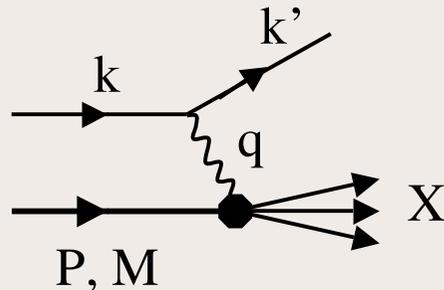


Lecture 4

Deep Inelastic Scattering: Current versus Constituent Quarks

Inclusive Lepton Scattering off the Nucleon



k (k') = incoming (outgoing) lepton momentum

$q = k - k' =$ four-momentum transfer

$P =$ initial four-momentum of the nucleon ($P^2 = M^2$)

Invariants: $\omega = \frac{q \cdot P}{M} =$ energy transfer in the lab frame (nucleon rest frame)

$Q^2 = -q \cdot q =$ squared four \square momentum transfer

$x = \frac{Q^2}{2M\omega} =$ Bjorken variable = fraction of the total nucleon momentum carried by the struck quark (parton model)

$y = \frac{q \cdot P}{k \cdot P} =$ fraction of the lepton energy loss

$W^2 = (P + q)^2 =$ squared mass of the system X

neutral current: exchange of virtual photon and Z-boson (and interference)

charged current: exchange of W^\pm bosons

the only one parity conserving

electron scattering (of interest for JLab):

- cross section:
$$\frac{d^2\sigma}{dx dy} = \frac{2y a_{em}^2}{Q^4} L_{\mu\nu} W^{\mu\nu}$$

- electron tensor:
$$L_{\mu\nu} = 2 \left(k_\mu k_\nu + k_\nu k_\mu - k \cdot k g_{\mu\nu} - i \epsilon_{\mu\nu\alpha\beta} k^\alpha k^\beta \right)$$

electron helicity (± 1)

- nucleonic tensor:
$$W_{\mu\nu} = \frac{1}{4} \int d^4z e^{iq \cdot z} \langle P, S | [J_\mu(z), J_\nu(0)] | P, S \rangle$$

$$= F_1(x, Q^2) g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} + F_2(x, Q^2) \frac{\hat{P}_\mu \hat{P}_\nu}{P \cdot q}$$

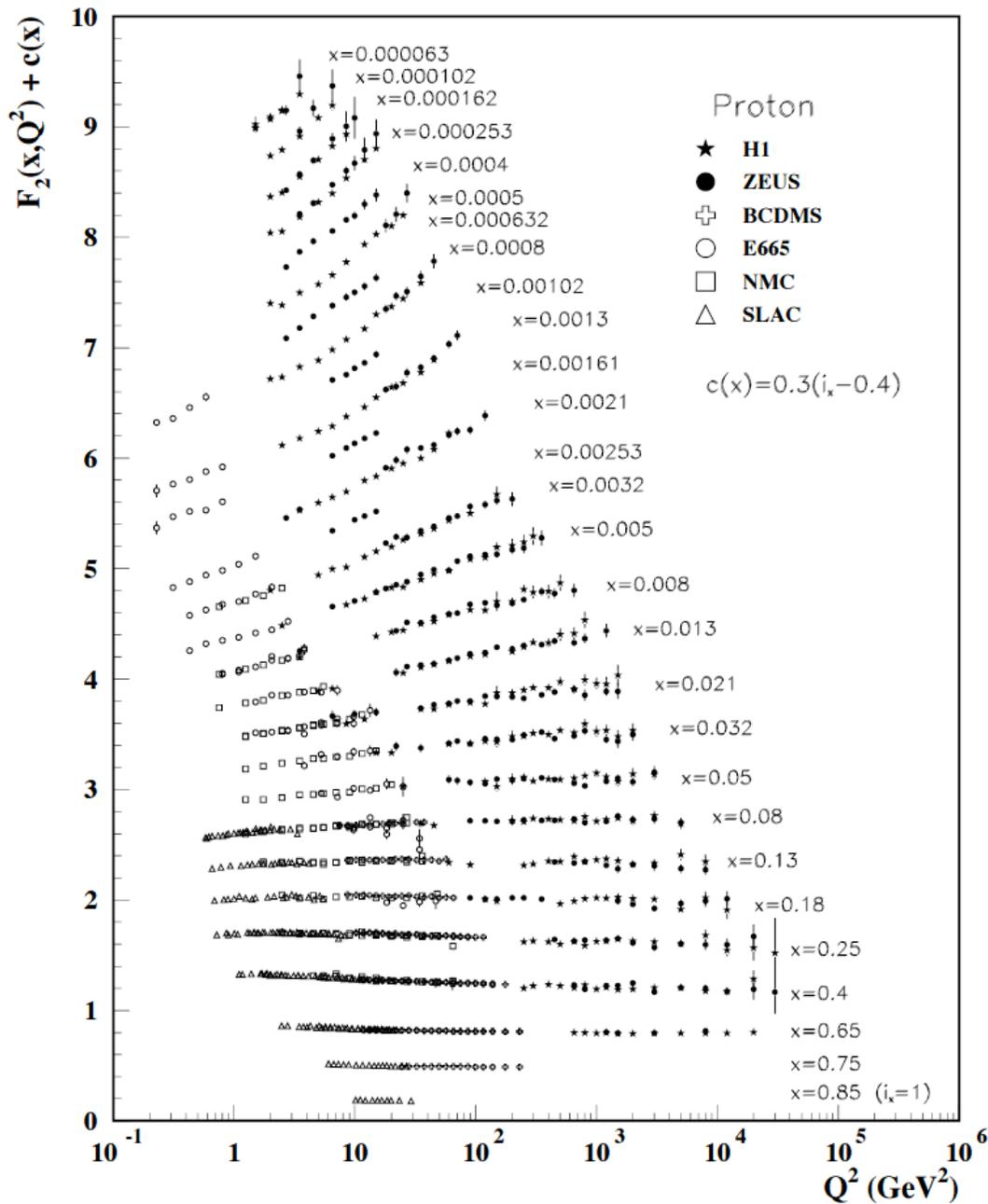
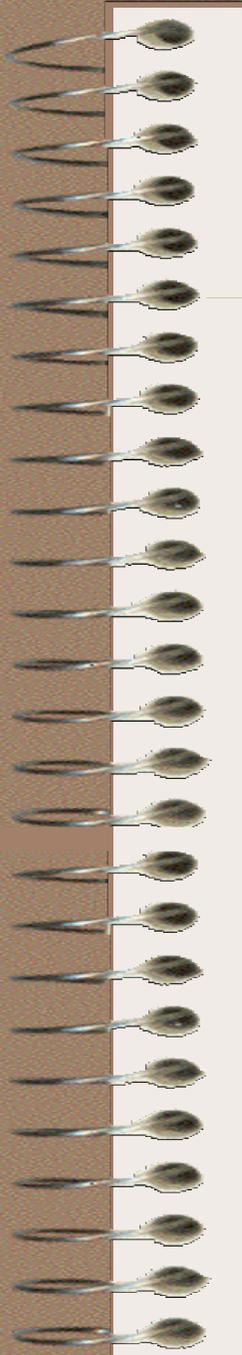
$$+ i \epsilon_{\mu\nu\alpha\beta} \frac{q^\alpha}{P \cdot q} g_1(x, Q^2) S^\beta + g_2(x, Q^2) S^\mu \frac{S \cdot q}{P \cdot q} P^\nu$$

nucleon spin 4-vector

F_1 and F_2 : **unpolarized** nucleon structure functions

$$\hat{P}_\mu = P_\mu - q_\mu \frac{P \cdot q}{q^2}$$

g_1 and g_2 : **polarized** nucleon structure functions



[PDG ('02)]

Deep ($Q^2 \gg M^2$)

Inelastic ($W^2 \gg M^2$)

