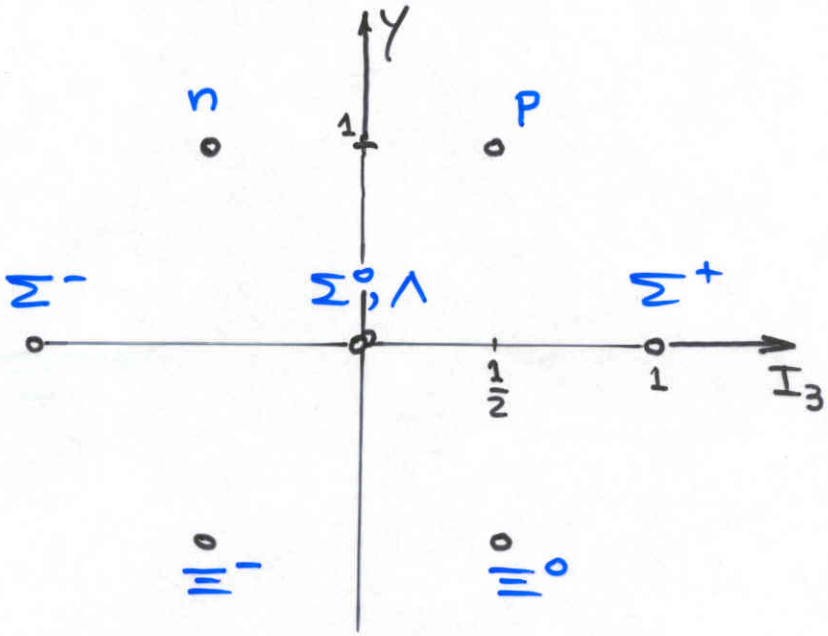
Lecture #5

BARYON

χPT

The stable baryons: baryon octet

Spin ½ baryons in 8 of SU(3)



Represented by 3x3 matrix

2 flavor world.

$$B = \begin{bmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{bmatrix}$$

Chiral Transformations of baryons

$SU_L(3) \times SU_R(3)$ transformations on Hermitian 3x3 matrices: non-linear realization [Callan, Coleman, Wess & Zumino]

$L \in SU_L(3)$ $\qquad$ $R \in SU_R(3)$

give $u \in SU(3)$ and define:

$$L u = u' \, h(L, R, u)$$

$$R u^\dagger = u'^\dagger \, h(L, R, u)$$

h provides a non-linear representation of $SU_L(3) \times SU_R(3)$

Exercise: show that

i) $h^\dagger(L,R,u) = h^{-1}(L,R,u)$

ii) $h^{-1}(L,R,u) = h(L^\dagger, R^\dagger, u')$

iii) $h(L_1 L_2, R_1 R_2, u) = h(L_1, R_1, u') \, h(L_2, R_2, u)$

where $L_2 u = u' h(L_2, R_2, u)$, $R_2 u^\dagger = u'^\dagger h(L_2, R_2, u)$

Take

$$u(x) = \sqrt{U(x)}$$

$$U(x) = \exp\left(i \frac{\pi^a \lambda^a}{2F_0}\right)$$

Non-linear transformation of u gives

$U \to R U L^\dagger$!

Then,

$$B(x) \xrightarrow{L,R} h(L,R,u(x)) B(x) h^\dagger(L,R,u(x))$$

defines a consistent transformation law.

For two flavors : $N = \begin{pmatrix} p \\ n \end{pmatrix}$

$$N(x) \xrightarrow{L,R} h(L,R,u(x)) N(x)$$

Building blocks :

- N or B
- Covariant derivative

2-flavors: $\mathcal{D}_\mu N = (\partial_\mu - i\Gamma_\mu) N$

3-flavors: $\mathcal{D}_\mu B = \partial_\mu B - i[\Gamma_\mu, B]$

$$\Gamma_\mu = \frac{i}{2}\left(u(\partial_\mu - i l_\mu)u^\dagger + u^\dagger(\partial_\mu - i r_\mu)u\right)$$

