Lecture # 6

MORE ON BARYONS

Baryon Masses: a problem of convergence

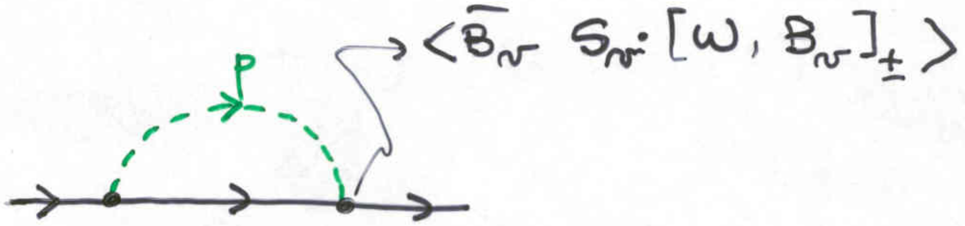
Deviation from G-M-O relation is very small. Expect small NNLO corrections to masses?. Let us see...

Masses @ $\mathcal{O}(p^3)$

Only contributions from 1-loop with $\mathcal{L}^{(1)}$.

$$m_B = m_0 + \mathcal{O}(m_q) + \mathcal{O}(m_q^{3/2}) + \cdots$$

Use HBχPT to calculate 1-loop contributions:



Counting: powers of p

4	+ 2	− 2	− 1	= 3
↑	↑	↑	↑	
d^4p	vertices	π-propagator	Baryon propagator	

$$\sim \frac{1}{F_0^2} \int \frac{d^d p}{(2\pi)^d} \frac{p_\mu p_\nu \, S_v^\mu S_v^\nu}{(p^2 - M_\pi^2 + i\varepsilon)(-p\cdot v + i\varepsilon)}$$

$$= \frac{i}{24\pi} \frac{M_\pi^3}{F_0^2} \left(g_{\mu\nu} - v_\mu v_\nu \right) S_v^\mu S_v^\nu$$

$$= -\frac{i}{32\pi} \frac{M_\pi^3}{F_0^2} \qquad \left(S_v \cdot S_v = -\frac{3}{4} \right)$$

Finite result $\propto m_q^{3/2}$.

$$m_N = m_0 + \delta m_N^{(2)} + \frac{M_K^3}{432\pi F_0^2}\left(-45D^2 - 4\sqrt{3}(D-3F)^2 + 54DF - 81F^2\right)$$

$$m_\Sigma = m_0 + \delta m_\Sigma^{(2)} + \left(-\frac{M_K^3}{216\pi F_0^2}\right)\left((27+8\sqrt{3})D^2 + 27F^2\right)$$

$$m_\Lambda = m_0 + \delta m_\Lambda^{(2)} + \left(-\frac{M_K^3}{216\pi F_0^2}\right)\left((9+8\sqrt{3})D^2 + 81F^2\right)$$

$$m_\Xi = m_0 + \delta m_\Xi^{(2)} + \frac{M_K^3}{432\pi F_0^2}\cdot\left(-45D^2 - 4\sqrt{3}(D+3F)^2 - 54DF - 81F^2\right)$$

$\mathcal{O}(p^3)$ corrections give modest deviation from GM-O, but :

$$\hspace{6em}\mathcal{O}(p^4)$$

$$m_N = m_0\,(1 + 0.34 - 0.35 \;\; \underset{\uparrow}{(+0.24)})$$

$$m_\Lambda = m_0\,(1 + 0.70 - 0.77 \;\; (+0.54))$$

$$m_\Sigma = m_0\,(1 + 0.81 - 0.70 \;\; (+0.44))$$

$$m_\Xi = m_0\,(1 + 1.10 - 1.16 \;\; (+0.78))$$

At $\mathcal{O}(p^4)$ corrections are also large (comparable to $\mathcal{O}(p^3)$) $\Rightarrow$ No good convergence in SU(3)

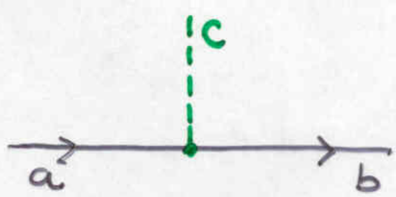
Possible improvements:

- Include spin-3/2 10-plet as explicit d.o.f.. This improves convergence.