Scattering

SLAC-MIT data

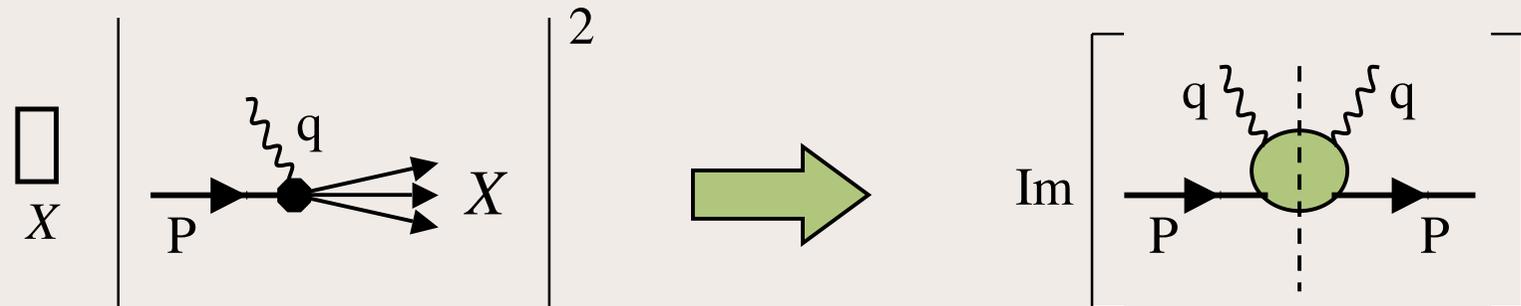


Bjorken scaling

$$\lim_{\substack{Q^2 \rightarrow \infty \\ x \text{ fixed}}} F_2(x, Q^2) \approx F_2(x)$$

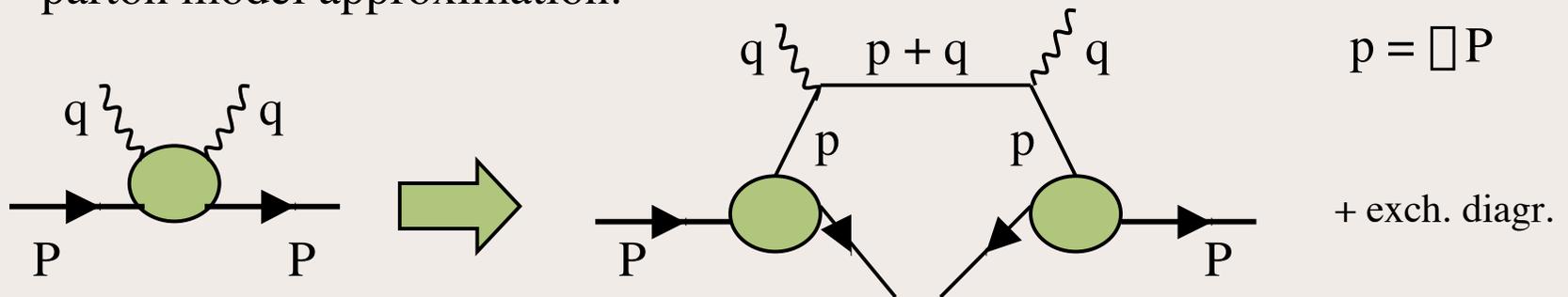
the naïve quark-parton model: [Bjorken, Feynman, ...]

Bj scaling arises from elastic scattering on free partons



$$T^{\square\square} = \text{forward Compton amplitude} = \int d^4z e^{iq \cdot z} \langle P, S | T [J^{\square}(z) J^{\square}(0)] | P, S \rangle$$

- parton model approximation:



- infinite momentum frame (neglect of transverse momenta):

$$T^{\square\square} \square \int_0^1 d^4 z e^{iq \cdot z} \int_0^1 d\square \int_j f_j(\square) \frac{1}{\square} \langle p, s | T [J^{\square}(z) J^{\square}(0)] | p, s \rangle \Big|_{p=\square P}$$

$f_j(\square)$ = distribution function of the parton j

\square = light-cone fraction of the parton j

- spin-1/2 partons:

$$T^{\square\square} \square \int_0^1 d\square \int_j f_j(\square) \frac{1}{\square} e_j^2 \bar{u}(p, s) \square \frac{i(\not{p} + \not{q})}{(p+q)^2 + i0} \square u(p, s) + \text{exch. diag.}$$

- unpolarized case:

$$T^{\square\square} \square \int_0^1 d\square \int_j f_j(\square) \frac{1}{\square} e_j^2 \frac{1}{2} \text{Tr} [\not{p} \square (p+q) \square] \frac{1}{2p \cdot q \square Q^2 + i0} \Big|_{p=\square P}$$

$$\text{Im}(T^{\square\square}) \square \int_0^1 d\square \int_j f_j(\square) \frac{1}{\square} e_j^2 \frac{1}{2} \text{Tr} [\not{P} \square (\not{P} + \not{q}) \square] \square (2\square P \cdot q \square Q^2)$$

$$\frac{d^2\sigma}{dx dy}(ep \rightarrow eX) = \frac{2\alpha\alpha_{em}^2}{xyQ^2} [1 + (1+y)^2] \bar{F}_2^p(x)$$

$$\bar{F}_2^p(x) = x \cdot \sum_j e_j^2 f_j(x)$$

$$\bar{F}_1^p(x) = \frac{1}{2x} \bar{F}_2^p(x)$$

- in terms of quark (antiquark) flavors:

$$\bar{F}_2^p(x) = x \cdot \sum_f e_f^2 [q_f(x) + \bar{q}_f(x)]$$

Callan-Gross relation

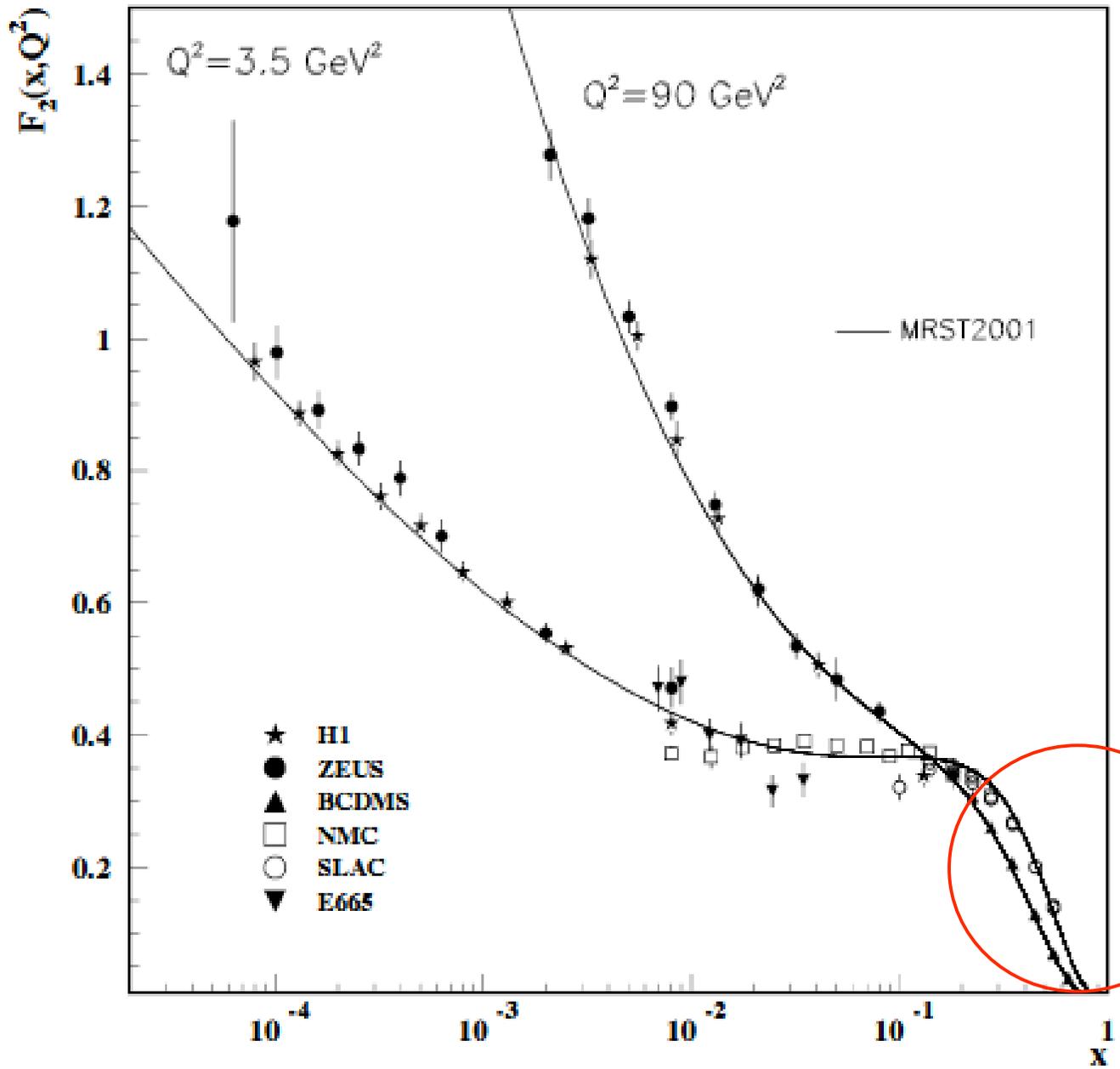
QCD-improved parton model: hard gluons can be radiated from quarks while gluons can convert into $q\bar{q}$ pairs



logarithmic scaling violations

$$q_f(x) \neq q_f(x, Q^2)$$

- as Q^2 increases: 1) softening of initial quark distributions,
- 2) growth of gluon and $q\bar{q}$ sea densities as x decreases