Exercise: show that

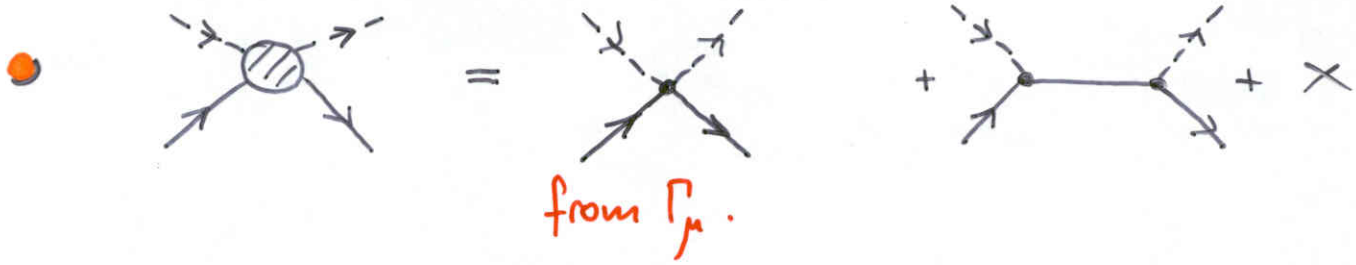
$$\mathcal{D}_\mu N \xrightarrow{L,R} h(L,R,u)\,\mathcal{D}_\mu N$$

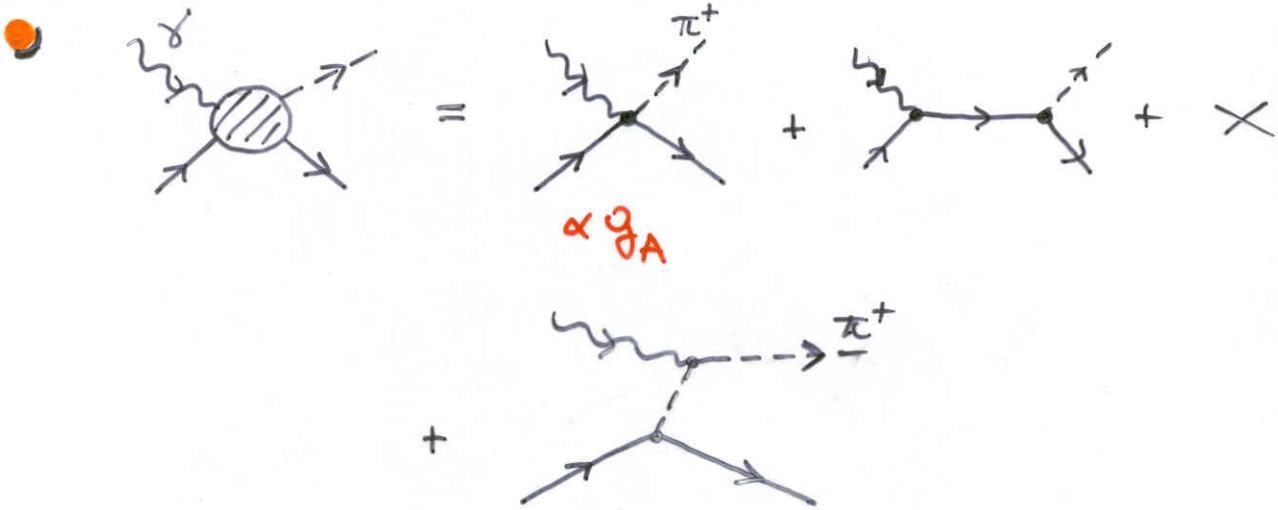
- Axial-vector connection

$$\omega_\mu = -\frac{i}{2}\left(u(\partial_\mu - i l_\mu)u^\dagger - u^\dagger(\partial_\mu - i r_\mu)u\right)$$

- $\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u$
- $F^\pm_{\mu\nu} = u^\dagger F^R_{\mu\nu} u \pm u F^L_{\mu\nu} u^\dagger$

$$(\omega_\mu;\ \chi_\pm;\ F^\pm_{\mu\nu}) \xrightarrow{L,R} h\,(\omega_\mu;\ \chi_\pm;\ F^\pm_{\mu\nu})\,h^\dagger$$





etc.

