Overview of low energy NN interaction and few nucleon systems

Renato Higa

Theory Group, Jefferson Lab

Cebaf Center, A223 (ext.6363)

higa@jlab.org

- Lecture II
  - ⋆ Basics on chiral EFT
  - ⋆ $\not\pi$ EFT

Chiral effective field theory

- Chiral symmetry plays a major role

  mid 60s - 70s: current algebra

- Low-lying meson spectra suggest spontaneous chiral symmetry breaking, with pions acting as Goldstone bosons

- Massless QCD: "Theoretical Paradise" - exhibits explicit, global, χ-symmetry

$$\mathcal{L}^{(0)}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + \bar{q}\, i\, \not{D}\, q \tag{1}$$

$$\mathcal{L}^{(v,a,s,p)}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + \bar{q}\, i\, \not{D}\, q + \bar{q}\gamma^\mu(v_\mu + a_\mu\gamma_5)q - \bar{q}(s - i\gamma_5 p)q \tag{2}$$

$$v, a, p \to 0, \qquad s \to \mathcal{M}, \qquad \mathcal{L}^{(v,a,s,p)}_{QCD} \to \mathcal{L}^{true}_{QCD} \tag{3}$$

- promote a global symmetry, with external sources, to a local one

  $\Rightarrow$ non-trivial relations among Green functions

$$e^{iZ[v,a,s,p]} = \int \mathcal{D}q\,\mathcal{D}\bar{q}\,\mathcal{D}G\,e^{i\int \mathcal{L}_{QCD}^{(v,a,s,p)}} \quad (4)$$

$$= \int \mathcal{D}U\,e^{i\int \mathcal{L}_{eff}(v,a,s,p,U)} \quad (5)$$

- H. Leutwyler, Ann. Phys. **235**, 165 (1994).

- $\mathcal{L}_{eff}$: includes all terms allowed by the relevant symmetries

- organization: derivative (momentum) expansion Q/Λ_χ

  where $\Lambda_\chi \sim 4\pi f_\pi \sim 2\pi\Lambda_{QCD} \Rightarrow$ power counting

- Callan *et.al.*, Coleman, Wess, and Zumino (69): Effective Lagrangians

  χ - symmetry is spontaneously broken: $\underbrace{SU(N_f)_L \times SU(N_f)_R}_{G} \to SU(N_f)_V$

$$
\begin{aligned}
U &\xrightarrow{G} g_R U g_L^\dagger \\
l_\mu = v_\mu - a_\mu &\xrightarrow{G} g_L l_\mu g_L^\dagger + i g_L \partial_\mu g_L^\dagger \\
r_\mu = v_\mu + a_\mu &\xrightarrow{G} g_R l_\mu g_R^\dagger + i g_R \partial_\mu g_R^\dagger \\
\chi = 2B(s + ip) &\xrightarrow{G} g_R \chi g_L^\dagger
\end{aligned}
\tag{6}
$$

$$D_\mu(U,\chi) = \partial_\mu(U,\chi) - i r_\mu(U,\chi) + i(U,\chi) l_\mu$$

(Same transformations of v, a, s, p which keeps $\mathcal{L}_{QCD}^{(v,a,s,p)}$ invariant)

$$F_{\mu\nu}^{L,R} = \partial_\mu(l_{\nu,r_\nu}) - \partial_\nu(l_{\mu,r_\mu}) - i[(l_{\mu,r_\mu}),(l_{\nu,r_\nu})] \tag{7}$$

- Invariants: $\langle A \rangle$ where $A \to g_L A g_L^\dagger$

$$\mathcal{L}_\pi^{(2)} = \frac{F^2}{4} \langle (D_\mu U)^\dagger D^\mu U + U^\dagger \chi + \chi^\dagger U \rangle \tag{8}$$

with $U = \frac{1}{F}\left\{\sqrt{F^2 - \boldsymbol{\pi}^2} + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}\right\}$ and $v, a, p \to 0$, $s \to \mathcal{M}$ one gets

$$\mathcal{L}_\pi^{(2)} = \frac{1}{2}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi} - \frac{m_\pi^2}{2}\boldsymbol{\pi}^2 + \frac{1}{2F^2}(\partial_\mu\boldsymbol{\pi}\cdot\boldsymbol{\pi})(\partial^\mu\boldsymbol{\pi}\cdot\boldsymbol{\pi}) - \frac{m_\pi^2}{8F^2}(\boldsymbol{\pi}^2)^2 + O(\pi^3) \tag{9}$$

(non-linear σ model)

- Observables are independent of the parametrization of U!

- Electroweak interactions $[SU(3)]$

$$\begin{aligned} v_\mu &\to eQA_\mu, & \mathcal{Q} &= \tfrac{1}{3}\,\mathrm{diag}(2,-1,-1), \\ a_\mu &\to \frac{e}{\sqrt{2}\sin\theta_W} W^\dagger T_+, & T_+ &= \begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{aligned} \tag{10}$$

$$\langle (D_\mu U)^\dagger D^\mu U \rangle \to (iU^\dagger r_\mu - i l_\mu U^\dagger)\partial^\mu U + \partial_\mu U^\dagger(-i r_\mu U + iU l_\mu) \tag{11}$$

$$
\begin{aligned}
\mathcal{L}_{\pi}^{(4)} = {} & L_1 \left\{ \langle D_\mu U (D^\mu U)^\dagger \rangle \right\}^2 + L_2 \, \langle D_\mu U (D_\nu U)^\dagger \rangle \langle D^\mu U (D^\nu U)^\dagger \rangle \\
& + L_3 \, \langle D_\mu U (D^\mu U)^\dagger D_\nu U (D^\nu U)^\dagger \rangle + L_4 \, \langle D_\mu U (D^\mu U)^\dagger \rangle \langle \chi U^\dagger + U \chi^\dagger \rangle \\
& + L_5 \, \langle D_\mu U (D^\mu U)^\dagger (\chi U^\dagger + U \chi^\dagger) \rangle + L_6 \left[\langle \chi U^\dagger + U \chi^\dagger \rangle \right]^2 \\
& + L_7 \left[\langle \chi U^\dagger - U \chi^\dagger \rangle \right]^2 + L_8 \, \langle U \chi^\dagger U \chi^\dagger + \chi U^\dagger \chi U^\dagger \rangle \\
& - i L_9 \, \langle F^R_{\mu\nu} D^\mu U (D^\nu U)^\dagger + F^L_{\mu\nu} (D^\mu U)^\dagger D^\nu U \rangle + L_{10} \, \langle U F^L_{\mu\nu} U^\dagger F^{\mu\nu}_R \rangle \\
& + H_1 \, \langle F^R_{\mu\nu} F^{\mu\nu}_R + F^L_{\mu\nu} F^{\mu\nu}_L \rangle + H_2 \, \langle \chi \chi^\dagger \rangle .
\end{aligned}
\tag{12}
$$