[PDG ('02)]

valence quark
region

evolution equations (Dokshitzer Gribov Lipatov Altarelli Parisi)

- non-singlet (NS) and singlet (S) quark distributions:

$$q_f^{NS} = q_f - q^S / 2N_f \qquad q^S = \sum_f (q_f + \bar{q}_f)$$

$$\frac{\partial q^{NS}}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\alpha} P_{qq} q^{NS}$$

$$\frac{\partial}{\partial \ln Q^2} \begin{pmatrix} q^S \\ G \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\alpha} \begin{pmatrix} P_{qq} & 2N_f P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \begin{pmatrix} q^S \\ G \end{pmatrix}$$

$$P_{ij} = \int_x^1 \frac{dy}{y} P(y) q \begin{pmatrix} x \\ y \end{pmatrix}$$

P_{ij} = splitting functions

- running of α_s (β function):

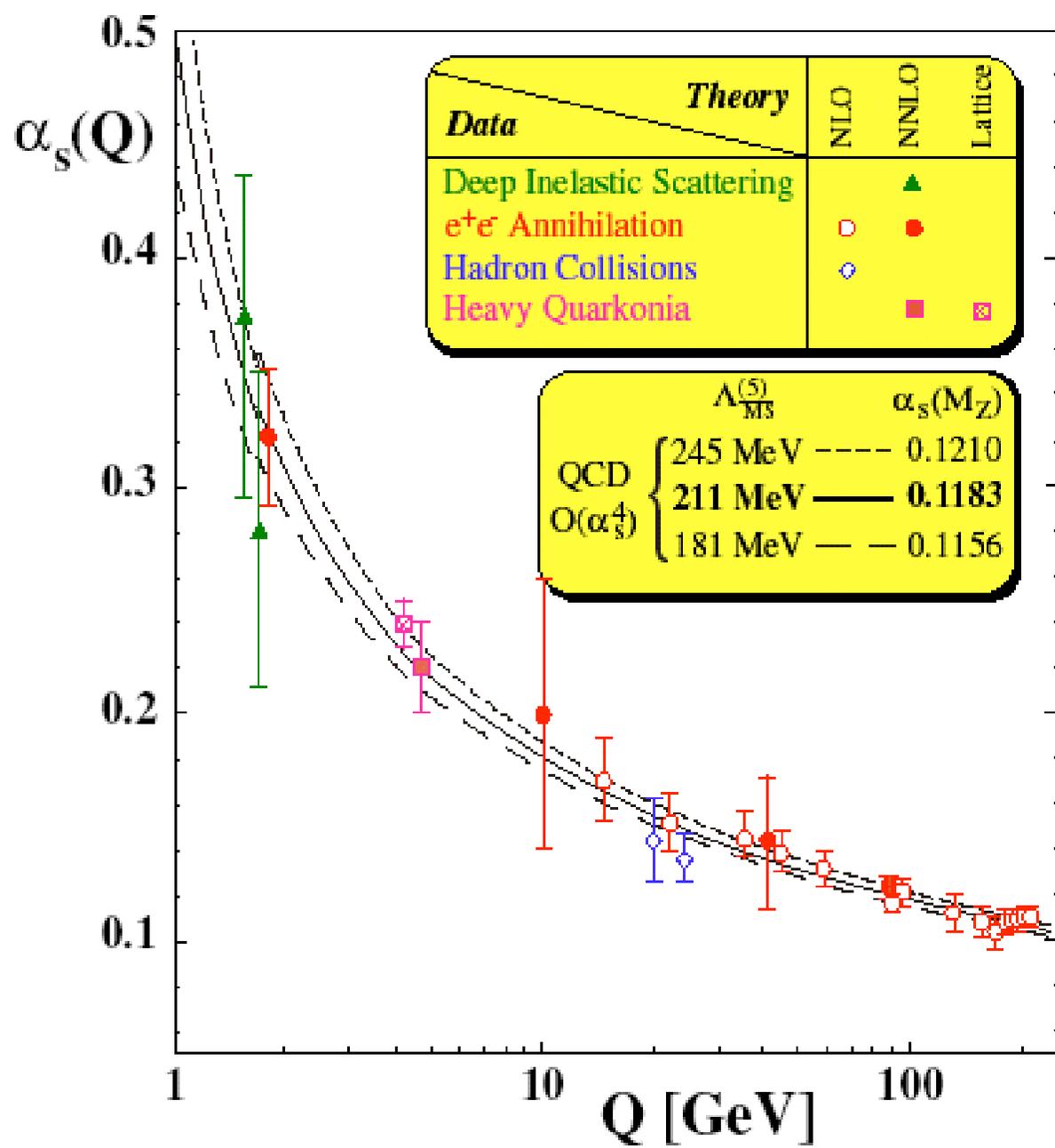
$$\frac{\partial \alpha_s(Q^2)}{\partial \ln Q^2} = \beta(\alpha_s) = -\frac{\beta_0}{4\alpha} \alpha_s^2 - \frac{\beta_1}{8\alpha^2} \alpha_s^3 + \dots$$

$\beta_0 > 0$ \Rightarrow asymptotic freedom

$$\beta_0 = 11 - \frac{2N_f}{3}$$

$$\beta_1 = 51 - \frac{19N_f}{3}$$

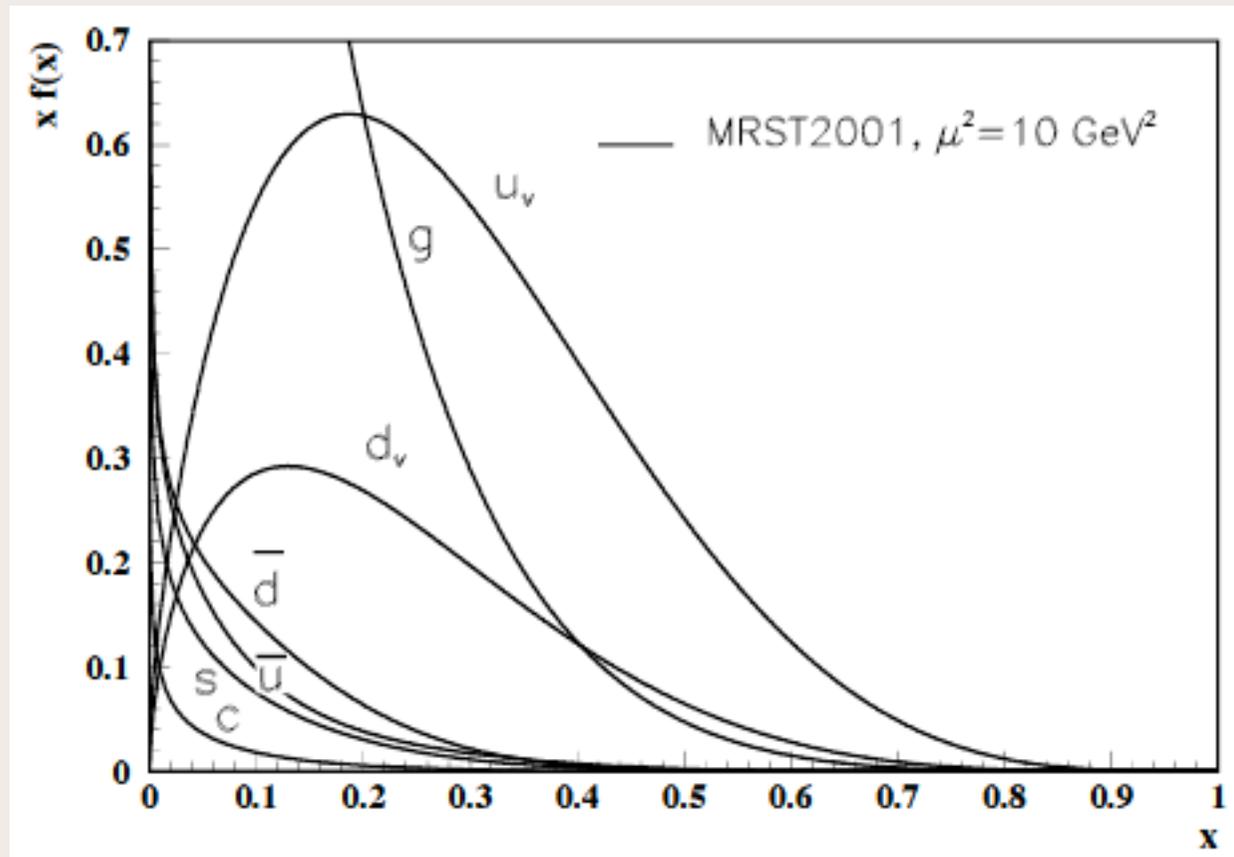
Bethke
(hep-ex/0211012)



running of α_s
is known

triumph of pQCD: explanation of scaling violations in DIS data

triumph of parton model: PDF's are universal, process independent



[PDG ('02)]

