

# Phenomenology at low $Q^2$

## Lecture IV

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HUGS @ JLAB  
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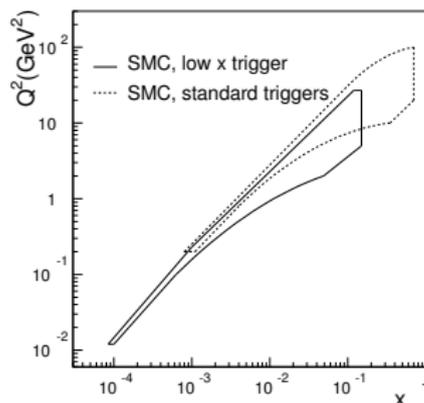
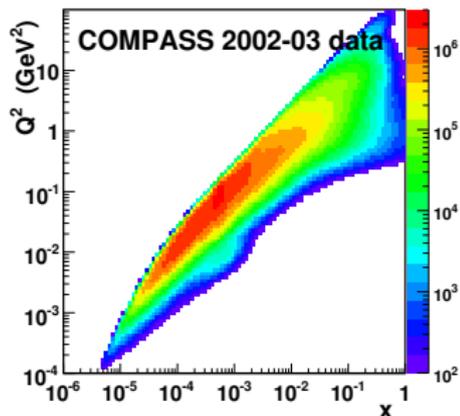
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  - Acceptance
  - Status of  $g_1$  measurements
- 2 Regge model predictions for  $g_1$
- 3 Low  $x$  implications from the pQCD
- 4  $\text{Ln}^2(1/x)$  corrections to  $g_1(x, Q^2)$ 
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# High energy DIS spin experiments

## Acceptance

- 1 E142, E143, E154, E155, E155X at SLAC; electrons of  $< 50$  GeV, targets: protons, deuterons, helium-3;
- 2 EMC, SMC, COMPASS at CERN; muons of 90 – 280 GeV, targets: protons, deuterons;
- 3 HERMES at DESY; electrons of 30 GeV, targets: protons, deuterons, (helium-3);
- 4 STAR, PHENIX at BNL;  $pp$  collider,  $\sqrt{s} = 200$  GeV;
- 5 Kinematic variables from incident and scattered leptons in 1, 2, 3; hadrons from target fragmentation often also measured and – in case of 2, 3 – identified if momenta larger than 1 and 2.5 GeV respectively;
- 6 background due to  $\mu e$  scattering (at  $x = 0.000545$ ) in 2, 3;



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# High energy DIS spin experiments...cont'd

## Status of $g_1$ measurements

Spin-dependent cross sections are a small part of the DIS cross section  $\implies$  c.s. asymmetries  $\implies$  getting  $A_{\parallel}$  then  $A_1$  then (using  $F_2$  and  $R$ )  $g_1$ . Practical matters in: R. Windmolders, in "Spin in physics", X Séminaire Rhodanien de Physique, eds Anselmino, Mila, Soffer, Frontier Group, 2002.

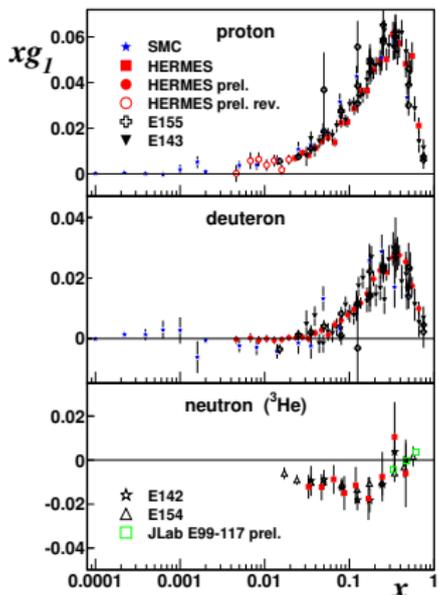


Figure from Stössllein, Acta Phys. Pol. B33 (2002) 2813

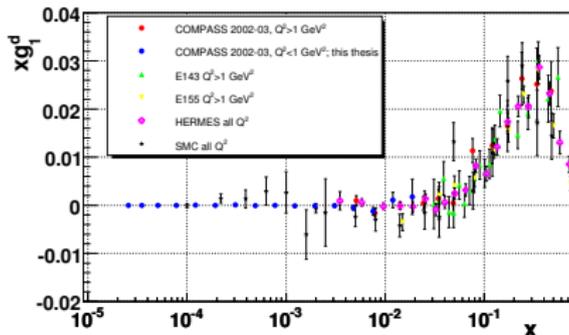


Figure comes from M. Stolarski, PhD, Warsaw University, 2006

All data at their quoted mean  $Q^2$ ;  
New precise results from COMPASS on  $g_1^d$ ,  
for  $x > \text{few} \times 10^{-6}$ ;

Lowest  $x$  for  $g_1^p$  from SMC,  $x > 0.00006$ ;

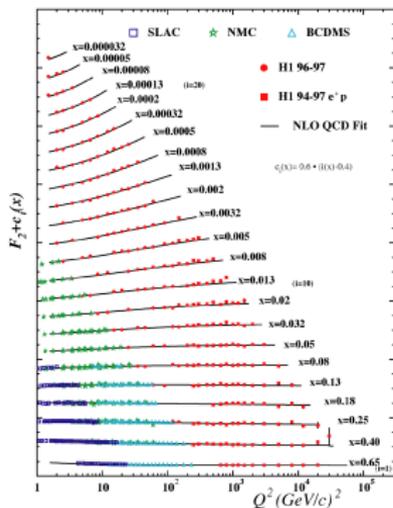
Direct measurements on neutron for  $x \gtrsim 0.02$ .

