Lecture #4

ANOMALIES + QUARK MASSES & $\langle \bar{q} q \rangle$

Something is missing...

Following processes are not accounted for so far:

$$\pi^0 \to \gamma\gamma$$

$$\eta \to \gamma\gamma$$

$$\eta \to \pi^+\pi^-\gamma$$

$$\gamma\pi^+ \to \pi^+\pi^0$$

$$K^+K^- \to \pi^+\pi^-\pi^0$$

Effects in spectrum of $\pi^+ \to \ell^+\nu_\ell\gamma$

What prevents these processes in $\mathcal{L}^{(2)}, \mathcal{L}^{(4)}$, etc?

- In presence of external gauge fields v_μ (with $a_\mu, p \to 0$), terms in $\mathcal{L}^{(n)}$ have **even** number of GB's.

- The other way around for $a_\mu \neq 0$, $v_\mu \to 0$.

Why?: $\mathcal{L}^{(n)}$ has extra symmetry

$$\mathcal{L}^{(n)} \xrightarrow{P_0 = (-1)^{\#GB + \#a_\mu + \#p}} \mathcal{L}^{(n)}$$

Lagrangians with this symmetry are called Natural Parity Lagrangians.

P_0 is <u>not</u> a symmetry of QCD !

All processes shown in previous transparency violate P_0 :

$$\underset{P_0 = (-1)}{\pi^0} \longrightarrow \underset{P_0 = (+1)}{\gamma\gamma}$$

$$\underset{P_0 = (+1)}{K^+ K^-} \longrightarrow \underset{P_0 = (-1)}{\pi^+ \pi^- \pi^0}$$

⋮

etc.

χ Lagrangians <u>odd</u> under P_0 are needed.

Anomalies

$$G \xrightarrow{\hbar \neq 0} G' \subset G$$

Classical level — Quantum level

- **Abelian Anomaly**

current associated with $U_A(1)$ transformations

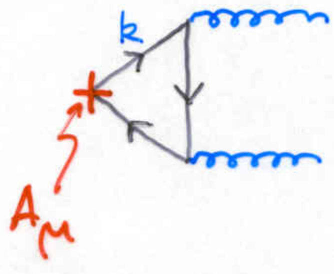
$$A_\mu = \bar{q}\,\gamma_\mu \gamma_5\, q$$

Assume massless quark

$$\partial^\mu A_\mu = 0 \qquad \text{Classical level}$$

$$\partial^\mu A_\mu = \frac{\alpha_s}{2\pi} N_f\, G\tilde{G} \qquad \text{Quantum level}$$

$$G\tilde{G} \equiv \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\, G_a^{\mu\nu} G^{a\rho\sigma} = 2\,\vec{E}\cdot\vec{H}$$



$$\sim \int \frac{d^4k}{(2\pi)^4}\,\frac{k^3}{k^6}$$

Diagram requires UV Reg., and this necessarily breaks $U_A(1)$

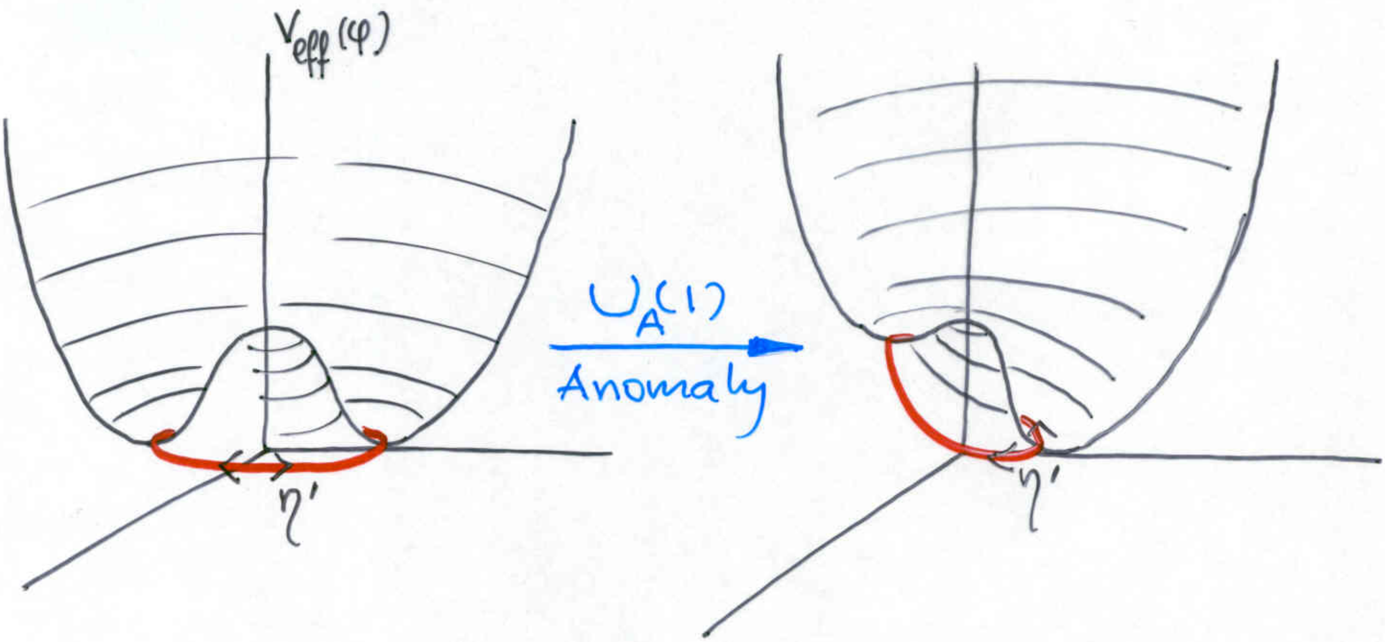
$U_A(1)$ problem:

If $U_A(1)$ is good symmetry, $\langle \bar{q} q \rangle \neq 0$ implies a GB associated with SB of $U_A(1)$. Only candidate η'

but

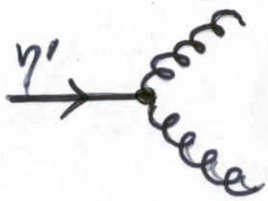
$$M_{\eta'} = 958 \text{ MeV} \;!$$

Clearly not a GB.



Mass formulas for η'

- Witten-Veneziano mass formula



$$= \frac{\alpha_s}{2\pi} \frac{N_f}{F_{\eta'}} G\tilde{G}$$

$$M_{\eta'}^2 = \frac{4N_f^2}{F_{\eta'}^2}\chi \; ; \qquad \chi \equiv \left(\frac{\alpha_s}{4\pi}\right)^2 \langle 0| G\tilde{G}\, G\tilde{G} |0\rangle \Big|_{\text{no quarks}}$$

χ: topological susceptibility

$\chi \equiv 0$ in perturbation theory.

From actual mass of η' (corrected by quark masses $\neq 0$):

$$\chi \sim (165 \text{ MeV})^4$$

From lattice QCD

$$\chi = (187 \pm 14 \pm 16 \text{ MeV})^4$$

[Smith & Tepper]

- From Anomaly Equation for $\langle 0|A_\mu A_\nu|0\rangle$

$N_f=3$, massless quarks

$$M^2_{\eta'} \simeq \frac{\sqrt{6}}{4\pi}\frac{\alpha_s}{F_{\eta'}}\langle \eta'|G\tilde{G}|0\rangle$$

In a world with N_c colors, for large N_c

$\alpha_s \sim 1/N_c$, $F_{\eta'}$, F_{π^0}, etc $\sim \sqrt{N_c}$, and

$\langle \eta'|G\tilde{G}|0\rangle \sim \sqrt{N_c} \Rightarrow M^2_{\eta'}\Big|_{\text{massless quarks}} \sim 1/N_c$.

Dealing with η' in χPT

- Consider it heavy and do not include as degree of freedom. η' controls value of L_7!

- Do $1/N_c$ expansion together with χPT. η' is included as a degree of freedom.

Both are consistent effective theories

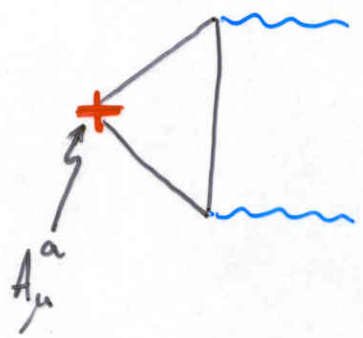
Non-Abelian Anomalies

Inclusion of external sources a_μ, v_μ generate Anomalies! Note, photon field contained among sources v_μ, W and Z Boson field also among the sources.

$SU(3)_L \times SU(3)_R$ currents

$$V_\mu^a = \frac{1}{2}\bar{q}\gamma_\mu \frac{\lambda^a}{2} q$$

$$A_\mu^a = \frac{1}{2}\bar{q}\gamma_\mu\gamma_5 \frac{\lambda^a}{2} q$$



$$\partial^\mu A^a_\mu = \frac{N_c}{8\pi^2} \langle \lambda^a \Big(\frac{1}{2} F_V^{\mu\nu} \tilde{F}_{V\mu\nu} + \frac{1}{6} F_A^{\mu\nu} \tilde{F}_{A\mu\nu}$$

$$+ i\frac{4}{3} \{ a_\mu a_\nu , \tilde{F}_V^{\mu\nu} \} + i \frac{4}{3} a_\mu \tilde{F}_V^{\mu\nu} a_\nu$$

$$- \frac{8}{3} \varepsilon_{\mu\nu\rho\sigma} a^\mu a^\nu a^\rho a^\sigma \Big) \rangle$$

$$F_V^{\mu\nu} \equiv \partial^\mu v^\nu - \partial^\nu v^\mu - i[v^\mu, v^\nu] - i[a^\mu, a^\nu]$$

$$F_A^{\mu\nu} \equiv \partial^\mu a^\nu - \partial^\nu a^\mu - i[v^\mu, a^\nu] - i[a^\mu, v^\nu]$$

[Bardeen]

Vector currents free of anomalies.