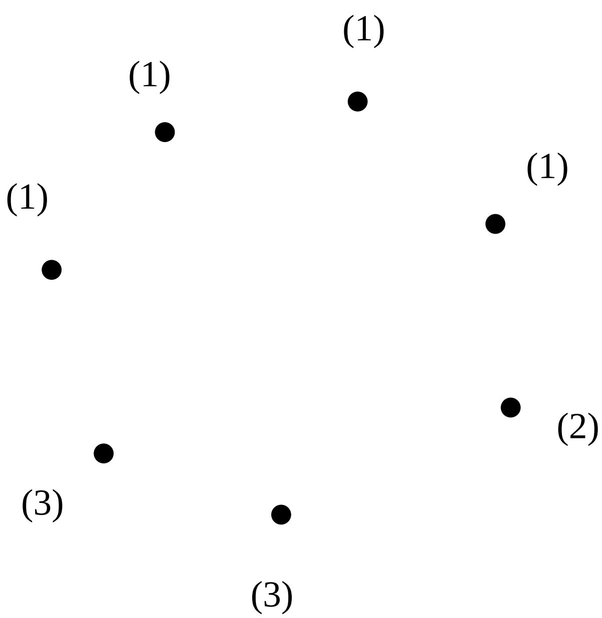
- Lagrangian expansion starts at $O(q^2)$
  - $\star$ $\mathcal{L}_\pi^{(n)} \Rightarrow$ vertices of $O(q^n)$
  - $\star$ pion propagator: $O(q^{-2})$
  - $\star$ each loop: $O(q^4)$

- $\nu \cdots$ chiral order of a particular Feynman diagram

$$
\begin{aligned}
&N_L \ \cdots \ \text{number of loops,}\\
&N_i \ \cdots \ \text{number of pion propagators,}\\
&d \ \cdots \ \text{number of derivatives on a particular vertex,}\\
&N_d \ \cdots \ \text{number of vertices with } d \text{ derivatives.}
\end{aligned}
\tag{13}
$$

- Lagrangian expansion starts at $O(q^2)$
  - $\star$ $\mathcal{L}_\pi^{(n)} \Rightarrow$ vertices of $O(q^n)$
  - $\star$ pion propagator: $O(q^{-2})$
  - $\star$ each loop: $O(q^4)$

- $\nu \cdots$ chiral order of a particular Feynman diagram



$$\sum_d dN_d = 1 \times 4 + 2 \times 1 + 3 \times 2 = 12$$

- Lagrangian expansion starts at $O(q^2)$
  - ⋆ $\mathcal{L}_\pi^{(n)} \Rightarrow$ vertices of $O(q^n)$
  - ⋆ pion propagator: $O(q^{-2})$
  - ⋆ each loop: $O(q^4)$

- $\nu \cdots$ chiral order of a particular Feynman diagram

N_L $\cdots$ number of loops,

N_i $\cdots$ number of pion propagators,

d $\cdots$ number of derivatives on a particular vertex,

N_d $\cdots$ number of vertices with d derivatives.

$$\nu = \sum_d dN_d + 4N_L - 2N_i = \sum_d dN_d + 4N_L - 2\Big[\sum_d N_d + (N_L - 1)\Big]$$
$$= 2 + 2N_L + \sum_d N_d(d-2) \tag{13}$$

- EM form factors

$$F_\phi(t) = 1 + \frac{1}{6}\langle r^2\rangle_\phi\, t + \cdots \tag{14}$$

$$\begin{aligned}\langle r^2\rangle_{\pi^\pm} &= \frac{12\, L_9^r(\mu)}{F^2} - \frac{1}{32\pi^2F^2}\left[2\ln\left(\frac{m_\pi^2}{\mu^2}\right) + \ln\left(\frac{m_K^2}{\mu^2}\right) + 3\right] \\ &= 0.439 \pm 0.008\,\text{fm}^2 \text{ (exp.)}\end{aligned} \tag{15}$$

★ L_9^r dependence on μ cancels explicit dependence in the logs
★ fixing L_9^r at $\mu = m_\rho$ determines the LO parameters of $\mathcal{L}_{res}$ in χPT
$\Rightarrow$ saturates almost all the values of the other LECs (resonance saturation)

$$\begin{aligned}\langle r^2\rangle_{K^0} &= -\frac{1}{16\pi^2F^2}\ln\left(\frac{m_K}{m_\pi}\right) = -0.04 \pm 0.03\,\text{fm}^2 \\ \text{exp.} &= -0.054 \pm 0.026\,\text{fm}^2\end{aligned} \tag{16}$$

Baryon χPT

$$
\begin{aligned}
N &\xrightarrow{G} hN, & h &= h(g_L, g_R, U) \\
u &\xrightarrow{G} g_R u h^\dagger = h u g_L^\dagger, & u^2 &= U
\end{aligned} \tag{17}
$$

- Invariants: $\bar{N}\,A\,N$ where $A \xrightarrow{G} h\,A\,h^\dagger$
- Building blocks:

$$
u_\mu = i u^\dagger D_\mu U u^\dagger, \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \quad f_{\mu\nu}^\pm = u F_{\mu\nu}^L u^\dagger \pm u^\dagger F_{\mu\nu}^R u. \tag{18}
$$

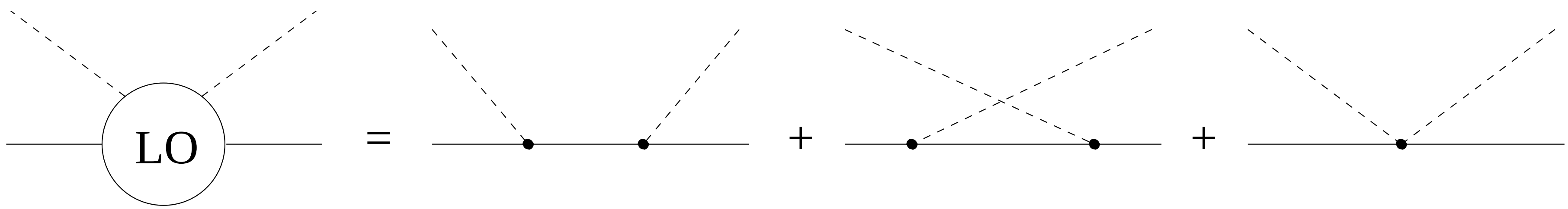
$$\mathcal{L}_{\pi N}^{(1)} = \bar{\psi}(i \not{D} - m)\psi + \frac{g_A}{2}\bar{\psi} \not{u}\gamma_5\psi \tag{19}$$

$$U = u^2 = \exp\left[\frac{\boldsymbol{\tau}\cdot\boldsymbol{\pi}}{f_\pi}\right], \quad u_\mu = i\{u^\dagger, \partial_\mu u\}, \quad D_\mu = \partial_\mu + \Gamma_\mu,$$

$$\Gamma_\mu = \frac{1}{2}[u^\dagger, \partial_\mu] - \frac{i}{2}u^\dagger(v_\mu + a_\mu)u - \frac{i}{2}u(v_\mu - a_\mu)u^\dagger.$$

exercise: expanding U in the pion fields obtain the result

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\psi}(i \not{\partial} - m)\psi - \frac{1}{4f_\pi^2}\bar{\psi}\gamma^\mu\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})\psi + \frac{g_A}{2f_\pi}\bar{\psi}\gamma_5\boldsymbol{\tau}\cdot\partial_\mu\boldsymbol{\pi}\psi + O(\pi^3) \tag{20}$$



Loops in Baryon χPT

- nucleon mass doesn't vanish in the χ-limit

  $\Rightarrow$ introduces another (large) scale on the theory

  $\Rightarrow$ the power counting is not transparent

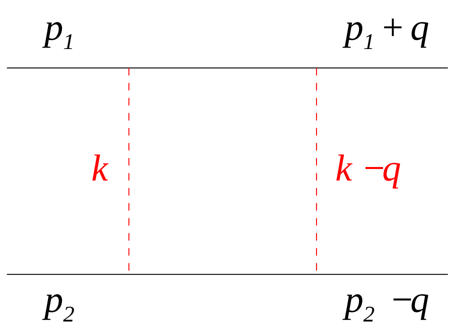
- considering the baryon infinitely heavy (HB-χPT) restores the power counting

- alternative: infrared regularization - keeps the covariant aspect of the theory

- power counting: $\nu = 1 + 2N_L + \sum_d N_d^\pi (d-2) + \sum_d N_d^N (d-1)$

2 nucleons

- 2 or more nucleons: χPT power counting is different from expected!