**No significant spin effects seen at lowest  $x$ !**

# High energy DIS spin experiments...cont'd

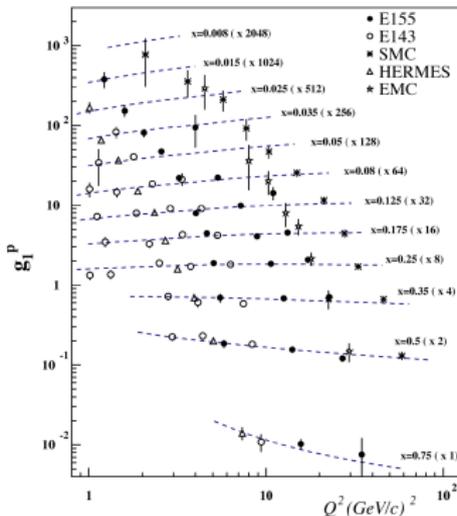
Status of  $g_1$  measurements...cont'd

## World data on $F_2^p$



→ 50% of momentum  
carried by gluons

## World data on $g_1^p$



→ 20% of proton spin  
carried by quark spin

Figure from R.Ent, DIS2006

Scaling violation in  $g_1(x, Q^2)$  is weak.

For  $g_1$ ,  $Q^2$  becomes  $> 1 \text{ GeV}^2$  at  $x \gtrsim 0.003$  for SMC, 0.03 for HERMES and for COMPASS.

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# Regge model predictions for $g_1$

- Remember:  $s \equiv W^2 = M^2 + Q^2(1/x - 1)$ ; thus low  $x$  behaviour of a structure function ( $F_2, g_1, \dots$ ) reflects the high energy behaviour of the virtual Compton scattering cross section with the cms energy squared,  $s$ . **This is the Regge limit of DIS.**
- Regge gives for  $x \rightarrow 0$  (i.e.  $Q^2 \ll W^2$ ):

$$g_1^i(x, Q^2) \sim \beta(Q^2)x^{-\alpha_i(0)} \quad (1)$$

where  $i$  = singlet (s), nonsinglet (ns):  $g_1^s = g_1^p + g_1^n$ ,  $g_1^{ns} = g_1^p - g_1^n$ .

- Possible trajectories:  $l=0$  ( $g_1^s$ ;  $f_1$  trajectory) and  $l=1$  ( $g_1^{ns}$ ;  $a_1$  trajectory).  
Expectations:  $\alpha_{s,ns}(0) \lesssim 0$  and  $\alpha_s(0) \approx \alpha_{ns}(0)$ .
- Consequence: for  $Q^2 \rightarrow 0$ ,  $g_1(W^2) \sim W^{2\alpha(0)}$ .
- At large  $Q^2$ : the DGLAP evolution and resummation of  $\ln^2(1/x)$  generate more singular  $x$  dependence than that implied by eq.(1) for  $\alpha_{s,ns}(0) \lesssim 0$ .
- Other Regge isosinglet contributions to  $g_1$  at low  $x$ :
  - a term  $\sim \ln x$ ;
  - a term  $\sim 2 \ln(1/x) - 1$ ;
  - a perverse term  $\sim 1/(x \ln^2 x)$  got invalidated.

**Perturbative QCD effects might modify the Regge expectations.** In case of  $g_1$  it creates a more singular low  $x$  behaviour than the (nonperturbative) Regge expectations.

# Regge model predictions for $g_1$ ...cont'd

Testing Regge behaviour of  $g_1$  through its  $x$  dependence:

- choose high  $W^2$ ;
- choose low  $x$  (i.e.  $Q^2 \ll W^2$  but not necessarily low  $Q^2$ );
- choose a bin of  $Q^2$  (i.e.  $Q^2 = \text{const}$ );
- fit the  $x$  dependence of  $g_1$ .

For the SMC:

- Testing not possible

For COMPASS:

- Testing not possible either
- **Observe:** assuming  $g_1 \sim x^0$  to get  $x \rightarrow 0$  extrapolation of  $g_1$  to extract  $g_1$  moments is **not correct!** Evolve  $g_1$  to a common  $Q^2$  before extrapolation!

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# Low $x$ implications from the pQCD

In the DGLAP the singular small  $x$  behaviour of the gluon and sea quark distributions (implied by the data) may originate from

- parametrization of the starting distributions at moderate  $Q_0^2$  (equal to about  $4 \text{ GeV}^2$  or so);
- evolution starting from non-singular “valence-like” parton distributions at a very low scale,  $\mu_0 \sim 0.35 \text{ GeV}^2$ . Glück, Reya, Vogt, Eur.