- Use different regularization method (cutoff). This reorders expansion. Difficult to keep symmetries in check, but it can be done.

A lucky case: GTR in SU(3)

LO: GTR holds exactly



$$g_A^{abc} \, m_0 = F_0 \, g^{abc}$$

Beyond LO:

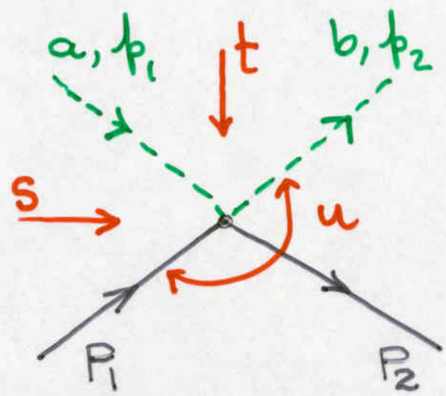
$$g_A^{abc}\,\frac{m_a+m_b}{2} = (1-\Delta^{abc})\,F_c\,g^{abc}$$

Δ^{abc} : GT Discrepancy

$$\Delta^{abc} = \underset{\text{from } \mathcal{L}^{(3)}}{\mathcal{O}(m_q)} + \underset{\text{1-loop with } \mathcal{L}^{(3)}\text{, and } \mathcal{L}^{(5)}}{\mathcal{O}(m_q^2 \times \log m_q^2)}$$

π-N Scattering @ $\mathcal{O}(p^2)$ and the σ- term. (SU(2))

Kinematics



convenient low energy variables

$$\nu = \frac{s-u}{4m_N} = \mathcal{O}(p)$$

$$\nu_B = \frac{2M_\pi^2 - t}{4m_N} = \mathcal{O}(p^2)$$

$$s = m_N(m_N + 2(\nu+\nu_B)) \qquad p_1 \cdot P_1 = m_N(\nu+\nu_B) - \frac{M_\pi^2}{2}$$

$$t = 2(M_\pi^2 - 2m_N\nu_B) \qquad p_1 \cdot P_2 = m_N(\nu-\nu_B) + \frac{M_\pi^2}{2}$$

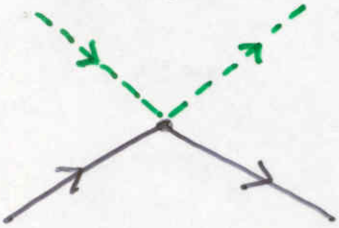
$$u = m_N(m_N + 2(\nu_B - \nu)) \qquad p_1 \cdot p_2 = 2m\nu_B$$

@ $\mathcal{O}(p^2)$ only tree level diagrams

$$\mathcal{L}^{(1)}_{\pi N \to \pi N} = \frac{1}{8F_0^2}\,\bar{N}\,[\Pi, \partial_\mu \Pi]\,\gamma_\mu N - \frac{g_A}{2F_0}\,\bar{N}\,\partial_\mu \Pi\,\gamma^\mu \gamma_5 N$$

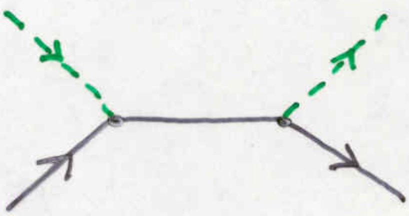
$$\mathcal{L}^{(2)}_{\pi N \to \pi N} = c_1\, 2M_\pi^2\,\bar{N}N - \frac{2c_2}{m_0^2 F_0^2}\langle \partial_\mu \Pi\, \partial_\nu \Pi\rangle\,\bar{N}\,\partial^\mu \partial^\nu N + \frac{2c_3}{F_0^2}\langle \partial_\mu \Pi\, \partial^\mu \Pi\rangle\,\bar{N}N - \frac{c_4}{F_0^2}\,\bar{N}\,\gamma^\mu\gamma^\nu [\partial_\mu \Pi, \partial^\nu \Pi] N$$

t-channel
iso vector



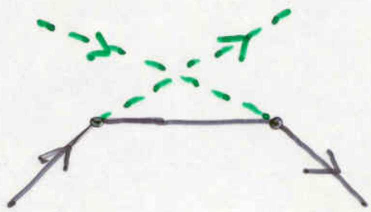
$\mathcal{O}(p)$

+



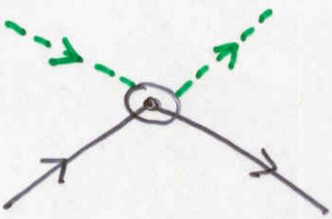
$\mathcal{O}(p^2)$

+



$\mathcal{O}(p^2)$

+



$\mathcal{O}(p^2)$

+ . . .