Profound theoretical structure
[See S. Adler : hep-th/0405040 (2004)
good collection of references]

Anomalies <u>must</u> be reproduced at Effective Theory level !

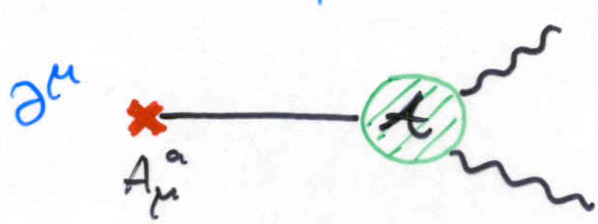
How can anomalies be included in χPT with only GB's ?

Veltman-Sutherland Theorem

$$= F \cdot \frac{p^2}{p^2 - M_{GB}^2} \cdot \mathcal{A} \xrightarrow[M_{GB}\to 0]{} 0$$

$$\Rightarrow \mathcal{A}(GB \to f) \xrightarrow[M_{GB}\to 0]{} 0$$

but, we have to reproduce anomalies !



$$\cdots \xrightarrow[M_{GB}\to 0]{} \neq 0$$

$$\Rightarrow \mathcal{A}(GB \to \text{"}\gamma\gamma\text{"}) \xrightarrow[M_{GB}\to 0]{} \neq 0$$

Need a Lagrangian that produces the right amplitudes for anomalies to be reproduced.

Wess-Zumino-Witten Lagrangian

[Wess & Zumino (1972); Witten (1984)]

$$U(x) \to \tilde{U}(x, x_4) \equiv \exp\left(x_4 \frac{i\pi^a(x)\lambda^a}{F_0}\right)$$

extra variable x_4 needed

$$\mathcal{L}_{WZW} = -i\frac{N_c}{240\pi^2}\Big[\varepsilon^{\mu_0\ldots\mu_4}\int_0^1 dx_4 \langle \tilde{U}^\dagger \partial_{\mu_0}\tilde{U}\ldots\tilde{U}^\dagger\partial_{\mu_4}\tilde{U}\rangle$$

$$+\,5\left(W(U, a_\mu, v_\mu) - W(\mathbb{1}, a_\mu, v_\mu)\right)\Big]$$

Properties of $\mathcal{L}_{WZW}$

- $\mathcal{O}(p^4)$
- No free parameters (no additional LECs)
- Does not depend on sources s, p.
- $\mathcal{L}_{WZW} \xrightarrow{P_0} -\mathcal{L}_{WZW}$
- Reproduces the anomalies.

Take only EM field.

$$W(U,A) = \varepsilon^{\mu\nu\alpha\beta}$$

$\pi^+\pi^-\pi^0\gamma$

$$\times\Big[-i\,\Sigma_\mu \Sigma_\nu \Sigma_\alpha A_\beta$$

$\pi^0 \to \gamma\gamma,\ \eta \to \gamma\gamma,\ \eta' \to \gamma\gamma$

$$+ \Sigma_\mu U^\dagger \partial_\nu A_\alpha U A_\beta + \Sigma_\mu A_\nu \partial_\alpha A_\beta + \Sigma_\mu \partial_\nu A_\alpha A_\beta$$

$$+ \tfrac{1}{2} \Sigma_\mu A_\nu \Sigma_\alpha A_\beta - \Sigma_\mu \Sigma_\nu U^\dagger A_\alpha U A_\beta$$

$\pi^+\pi^-\pi^0\gamma\gamma$

$$+ i\big(U\partial_\mu A_\nu A_\alpha U^\dagger A_\beta$$

$\pi^+\pi^-\pi^0 3\gamma$

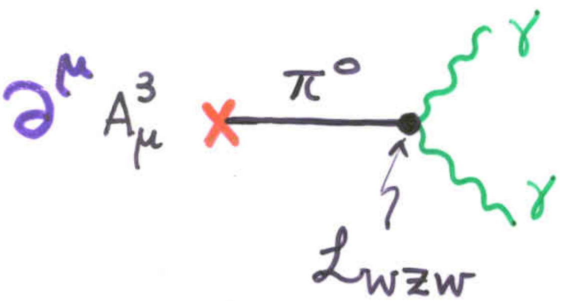
$$+ \partial_\mu A_\nu U A_\alpha U^\dagger A_\beta - \Sigma_\mu A_\nu U^\dagger A_\alpha U A_\beta \big)$$