Phys.J.C5 (1998) 461

Then

$$g_1(x, Q^2) \sim \exp \left[ A \sqrt{\xi(Q^2) \ln(1/x)} \right] \quad (2)$$

where

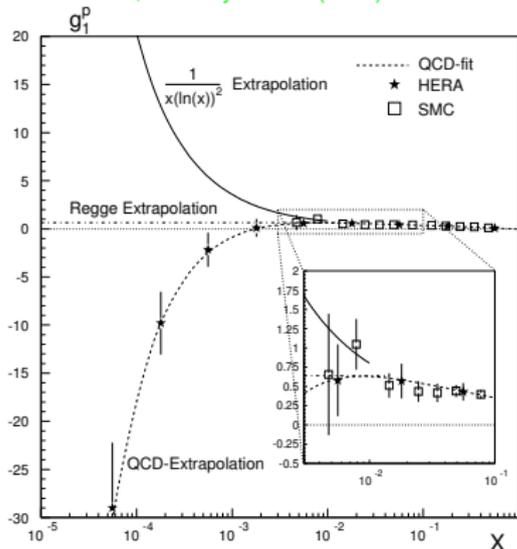
$$\xi(Q^2) = \int_{\mu_0^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \quad (3)$$

and  $A$  is different for the singlet and non-singlet case.

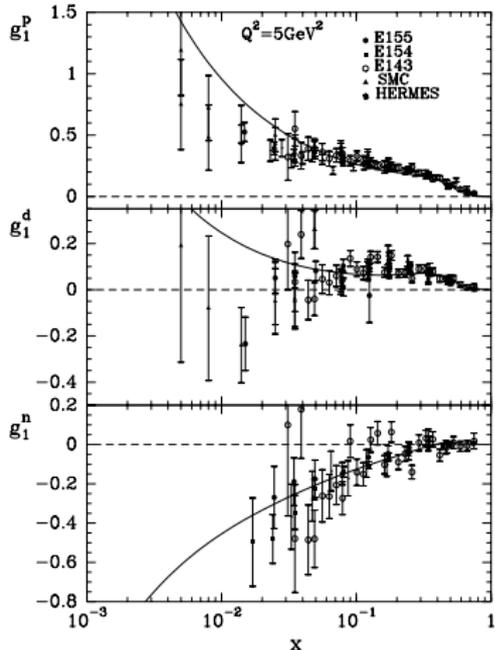
# Low $x$ implications from the pQCD

World data on  $g_1^p$  and  $g_1^d$  were NLO QCD analysed but at low  $x$  neither measurements nor reliable calculations exist.

De Roeck et al., Eur. Phys. J. C6 (1999)121



Bourrely, Soffer, Buccella, Eur. Phys. J. C23(2002) 487.



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# $\text{Ln}^2(1/x)$ corrections to $g_1(x, Q^2)$

## General

- **Low  $x \equiv$  large parton densities  $\implies$  new dynamics?**
- Small  $x$  behaviour of both  $g_1^S$  and  $g_1^{NS}$  is controlled by terms corresponding to powers of  $\alpha_s \text{Ln}^2(1/x)$  [Bartels, Ermolaev, Ryskin, Z.Phys. C70 \(1996\) 273; Z.Phys. C72 \(1996\) 627.](#)
- These terms generate the leading small  $x$  behaviour of  $g_1$ .
- They go **beyond** the standard QCD evolution of spin dependent parton densities which does not generate the double but only the single  $\text{Ln}(1/x)$  terms.
- They **may be included** in the QCD evolution; one of the methods: a formalism based on unintegrated parton distributions,  $f(x, k^2)$ , where the conventional parton distributions  $p(x, Q^2)$  are

$$p(x, Q^2) = \int^{Q^2} \frac{dk^2}{k^2} f(x, k^2) \quad (4)$$

and  $k^2$  is a transverse momentum squared of the partons.

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- $\text{Ln}^2(1/x)$  corrections to  $g_1^{ns}$  are generated by ladder diagrams  $\implies$  or mathematically by an equation:

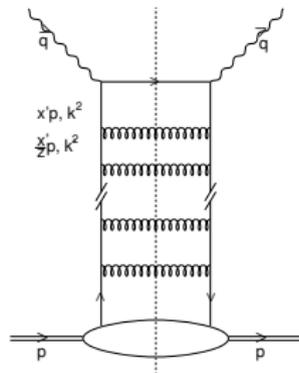
$$f(x', k) = f^{(0)}(x', k) + \bar{\alpha}_s(k^2) \int_{x'}^1 \frac{dz}{z} \int_{k_0^2}^{k^2/z} \frac{dk'^2}{k'^2} f\left(\frac{x'}{z}, k'^2\right)$$

and

$$g_1^{ns}(x, Q^2) = g_1^{(0)}(x) + \int_{k_0^2}^{W^2} \frac{dk^2}{k^2} f(x' = x(1 + \frac{k^2}{Q^2}), k^2)$$

where  $\bar{\alpha}_s(k^2) = 2\alpha_s(k^2)/3\pi$  and

$g_1^{(0)}(x)$  is a nonperturbative part, corresponding to  $k^2 < k_0^2$ .