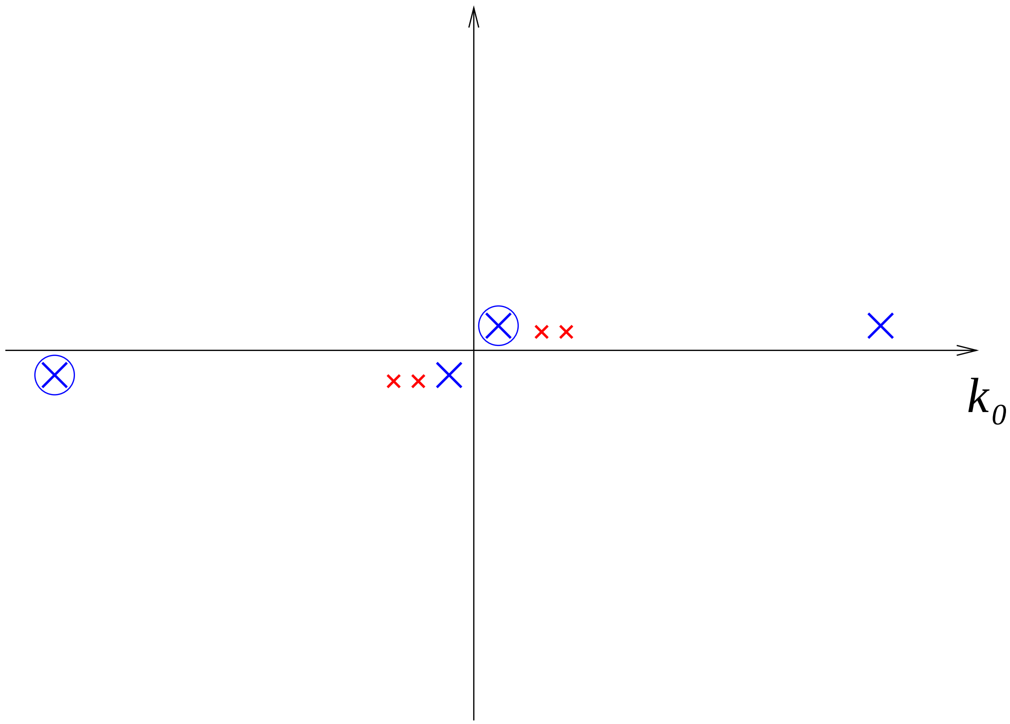
$$
\begin{aligned}
&= \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_\pi^2} \frac{1}{(k-q)^2 - m_\pi^2} \frac{1}{(p_1+k)^2 - m_N^2} \frac{1}{(p_2-k)^2 - m_N^2} \\
&= \int \frac{d^3k}{(2\pi)^3} \frac{dk_0}{2\pi} \frac{1}{k_0^2 - \omega_1^2} \frac{1}{(k_0 - q_0)^2 - \omega_2^2} \\
&\qquad \times \frac{1}{(p_0^{(1)} + k_0)^2 - \sigma_2^2} \frac{1}{(p_0^{(2)} - k_0)^2 - \sigma_2^2}
\end{aligned}
\tag{21}
$$

$\omega_1^2 = \vec{k}^2 + m_\pi^2$, $\omega_2^2 = (\vec{k} - \vec{q})^2 + m_\pi^2$,

$\sigma_1^2 = (\vec{p_1} + \vec{k})^2 + m_N^2$, $\sigma_2^2 = (\vec{p_2} - \vec{k})^2 + m_N^2$.

poles: $\pm\omega_1$, $\pm\omega_2$, $-p_0^{(1)} + \sigma_1$, $-p_0^{(1)} - \sigma_1$, $p_0^{(2)} + \sigma_2$, $p_0^{(2)} - \sigma_2$.

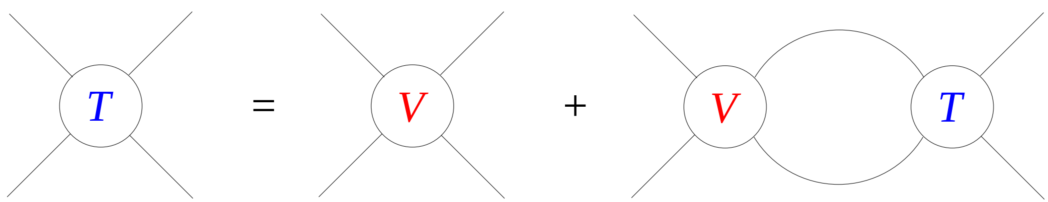
2 nucleons



poles: $\pm\omega_1$, $\pm\omega_2$, $-p_0^{(1)} + \sigma_1$, $-p_0^{(1)} - \sigma_1$, $p_0^{(2)} + \sigma_2$, $p_0^{(2)} - \sigma_2$.

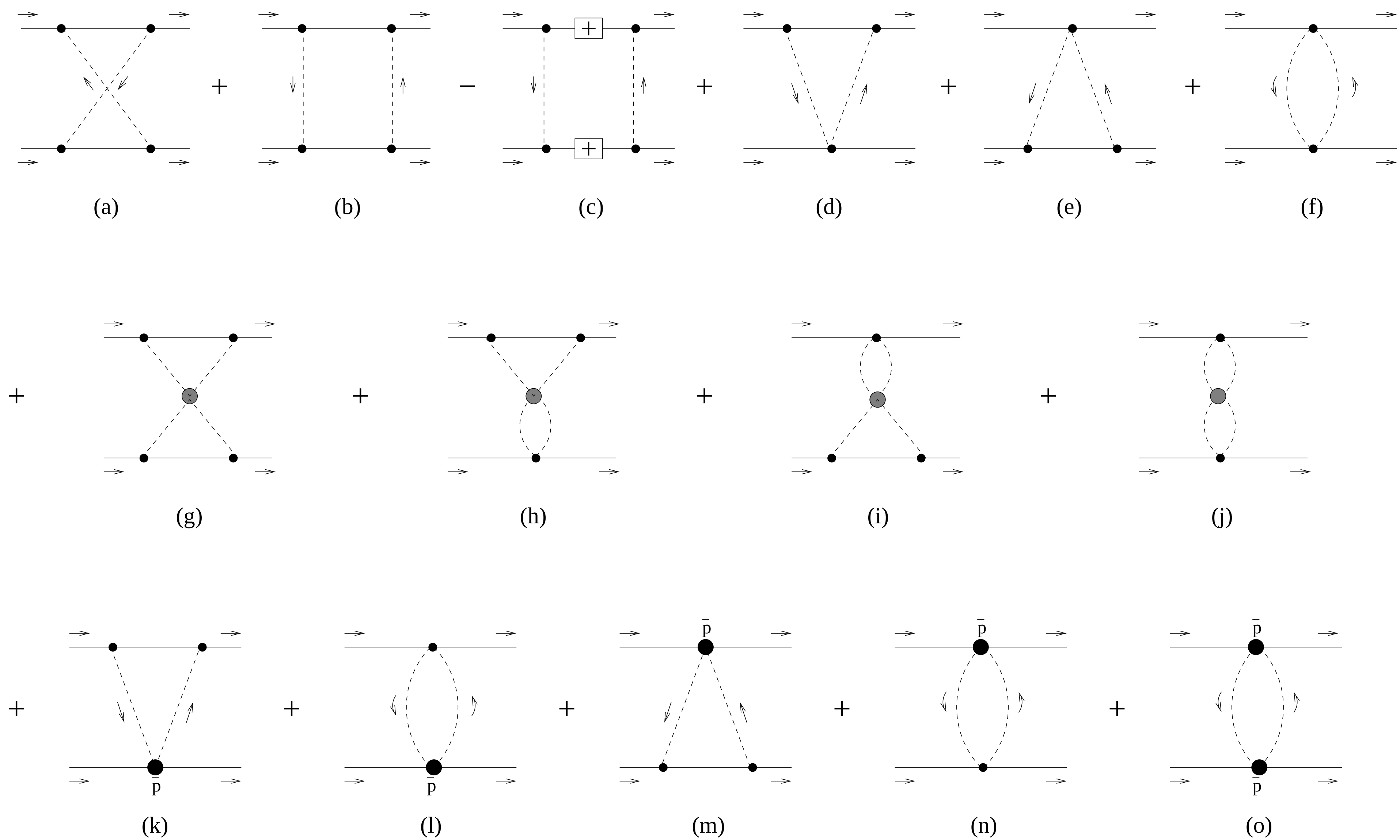
- Weinberg (90, 91): 2 (and more) nucleons ⇒ non-perturbative problem
  - ⋆ solution: χPT power counting rules applied to the potential, and solve the scattering with a LS equation