Cornwall-Norton moments: $M_n^{(CN)}(Q^2) = \int_0^1 dx x^{n-2} F_2(x, Q^2)$

$$F_2(x, Q^2) \stackrel{DIS}{=} F_2^{NS}(x, Q^2) + F_2^S(x, Q^2)$$

$$F_2^{NS}(x, Q^2) = x \cdot \sum_f e_f^2 [q_f^{NS}(x, Q^2) + \bar{q}_f^{NS}(x, Q^2)] \quad F_2^S(x, Q^2) = x \cdot \sum_f \frac{1}{N_f} \sum_f e_f^2 q^S(x, Q^2)$$

$$M_n^{(CN)}(Q^2) \stackrel{DIS}{=} M_n^{NS}(Q^2) + M_n^S(Q^2)$$

$$M_n^{NS}(Q^2) \stackrel{NLO}{=} a_n^{NS} [\Delta_s(Q^2)]^{\Delta_n^{NS}} + \frac{\Delta_s(Q^2)}{4\Delta} R_n^{NS}$$

$$M_n^S(Q^2) \stackrel{NLO}{=} a_n^+ [\Delta_s(Q^2)]^{\Delta_n^+} + \frac{\Delta_s(Q^2)}{4\Delta} R_n^+ + a_n^- [\Delta_s(Q^2)]^{\Delta_n^-} + \frac{\Delta_s(Q^2)}{4\Delta} R_n^-$$

$\Delta_n^{NS}, \Delta_n^\pm$ = anomalous dimensions

R_n^{NS}, R_n^\pm = NLO coefficients \rightarrow known from QCD

a_n^{NS}, a_n^\pm to be determined from data

Operator Product Expansion [Wilson ('64), Shifman ('79), Shuryak&Vainshtein ('82), ...]

$$T^{\square\square} = \int d^4 z e^{iq \cdot z} \langle P, S | T [J^{\square}(z) J^{\square}(0)] | P, S \rangle$$

- expand the T-product as a series of local operators

$$T [J^{\square}(z) J^{\square}(0)] = \sum_{m, \square} C_n^{\square} (\mu^2 z^2 + i \square) z^{\square_1} \dots z^{\square_m} O_{\square_1 \dots \square_m}^{\square}(0)$$

short-distance coefficients
calculable in pQCD

symmetric traceless
operators

- twist expansion of the moments:

$$M_n(Q^2) = \sum_{\square \geq 2, \text{even}} E_n^{(\square)}(\mu, Q^2) O_n^{(\square)}(\mu) \frac{\mu^{\square} Q^{2-\square}}{Q^2} \quad \mu = \text{renormalization scale}$$

$$= E_n^{(2)}(\mu, Q^2) O_n^{(2)} + E_n^{(4)}(\mu, Q^2) O_n^{(4)} \frac{\mu^4}{Q^2} + E_n^{(6)}(\mu, Q^2) O_n^{(6)} \frac{\mu^6}{Q^2} + \dots$$

parton model → leading twist + twist-4 + twist-6 + ...

CQ's as quasi-particles: dressing of valence quarks with gluons and quark-antiquark pairs

CQ's as intermediate structures between current quarks and hadrons



two-stage model

Altarelli et al. ('74)

Cabibbo et al. ('78)

Hwa ('80)

hadrons are composed by a finite number of CQ's having a partonic structure

try to explain **DIS data** with two main ingredients:

1) partonic structure of the CQ's (**short-distance, high- Q^2 physics**)

$$F_2^{CQ}(x/z, Q^2) = \text{CQ structure function (independent of H)}$$

2) motion of the CQ's inside the hadron (**long-distance, low- Q^2 physics**)

$$f_{CQ}^H(z) = \text{light-front CQ momentum distribution inside the hadron H}$$

convolution formula in DIS: $F_2^H(x, Q^2) = \sum_{j=U, D, \dots} \int_x^1 dz f_j^H(z) F_2^j\left(\frac{x}{z}, Q^2\right)$

- same convolution for parton distributions:

$$q_f^H(x, Q^2) = \sum_{j=U, D, \dots} \int_x^1 dz f_j^H(z) q_f^j\left(\frac{x}{z}, Q^2\right)$$

- nucleon structure function:

$$F_2^p = \frac{4}{9} f_U \quad F_2^U + \frac{1}{9} f_D \quad F_2^D$$

$$F_2^n = \frac{1}{9} f_U \quad F_2^U + \frac{4}{9} f_D \quad F_2^D$$

with $f_U = f_U^p$ and $f_D = f_D^p$

- valence dominance at large x: $F_2^U \approx F_2^D \approx F_2^{CQ}$ [SU(2) symmetry]

$$\frac{F_2^n}{F_2^p} \approx \frac{(f_U + 4f_D)}{(4f_U + f_D)} \frac{F_2^{CQ}}{F_2^{CQ}} \approx \frac{1 + 4f_D/f_U}{4 + f_D/f_U}$$

light-front CQ momentum distributions in the proton (Q=U, D):

$$f_Q(x) = \frac{3}{2} \int_{\Omega_p} \int [d\Omega_i d\vec{k}_{i\Omega}] \int_{\{\Omega_i\}} \Omega(x, \Omega_1) \Omega_{Q\Omega} \left| \left\langle \left\{ \Omega_i \vec{k}_{i\Omega}; \Omega_i \Omega_i \right\} \left| \Omega_p \right\rangle \right|^2$$

LF proton wave function

- normalizations:

$$\int_0^1 dx f_U(x) = 2$$

$$\int_0^1 dx f_D(x) = 1$$

- momentum sum rule: $\int_0^1 dx x \cdot [f_U(x) + f_D(x)] = 1$

- LF wave function: $\Omega_p = \int_{\Omega\Omega} \left(\Omega_i, \vec{k}_{i\Omega}; \Omega_i \right) = \int_{\Omega\Omega} \langle \Omega_i | R^+ | \Omega\Omega \rangle \Omega^{(c)}(\vec{k}_i; \Omega\Omega)$

R = Melosh rotation (it does not affect spin-averaged quantities)

$\Omega^{(c)}$ = canonical wave function



$$\chi^{(c)} = a_S w_S(\vec{p}_{12}, \vec{p}) \chi_S(S_{12}, T_{12}) + \frac{a_{S\Box}}{\sqrt{2}} \left\{ w_{S\Box}(\vec{p}_{12}, \vec{p}) \chi_{S\Box}(S_{12}, T_{12}) + w_{S\sqcup}(\vec{p}_{12}, \vec{p}) \chi_{S\sqcup}(S_{12}, T_{12}) \right\} \\ + a_A w_A(\vec{p}_{12}, \vec{p}) \chi_A(S_{12}, T_{12})$$

Jacobian momenta:

$$\begin{cases} \vec{p}_{12} \equiv \vec{p}_1 \Box \vec{p}_2 \\ \vec{p} \equiv \vec{p}_3 \\ [\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0] \end{cases}$$

$$\chi_S(S_{12}, T_{12}) = \frac{1}{\sqrt{2}} \left\{ |S_{12} = 0, T_{12} = 0\rangle + |S_{12} = 1, T_{12} = 1\rangle \right\} = \text{completely symmetric}$$

$$\chi_{S\Box}(S_{12}, T_{12}) = \frac{1}{\sqrt{2}} \left\{ |S_{12} = 0, T_{12} = 0\rangle \Box |S_{12} = 1, T_{12} = 1\rangle \right\} = \text{mixed symmetry (1 } \Box \text{ 2 symmetric)}$$

$$\chi_{S\sqcup}(S_{12}, T_{12}) = \frac{1}{\sqrt{2}} \left\{ |S_{12} = 0, T_{12} = 1\rangle + |S_{12} = 1, T_{12} = 0\rangle \right\} = \text{mixed symmetry (1 } \Box \text{ 2 antisymmetric)}$$