Two flavor Lagrangian: LO

$$\mathcal{L}^{(1)} = \bar{N}(i\not{D} - m_0)N + \frac{g_A}{2}\bar{N}\not{\omega}\gamma_5 N$$

Counting:

$$\omega_\mu = \mathcal{O}(p) \qquad \Gamma_\mu = \mathcal{O}(p)$$

$$\omega_\mu = -\frac{1}{F_0}\partial_\mu \pi + 2a_\mu + \dots \qquad \Gamma_\mu = \frac{i}{2F_0}[\partial_\mu, \pi] + \frac{1}{8F_0^2}[\pi, \partial_\mu \pi] + \dots$$

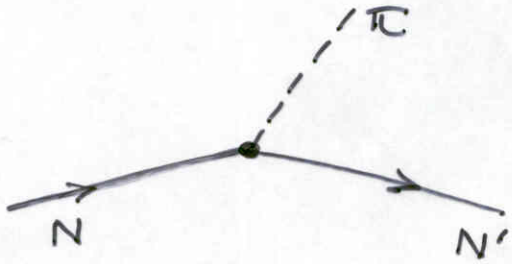
Equation of motion:

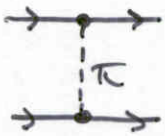
$$(\not{\partial} - m_0)N = (i\Gamma_\mu - \frac{g_A}{2}\not{\omega}\gamma_5)N$$

$$\Downarrow$$

$$(\not{\partial} - m_0) = \mathcal{O}(p)$$

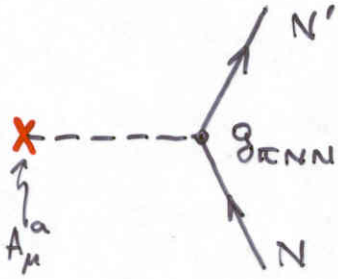
Possible processes:

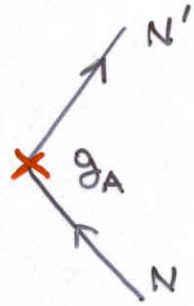
- 

$\propto g_A \cdot \Big($ 

$\Big)$

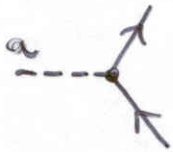
LO Low Energy Theorem: Goldberger-Treiman Relation

$\partial^\mu \Big($ 

$+$ 

$\Big) = 0$

in χ-limit



$= g_{\pi NN} \; \bar{N} \gamma_5 \tau^a N$

$= g_A \; \bar{N} \gamma_\mu \gamma_5 \tau^a N$

$$p^\mu F_0 \frac{p_\mu}{p^2} g_{\pi NN} - m_0 g_A = 0$$

$$g_{\pi NN} = \frac{g_A m_N}{F_\pi}$$

$$g_A = -1.267 \pm 0.003$$

from $n \to p e^- \bar{\nu}_e$ (β-decay)

$$g_{\pi NN} = \begin{cases} 13.65 \pm 0.30 & \text{(Karlsruhe-Helsinki)} \\ 13.05 \pm 0.08 & \text{(DeSwart et al)} \\ 13.21 \,{}^{+0.11}_{-0.05} & \text{(PSI: pionic H and D)} \end{cases}$$

Deviation from GTR: $\begin{cases} 6\% \\ 1.5\% \\ 2.7\% \end{cases}$

Good test of chiral symmetry!

GTR automatically satisfied by $\mathcal{L}^{(1)}$!

Three-flavor Lagrangian

$$\mathcal{L}_0^{(1)} = \langle \bar{B}(i\not{D} - m_0) B \rangle$$
$$- D \langle \bar{B} \gamma_5 \{ \not{u}, B \} \rangle$$
$$- F \langle \bar{B} \gamma_5 [\not{u}, B] \rangle$$

$g_A = F + D$

F & D from β-decay including Hyperons.

$F \sim 0.5 \qquad D = g_A - F$

Cancellations : at threshold

$$s = (m_N + M_\pi)^2 \,, \quad u = (m_N - M_\pi)^2$$

$$\not{p} + \not{p}' \to 2 M_\pi$$

$\mathcal{O}(p)$ terms cancel in T^+:

$$T^+ = -\frac{g_{\pi NN}^2 M_\pi^2}{4 m_N^3} = \mathcal{O}(p^2)$$

$$T^- = \frac{M_\pi}{2 F_\pi^2} = \mathcal{O}(p)$$

Scattering lengths:

$$T^\pm = 4\pi \left(1 + \frac{M_\pi}{m_N}\right) a^\pm$$

$$a^+ = \mathcal{O}(p^2)$$

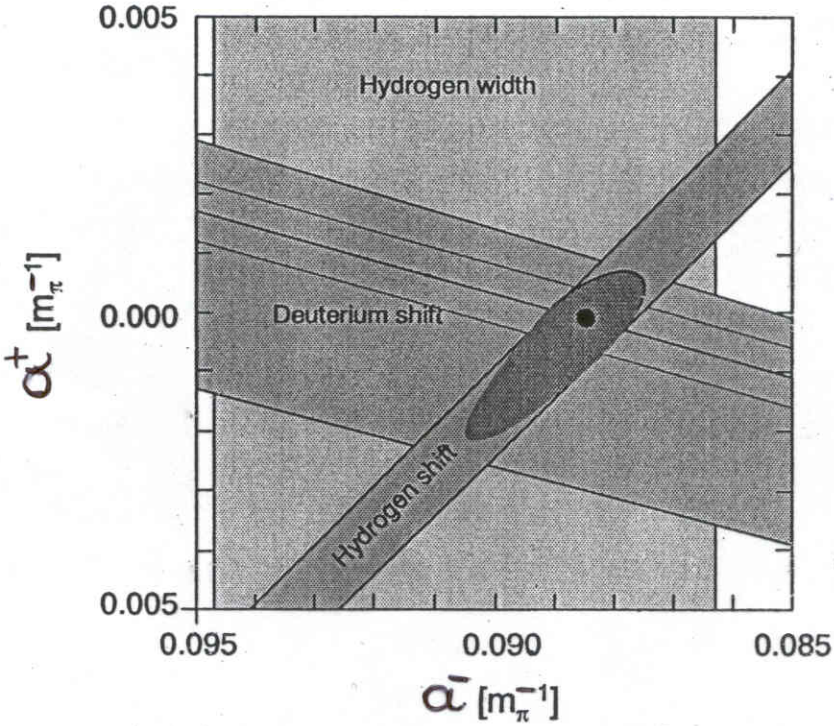
$$a^- = \frac{M_\pi}{8\pi\left(1+\frac{M_\pi}{m_N}\right) F_\pi^2} = 0.078 \, \frac{1}{M_\pi}$$

From PSI π-onic H and D:

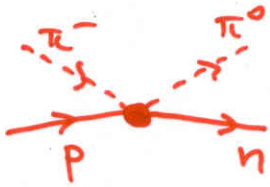
$$a^+\Big|^{Exp} = -0.1^{+0.9}_{-2.1} \; 10^{-3} \, \frac{1}{M_\pi}$$

$$a^-\Big|^{Exp} = 0.0885^{+0.0010}_{-0.0021} \, \frac{1}{M_\pi}$$

PSI pionic Hydrogen and Deuterium Experiment [2001]



H width:



Energy shifts: Deser's formula.

NLO Lagrangian: $\mathcal{O}(p^2)$

Two flavors, no isospin breaking

$$\mathcal{L}^{(2)} = c_1 \langle \chi_+ \rangle \bar{N} N - \frac{c_2}{m_0^2} \langle \omega_\mu \omega_\nu \rangle (\bar{N} \mathcal{D}^\mu \mathcal{D}^\nu N + h.c.)$$
$$+ 2c_3 \langle \omega_\mu \omega^\mu \rangle \bar{N} N - c_4 \bar{N} \gamma^\mu \gamma^\nu [\omega_\mu, \omega_\nu] N$$
$$+ \dots$$

... contains terms reproducing the p & n anomalous magnetic moments.

σ-term: c_1-term shifts m_N proportionally to $m_{u,d}$.

$$\langle \chi_+ \rangle \to 4 M_\pi^2$$

$$\sigma \equiv \hat{m} \frac{\partial m_N}{\partial \hat{m}} = -4 c_1 M_\pi^2$$

σ-term can be extracted via πN scattering, or by lattice calculation.

$$\sigma = \hat{m} \langle N | \bar{q} q | N \rangle$$

Through σ-term a test of χPT can result from the independent evaluation of σ via πN scattering and via Lattice QCD.