$$T = V + V\,G\,T$$

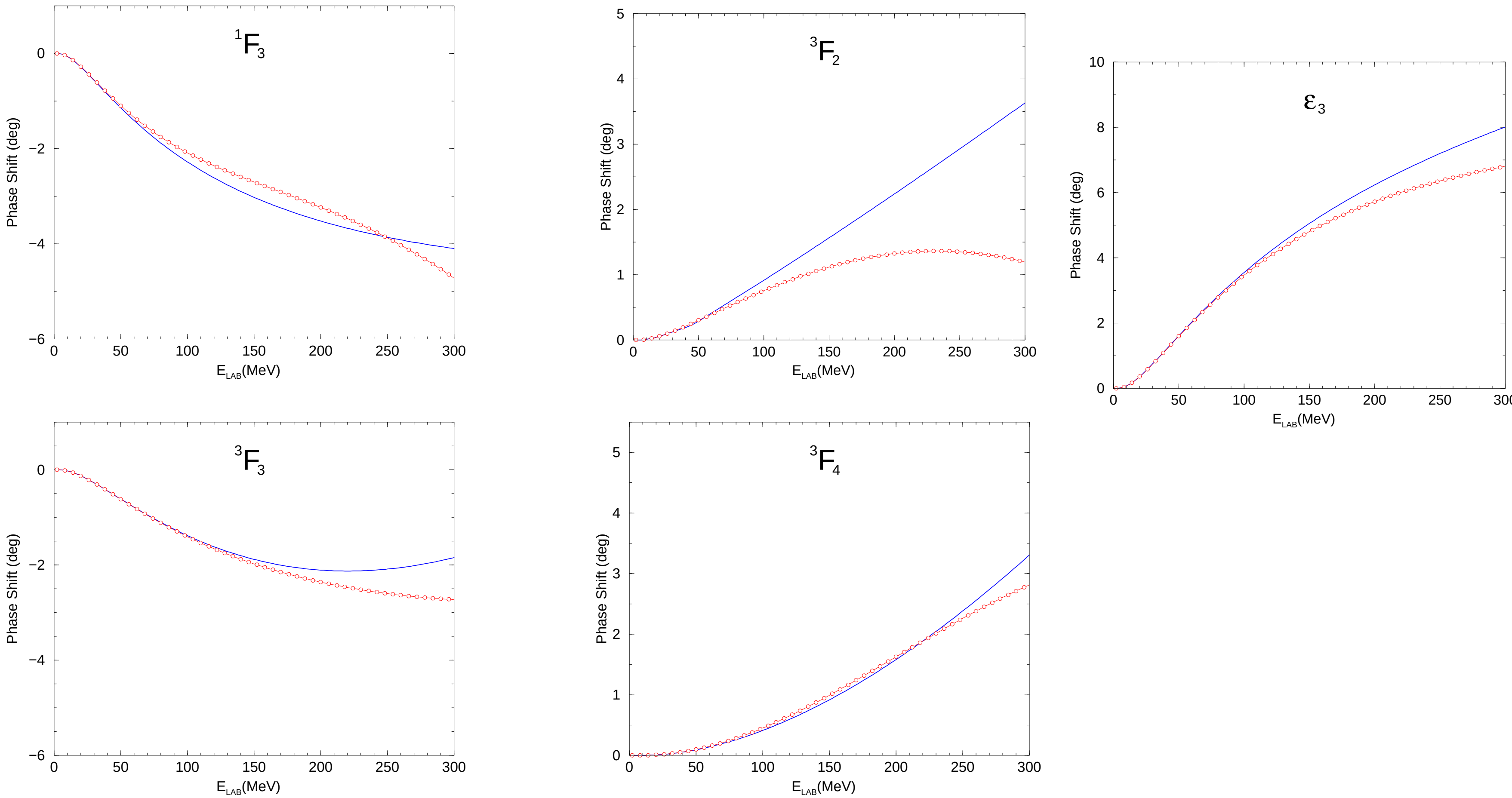


- Ordoñez, Ray, van Kolck (92-96) - potential up to $O(q^2)$

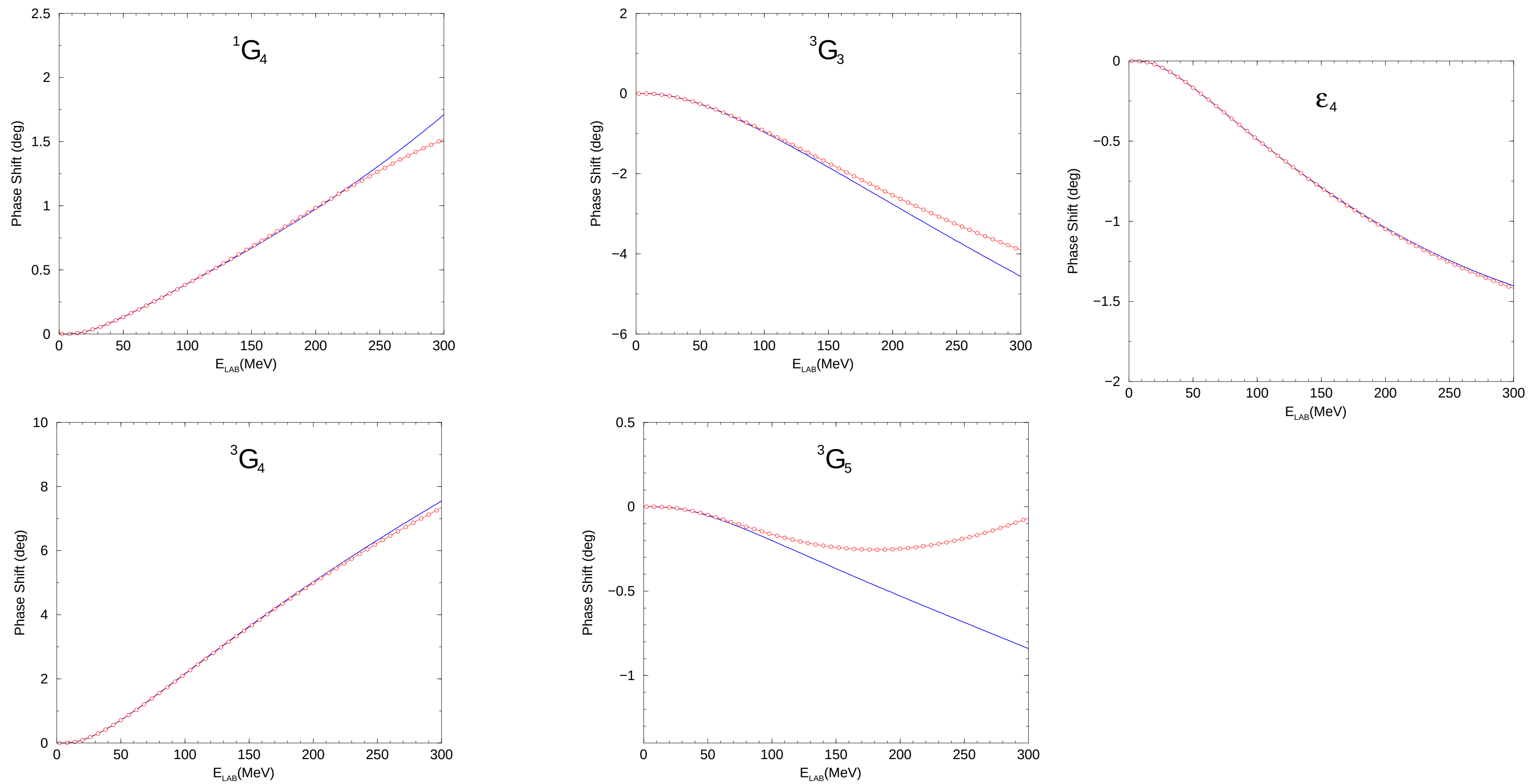
- 02-present: NN potential up to $O(q^4)$
  - ⋆ D.R. Entem, R. Machleidt, PRC 66, 014002
  - ⋆ R.H., M.R. Robilotta, PRC 68, 024004, + C.A. Rocha, PRC 69, 034009