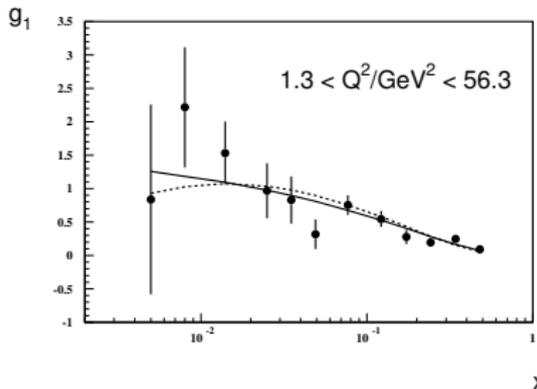


- $\text{Ln}^2(1/x)$  terms originate from the  $z$ -dependent limit of the  $\int dk'^2/k'^2$  and  $x$ -dependent limit in  $W^2(x)$ .
- They create a leading small  $x$  behaviour of  $g_1^{ns}$  if  $g_1^{ns(0)}$  and  $f^{(0)}$  are non-singular at  $x \rightarrow 0$ .
- DGLAP evolution is incomplete at low  $x$ ; only  $\text{Ln}(1/x)$  terms are present, originating from  $\int_{k_0^2}^{k^2} dk'^2/k'^2$ .
- For fixed (i.e. non-running)  $\bar{\alpha}_s(k^2) \rightarrow \tilde{\alpha}_s$ , small  $x$  behaviour is  $g_1^{ns}(x, Q^2) \sim x^{-\lambda}$  where  $\lambda = 2\sqrt{\tilde{\alpha}_s}$

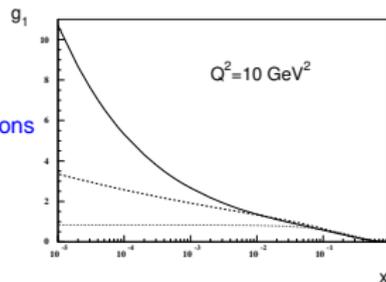
# $\ln^2(1/x)$ corrections to $g_1(x, Q^2)$ ...cont'd

## Predictions for $g_1^{ns}$ ...cont'd

- A unified equation which incorporates the complete LO DGLAP at finite  $x$  and  $\ln^2(1/x)$  effects at  $x \rightarrow 0$  was formulated.
- Potentially large  $\ln Q^2$  and  $\ln(1/x)$  treated **on equal footing**.
- For the numerical results it was assumed that  $g_1^{ns(0)} = 2g_A(1-x)^3/3$  where  $g_A = 1.257$  (axial vector coupling). At  $x \rightarrow 0$ ,  $g_1^{ns(0)} \rightarrow \text{const}$ , in agreement with the Regge expectation.
- The  $g_1^{ns(0)}$  satisfies the Bjorken sum rule at LO:  $\int_0^1 dx g_1^{ns(0)}(x) = g_A/6$ .
- Parameter  $k_0^2 = 1 \text{ GeV}^2$ .
- To compare the  $g_1^{ns}$  to the (SMC) data it was assumed:  
 $g_1^{ns} \equiv g_1^p - g_1^n = 2 [g_1^p - g_1^d / (1 - \omega_D/3)]$ ;  $\omega_D = 0.05$  (D-state probability in the deuteron).



$g_1^{ns}$  vs  $x$   
continuous – full calculations  
broken – LO DGLAP



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Low  $Q^2$  extrapolation of  $g_1^{ns}$

Badelek, Kwiecinski, Phys. Lett. B418 (1998) 229

- For  $Q^2 \rightarrow 0$  (for fixed  $W^2$ ),  $g_1$  should be a finite function of  $W^2$ , free from kinematical singularities or zeroes at  $Q^2 = 0$ .
- $g_1^{ns}$  from the above formalism and the above  $g_1^{ns(0)}$  fulfill this.
- If  $g_1^{ns(0)}(x)$  has a singularity then it should be replaced by  $g_1^{ns(0)}(\bar{x})$  where  $\bar{x} = x(1 + k_0^2/Q^2)$ . Remaining parts left unchanged.
- Then  $g_1^{ns}$  can be extrapolated to the low  $Q^2$  for fixed  $2M\nu = Q^2/x$  including  $Q^2 = 0$ . **Observe!** That is just the partonic contribution to the low  $Q^2$  region; it may not be the only one there.

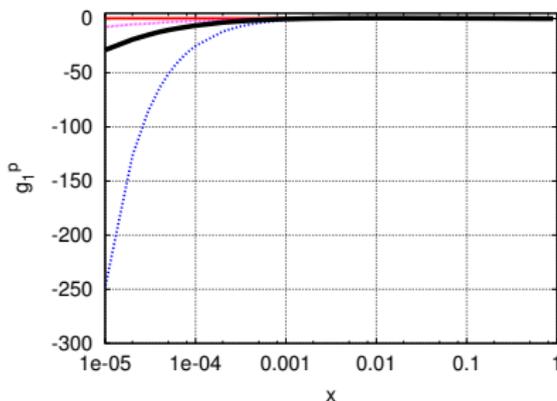
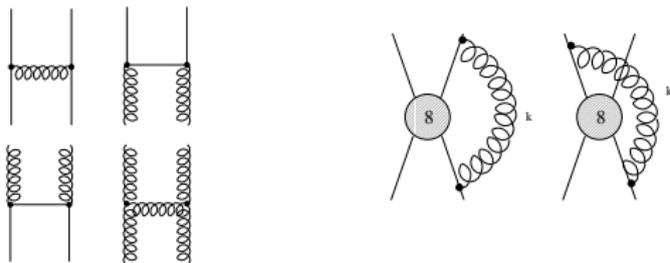
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Predictions for  $g_1$  Kwiecinski, Ziaja, Phys. Rev. D60 (1999) 054004

