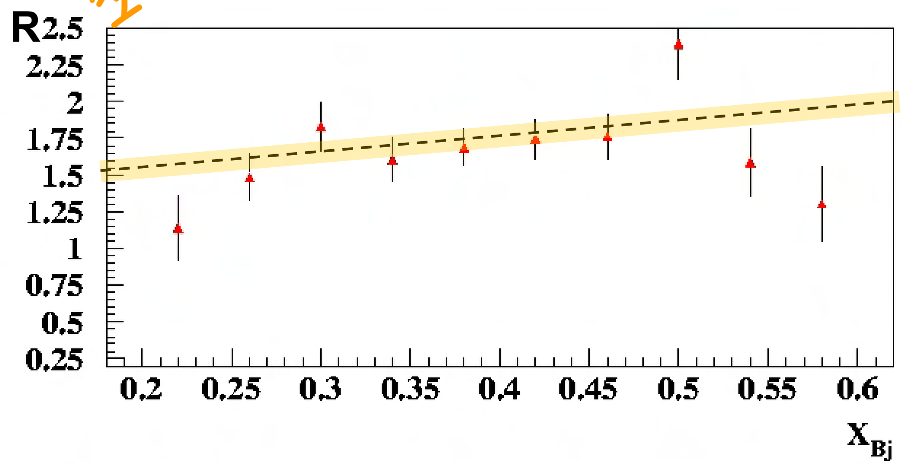
E00-108, Spokespersons: Hamlet Mkrtchyan (Yerevan), Gabriel Niculescu (JMU), Rolf Ent (Jlab)

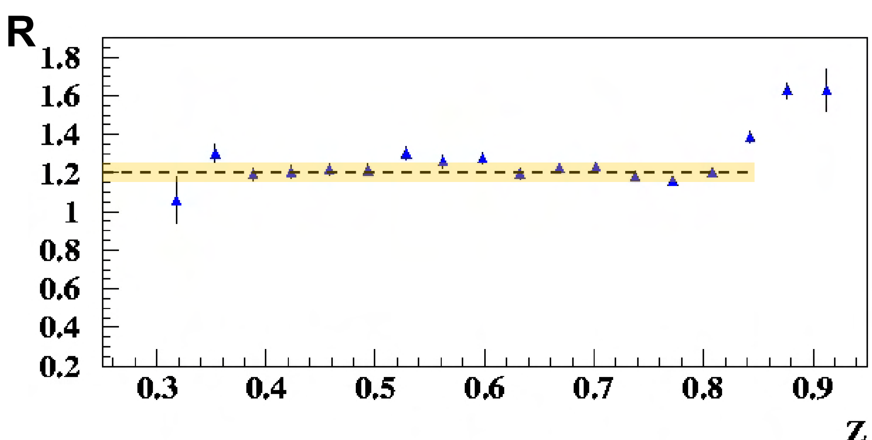
$$R = \frac{\sigma_p(\pi^+)+\sigma_p(\pi^-)}{\sigma_d(\pi^+)+\sigma_d(\pi^-)}$$

Still Preliminary

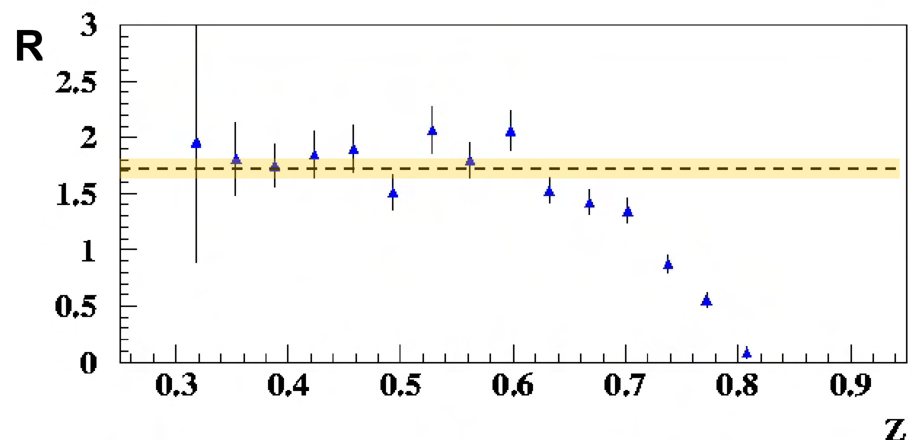
$$R = \frac{\sigma_p(\pi^+)-\sigma_p(\pi^-)}{\sigma_d(\pi^+)-\sigma_d(\pi^-)}$$