  - ⋆ E. Epelbaum, W. Glöckle, U-G. Meißner, NPA 747, 362



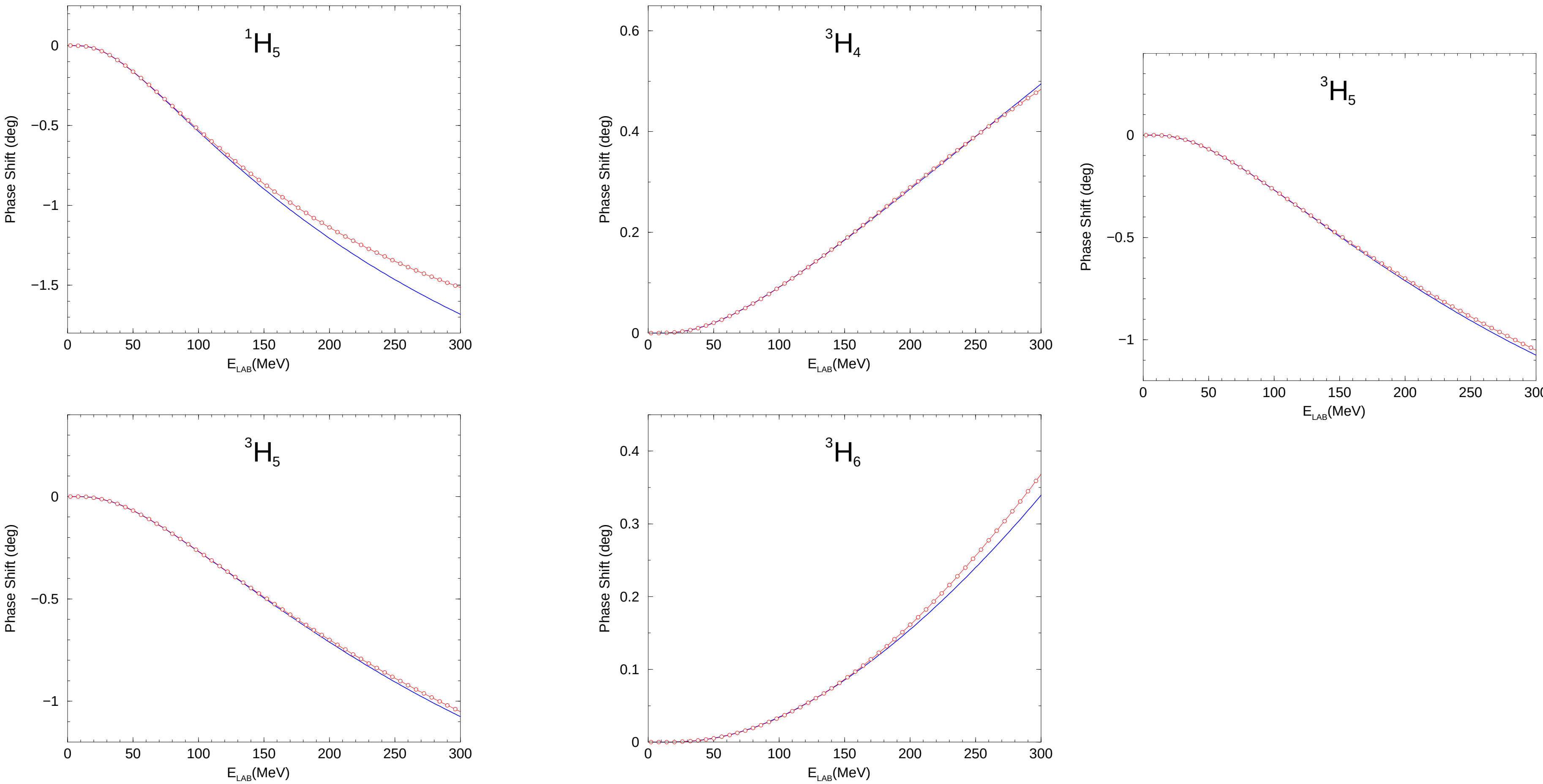
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
(j)
(k)
(l)
(m)
(n)
(o)



1F3
3F2
ε3
3F3
3F4
Phase Shift (deg)
E_LAB(MeV)



$^1G_4$
$^3G_3$
ε_4
$^3G_4$
$^3G_5$
Phase Shift (deg)
E_{LAB}(MeV)