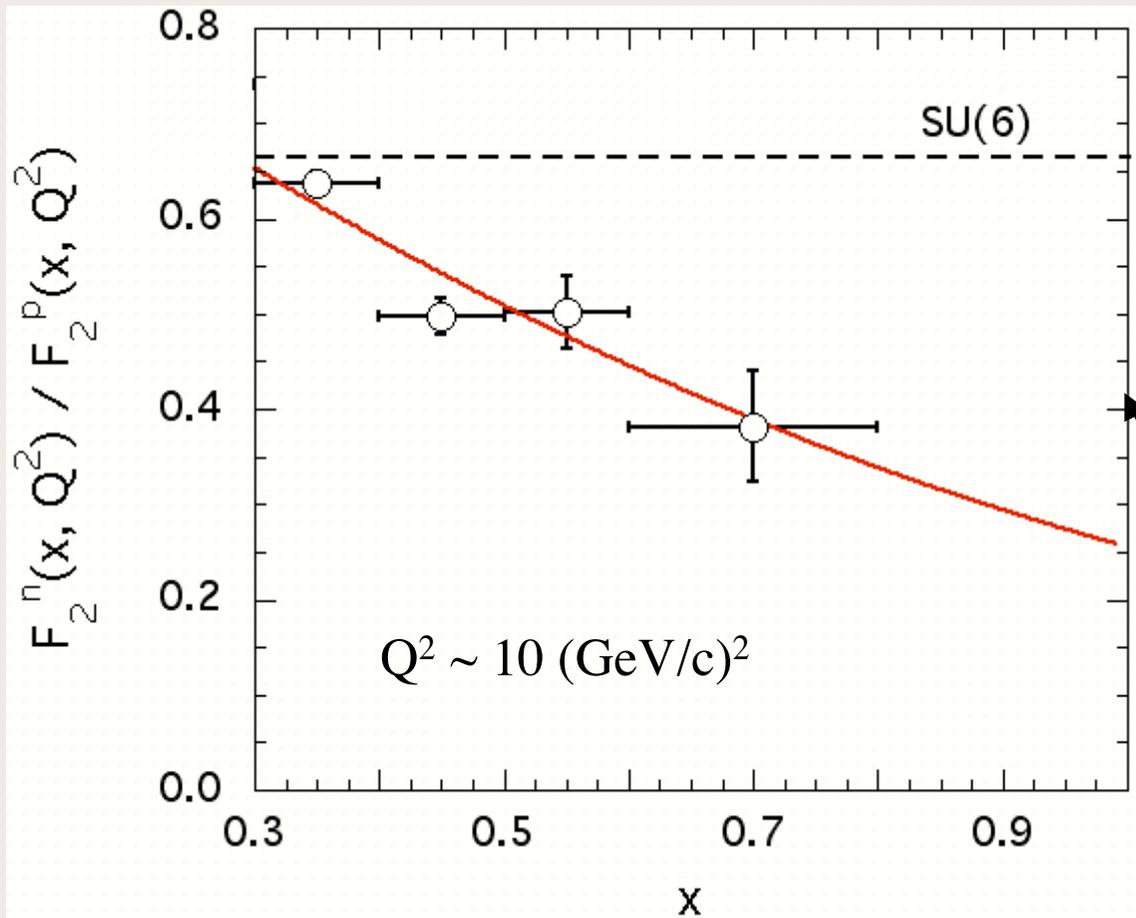
$$\chi_A(S_{12}, T_{12}) = \frac{1}{\sqrt{2}} \left\{ |S_{12} = 0, T_{12} = 1\rangle \Box |S_{12} = 1, T_{12} = 0\rangle \right\} = \text{completely antisymmetric}$$

SU(6) symmetry $\Rightarrow a_{S\Box} = a_A = 0$

spin-spin interaction $\Rightarrow a_{S\Box} \neq 0$

CQ potential models $\Rightarrow a_A \Box 0$

SU(6) symmetry implies $f_D = 0.5 f_U$, which means $\frac{F_2^n}{F_2^p} \approx \frac{2}{3}$



- NMC data ('94)
- SLAC parm. of F_2
 - d/u \approx 0
 - n/p \approx 1/4
- [Feynman, Isgur, Close]
- pQCD: d/u \approx 1/5
- n/p \approx 3/7
- [Brodsky, Farrar, Jackson]

$f_D / f_U \ll 0$ is a consequence of the dominance of the $S_{12} = 0$ component

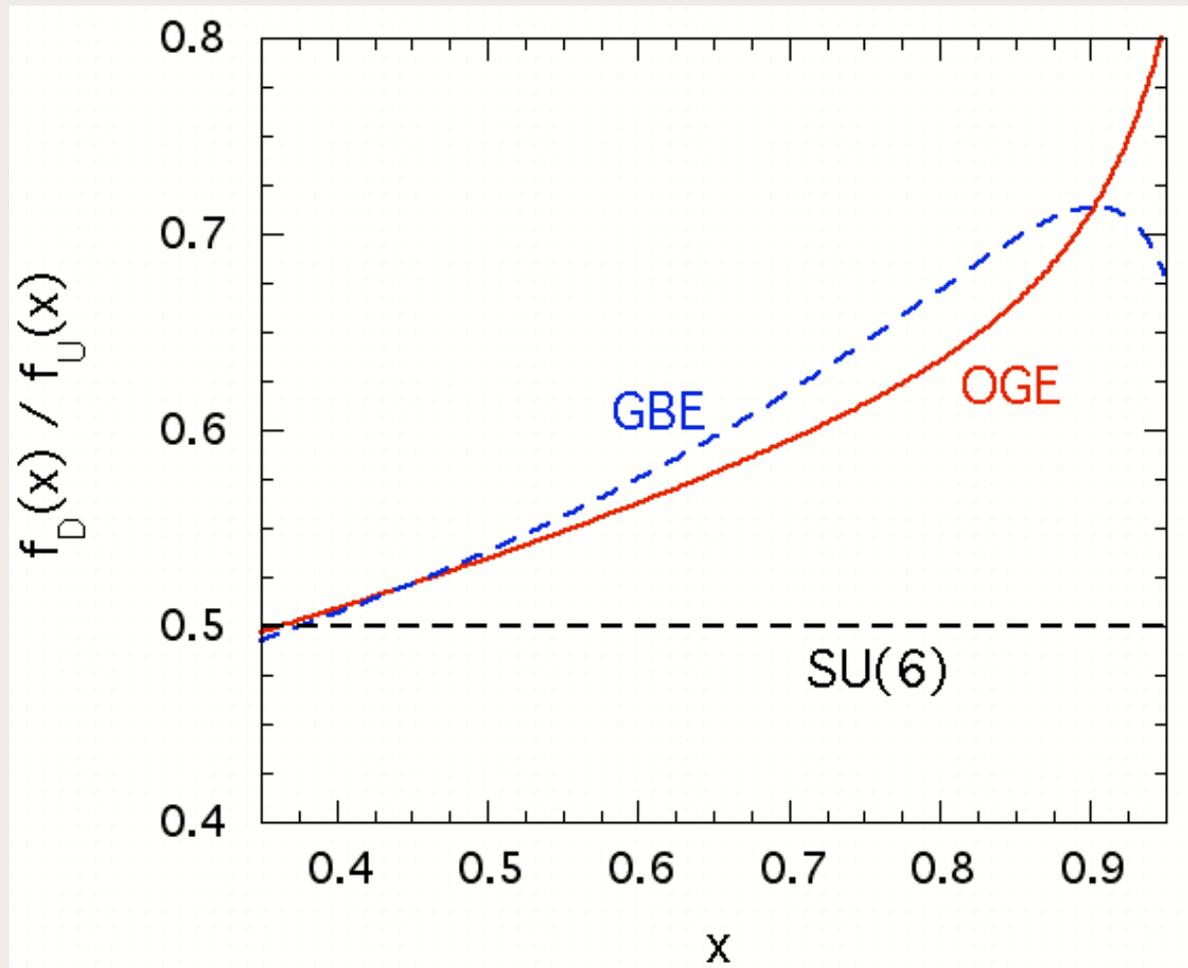
$$\begin{aligned} \chi^{(c)} &= \chi_S(\vec{p}_{12}, \vec{p}) \frac{1}{\sqrt{2}} \left\{ |S_{12} = 0, T_{12} = 0\rangle + |S_{12} = 1, T_{12} = 1\rangle \right\} \\ &+ \chi_{S\pi}(\vec{p}_{12}, \vec{p}) \frac{1}{\sqrt{2}} \left\{ |S_{12} = 0, T_{12} = 0\rangle - |S_{12} = 1, T_{12} = 1\rangle \right\} \end{aligned}$$



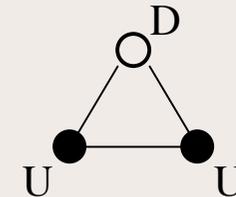
$$\begin{aligned} f_D &= \underbrace{\chi_S^2}_{\text{high momenta}} - 2\chi_S \cdot \chi_{S\pi} \\ f_U &= \underbrace{2\chi_S^2}_{\text{high momenta}} + 2\chi_S \cdot \chi_{S\pi} \end{aligned}$$

if $\chi_S \cdot \chi_{S\pi} > 0$, then $f_D / f_U < 0.5$

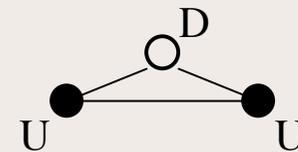
however, effects from spin-spin interaction leads to $f_D > 0.5 f_U$!!!



