Lecture #1

QCD
&
Its Symmetries

Outline

L1) QCD & its symmetries.

L2) Elements of χPT.

L3) χPT at NLO.

L4) Anomalies. Quark masses.

L5) Baryon χPT.

L6) - Some results in BχPT.

- Miscellaneous remarks.

HUGS 2004

χPT

CHIRAL PERTURBATION THEORY

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Brief History : trail to QCD

- Global color symmetry

Quark model of hadrons :

3 spin $\frac{1}{2}$ quarks u, d, s

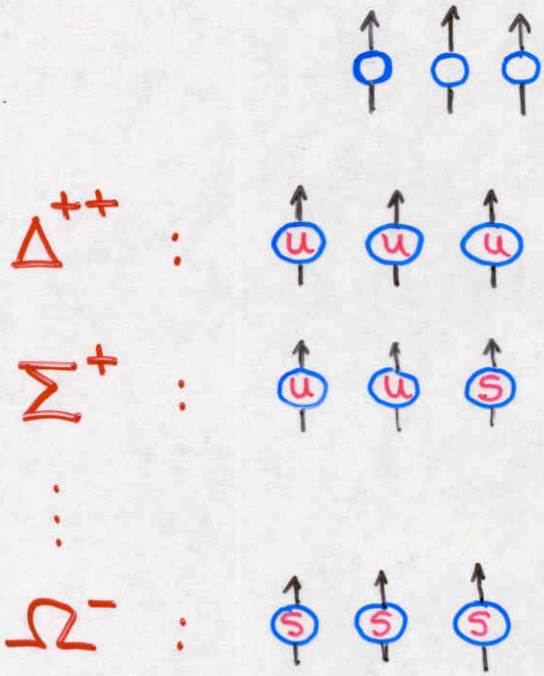
Succesful description of hadronic states and relations among their static properties : $SU_f(3)$ flavor symmetry.

Mesons	Baryons
$q\bar{q}$ states	qqq $(\bar{q}\bar{q}\bar{q})$
8 & 1 of $SU_f(3)$	1, 8, 10 $(\overline{10})$ of $SU_f(3)$

Statistics puzzle in QM

Baryon 10 (states with spin 3/2)

Assumption (consistent with number of states): quarks in s-waves



Problem with Pauli principle!

Solution to puzzle (Nambu; Greenberg)

Quarks carry additional quantum number

"color"

$u_\alpha \; , \; d_\alpha \; , \; s_\alpha \qquad \alpha = 1, 2, 3$

Baryons have wave function antisymmetrized in color!

$\Omega^- : \quad \varepsilon_{\alpha\beta\gamma} \; s_\alpha^\uparrow \, s_\beta^\uparrow \, s_\gamma^\uparrow$

Consistency with Pauli principle

Picture of hadrons in QM with color

Mesons	Baryons
$q_\alpha \, \bar{q}^\alpha$	$\varepsilon_{\alpha\beta\gamma} \quad q_\alpha \, q_\beta \, q_\gamma$

Color symmetry:

$$q \longrightarrow U q \qquad U \in SU_c(3)$$

leaves hadrons unchanged

Remaining and new puzzles:

- What interaction keeps quarks together?
- Why color is not observable degree of freedom? All hadrons are color singlets! (Color confinement)

It seems clear that color must be fundamentally related to interaction between quarks.

Gauge theory of color: QCD

Gauge Symmetry in QED

$\psi(x)$ wave function of e^-. $x = (x_0, \vec{x})$

Hamiltonian (and Lagrangian) of ED invariant under local $U(1)$ transformations

$$\psi(x) \longrightarrow e^{i\alpha(x)}\psi(x)$$

$$A_\mu(x) \longrightarrow A_\mu(x) + \frac{1}{e}\partial_\mu \alpha(x)$$

A_μ : gauge field (photon)

QED Lagrangian

$$\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}\left(i\gamma^\mu D_\mu(A) - m_e\right)\psi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$D_\mu(A) = \partial_\mu - ieA_\mu$$

γ_μ : Dirac 4x4 matices $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$

Yang-Mills Theory (Yang & Mills (1954))

Generalize gauge principle to non-Abelian groups.

QCD : gauge theory with $SU_c(3)$ group!