$^1H_5$
$^3H_4$
$^3H_5$
$^3H_5$
$^3H_6$
Phase Shift (deg)
E_{LAB}(MeV)
0
50
100
150
200
250
300
−0.5
−1
−1.5
0.2
0.4
0.6
0.1
0.3

$$
\begin{aligned}
V_{\rm cont} &= V^{(0)}_{\rm cont} + V^{(2)}_{\rm cont} + V^{(4)}_{\rm cont} \;, \\
V^{(0)}_{\rm cont} &= C_S + C_T\, \boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)} \;, \\
V^{(2)}_{\rm cont} &= C_1 \boldsymbol{q}^2 + C_2 \boldsymbol{z}^2 + (C_3 \boldsymbol{q}^2 + C_4 \boldsymbol{z}^2)(\boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)}) + i C_5 \frac{1}{2} (\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)}) \cdot (\boldsymbol{q} \times \boldsymbol{z}) \\
&\quad + C_6 (\boldsymbol{q} \cdot \boldsymbol{\sigma}^{(1)})(\boldsymbol{q} \cdot \boldsymbol{\sigma}^{(2)}) + C_7 (\boldsymbol{z} \cdot \boldsymbol{\sigma}^{(1)})(\boldsymbol{z} \cdot \boldsymbol{\sigma}^{(2)}) \;, \\
V^{(4)}_{\rm cont} &= D_1 \boldsymbol{q}^4 + D_2 \boldsymbol{z}^4 + D_3 \boldsymbol{q}^2 \boldsymbol{z}^2 + D_4 (\boldsymbol{q} \times \boldsymbol{z})^2 \\
&\quad + \Big(D_5 \boldsymbol{q}^4 + D_6 \boldsymbol{z}^4 + D_7 \boldsymbol{q}^2 \boldsymbol{z}^2 + D_8 (\boldsymbol{q} \times \boldsymbol{z})^2 \Big) (\boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)}) \\
&\quad + i \left(D_9 \boldsymbol{q}^2 + D_{10} \boldsymbol{z}^2 \right) \frac{\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)}}{2} \cdot (\boldsymbol{q} \times \boldsymbol{z}) + \left(D_{11} \boldsymbol{q}^2 + D_{12} \boldsymbol{z}^2 \right) (\boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{q})(\boldsymbol{\sigma}^{(2)} \cdot \boldsymbol{q}) \\
&\quad + \Big(D_{13} \boldsymbol{q}^2 + D_{14} \boldsymbol{z}^2 \Big) (\boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{z})(\boldsymbol{\sigma}^{(2)} \cdot \boldsymbol{z}) + D_{15} \Big(\boldsymbol{\sigma} \cdot (\boldsymbol{q} \times \boldsymbol{z})\, \boldsymbol{\sigma}^{(2)} \cdot (\boldsymbol{q} \times \boldsymbol{z}) \Big)
\end{aligned}
\tag{22}
$$

with $\boldsymbol{q} = \boldsymbol{p}\,' - \boldsymbol{p}$ and $\boldsymbol{z} = (\boldsymbol{p} + \boldsymbol{p}\,')/2$.

$\not\pi$ EFT

$\Rightarrow$ **very** low energies : $m_\pi \sim \Lambda$

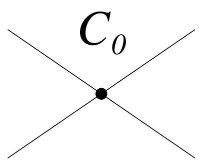
$$T = \frac{4\pi}{m_N} \frac{1}{k \cot\delta - ik} \tag{23}$$

$$k \cot\delta = -\frac{1}{a} + \frac{\Lambda^2}{2} \sum_{n=0}^{\infty} r_n \left(\frac{k^2}{\Lambda^2}\right)^{n+1} \qquad (ERE) \tag{24}$$

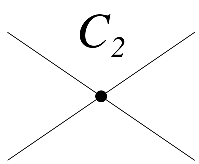
$$\begin{aligned}\mathcal{L} &= N^\dagger \left(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2m_N}\right) N \\ &\quad + (\mu/2)^{4-D} \left\{ -C_0 (N^\dagger N)^2 + \frac{C_2}{8} \left[(NN)^\dagger N (\overleftrightarrow{\boldsymbol{\nabla}})^2 N + h.c. \right] + \cdots \right\}\end{aligned} \tag{25}$$

$$(\overleftrightarrow{\boldsymbol{\nabla}})^2 \equiv (\overleftarrow{\boldsymbol{\nabla}})^2 - 2\overleftarrow{\boldsymbol{\nabla}} \cdot \overrightarrow{\boldsymbol{\nabla}} + (\overrightarrow{\boldsymbol{\nabla}})^2 , \tag{26}$$

⋆ tree graphs:



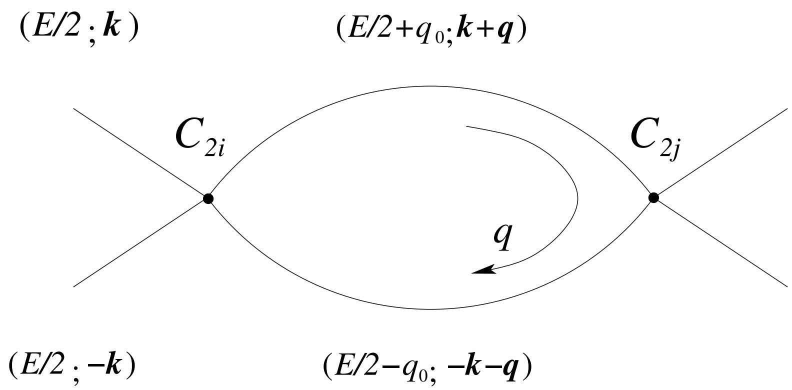
$$= \quad i(\mu/2)^{4-D}(-C_0) = -i(\mu/2)^{4-D}C_0\,, \qquad (27)$$



$$\begin{aligned} &= \quad i(\mu/2)^{4-D}\,\frac{C_2}{8}\Big\{\Big[(-i\boldsymbol{k})^2 - 2(-i\boldsymbol{k})\cdot(i\boldsymbol{k}) + (i\boldsymbol{k})^2\Big] \\ &\qquad + \Big[(-i\boldsymbol{k'})^2 - 2(-i\boldsymbol{k'})\cdot(i\boldsymbol{k'}) + (i\boldsymbol{k'})^2\Big]\Big\} \\ &= \quad -i(\mu/2)^{4-D}\,\frac{C_2}{2}\Big(\boldsymbol{k}^2 + \boldsymbol{k'}^2\Big) = -i(\mu/2)^{4-D}\,\frac{C_2}{2}\,k^2\,. \qquad (28) \end{aligned}$$

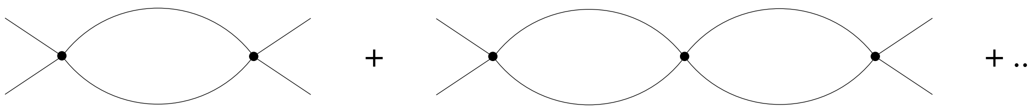
$$\begin{aligned} iT_{tree} &= -i(\mu/2)^{4-D}\,C_0 - i(\mu/2)^{4-D}\,C_2 k^2 + \cdots \\ &= -i(\mu/2)^{4-D}\sum_{n=0}^{\infty} C_{2n}k^{2n}\,. \qquad (29) \end{aligned}$$