SU(6) symmetry



spin-spin term



D is closer to the c.m.



higher momenta

alternative: mix the dominant S-wave with a P-wave [Altarelli et al. ('74)]

$$\chi^{(c)} = b_S \chi_S + b_P \chi_P$$

χ_S belongs to the 56^+ multiplet of $SU(6)$
 χ_P belongs to the 70^- multiplet of $SU(6)$

- it works with $|b_P|^2 \sim 10\%$, however:

a P-wave can be generated in the nucleon w.f. only by a

spin-orbit interaction term



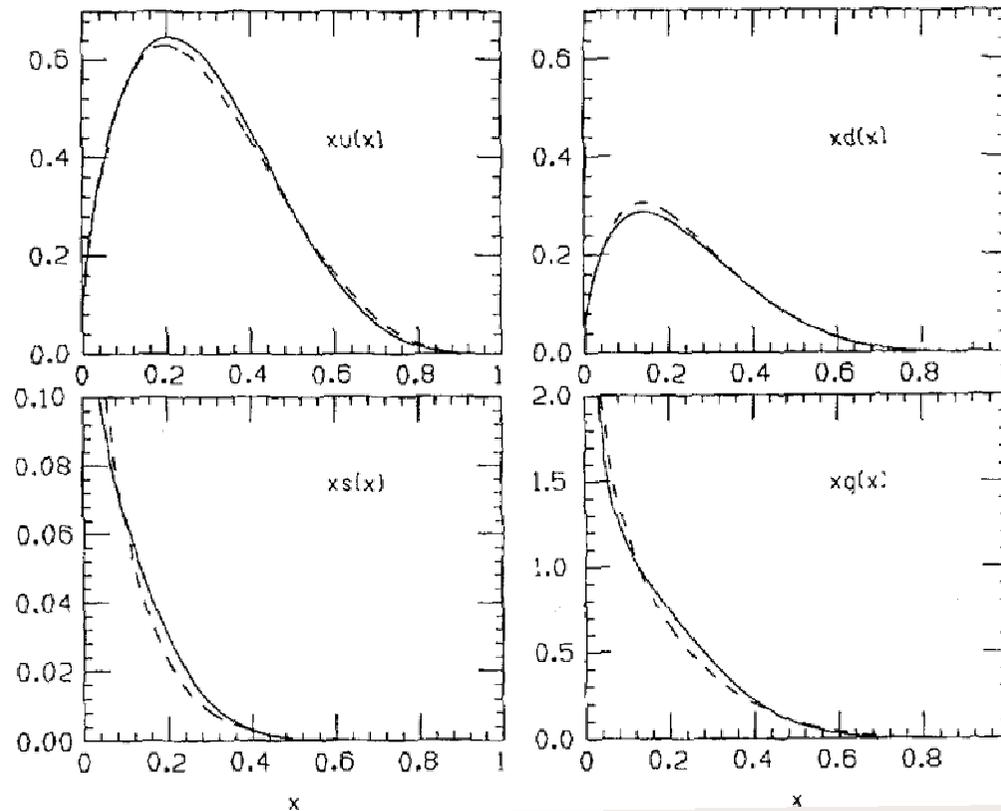
absence of spin-orbit splitting in baryons and $|b_P|^2 \sim 10\%$ are totally inconsistent

D / U puzzle !!!

reproduce PDF's at a given scale Q_0^2 : $q_f^N(x, Q_0^2) = \int_{j=U,D} \int_x^1 dz f_j^N(z) q_f^j\left(\frac{x}{z}, Q_0^2\right)$

evolution equations are not affected by the two-stage convolution

G. Altarelli et al. / Physics Letters B 373 (1996) 200–206



$$Q_0^2 = 4 \text{ GeV}^2$$

$$q_f^j \sim x^a (1-x)^b$$

[phen. param.]

———— two-stage model

----- MRS(G) PDF set

- given a reasonable model for $f_j^H(z)$ the CQ structure functions $F_2^j(x/z, Q^2)$ can be obtained by deconvolution of known hadron data on $F_2^H(x, Q^2)$
- then, using $F_2^j(x/z, Q^2)$ and a reasonable model for $f_j^H(z)$ one can predict the structure function of another hadron H' $F_2^{H'}(x, Q^2)$

this procedure has been applied in [Altarelli et al. \('96\)](#): within the two-stage model the pion structure function has been predicted from nucleon data



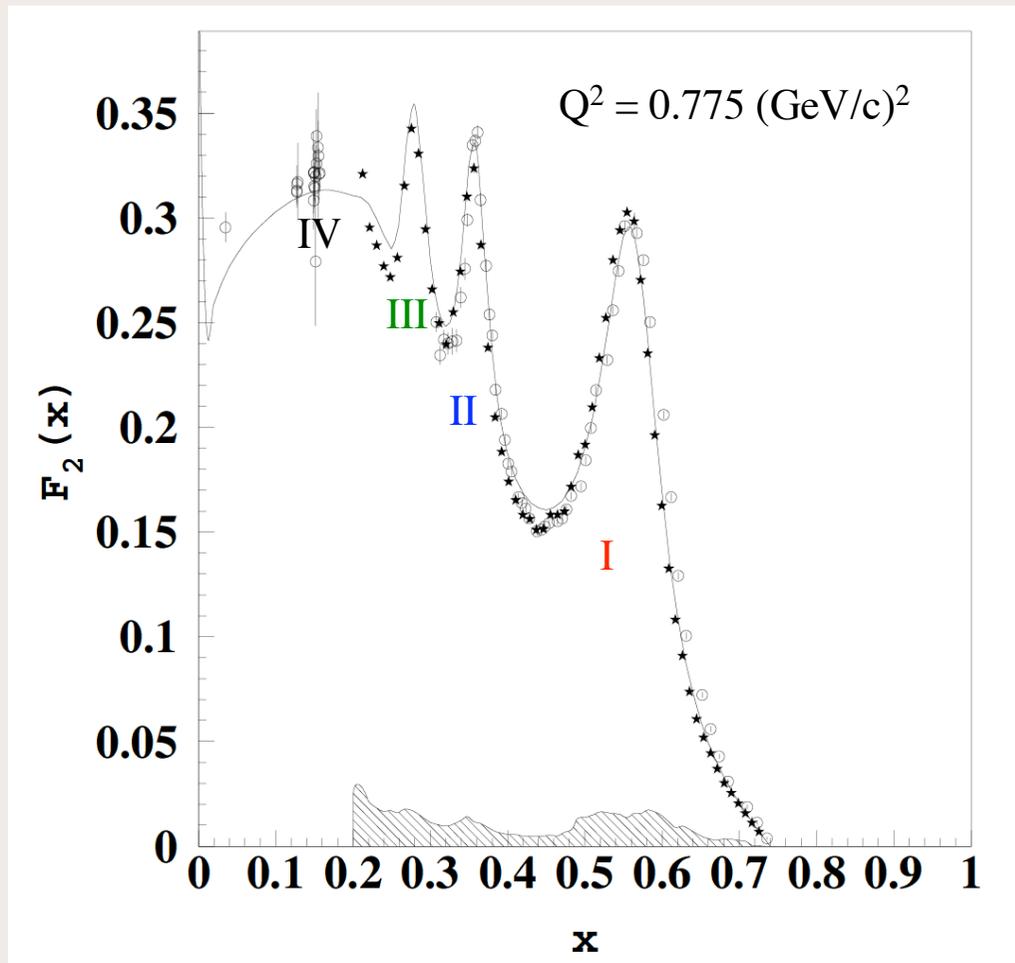
two-stage model ~ O.K., at least as a first approximation

the idea of CQ's with a partonic structure has been applied to polarized structure functions, off-forward parton distributions, ... [[Hwa](#), [Hwa&Yang](#), [Arash&Khorramian](#), [Vento&Scopetta](#), [Traini&Scopetta](#), ...]

overall consistency with DIS data

how is the structure function F_2 away from DIS kinematics ?