Results of full  $g_1(x, Q^2)$  calculations at  $Q^2 = 10 \text{ GeV}^2$ . At low  $x$ , the singlet part,  $g_1^S$  dominates  $g_1^{nS}$ . Apart of the "standard" ladder diagram, the following ones were taken into account for  $g_1^{nS}(x, Q^2)$ :



$\Leftarrow g_1^p(x, Q^2)$  vs  $x$  at  $Q^2 = 10 \text{ GeV}^2$

red – nonperturbative input,  $g_1^{(0)}$

pink – only LO DGLAP

thick black – full  $g_1$

blue – LO DGLAP + ladder  $\text{Ln}^2(1/x)$

At low  $x$ ,  $g_1 \rightarrow x^{-\lambda}$  with  $\lambda \sim 0.4$  for  $g_1^{nS}$   
and  $\lambda \sim 0.8$  for  $g_1^S$ .

More singular than Regge expectations!

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# $\ln^2(1/x)$ corrections to $g_1(x, Q^2)$ ...cont'd

## Low $x$ contributions to $g_1$ moments

- Fundamental tools: sum rules (Ellis–Jaffe, Bjorken, DHG, ...) which involve first moments of  $g_1$ , i.e. integrations over  $dx$  from 0 to 1, e.g.  $\Gamma_1 = \int_0^1 g_1 dx$ .
- Unmeasured regions:  $[0, x_{min}]$ ,  $[x_{max}, 1]$ .
- The  $[x_{max}, 1]$  not critical but  $[0, x_{min}]$  is very important.
- $x_{min}$  depends on  $\nu_{max}$  accessed in experiments at a given  $Q_0^2$ , e.g.
  - SMC at 200 GeV and  $Q_0^2 = 1 \text{ GeV}^2 \implies x_{min} \approx 0.003$ ;
  - COMPASS at 160 GeV and  $Q_0^2 = 1 \text{ GeV}^2 \implies x_{min} \approx 0.002$ .
- Contribution to moments from the  $0 \leq x \lesssim 0.003$  has to be estimated phenomenologically.
- LO DGLAP +  $\ln^2(1/x)$  resummation used to extrapolate polarised parton distributions and structure functions down to  $x \sim 10^{-5}$  to calculate contributions to moments from  $10^{-5} < x < 10^{-3}$ . In  $2 < Q^2 < 15 \text{ GeV}^2$  interval, contributions to  $\Gamma_1^p$  was 2% and 8% for  $\Gamma_1^\eta$  (however calculations of  $\Gamma_1^\eta$  were below the data in the overlap region). Contributions  $\nearrow$  with  $Q^2 \nearrow$ . Also estimated that  $10^{-5} < x < 10^{-3}$  interval contributes 1% and 2% to the Bjorken and Ellis–Jaffe s.r.
- Same formalism gave a contribution of 0.0080 to the Bjorken integral from the unmeasured region,  $0 \leq x < 0.003$  at  $Q^2 = 10 \text{ GeV}^2$  (LO DGLAP only gave 0.0057 and assuming  $g_1 = \text{const}$  resulted in 0.004). [Kwiecinski, Ziaja, Phys. Rev., 60 \(1999\)054004](#)
- Extrapolation of the NLO DGLAP fits to the world data: in  $0 \leq x < 0.003$  is 10% of  $\Gamma_1^p$ . NLO DGLAP for the SMC data at  $Q^2 = 10 \text{ GeV}^2$  gave 10% contribution to the Bjorken integral. [SMC, Phys. Rev., D58 \(1998\) 112002](#) (obs! assumptions!).

# Nonperturbative effects in $g_1$

- Data on  $g_1(x, Q^2)$  extend to low  $Q^2 \sim 0.0001 \text{ GeV}^2$ .
- Nonperturbative mechanisms dominate the particle dynamics there; transition from “soft” to “hard” physics may be studied.
- Partonic contribution to  $g_1$  has to be suitably extrapolated to low  $Q^2$  and complemented by a nonperturbative component.
- Low  $Q^2$ , spin-independent electroproduction well described by the GVMD  $\implies$  GVMD should be used to describe the  $g_1$ .
- Two attempts tried to extract from the data a contribution of nonperturbative effects at low  $x$ , low  $Q^2$ .