(Fritzsch, Gell-Mann & Leutwyler; Weinberg; Gross & Wilczek; Politzer; ...)

q : Dirac spinor for quark (single flavor)

Local color transformation

$$q(x) \longrightarrow U(x)\, q(x)$$

$$U(x) \in SU_c(3)$$

$$U(x) = \exp(i\, \alpha_a(x)\, \lambda^a)$$

λ^a : generators of $SU(3)$, 3x3 traceless and Hermitian matices

$a = 1, \dots, 8$

Gauge invariance $\Rightarrow$ vector fields G_μ^a, one per group generator

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \bar{q}(i\gamma^\mu D_\mu(G) - m_q) q$$

$$D_\mu(G) = \partial_\mu - ig \overbrace{G^a_\mu \frac{\lambda^a}{2}}^{G_\mu}$$

$$G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu - ig[G_\mu, G_\nu]$$

g: gauge coupling of QCD

$\mathcal{L}_{QCD}$ invariant under $SU_c(3)$ gauge transformation

$$q(x) \longrightarrow U(x)\, q(x)$$

$$G_\mu(x) \longrightarrow U(x)\left(G_\mu(x) + \frac{i}{g}\partial_\mu\right) U^\dagger(x)$$

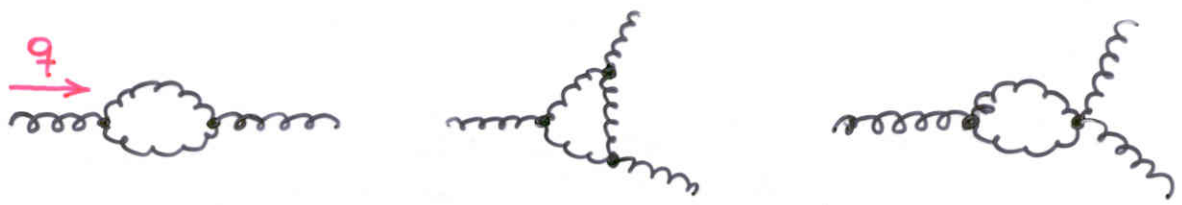
G_μ : gluon field

Key feature: gluons interact with each other

$\Downarrow$

Quantum effects are paramount in QCD

- Asymptotic freedom



...

$$\beta(g) = \mu \frac{\partial}{\partial \mu} g(\mu)$$

New scale Λ_{QCD} required to define theory at quantum level

$$\beta(g) = -\frac{g^3}{16\pi^2} b \quad ; \quad b = 11 - \frac{2}{3} N_f$$

!!!

$$\alpha_s(Q^2) \equiv \frac{g^2(\sqrt{Q^2})}{4\pi} = \frac{\alpha_s(\mu^2)}{1 + 4\pi b\, \alpha_s(\mu^2) \log(Q^2/\mu^2)}$$