$$+ U A_\mu A_\nu A_\alpha U^\dagger A_\beta + \tfrac{1}{4} U A_\mu U^\dagger A_\nu U A_\alpha U^\dagger A_\beta \Big]$$

$$- \; U \leftrightarrow U^\dagger$$

$\pi^+\pi^-\pi^0 4\gamma$

$$\Sigma_\mu = U^\dagger \partial_\mu U$$



$$= \frac{\alpha N_c}{12\pi} F\tilde{F}$$

$\pi^0, \eta, \eta' \to \gamma\gamma$ Decays

- $\pi^0 \to \gamma\gamma$, γe^+e^- , $e^+e^-\ e^+e^-$

  98.8 % 1.2 % 3.1×10^{-5}

  $\tau_{\pi^0} = (8.4 \pm 0.6) \times 10^{-17}$ sec

Parameter free prediction from $\mathcal{L}_{WZW}$:

$$A_{\pi^0 \to \gamma\gamma} = -i \frac{\alpha}{4\pi F_\pi} F\tilde{F} \times \frac{N_c}{3}$$

$F_\pi = 92.42$ MeV $\qquad \alpha = 1/137$

$$\Gamma^{\chi PT}_{\pi^0 \to \gamma\gamma} = 7.73 \text{ eV}$$

$$\Gamma^{Exp}_{\pi^0 \to \gamma\gamma} = (7.73 \pm 0.59) \text{ eV}$$

Exp. precission not that great, but enough to make sure what the mechanismm of decay is !.

Expectation if Veltman-Sutherland would apply

$$\Gamma \sim \frac{M_\pi^4}{\Lambda_\chi^4} \cdot 7.73 \text{ eV}$$

- $\eta, \eta' \to \gamma\gamma$

$$\Gamma^{\chi PT}_{\eta\to\gamma\gamma} = 170\ \mathrm{eV} \qquad \Gamma^{Exp}_{\eta\to\gamma\gamma} \cong 464 \pm 45\ \mathrm{eV}$$

Explanation for discrepancy: SU(3) breaking by $(m_s - \hat{m})$. Good case to keep η'!

State mixing driven by $(m_s - \hat{m})$:

SU(3) eigenstates	Physical States
π_8	$\eta = \cos\theta\,\pi_8 - \sin\theta\,\pi_0$
π_0	$\eta' = \sin\theta\,\pi_8 + \cos\theta\,\pi_0$

use $\Gamma^{Exp}_{\eta'\to\gamma\gamma} = 4.28 \pm 0.34\ \mathrm{keV}$ and fit $F_{\eta'}$ and θ.

$$\theta \sim -20^\circ \quad \propto B_0 \frac{(m_s - \hat{m})}{M^2_{\eta'}}$$

- More precision for $\pi^0 \to \gamma\gamma$

π^0 contains admixture of π_8 and π_0 driven by isospin breaking:

$$\frac{m_d - m_u}{m_s - \hat{m}} \quad \text{and} \quad B_0 \frac{(m_d - m_u)}{M_{\eta'}^2}.$$

Detailed analysis gives more precise prediction:

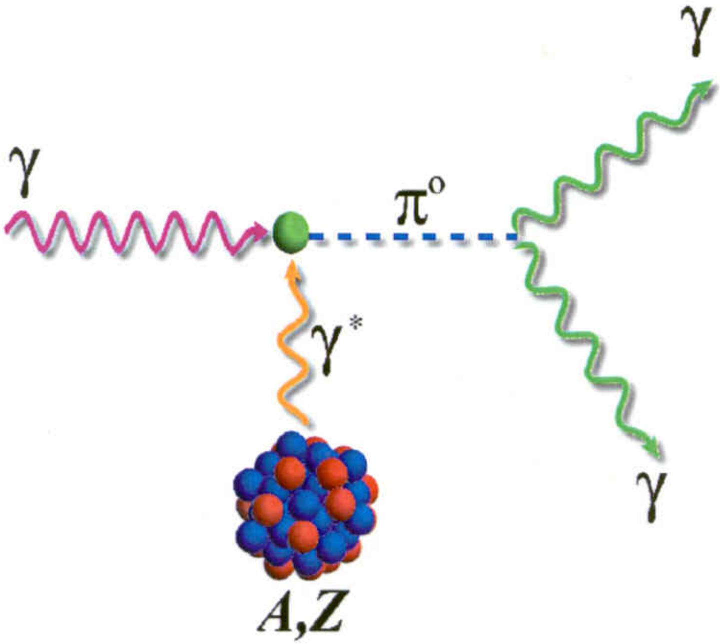
$$\Gamma^{\chi PT}_{\pi^0 \to \gamma\gamma} = 8.08 \text{ eV}$$

A more precise measurement can confirm this mixing effect, being this a further test of χPT.