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  - $g_1$  at low  $Q^2$ , method II
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# Nonperturbative effects in $g_1 \dots \text{cont'd}$

$g_1$  at low  $Q^2$ , method I

Badelek, Kiryluk, Kwieciński, Phys. Rev. D61 (2000) 014009

The following representation of  $g_1$  was assumed:

$$g_1(x, Q^2) = g_1^{\text{VMD}}(x, Q^2) + g_1^{\text{part}}(x, Q^2) \quad (5)$$

$g_1^{\text{part}}$  at low  $x$  is controlled by the  $\ln^2(1/x)$  terms; it was parametrised as discussed in Kwieciński, Ziaja, Phys. Rev. D60 (1999) 054004.  $g_1^{\text{VMD}}(x, Q^2)$  was represented as:

$$g_1^{\text{VMD}}(x, Q^2) = \frac{M\nu}{4\pi} \sum_{V=\rho,\omega,\phi} \frac{M_V^4 \Delta\sigma_V(W^2)}{\gamma_V^2(Q^2 + M_V^2)^2} \quad (6)$$

The unknown cross sections  $\Delta\sigma_V(W^2)$  are combinations of the total cross sections for the scattering of polarised vector mesons and nucleons. At high  $W^2$ :  $\Delta\sigma_V = (\sigma_{1/2} - \sigma_{3/2})/2$   
Assume:

$$\frac{M\nu}{4\pi} \sum_{V=\rho,\omega} \frac{M_V^4 \Delta\sigma_V}{\gamma_V^2(Q^2 + M_V^2)^2} = \text{C} \left[ \frac{4}{9} \left( \Delta u_{\text{val}}^0(x) + 2\Delta \bar{u}^0(x) \right) + \frac{1}{9} \left( \Delta d_{\text{val}}^0(x) + 2\Delta \bar{d}^0(x) \right) \right] \frac{M_\rho^4}{(Q^2 + M_\rho^2)^2}, \quad (7)$$

$$\frac{M\nu}{4\pi} \frac{M_\phi^4 \Delta\sigma_{\phi p}}{\gamma_\phi^2(Q^2 + M_\phi^2)^2} = \text{C} \frac{2}{9} \Delta \bar{s}^0(x) \frac{M_\phi^4}{(Q^2 + M_\phi^2)^2}, \quad (8)$$

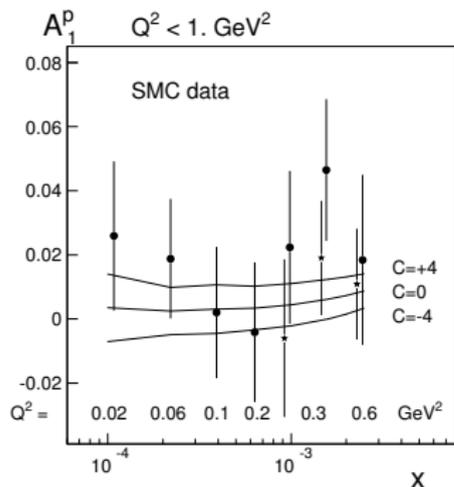
# Nonperturbative effects in $g_1 \dots \text{cont'd}$

$g_1$  at low  $Q^2$ , method I...cont'd

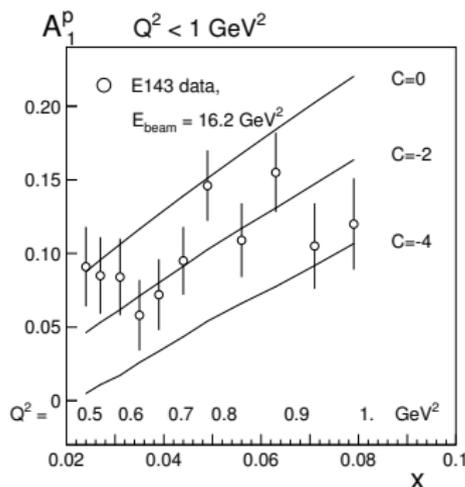
Each  $\Delta p_j^0(x) \rightarrow x^0$  for  $x \rightarrow 0$ . Thus  $\Delta\sigma_V \rightarrow 1/W^2$  at large  $W^2$ , i.e. zero intercept of the appropriate Regge trajectories.

Results for the spin asymmetry,  $A_1 = g_1/F_1$ , for the proton, and for different  $C$ :

$C ??$



$C < 0 ?$



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# Nonperturbative effects in $g_1 \dots \text{cont'd}$

$g_1$  at low  $Q^2$ , method II

Badelek, Kwiecinski, Ziaja, Eur. Phys. J. C26 (2002) 45

The following representation of  $g_1$  was assumed, valid for fixed  $W^2 \gg Q^2$ , i.e. small  $x = Q^2/(Q^2 + W^2 - M^2)$ :

$$g_1(x, Q^2) = g_1^L(x, Q^2) + g_1^H(x, Q^2) = \frac{M_V}{4\pi} \sum_V \frac{M_V^4 \Delta\sigma_V(W^2)}{\gamma_V^2 (Q^2 + M_V^2)^2} + g_1^{\text{AS}}(\bar{x}, Q^2 + Q_0^2). \quad (9)$$

The first term sums up contributions from light vector mesons,  $M_V < Q_0$ ,  $Q_0^2 \sim 1 \text{ GeV}^2$ . The unknown  $\Delta\sigma_V$  are expressed through the combinations of nonperturbative parton distributions, evaluated at fixed  $Q_0^2$ , similar to method I.