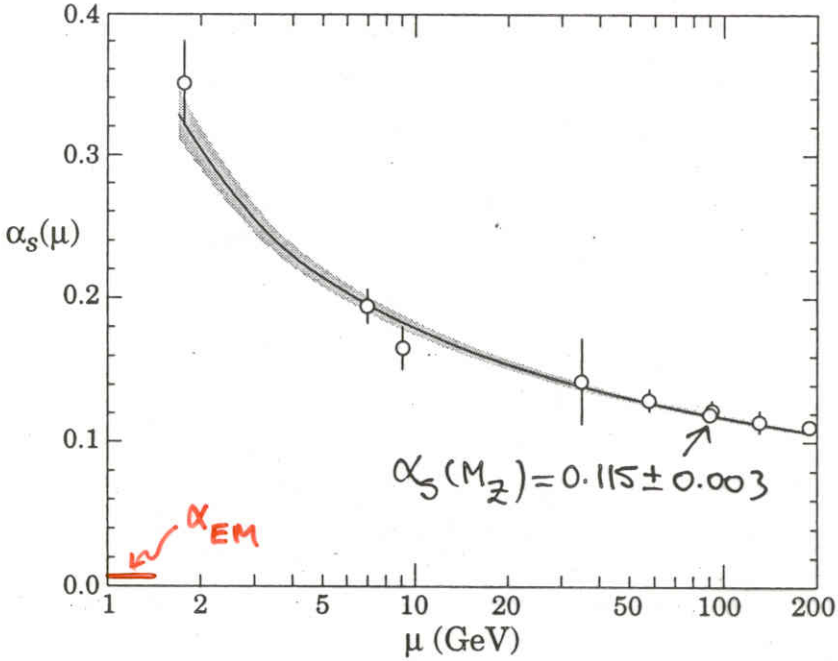


Figure 9.2: Summary of the values of $\alpha_s(\mu)$ at the values of μ where they are measured. The lines show the central values and the $\pm 1\sigma$ limits of our average. The figure clearly shows the decrease in $\alpha_s(\mu)$ with increasing μ. The data are, in increasing order of μ, τ width, deep inelastic scattering, Υ decays, e^+e^- event rate at 25 GeV, event shapes at TRISTAN, Z width, e^+e^- event shapes of M_Z, 135, and 189 GeV.

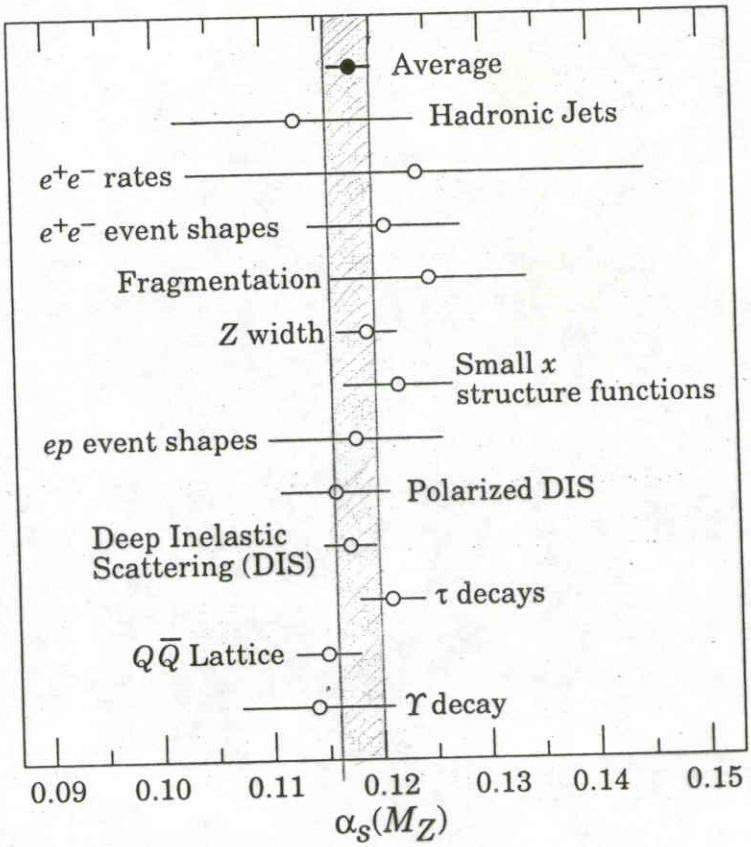
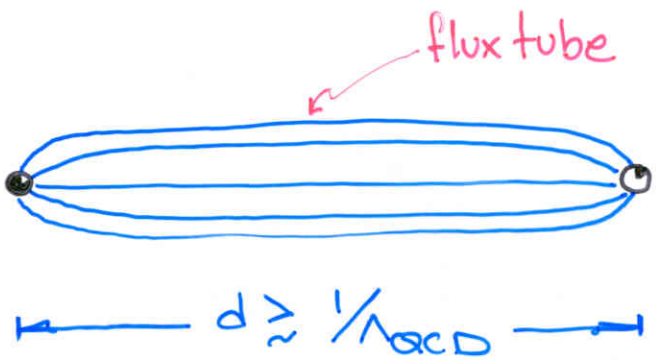


Figure 9.1: Summary of the values of $\alpha_s(M_Z)$ and $\Lambda^{(5)}$ from various processes. The values shown indicate the process and the measured value of α_s extrapolated up to $\mu = M_Z$. The error shown is the *total* error including theoretical uncertainties.