the most striking feature is the appearance of bumps due to resonance production and subsequent decay in the final state



four regions

I region: $\Delta(1232)$

II region: $D_{13}(1520)$, $S_{11}(1535)$

III region: mainly $F_{15}(1680)$

IV region: higher resonances

★ CLAS data

○ previous exp.'s

[Osipenko et al. ('03)]

investigation of resonant and non-resonant processes (exclusive channels)

Bloom-Gilman (local quark-hadron) **duality** [Bloom&Gilman ('70)]

- empirical observation made on SLAC data

The smooth scaling curve measured in the DIS region at high Q^2 represents the average over the resonance bumps seen at low Q^2 at corresponding values of the improved empirical variable $x' = x / (1 + M^2 x^2 / Q^2)$

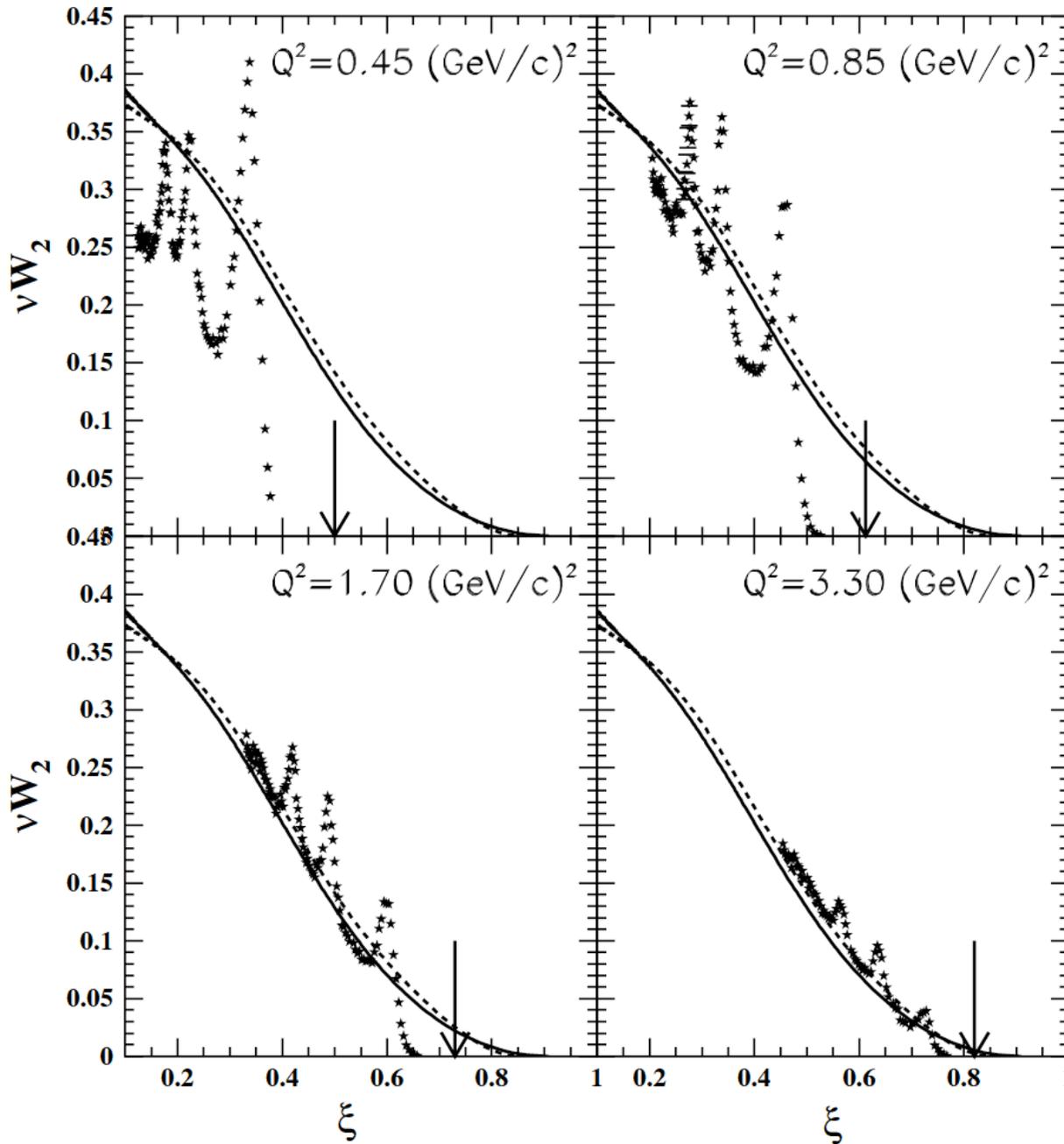
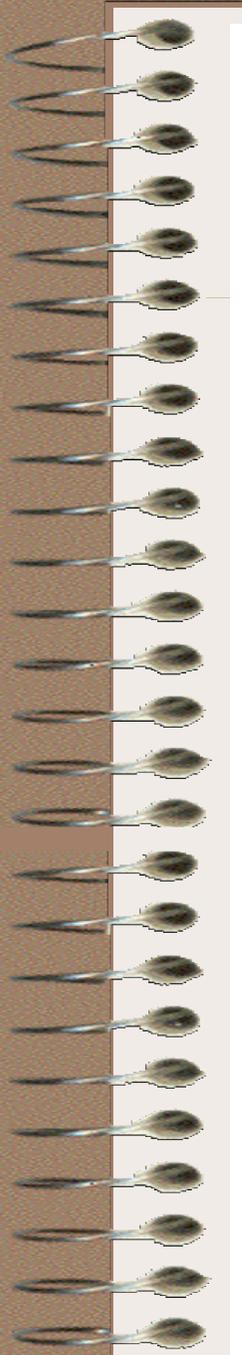
onset of BG duality at $Q^2 \sim 1 \div 2 \text{ (GeV/c)}^2$

the BG duality implies that there should be a relation between the physics in the nucleon-resonance region and the one in the DIS kinematics

such a relation is not yet fully understood

new precise measurements from JLab





Hall C data
 [Niculescu et al. ('00)]

$$\square W_2 = F_2$$

$$\square = \frac{2x}{1 + \sqrt{1 + 4M^2 x^2 / Q^2}}$$

NMC fit
 - - - $Q^2 = 5$ (GeV/c)²
 — $Q^2 = 10$ (GeV/c)²

↓ elastic peak location

a partial explanation based on QCD was proposed by **De Rujula, Georgi and Politzer ('77)**

- consider the moments of the structure function F_2 :

$$M_n(Q^2) = \int_0^1 dx x^{n-2} F_2(x, Q^2) \quad x = \frac{2x}{1 + \sqrt{1 + 4M^2 x^2 / Q^2}}$$

x is the Nachtmann variable (which takes into account target-mass corrections to the Bjorken scaling variable x)

$$x \approx \frac{2x}{1 + \sqrt{1 + 4M^2 x^2 / Q^2}} \approx x + O\left(\frac{M^4}{Q^4}\right)$$

- using OPE and some generic “democratic” arguments

$$M_n(Q^2) = A_n(\ln Q^2) + \sum_{k=1}^{\infty} \frac{Q^{-2k}}{Q^2} B_{nk}(\ln Q^2) = \text{leading twist} + \text{higher twists}$$

$$Q^2 \sim \langle k_{\perp}^2 \rangle \sim \Lambda_{QCD}^2 \sim 0.1 \div 0.2 \text{ GeV}^2 \quad \Rightarrow \quad M_n(Q^2) \approx \sum_{Q^2 > M^2} A_n(\ln Q^2)$$

precocious dominance of the leading twists for $Q^2 > M^2 \sim 1 \text{ (GeV/c)}^2$