The second term,  $g_1^H(x, Q^2)$ , represents the contribution of heavy ( $M_V > Q_0$ ) vector mesons to  $g_1(x, Q^2)$  can also be treated as an extrapolation of the QCD improved parton model structure function,  $g_1^{\text{AS}}(x, Q^2)$ , to arbitrary values of  $Q^2$ :  $g_1^H(x, Q^2) = g_1^{\text{AS}}(\bar{x}, Q^2 + Q_0^2)$ . The scaling variable  $x$  is replaced by  $\bar{x} = (Q^2 + Q_0^2)/(Q^2 + Q_0^2 + W^2 - M^2)$ . It follows that at large  $Q^2$ ,  $g_1^H(x, Q^2) \rightarrow g_1^{\text{AS}}(x, Q^2)$ . Thus:

$$g_1(x, Q^2) = C \left[ \frac{4}{9} (\Delta u_{\text{val}}^0(x) + 2\Delta \bar{u}^0(x)) + \frac{1}{9} (\Delta d_{\text{val}}^0(x) + 2\Delta \bar{d}^0(x)) \right] \frac{M_\rho^4}{(Q^2 + M_\rho^2)^2} + C \left[ \frac{1}{9} (2\Delta \bar{s}^0(x)) \right] \frac{M_\phi^4}{(Q^2 + M_\phi^2)^2} + g_1^{\text{AS}}(\bar{x}, Q^2 + Q_0^2). \quad (10)$$

# Nonperturbative effects in $g_1$ ...cont'd

$g_1$  at low  $Q^2$ , method II...cont'd

Now fixing  $C$  in the photoproduction limit via the DHGHY sum rule.

## Digression: the DHGHY sum rule

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The  $\gamma^*p$  scattering amplitude fulfills the dispersion relation:

$$S_1(\nu, q^2) = 4 \int_{-q^2/2M}^{\infty} \nu' d\nu' \frac{G_1(\nu', q^2)}{(\nu')^2 - \nu^2} \quad (11)$$

where

$$G_1(\nu, q^2) = \frac{M}{\nu} g_1(x, Q^2) \quad (12)$$

in the  $Q^2, \nu \rightarrow \infty$  limit. As a result of Low's theorem:  $S_1(0, 0) = -\kappa_{p(n)}^2$ ,  $G_1$  in the  $Q^2 \rightarrow 0$  limit fulfills the DHGHY sum rule:

$$\int_0^{\infty} \frac{d\nu}{\nu} G_1(\nu, 0) = -\frac{1}{4} \kappa_{p(n)}^2. \quad (13)$$

End of digression

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# Nonperturbative effects in $g_1 \dots \text{cont'd}$

$g_1$  at low  $Q^2$ , method II...cont'd

## First moment $\Gamma_1$ for the proton and deuteron

$$\Gamma_1 = \int_{x=0.001}^{x_{\min}} g_1 dx + \int_{x_{\min}}^{x(W=1.07)} g_1 dx$$

DIS (unmeasured)  
Parameterization  
of world data

without elastic  
contribution

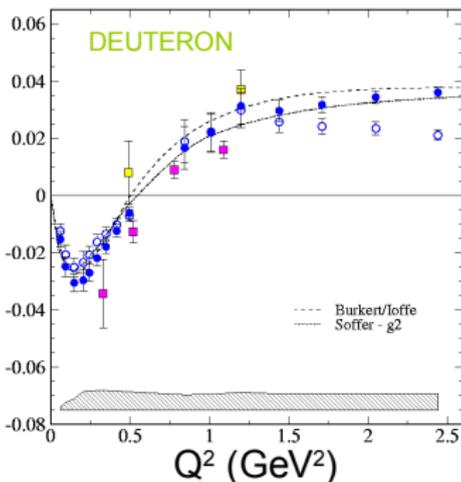
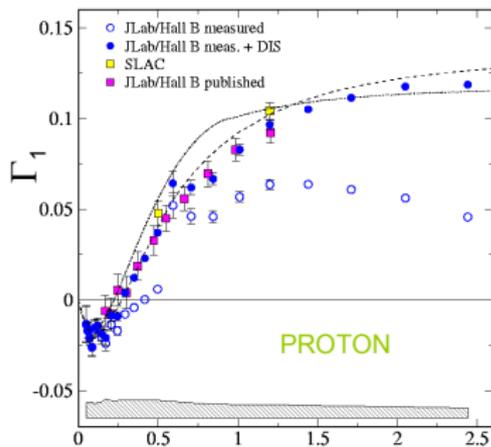
### Phenomenological Models

#### Burkert/Ioffe

Resonance contribution pion electroproduction analysis

#### Soffer/Teryaev

Interpolation of the integral  $\int (g_1 + g_2) dx$



V. Dharmawardane, DIS2006

# Nonperturbative effects in $g_1$ ...cont'd

$g_1$  at low  $Q^2$ , method II...cont'd

At  $\nu \rightarrow 0$ , eq.(11) is:

$$S_1(0, q^2) = 4M \int_{Q^2/2M}^{\infty} \frac{d\nu}{\nu^2} g_1(x(\nu), Q^2). \quad (14)$$