$\mathcal{O}(p^3)$

Getting c_1 with π-N scattering.

Need t-channel isosinglet amplitude

$$T^{ab}_{\pi N} = \bar{N}'\left(T^{+}\delta^{ab} + T^{-}\tfrac{1}{2}[\tau^a,\tau^b]\right)N$$

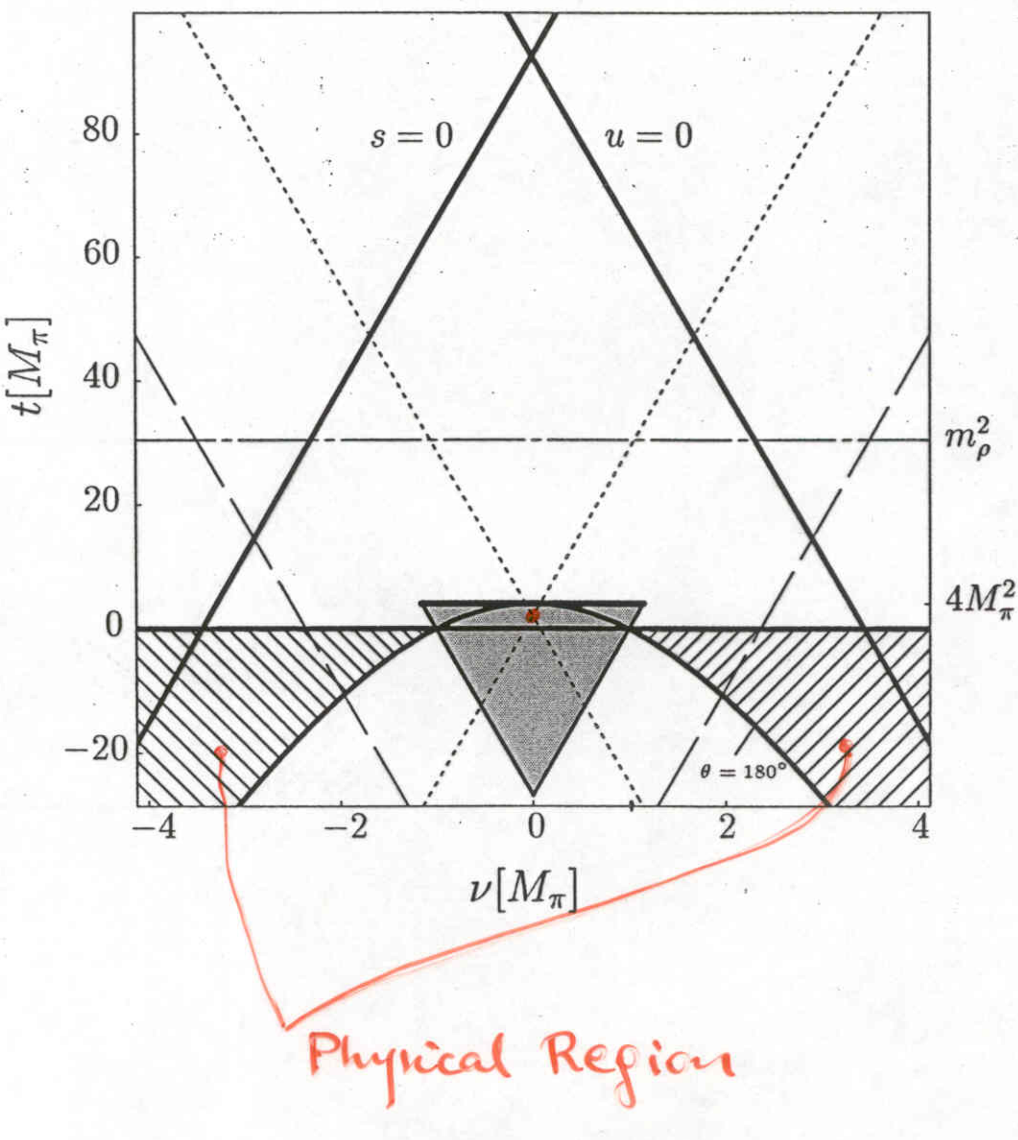
$$T^{+} = A^{+}(\nu,\nu_B) + \tfrac{1}{2}(\not{p}+\not{p}')B^{+}(\nu,\nu_B)$$