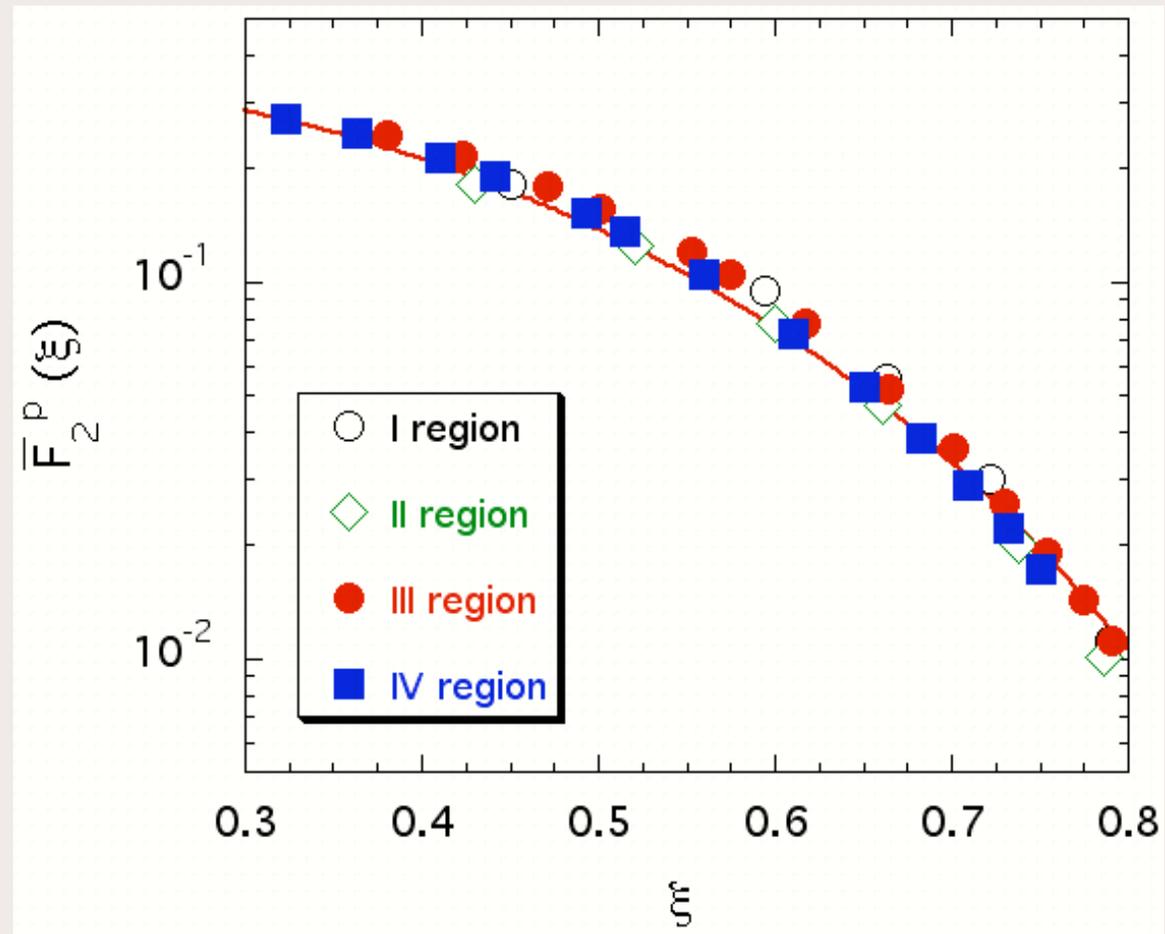
~ onset of BG duality

$$\bar{F}_2^P(\xi = \xi_R) = \frac{1}{\xi_U \xi_L} \int_{\xi_L}^{\xi_U} d\xi \xi F_2^P(\xi, Q^2)$$

$$\xi_L = \xi(W_U, Q^2)$$

$$\xi_U = \xi(W_L, Q^2)$$

$$\xi_R = \xi(W_R, Q^2)$$



W_L and W_U in GeV

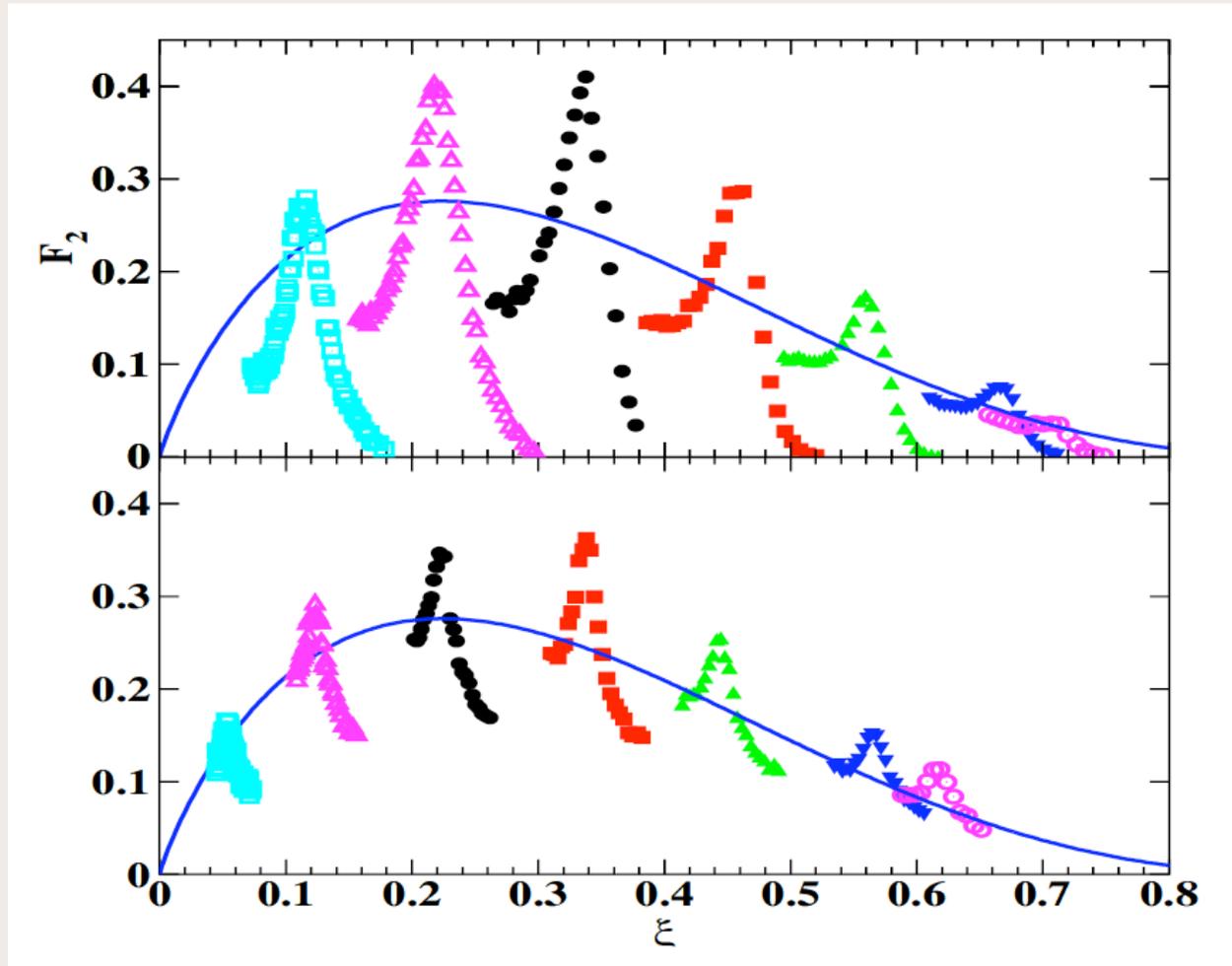
region	W _L	W _U
I (□)	1.14	1.38
II (S ₁₁)	1.38	1.58
III (F ₁₅)	1.58	1.76
IV	1.76	1.97

— NMC DIS fit
 $Q^2 = 10 \text{ (GeV/c)}^2$

[Niculescu et al. ('00),
 Simula ('01)]

averages over resonance bumps define the **dual curve** $\bar{F}_2^P(\square)$

$$0.07 < Q^2 \text{ (GeV/c)}^2 < 3$$



[Armstrong et al. ('01)]

I region: $\square(1232)$

II region: $S_{11}(1535)$

— dual curve

what's the origin of the BG duality ? Not yet understood.

see: F. Close and N. Isgur: PLB 509, 81 (2001).

N. Isgur et al.: PRD 64, 054005 (2001)

intense theoretical and experimental activity at JLab on the BG duality:

- can the local quark-hadron duality hold as well for the unpolarized longitudinal and/or polarized structure functions ?
- can duality be extended to exclusive processes ?
- can duality be beneficial for investigating the behavior of structure functions at $x \ll 1$?

some approved experiments at JLab

00-002 (Keppel, Niculescu)

00-116 (Keppel)

BG duality in nuclei ? Its onset appears to be anticipated !!!

[Ricco et al. : PRC 57, 356 (1997)]

← first paper on BG duality after many years

SUMMARY

- # CQ's as quasi-particles: dressing of valence quarks with gluons and $q\bar{q}$ pairs
- # CQ's as intermediate structures between current quarks and hadrons
- # hadrons are composed by a finite number of CQ's having a partonic structure

two-stage model

- # O.K. with DIS data, except for the d/u ratio at large x

a nucleon P-wave seems to be required to get $d/u < 0.5$, but baryon spectroscopy requires mainly an SU(6) breaking due to the spin-spin contact term which produces $d/u > 0.5$

- # away from DIS regions the **local quark-hadron duality** is one of the most interesting phenomenon
- # extension of the two-stage model to low momentum transfer is mandatory



next lecture