⋆ loops:



$$
\begin{aligned}
&= \Big[-i(\mu/2)^{4-D} C_{2i}\Big]\Big[-i(\mu/2)^{4-D} C_{2j}\Big] \int \frac{d^{D-1}q}{(2\pi)^{D-1}}\,\frac{dq_0}{2\pi}\,(\boldsymbol{k}+\boldsymbol{q})^{2i+2j} \\
&\quad \times i S_F(E/2+q_0;\boldsymbol{k}+\boldsymbol{q})\, i S_F(E/2-q_0;-\boldsymbol{k}-\boldsymbol{q}) \\
&= \Big[-i(\mu/2)^{4-D} C_{2i}\Big]\Big[-i(\mu/2)^{4-D} C_{2j}\Big] \int \frac{d^{D-1}q}{(2\pi)^{D-1}}\,\frac{dq_0}{2\pi}\,\boldsymbol{q}^{2i+2j} \\
&\quad \times i S_F(E/2+q_0;\boldsymbol{q})\, i S_F(E/2-q_0;-\boldsymbol{q}) \\
&= -i(\mu/2)^{4-D} C_{2i}\, C_{2j}\, I_{i+j}\,. \qquad (30)
\end{aligned}
$$

$$
\begin{aligned}
I_n &= -i(\mu/2)^{4-D} \int \frac{d^{D-1}q}{(2\pi)^{D-1}} \frac{dq_0}{2\pi} \\
&\quad \times \boldsymbol{q}^{2n} \left[\frac{i}{E/2 + q_0 - \boldsymbol{q}^2/2m_N + i\epsilon} \right] \left[\frac{i}{E/2 - q_0 - \boldsymbol{q}^2/2m_N + i\epsilon} \right] \\
&= (\mu/2)^{4-D} \int \frac{d^{D-1}q}{(2\pi)^{D-1}} \frac{\boldsymbol{q}^{2n}}{E - \boldsymbol{q}^2/m_N + i\epsilon} \\
&= -m_N (m_N E)^n (-m_N E - i\epsilon)^{(D-3)/2} \, \Gamma\left(\frac{3-D}{2} \right) \frac{(\mu/2)^{4-D}}{(4\pi)^{(D-1)/2}} \\
&= -i \left(\frac{m_N}{4\pi} \right) k^{2n+1}
\end{aligned}
\tag{31}
$$



$$= iT_{loops}$$

$$= -i(\mu/2)^{4-D} \sum_{l,m} C_{2l}\, C_{2m}\, I_{l+m} - i(\mu/2)^{4-D} \sum_{l,m,n} C_{2l}\, C_{2m}\, C_{2n}\, I_{l+m}\, I_{m+n} + \cdots$$

$$= -i(\mu/2)^{4-D} \left\{ \sum_{l,m} C_{2l}\, C_{2m} \left(-i\, \frac{m_N}{4\pi}\, k\right) k^{2l+2m} \right.$$
$$\left. + \sum_{l,m,n} C_{2l}\, C_{2m}\, C_{2n} \left(-i\, \frac{m_N}{4\pi}\, k\right)^2 k^{2l+2m+2n} + \cdots \right\} \tag{32}$$

$$T = T_{tree} + T_{loops} = -\frac{\sum_n C_{2n} k^{2n}}{1 + i\frac{m_N}{4\pi} k \sum_n C_{2n} k^{2n}}\,. \tag{33}$$

- the natural case

  Scales : M_{lo}, M_{hi} , variables: k , a , r_n

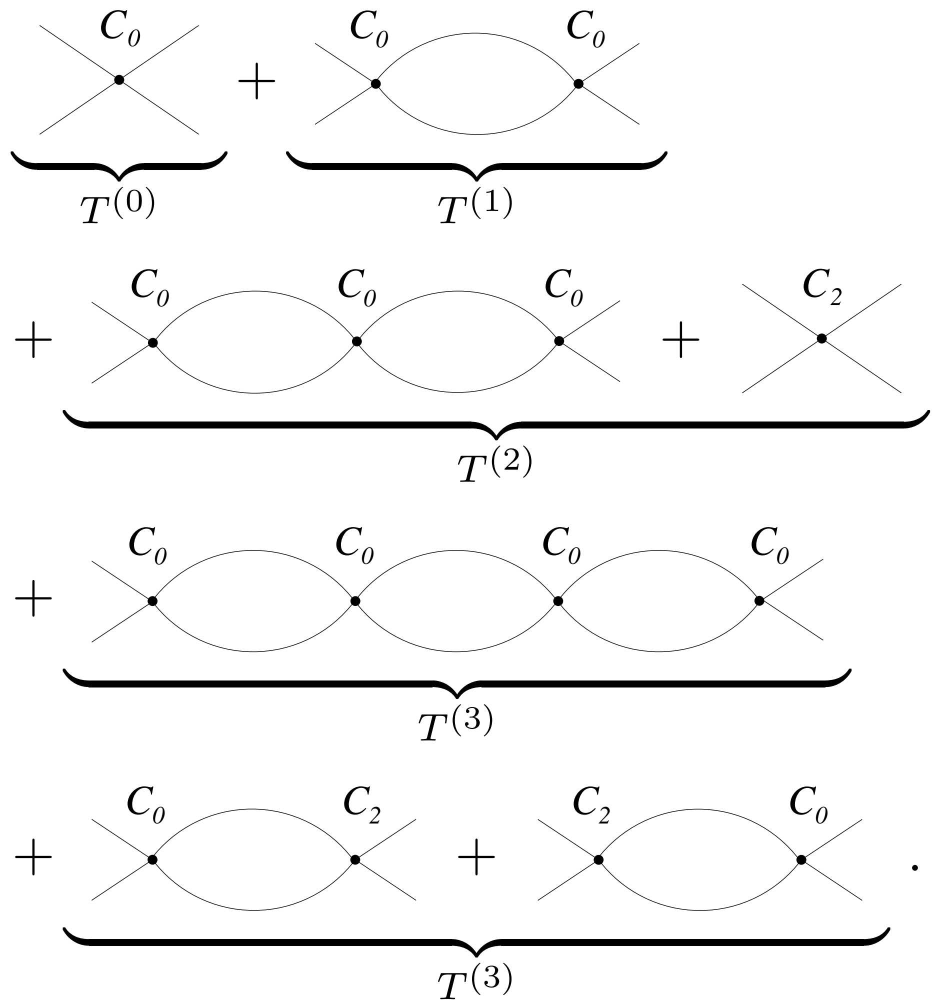
  a, $r_n \sim 1/\Lambda$

$$C_{2n} \sim \frac{4\pi}{m_N} \frac{1}{M_{hi}^{2n+1}} \tag{34}$$

$$C_0 \, I_0 \sim C_0 \, \frac{m_N}{4\pi} \, Q \sim \frac{Q}{M_{hi}} \tag{35}$$

  ⋆ loops: suppressed at least by Q/M_{hi}
  ⋆ higher order contact terms: suppressed by $(Q/M_{hi})^2$

$T =$



(36)

$$
\begin{aligned}
T_{ERE} &= -\frac{4\pi}{m_N}\,a + i\,\frac{4\pi}{m_N}\,a^2 k + \frac{4\pi}{m_N}\,a\left[a^2 - \frac{a r_0}{2}\right]k^2 + i\,\frac{4\pi}{m_N}\,a\left[a^2 r_0 - a^3\right]k^3 \\
&\quad + \left[-\frac{4\pi}{m_N}\,a^5 + \frac{6\pi}{m_N}\,r_0 a^4 - \frac{\pi}{m_N}\,r_0^2 a^3 - \frac{2\pi}{\Lambda^2 m_N}\,r_1 a^2\right]k^4 + \cdots\,, \qquad (37)
\end{aligned}
$$

$$
\begin{aligned}
T_{EFT} &= -C_0 + i\,\frac{m_N}{4\pi}\,C_0{}^2 k + \left[\left(\frac{m_N}{4\pi}\right)^2 C_0^3 - C_2\right]k^2 + i\,\frac{m_N}{4\pi}\,C_0\left[2C_2 - \left(\frac{m_N}{4\pi}\right)^2 C_0\right]k^3 \\
&\quad + \left[-\left(\frac{m_N}{4\pi}\right)^4 C_0^5 + 3\left(\frac{m_N}{4\pi}\right)^2 C_2 C_0^2 - C_4\right]k^4 + \cdots\,, \qquad (38)
\end{aligned}
$$

$$
C_0 = \frac{4\pi}{m_N}\,a\,, \qquad C_2 = C_0\,\frac{a r_0}{2}\,, \qquad C_4 = \frac{C_2^2}{C_0} + \frac{C_0}{\Lambda^2}\,\frac{a r_1}{2}\,. \qquad (39)
$$