- Color confinement

No free color charges observed



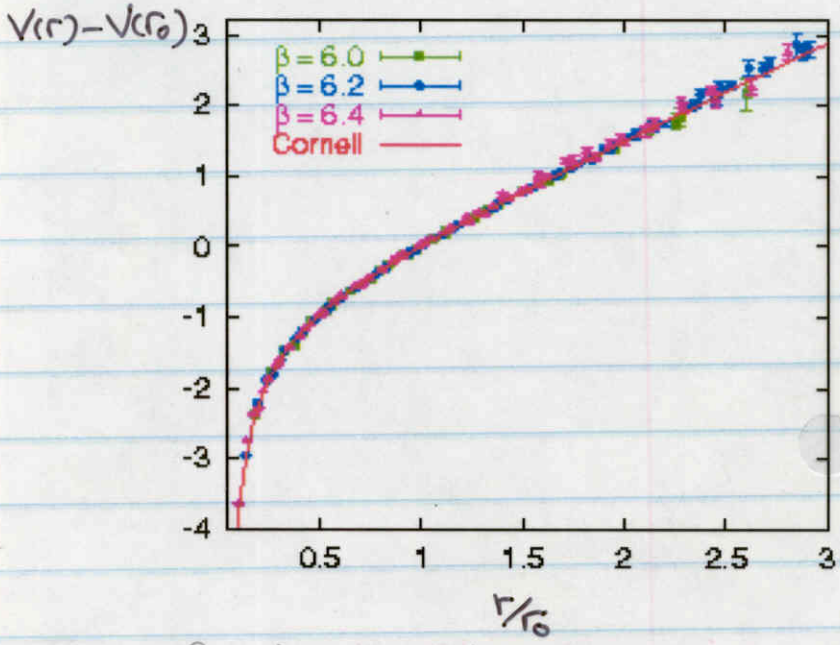
$$\sigma \sim 1.2 \frac{\text{GeV}}{\text{fm}} = 19 \text{ Tons} \quad !$$

No mathematical proof of confinement exists, but plenty of evidence

- Experimental evidence : all searches for unconfined quarks are negative.

- Lattice QCD

q-$\bar{q}$ Potential from Lattice QCD



Pictures with values of α_s and with run of α_s.

Symmetries of QCD

- Pure gluodynamics

$$\mathcal{L}_{YM} = -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + \frac{\theta}{(4\pi)^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

$$\tilde{G}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} G^{\rho\sigma}$$

Symmetries of $\mathcal{L}_{YM}$

- gauge symmetry
- charge conjugation
- parity if $\theta = 0$ $\left(\theta < (1-10) \times 10^{-10}\right)^*$
- Scale invariance

Quantum level

All the above except scale invariance which is broken $\propto \hbar$ due to requirement of scale Λ_{QCD} (scale anomaly).

* from EDM of neutron $d_n < 0.63 \times 10^{-25}$ e cm

- Quark sector : only light quarks considered

$$q_\alpha^f \qquad f=1,2,3 \quad \text{flavor index}$$
$$\alpha=1,2,3 \quad \text{color index}$$

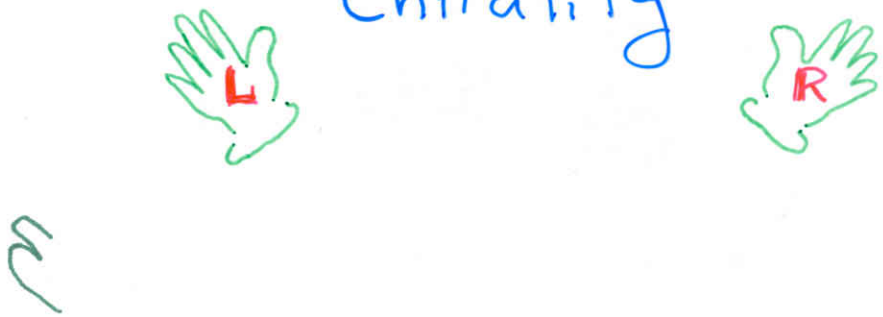
$$\mathcal{L}_q = \bar{q}\,(i\gamma^\mu D_\mu(G) - \mathcal{M})\,q$$

$\mathcal{M}$: "current" quark mass matrix.