Now we define the DHGHY moment,  $I(Q^2)$  as:

$$I(Q^2) = S_1(0, q^2)/4 = M \int_{Q^2/2M}^{\infty} \frac{d\nu}{\nu^2} g_1(x(\nu), Q^2). \quad (15)$$

Before taking the  $Q^2 \rightarrow 0$  limit of (14), observe that it is valid only down to some threshold value of  $W$ ,  $W_{th} \lesssim 2$  GeV (above resonances). Requirement  $W > W_{th}$  gives the lower limit for integration over  $\nu$  in (14), where  $\nu_t(Q^2) = (W_t^2 + Q^2 - M^2)/2M$ :

$$I(Q^2) = I_{res}(Q^2) + M \int_{\nu_t(Q^2)}^{\infty} \frac{d\nu}{\nu^2} g_1(x(\nu), Q^2). \quad (16)$$

Here  $I_{res}$  = contribution of resonances. The DHGHY sum rule now implies:

$$I(0) = I_{res}(0) + M \int_{\nu_t(0)}^{\infty} \frac{d\nu}{\nu^2} g_1(x(\nu), 0) = -\kappa_{p(n)}^2/4. \quad (17)$$

# Nonperturbative effects in $g_1 \dots \text{cont'd}$

$g_1$  at low  $Q^2$ , method II...cont'd

Thus action plan for extracting  $C$  in eq.(10):

- take  $g_1(x(\nu), 0)$ , eq.(10);  $C$  is the only free parameter,
- put it into eq.(17),
- get  $I_{res}(0)$  from measurements,
- extract  $C$  from (17).

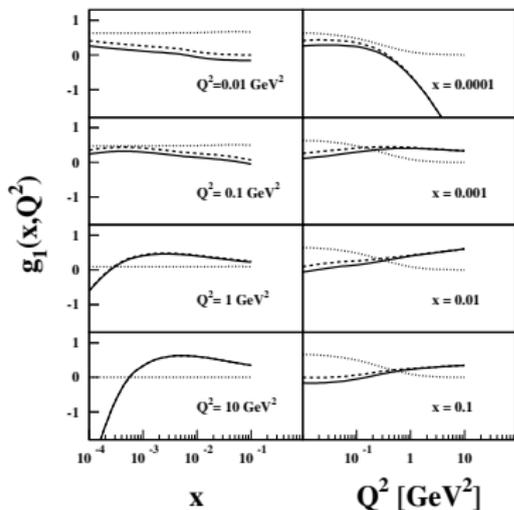
Taking:

- $I_{res}(0)$  from photoproduction,  $W_t=1.8$  GeV [GDH, Nucl. Phys. 105 \(2002\) 113](#),
- $g_1^{AS}$  parametrized by NLO GRSV2000 [Phys.Rev. D63 \(2001\) 094005](#)
- nonperturbative  $\Delta p_j^{(0)}(x)$  at  $Q^2 = Q_0^2 = 1.2 \text{ GeV}^2$  from
  - 1 GRSV2000  $\implies C = -0.30$
  - 2 "flat"  $\Delta p_j^{(0)}(x) = N_i(1-x)^{\eta_i}$   $\implies C = -0.24$ .

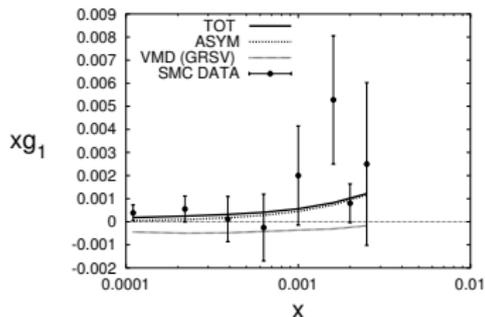
# Nonperturbative effects in $g_1 \dots \text{cont'd}$

$g_1$  at low  $Q^2$ , method II...cont'd

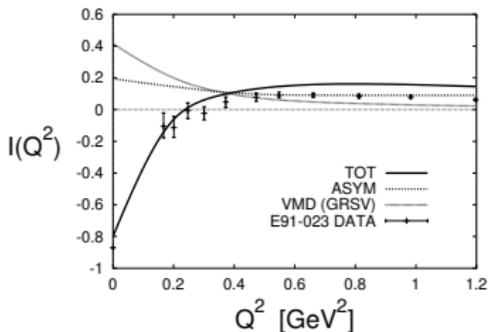
Byproducts:  $g_1$  from eq.(10) and the DHGHY moment,  $I(Q^2)$ , eq.(15). Results for the proton:



broken lines –  $g_1^{AS}$ , dotted –  $g_1^L$ ,  
continuous – total  $g_1$



data at  $Q^2 < 1 \text{ GeV}^2$



points mark  
resonances  
at  $W < W_t(Q^2)$

Figures from: Badelek, Kwicinski, Ziaja, Eur. Phys. J. C26 (2002)45.

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- At high energies, low  $Q^2$  region correlated with low  $x$ .
- Very important for understanding the nucleon structure is the transition from photoproduction to DIS; also for practical purposes.
- Several theoretical concepts relevant there.

Both perturbative and nonperturbative contributions to the nucleon structure are present everywhere in  $Q^2$