$a, r_n \sim 1/\Lambda \sim 1/M_{hi}$ consistent with $C_{2n} \sim 1/M_{hi}^{2n+1}$

- the unnatural case

new scale : $1/a \Rightarrow \aleph \sim M_{lo}$,

$$C_{2n} \sim \frac{4\pi}{m_N} \frac{1}{\aleph(\aleph M_{hi})^n} \,. \tag{40}$$

⋆ higher order contact interactions still suppressed,

$$\frac{C_{2(n+1)} k^{2(n+1)}}{C_{2n} k^{2n}} \sim \frac{k^2}{\aleph \Lambda} \sim \frac{Q}{M_{hi}} \tag{41}$$

⋆ loops:

$$C_{2n}\, I_{2n} \sim \frac{k^{2n+1}}{\aleph(\aleph\Lambda)^n} \sim \left(\frac{Q}{M_{hi}}\right)^n \tag{42}$$

loops with C_0 only are not suppressed $\Rightarrow$ **have to be resummed to all orders!**

- LO: bubble sum with C_0 vertices

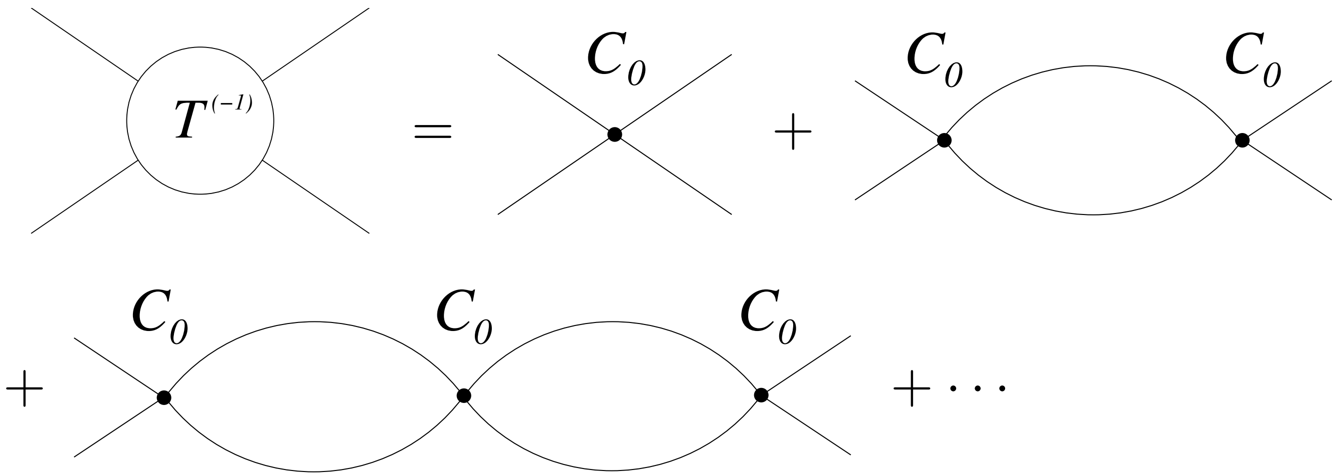
$$T^{(-1)} = -\frac{C_0}{1 + ik\frac{m_N}{4\pi}C_0}\,. \tag{43}$$

- NLO: tree graph with C_2,

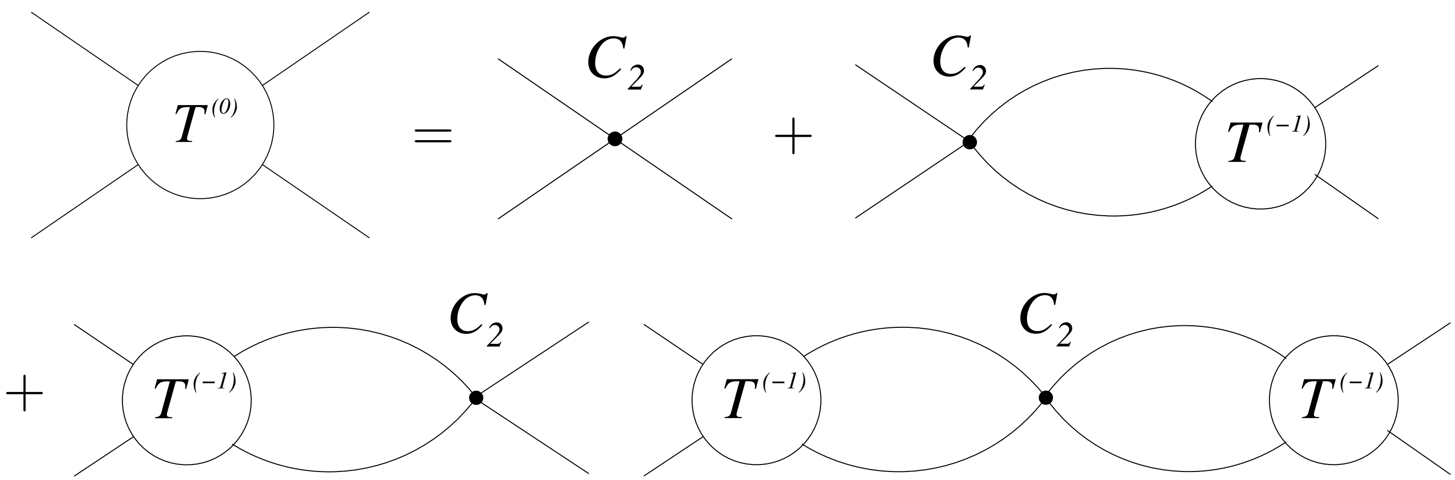
  plus bubble diagrams with all C_0 vertices but one with C_2

$$T^{(0)} = -\frac{C_2 k^2}{\left[1 + ik\frac{m_N}{4\pi}C_0\right]^2}\,. \tag{44}$$

exercise: obtain the above result.



(45)



(46)

$$
\begin{aligned}
T_{ERE} &= \frac{4\pi}{m_N}\left[\underbrace{-ik-\frac{1}{a}}_{O(\aleph)}+\underbrace{\frac{1}{2}r_0k^2}_{O(\aleph^2/M_{hi})}+\underbrace{\frac{1}{2}r_1\frac{k^4}{\Lambda^2}}_{O(\aleph^4/M_{hi}^3)}+\cdots\right]^{-1} \\
&= -\frac{4\pi}{m_N}\left[\underbrace{\frac{1}{(1/a+ik)}}_{T^{(-1)}}+\underbrace{\frac{r_0k^2}{2(1/a+ik)^2}}_{T^{(0)}}+\underbrace{\frac{r_0^2k^4}{4(1/a+ik)^3}}_{T^{(1)}}+\underbrace{\frac{r_1k^4}{2\Lambda^2(1/a+ik)^2}}_{T^{(2)}}+\cdots\right]
\end{aligned}
$$

Summary

- χPT for mesons: most successful application of EFT ideas
- χPT for baryons: slight complication due to the baryon mass (doesn't vanish in the χ-limit) but can still be treated "perturbatively"
- χEFT for nuclear systems: χPT rules applied to the potential

  (non-perturbative problem: LS equation)
- $\not\pi$ EFT: much simpler theory, 1st step towards χEFT