Symmetries of $\mathcal{L}_q$

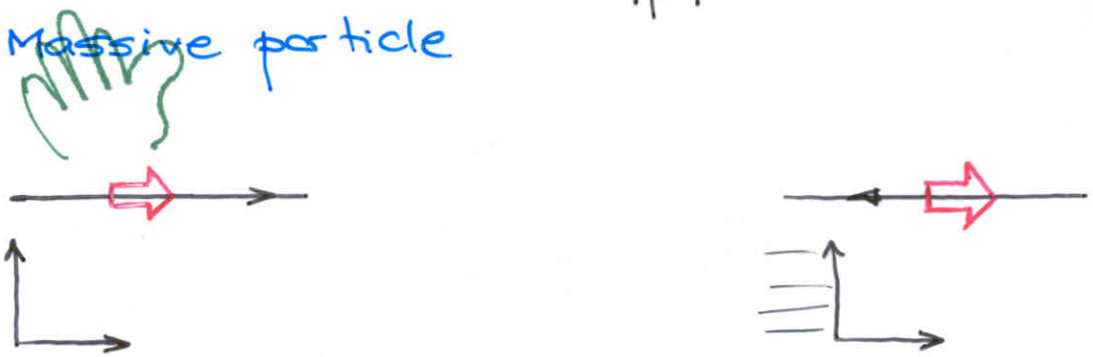
- gauge symmetry
- Baryon number symmetry : $q \to e^{i\alpha} q$
- For $\mathcal{M} \propto I$, $SU_f(3)$ symmetry.
- For $\mathcal{M}=0$, Chiral Symmetry

Chirality



$$\lambda = \frac{\vec{S}\cdot\vec{P}}{|\vec{P}|}$$ λ: helicity

Massive particle



λ positive λ negative

λ is frame dependent

Massless particle

No frame boost can reverse $\vec{p}$; λ independent of frame. λ: intrinsic property of particle. In this case λ is called <u>chirality</u>

Chirality for spin 1/2 Fermions

γ_μ : Dirac matices $\quad \{\gamma_\mu, \gamma_\nu\} = 2 g_{\mu\nu}$

$$\gamma_5 \equiv i\,\gamma^0 \gamma^1 \gamma^2 \gamma^3 \qquad \{\gamma_5, \gamma_\mu\} = 0$$

$$\gamma_5^\dagger = \gamma_5 \qquad \gamma_5^2 = \mathbb{1}$$

Dirac Eqn. for massless Fermion

$$i\not{\partial}\psi = 0$$

$$0 = \gamma_5\, i\not{\partial}\psi = -i\not{\partial}\gamma_5\psi$$

if ψ satisfies Eqn. so does $\gamma_5\psi$

Chirality projectors:

$$P_\pm = \frac{1}{2}(1 \pm \gamma_5)$$

$$P_+P_+ = P_+ \,, \quad P_-P_- = P_- \,, \quad P_+P_- = P_-P_+ = 0$$

$$\psi_L \equiv P_-\psi \qquad \psi_R \equiv P_+\psi$$

Spin operator: (Dirac basis)

$$\vec{S} = \frac{1}{2}\vec{\Sigma} = \frac{1}{2}\begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$$

$$\gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \qquad \vec{\alpha} = \gamma_0 \vec{\gamma}$$

$$\vec{\Sigma} = -\vec{\alpha}\,\gamma_5$$

$$\lambda = \frac{\vec{S}\cdot\vec{p}}{|\vec{p}|} \qquad \lambda \psi_L = \frac{1}{2}\frac{\vec{\alpha}\cdot\vec{p}}{|\vec{p}|}\psi_L$$

Dirac Eqn. $\Rightarrow$ $\vec{\alpha}\cdot\vec{p}\,\psi_L = \frac{1}{2}|\vec{p}|\,\psi_L$

so $\lambda\psi_L = \frac{1}{2}\psi_L$

Exercise: check $\lambda\psi_R = -\frac{1}{2}\psi_R$

Chirality $\longleftrightarrow$ Helicity

Chirality and gauge couplings

$$\bar{\psi}\, \gamma_\mu\, (\gamma_5)(\Lambda)\, \psi$$

$$= \bar{\psi}_L\, \gamma_\mu (\gamma_5)(\Lambda)\, \psi_L + \bar{\psi}_R\, \gamma_\mu (\gamma_5)(\Lambda)\, \psi_R$$

Gauge couplings preserve chirality

Chirality and mass

$$\bar{\psi}\, \mathcal{M}\, \psi = \bar{\psi}_L\, \mathcal{M}\, \psi_R + \bar{\psi}_R\, \mathcal{M}\, \psi_L$$

Mass term connects L and R chiralities

$\Rightarrow$ non conservation of chirality.