New measurement in the works @ JLab

PRIMEX

PRIMEX @ JLab*

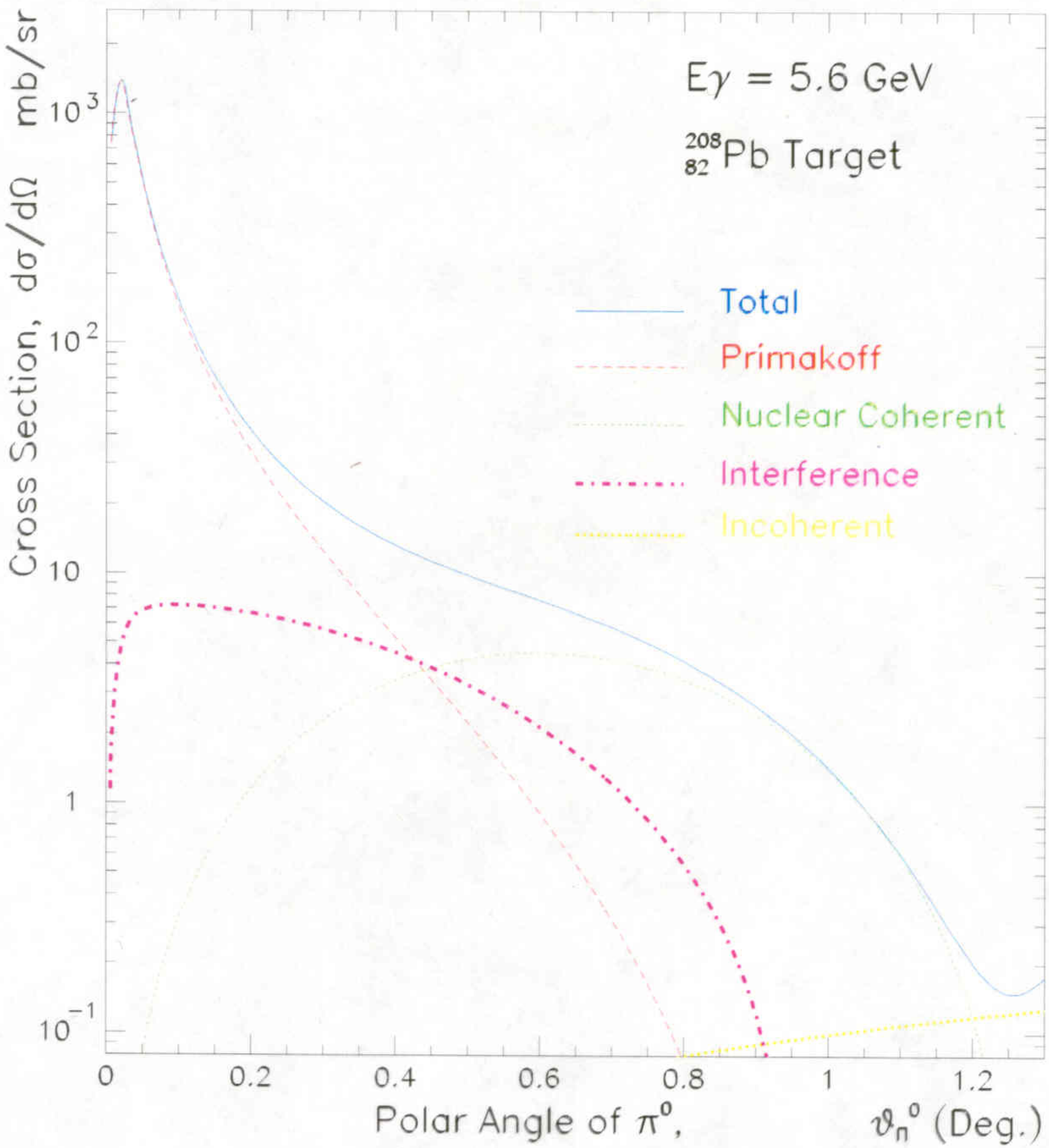


$$\frac{d\sigma_P}{d\Omega_\pi} = \Gamma_{\pi^0\to\gamma\gamma}\frac{8\alpha Z^2}{M_\pi^3}\frac{\beta_\pi^3 E_\gamma^4}{Q^4}|F_{e.m.}(Q)|^2 \sin^2\theta_\pi$$