Three flavors

$$\mathcal{L}^{(2)} = c_1 \langle \chi_+ \rangle \langle \bar{B} B \rangle + c_D \langle \bar{B} \{ \chi_+, B \} \rangle$$

$$+ c_F \langle \bar{B} [\chi_+, B] \rangle$$

$$+ b_1 \langle \bar{B} [\omega_\mu, [\omega^\mu, B]] \rangle$$

$$+ b_2 \langle \bar{B} [\omega_\mu, \{ \omega^\mu, B \}] \rangle$$

$$+ b_3 \langle \bar{B} \{ \omega_\mu, \{ \omega^\mu, B \} \} \rangle$$

$$+ b_8 \langle \omega^\mu \omega_\mu \rangle \langle \bar{B} B \rangle$$

$$+ \cdots$$

Test of SU(3) breaking by quark masses: Gell-Mann–Okubo relation

$$m_{\text{Baryon}} = m_0 + c\, m_q + \dots$$

Masses to $\mathcal{O}(p^2)$:

$$m_N = m_0 - 4B_0\left(c_1(m_s+2\hat{m}) + c_D(m_s+\hat{m}) + c_F(\hat{m}-m_s)\right)$$

$$m_\Sigma = m_0 - 4B_0\left(\quad " \quad + c_D\, 2\hat{m}\right)$$

$$m_\Lambda = m_0 - 4B_0\left(\quad " \quad + c_D\, \tfrac{2}{3}(\hat{m}+2m_s)\right)$$

$$m_\Xi = m_0 - 4B_0\left(\quad " \quad + c_D(m_s+\hat{m}) - c_F(\hat{m}-m_s)\right)$$

$$\Downarrow$$

$$\underbrace{m_N + m_\Xi}_{2.25\ \text{GeV}} = \underbrace{\frac{m_\Sigma + 3m_\Lambda}{2}}_{2.23\ \text{GeV}} \quad !$$

Why is GM-O relation so good?!

As shown later, quite a bit of a mystery.

Use masses and ratios of quark masses to fix 3 out of 4 unknowns (m_0, c_1, c_D, c_F).

$c_F \sim -0.2\,\text{GeV}^{-1}$ $\quad c_D \sim 0.06\,\text{GeV}^{-1}$

NNLO : $\mathcal{O}(p^3)$

Low Energy Expansion for baryons hopps by one unit at the time. Likely to be slowly convergent.

At $\mathcal{O}(p^3)$ loops become important.

The problem with loops

$$I = \int \frac{d^d p}{(2\pi)^d} \frac{i}{p^2 - M_\pi^2} \frac{i}{(P-p)^2 - m_N^2} \left\{ 1 ; P_\mu P_\nu \right\} \Big|_{P^2 = m_N^2}$$

$$= \frac{i}{16\pi^2} \Big\{ \frac{1}{\varepsilon} - \gamma + \log 4\pi + 1 - \log m_N^2 + \mathcal{O}(M_\pi^2) ;$$

$$\frac{g_{\mu\nu}}{6} m_N^2 \cdot \left(\frac{1}{\varepsilon} - \gamma + \log 4\pi + \frac{5}{3} - \log m_N^2 \right)$$

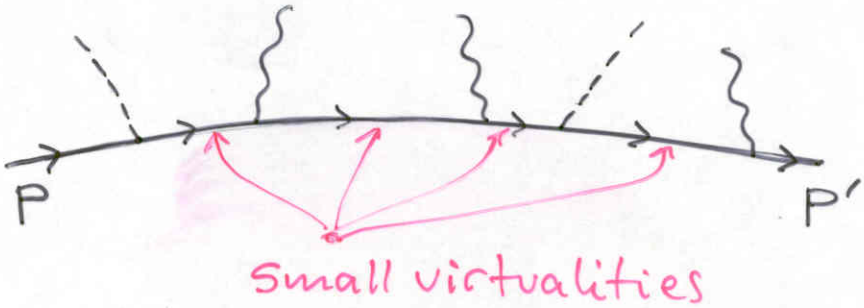
$$+ P_\mu P_\nu \cdot \left(\frac{1}{\varepsilon} - \gamma + \log 4\pi + \frac{2}{3} - \log m_N^2 \right) \Big\} + \mathcal{O}(M_\pi^2)$$

Give terms $\propto \log m_N^2$ or $m_N^2 \log m_N^2$

No LE power counting for loop !