Chiral Symmetry (χS)

Chiral transformations

$$q_L \to L\, q_L \qquad L \in U_L(3)$$

$$q_R \to R\, q_R \qquad R \in U_R(3)$$

If $\mathcal{M}=0$, $\mathcal{L}_q$ is left unchanged by these transformations

$$U_L(3) \times U_R(3) = U_B(1) \times U_A(1) \times SU_L(3) \times SU_R(3)$$

$$U_B(1): \quad q_{L,R} \to e^{i\alpha} q_{L,R}$$

$$U_A(1): \quad q_{L,R} \to e^{\pm i\alpha} q_{L,R}$$

Noether currents and charges

Any continuous symmetry implies a corresponding conserved current

$U_B(1)$: $\quad V^0_\mu \equiv \bar{q}\,\gamma_\mu\, q \qquad \partial^\mu V^0_\mu = 0$

Best determination of this conservation law is by bounds on proton lifetime : $\tau_p \geq$ $1.6\,10^{25}$ years, 10^{31} years

$SU_L(3) \times SU_R(3)$:

$$L^a_\mu = \bar{q}_L\,\gamma_\mu\,\frac{\lambda^a}{2}\,q_L \qquad \text{left handed}$$

$$R^a_\mu = \bar{q}_R\,\gamma_\mu\,\frac{\lambda^a}{2}\,q_R \qquad \text{right handed}$$

other convenient currents :

$$* \quad V^a_\mu = \bar{q}\,\gamma_\mu\,\frac{\lambda^a}{2}\,q \qquad \text{vector}$$

$$** \quad A^a_\mu = \bar{q}\,\gamma_\mu\gamma_5\,\frac{\lambda^a}{2}\,q \qquad \text{axial-vector}$$

* Conserved if $\mathcal{M} \propto \mathbb{1}$

** Conserved if $\mathcal{M} \equiv 0$

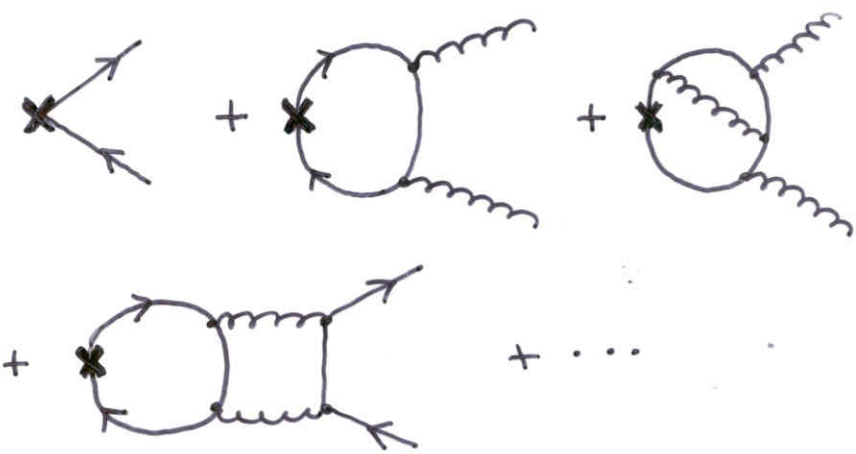
$U_A(1)$:

$$A^0_\mu = \bar{q}\,\gamma_\mu\,\gamma_5\,q$$

if $m \equiv 0$,

$$\partial^\mu A^0_\mu = 0 \qquad \text{Classically } (\hbar^0)$$

$U_A(1)$ Symmetry broken by quantum corrections in presence of gluons

$A^0_\mu =$ 

$$\partial^\mu A^0_\mu = \underbrace{N_f \frac{\alpha_s}{4\pi} G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a}_{\text{Chiral Anomaly}}$$