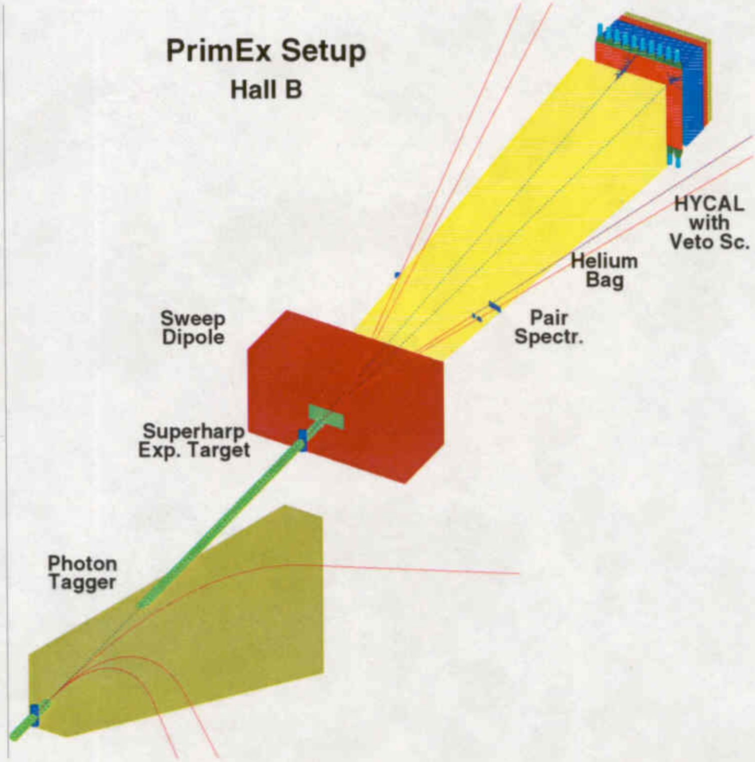
All production mechanisms

$$\frac{d\sigma_{\text{Total}}}{d\Omega_\pi} = \frac{d\sigma_P}{d\Omega_\pi} + \frac{d\sigma_C}{d\Omega_\pi} + \frac{d\sigma_I}{d\Omega} + \frac{d\sigma_{P-C}}{d\Omega_\pi}$$

* [see talk in GB working group talk by A. Gasparian]



Eγ = 5.6 GeV
$^{208}_{82}$Pb Target
Total
Primakoff
Nuclear Coherent
Interference
Incoherent
Cross Section, dσ/dΩ mb/sr
10^3
10^2
10
1
10^{-1}
0
0.2
0.4
0.6
0.8
1
1.2
Polar Angle of π^0, ϑ_{π^0} (Deg.)