$$D^{+} \equiv A^{+} + \nu B^{+} = \mathcal{O}(p^2)$$

Need to eliminate terms from D^+ to isolate contribution $\propto c_1$.

- Eliminate Born terms : $D^+ \to \bar{D}^+$

- Evaluate $\bar{D}^+$ at $\nu = \nu_B = 0$ (Cheng-Dashen point). Up to $\mathcal{O}(p^4)$ corrections, this isolates term $\propto c_1$.



80
60
40
20
0
−20
$s=0$
$u=0$
m_ρ^2
$4M_\pi^2$
$\theta = 180°$
−4
−2
0
2
4
$t[M_\pi]$
$\nu[M_\pi]$
Physical Region

LE Theorem: $\Sigma \equiv F_\pi^2 \bar{D}^+ \big|_{C-D}$,

$\sigma(t) \equiv \hat{m} \langle N | \bar{q}q(t) | N \rangle$.

Then:

$$\sigma(t = 2M_\pi^2) = \Sigma + \mathcal{O}(p^4)$$

@ $\mathcal{O}(p^2)$ σ is t-independent. NLO corrections are $\mathcal{O}(p^3)$.

$$\sigma(t=0) = \underset{\substack{\uparrow \\ \mathcal{O}(p^2)}}{\overset{\substack{c_1 \\ \uparrow}}{\Sigma}} + \underbrace{\underset{\sim -7\,\text{MeV}}{\mathcal{O}(p^3)} + \mathcal{O}(p^4) + \cdots}_{\sim -15\,\text{MeV}}$$

From π-N data to Σ:

i) Determine $\bar{D}^+$ @ threshold ($t=0$, $\nu = M_\pi$) via dispersion relation techniques (all amplitudes $\bar{D}^+$, D^-, $B^\pm$ involved here). Data is input at this stage.

ii) Extrapolate from threshold to CD point ($t = 2M_\pi^2$, $\nu = 0$). χPT to $\mathcal{O}(p^4)$ has been used here.

$$\Sigma \sim 60\,\text{MeV}$$

[Leutwyler et al].

$$\Downarrow$$

$$\sigma \cong (\underbrace{60}_{\mathcal{O}(p^2)} - \underbrace{15}_{\text{higher orders}})\,\text{MeV}$$

$\mathcal{O}(p^2) \Rightarrow c_1 \sim -0.8/m_N$

What does $\sigma \sim 45\,\text{MeV}$ mean?

$$m_n - m_p = 1.293\,\text{MeV} \simeq \underbrace{2\,\text{MeV}}_{m_d - m_u} - \underbrace{0.7\,\text{MeV}}_{\text{EM}}$$

Valence quark picture:

$$m_p = m_0 + \xi\,(2m_u + m_d)$$

$$m_n = m_0 + \xi\,(2m_d + m_u)$$

$$m_n - m_p = \xi\,(m_d - m_u)$$

$$\sigma_{\text{val.}} = \frac{1}{2}\hat{m}\,\frac{\partial}{\partial \hat{m}}(m_p + m_n) = 3\xi\hat{m}$$

using $\frac{m_d - m_u}{\hat{m}} \sim 0.57 \Rightarrow \sigma_{\text{val}} \sim 10\,\text{MeV} \ll \sigma$

Topics Left Out

Mesons:

- Weak non-leptonic decays.
- EM radiative corrections.
- Extension of χPT to heavy mesons.
- χPT at finite T.

Baryons:

- γ- and e-production (cover in part by Marc Vanderhaeghen) and Compton.
- IR regularization as alternative to HBχPT.
- Magnetic moments, etc.
- NN interactions (whole new field)
- Weak decays.