Implementing LE counting in loop: HBχPT

$$m_0 \gtrsim \Lambda_\chi$$



Expand propagators in powers of $1/m_0$:

$$\frac{1}{(P-p)^2-m_0^2} = \frac{1}{P^2-m_0^2-2P\cdot p+p^2}$$

p: small momentum

$P^2=m_0^2 \Rightarrow P_\mu = m_0 v_\mu$ $\quad v_\mu$: 4-velocity

thus:

$$\frac{1}{(P-p)^2-m_0^2} = -\frac{1}{2m_0 v\cdot p} + \mathcal{O}\left(\frac{p^2}{m_0^2}\right)$$

Back to our integrals:

replace $\dfrac{1}{(P-p)^2-m_0^2+i\varepsilon} \longrightarrow -\dfrac{1}{2m_0\, v\cdot p - i\varepsilon}$

$$\bar{I} = \int \frac{d^d p}{(2\pi)^d} \frac{i}{p^2-M_\pi^2} \frac{(-i)}{2m_0\, v\cdot p} \{1;\ p_\mu p_\nu\}$$

Sample calculation:

$$\int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2-M_\pi^2+i\varepsilon} \frac{1}{-v\cdot p+i\varepsilon}$$

$$= 2\int_0^\infty d\lambda \int \frac{d^d p}{(2\pi)^d} \frac{1}{(p^2-M_\pi^2-\lambda\, p\cdot v+i\varepsilon)^2}$$

shift $p \to p-\lambda v$

$$= 2\int_0^\infty d\lambda \int \frac{d^d p}{(2\pi)^d} \frac{1}{(p^2-M_\pi^2-\lambda^2+i\varepsilon)^2}$$

DR ⊕ Wick rotation

$$= 2i\,\frac{\Gamma(\varepsilon)}{(4\pi)^{2-\varepsilon}} \frac{1}{1-2\varepsilon}(-\varepsilon\pi M_\pi)$$

$$= \frac{i}{8\pi} M_\pi$$ (no UV divergence for this one)

Exercise: show that

$$\int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2 - M_\pi^2 + i\varepsilon} \frac{p_\mu p_\nu}{-p\cdot v + i\varepsilon}$$

$$= \frac{1}{3} \frac{i}{8\pi} (g_{\mu\nu} - v_\mu v_\nu) M_\pi^3$$

Expanding propagators in $1/m_0$ solves the counting problem.

Need to implement $1/m_0$ expansion throughout. Start with fields and Lagrangians: steps correspond to Foldy-Wuythuisen transformation.

Choose 4-velocity v_μ

$$B_v(x) = \underbrace{e^{i m_0 v\cdot x}}_{\text{removes rest mass dependence from field.}} \underbrace{\frac{1+\not{v}}{2}}_{\text{"+ energy projector"}} B(x)$$

Reductions:

$$\bar{B}_v \gamma_5 B_v = 0 \qquad \bar{B}_v \gamma_\mu B_v = v_\mu \bar{B} B$$

$$\bar{B}_v \gamma^\mu \gamma_5 B_v = 2 \bar{B}_v S_v^\mu B_v$$

S_v^μ: covariant spin

$$S_v^\mu = -\frac{i}{2} \gamma_5 v_\nu \sigma^{\nu\mu}$$

$$v \cdot S_v = 0$$

LO Lagrangian

$$\mathcal{L}_v^{(1)} = \bar{N}_v (i v \cdot \mathcal{D} + g_A \, \omega \cdot S_v) N_v$$

Terms $\mathcal{O}(1/m_0)$ and higher shifted to higher order Lagrangians.

Equation of motion:

$$(i v \cdot \mathcal{D} + g_A \, \omega \cdot S_v) N_v = 0$$

Change to NR normalization: $N_v \rightarrow \frac{1}{\sqrt{2m_0}} N_v$

Gives desired propagator of heavy particle:

$$\frac{i}{-p\cdot v + i\varepsilon}$$

NLO Lagrangian

$$\mathcal{L}_v^{(2)} = \bar{N}_v \Big\{ \frac{1}{2m_0}(v\cdot\mathcal{D})^2 - \frac{\mathcal{D}^2}{2m_0} - i\frac{g_A}{m_0}\{S_v\cdot\mathcal{D}, v\cdot\omega\}$$

$$+ c_1 \langle \chi_+ \rangle + \left(c_2 - \frac{g_A^2}{8m_0}\right)(v\cdot\omega)^2 + c_3\, \omega\cdot\omega$$

$$+ \left(c_4 + \frac{1}{4m_0}\right)[S_v^\mu, S_v^\nu]\,\omega_\mu\omega_\nu + \cdots \Big\} N_v$$

Terms showing powers of $1/m_0$ come from expansion of $\mathcal{L}^{(1)}$.