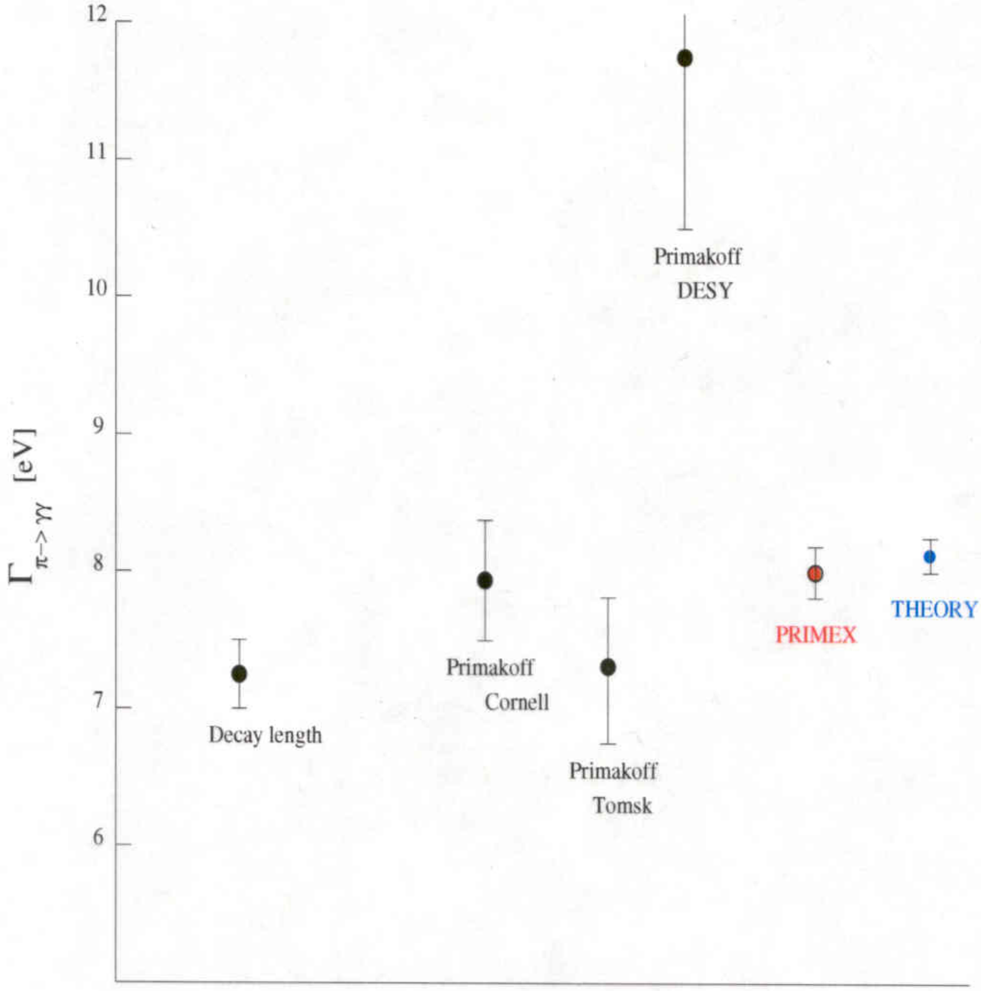


- Coherent Bremmstrahlung tagged photon beam. Uncertainties: 1% in intensity, 10^{-3} resoln. in energy

- Targets: spin zero nuclei: ^{12}C, ^{120}Sn and ^{208}Pb.

- Calorimeter: HYCAL Pb glass and $PbWO_4$ crystals. Characteristics: High spatial (1.4 mm) and energy (1%) resolutions for γ detection. Crucial for determining θ_π: $\delta\theta_\pi \sim 0.01^o - 0.02^o$.

Experimental Status and PRIMEX



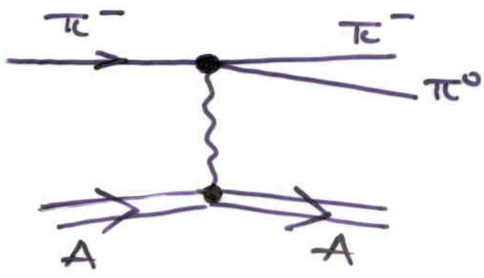
- $\gamma\pi^- \to \pi^-\pi^0$

$$A_{\gamma\pi^-\to\pi^-\pi^0} = F^{3\pi}(s,t,u)\, \tilde{F}_{\mu\nu}\, p^{+\mu} p^{0\nu}$$

χ limit: $$F^{3\pi} = \frac{e N_c}{12\pi^2 F_0^3}$$

$$= 9.72 \pm 0.09 \ \text{GeV}^{-3}$$

Experiment: [Antipov et al.]



$F^{3\pi}_{Exp} = 12.9 \pm 0.9 \pm 0.5 \ \text{GeV}^{-3}$ No unitarity corrections

$F^{3\pi}_{Exp} = 11.4 \pm 1.3 \ \text{GeV}^{-3}$ With unitarity corrections [Bijnens, Bramon, Cornet; Holstein; Hanna]

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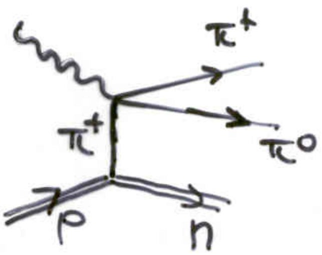


Beyond Effective Theory

Very important question

What is the value of $\langle \bar{q}q \rangle$?

- Important step towards understanding the dynamical mechanism of SXSB.

- $\langle \bar{q}q \rangle \oplus \chi PT \Rightarrow$ values of $m_{u,d,s}$!

From χPT ratios of quark masses are reliably known.

$$\left(\frac{m_u}{m_d}\right)^2 + \frac{1}{Q^2}\left(\frac{m_s}{m_d}\right)^2 = 1$$

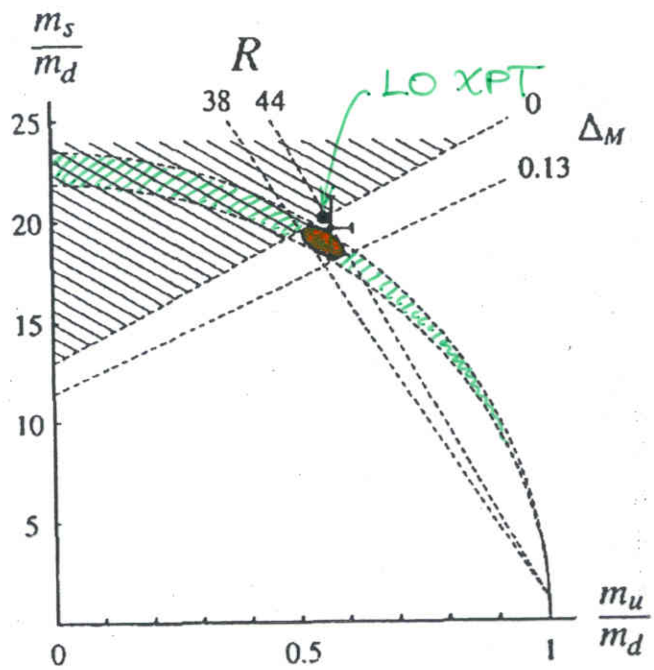


Fig. 2. Quark mass ratios. The dot corresponds to Weinberg's values, while the cross represents the estimates given in Ref. [2]. The hatched region is excluded by the bound $\Delta_M > 0$. The error ellipse shown is characterized by the constraints $Q = 22.7 \pm 0.8$, $\Delta_M > 0$, $R < 44$, which are indicated by dashed lines.