Introducing $\hbar$ explicitly, RHS $\propto \hbar$

More History: the trail to χS

Mass Puzzles

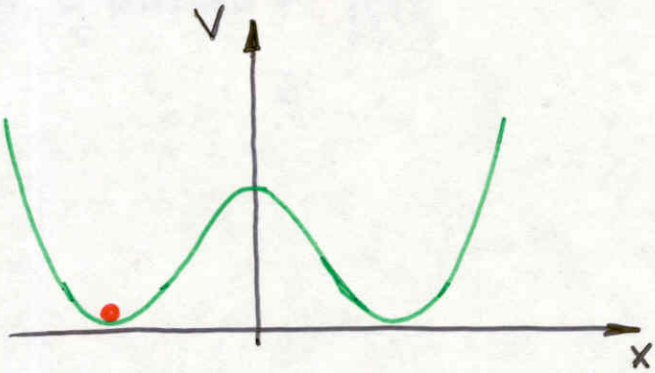
If QM would be accurate,

Expect	Real World
$m_p \sim 3\tilde{m}_q$	$m_p = 938$ MeV
$m_\pi \sim 2\tilde{m}_q$	$m_{\pi^+} = 139$ MeV
$m_\pi / m_p \sim 2/3$	$m_{\pi^+}/m_p = 0.15$
$m_\Lambda - m_p$	$m_\Lambda - m_p = 177$ MeV
$\sim m_K - m_\pi$	$m_K - m_\pi = 360$ MeV

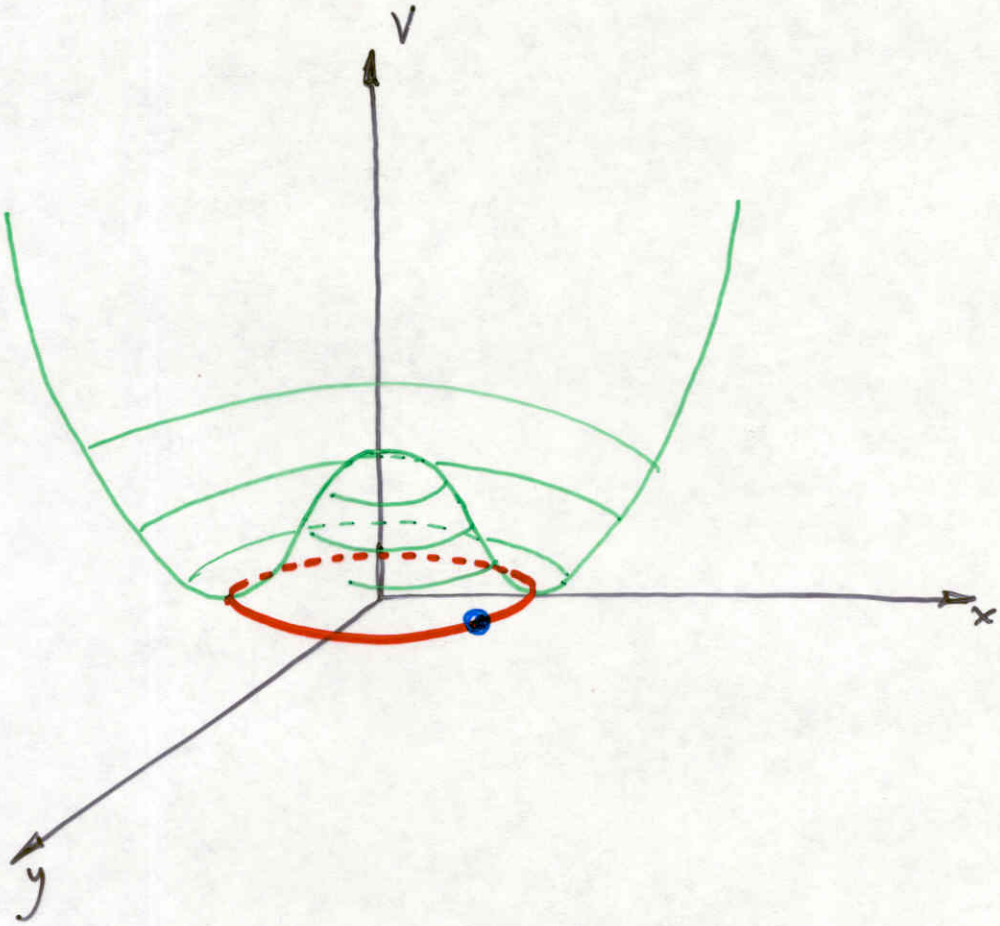
or similarly

$m_{D_s} - m_D \sim m_{B_s} - m_B$	$m_{D_s} - m_D = 100$ MeV
$\sim m_K - m_\pi$	

Spontaneous Symmetry Breaking



Spontaneous breaking of discrete symm.
(parity)