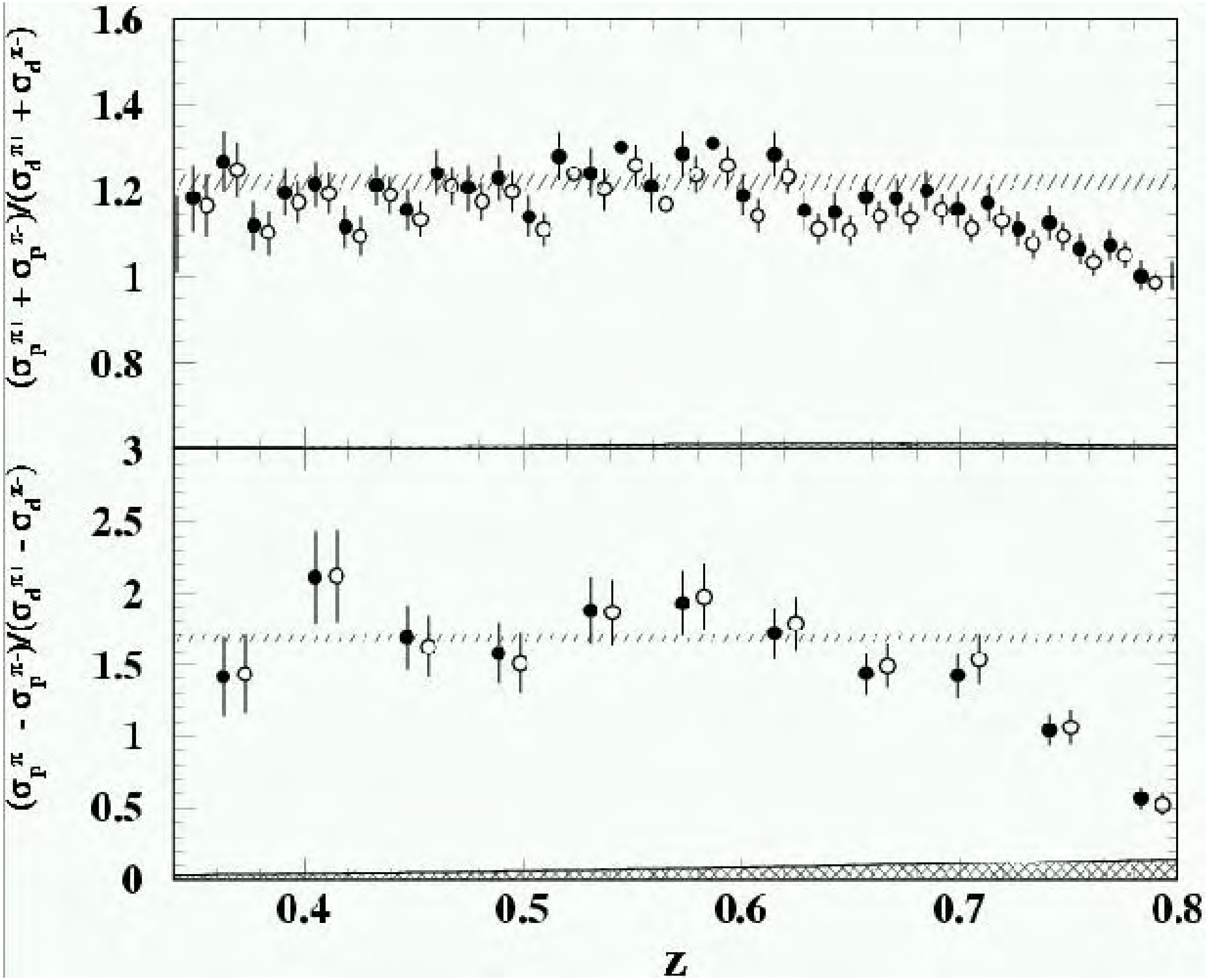


"Works" better

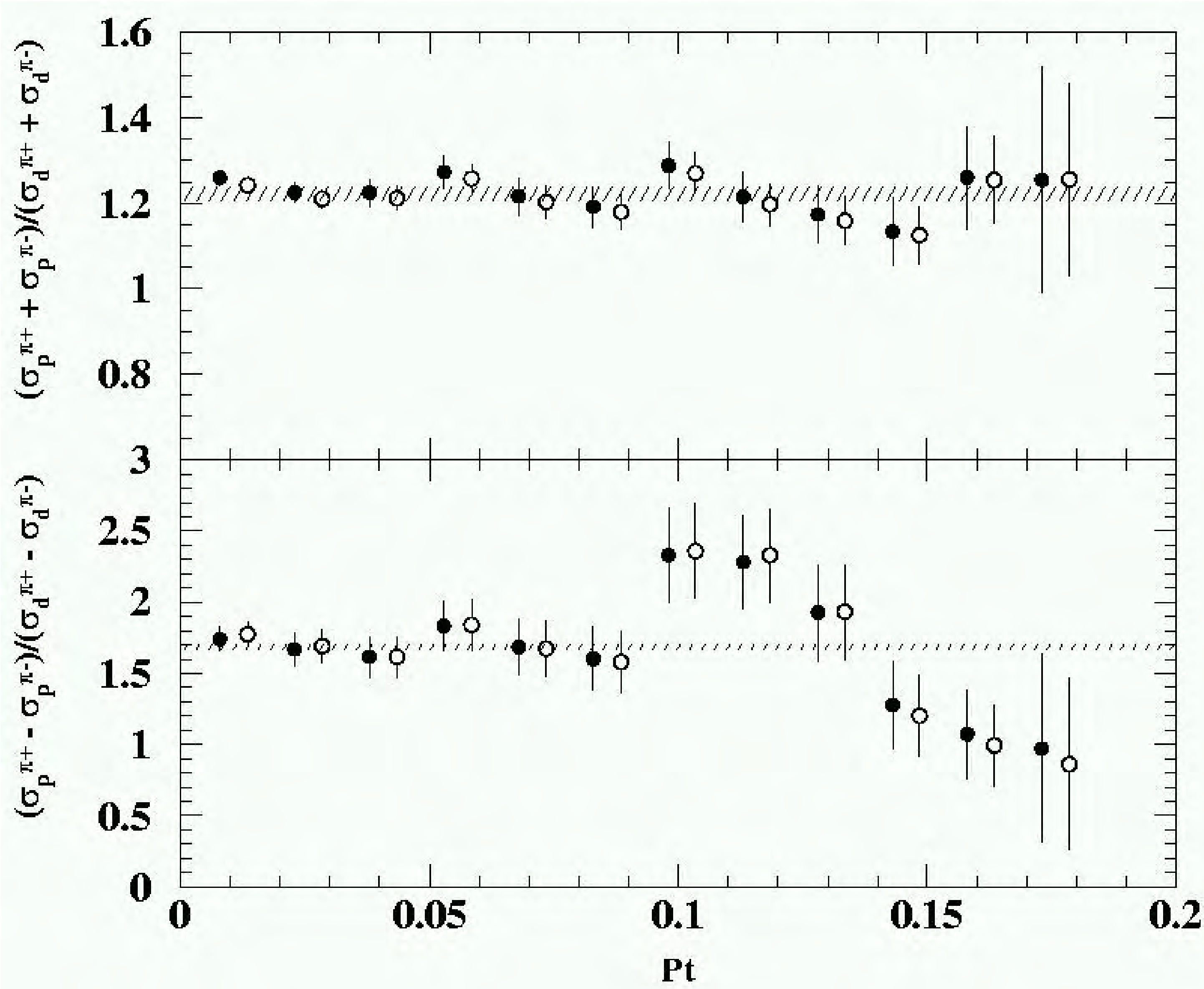


"Works" worse

Final data look good! … Dependence on z



Final data look good! ... Dependence on p_t



Data consistent with factorization!

Conclusion

High quality hadronic structure function data at JLab at 6 GeV have been accumulated spanning the nucleon resonance and low-W^2 deep inelastic region. The data indicate a surprisingly smooth transition from "Quark Model physics" to "Parton Model physics" at relatively low Q^2.

- Onset of quark-hadron duality in pion electroproduction seems unambiguously verified.
- Berger criterion may be applicable for $z > 0.4$ at JLab energies.
- Factorization tests work surprisingly well for $0.4 < z < 0.7$, $p_T < 1$ GeV, even for $W' < 2$ GeV (another evidence of quark-hadron duality).
- p_T dependence seems consistent with data from higher-energy experiments (HERMES)
- Breakdown of factorization at high z may be due to Delta excitation ($z > 0.7$) or diffraction vector meson production (still ongoing analysis).
- Still a lot of work to do to quantify low-energy factorization…