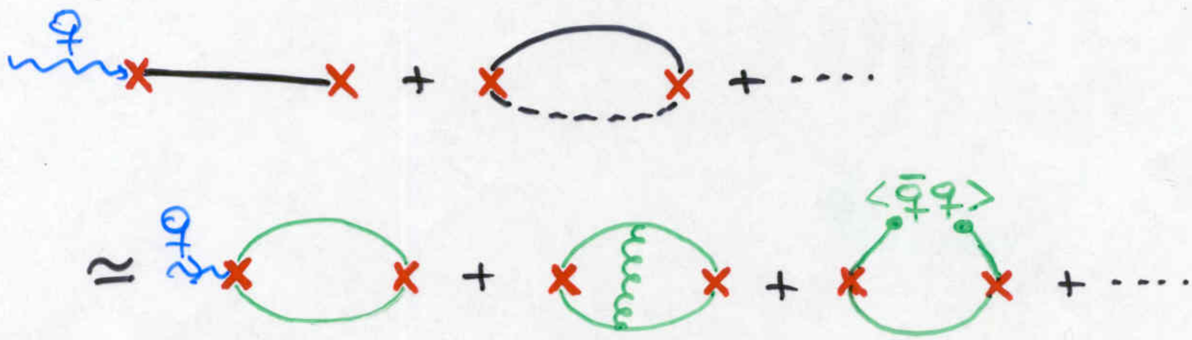
from H. Leutwyler

Phys. Lett. B378 (1996) 313

Two trails to $\langle \bar{q}q \rangle$

- **QCD Sum Rules (QCDSR)**

Basic idea: match correlation functions calculated at hadronic level and at quark-gluon level



For q^2 in an intermediate domain, non-perturbative effects in the quark-gluon picture are contained in various condensates:

$$\langle \bar{q}q \rangle$$

$$\langle G_{\mu\nu} G^{\mu\nu} \rangle$$

$$\langle \bar{q}\, \gamma_\mu \gamma_\nu G^{\mu\nu} q \rangle$$

etc.

$$\langle \bar{q} q \rangle_{QCDSR} (\mu \sim 1 \text{GeV})$$

$$\sim -(230 \pm 20)^3 \text{ MeV}^3$$

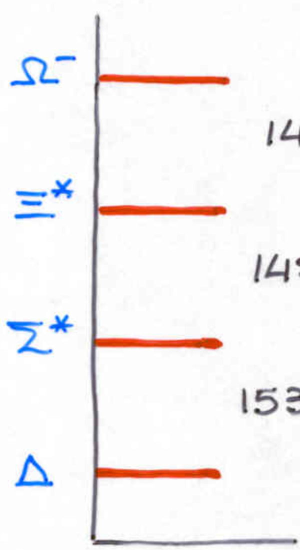
$$\Downarrow$$

$$m_s \sim 150 \text{ MeV}$$

$$\hat{m} \sim 6 \text{ MeV}$$

Important systematic uncertainties, but very likely to be correct at the 30% level. level.

m_s :



- Lattice QCD

Look at m_q dependence of various masses : $M_{\pi,K}$, M_ρ . Fix m_q in units of $1/\underline{a}$ $\underline{a}$: lattice spacing.

Determine $\underline{a}$ via M_ρ in units of $1/\underline{a}$.

Various uncertainties:

- Discretization effects ($1/\underline{a}$ corrections)
- Need for extrapolation of quark masses (simulation made with larger m_q's)
- Quenching uncertainties (only for two flavor some unquenched simulations)

From lattice:

$$\hat{m}(\mu = 2\,GeV) = 4.2 \pm 1.0\ MeV$$

$$m_s(\quad " \quad) = 105 \pm 25\ MeV$$

$$\langle \bar{q}q \rangle (\mu = 2\,GeV) = -(264\ MeV)^3$$

$$\langle \bar{q}q \rangle (\mu = 1\,GeV) \simeq -(245\ MeV)^3$$

• Implication

$\langle \bar{q}q \rangle$ is the dominant order parameter of SXSB.

q_L q_R

$$\mathcal{L} = \mathcal{L}_L + \mathcal{L}_R$$

$q_L \to U q_L$

$U_L(3)$

$\underline{U(3)} = U(1) \times \underline{SU(3)}$

$U_L(3) \times U_R(3)$

$= \underline{U_B(1)} \times U_A(1) \times SU_L(3) \times SU_L(3)$