Spontaneous breaking of continuous symmetry
U(1)

Field Theory Example :
Abelian Higgs model

φ : complex scalar field

$$\mathcal{L} = \partial_\mu \varphi^* \partial^\mu \varphi - \left(\mu^2 - \frac{\lambda}{2}\varphi^*\varphi\right)\varphi^*\varphi$$

$$\mu^2 < 0$$

U(1) symmetry : $\varphi \to e^{i\alpha}\varphi$

Ground state:

$$\varphi^*\varphi = -\frac{\mu^2}{\lambda} = v^2$$

Any $\langle\varphi\rangle = \text{const.} = e^{i\alpha} v$ is a GS.

$$\varphi(x) = \left(v + \frac{\sigma(x)}{\sqrt{2}}\right) e^{i\alpha(x)/\sqrt{2}v}$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\alpha\,\partial^\mu\alpha + \frac{1}{2}\partial_\mu\sigma\,\partial^\mu\sigma - V(\sigma)$$

$\alpha(x)$ describes massless particle

Goldstone's Theorem

Let J_μ be a conserved current. If the corresponding conserved charge:

$$Q = \int d^3x \, J_0(x)$$

does not leave the ground state invariant, i.e.

$$Q\,|0\rangle \neq 0 \,,$$

then there must be a massless particle in the spectrum.*

The massless particle is called a Goldstone particle. Its quantum numbers are those of the charge Q.

* This formulation is only valid if $|0\rangle$ is Lorentz invariant.

A remarkable hypothesis

(Nambu; Gell-Mann & Levy) (1960)

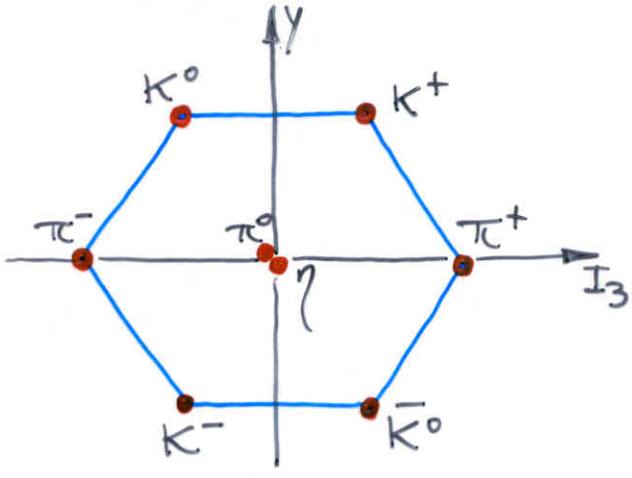
$$Q_A^a = \int d^3x \; A_0^a(x)$$

$$Q_V^a = \int d^3x \; V_0^a(x)$$

$$Q_V^a|0\rangle = 0 \qquad\qquad Q_A^a|0\rangle \neq 0$$

If $[H_{strong}, Q_A^a] = 0$, then Goldstone's theorem implies one massless spin 0 negative parity particle for each Q_A^a

Octet of pseudo-scalar mesons fits the bill



As shown later, **SXSB** solves mass puzzle.

SXSB in QCD

- What is known
  - Confinement $\Rightarrow$ SXSB
    ('tHooft anomaly matching criterion to prove this)
  - If $\theta = 0$, $Q_V^a |0\rangle = 0$
    (Vafa & Witten)
  - Order parameters : many in principle, but $\langle \bar{q} q \rangle \neq 0$ necessarily
    (Kogan, Kovner & Shifman)
- What is not (well) known
  Detailed mechanism for SXSB
  Different models can give hints
  - Schwinger-Dyson Equations with some approximations show SXSB.
  - Models for QCD vacuum
  - Instantons as triggers of SXSB.
  - Lattice QCD