Lecture 3

Hadron Form Factors

within

the Covariant Light-Front CQ Model

loss of rotational covariance in the light-front formalism:

- angular condition among matrix elements of I^+
- form factors may depend upon the component of the current used
- impact on the estimate of the CQ size

two approaches have been proposed

$\mathbf{q^+ \neq 0}$	$\mathbf{q^+ = 0}$
[Lev, Pace, Salmè ('98)]	[Karmanov ('96), Melikhov & Simula ('02)]

- transformation between frames at $q^+ \neq 0$ and $q^+ = 0$ is interaction dependent $\longrightarrow$ non-equivalence of the two approaches

approach at $q^+ \neq 0$: q is along the spin-quantization axis $\hat{n}$

$$Q^2 = -q \bullet q$$

special Breit frame: $q^+ = -q^- = Q$ and $\vec{q}_\perp = 0$

- covariance and hermiticity: $I^\mu = \frac{1}{2}\left\{ C^\mu + L^{\mu\nu}\left[r_x(-\pi)\right] e^{i\pi S_x}\, C^*_\nu\, e^{-i\pi S_x} \right\}$

S_x = x-component of the LF spin operator

$r_x(-\pi)$ = rotation around the x-axis of $(-\pi)$

L = element of the Lorentz group

choice of C^μ:

$$C^+ = I^+_{(1)}$$

$$\vec{C}_\perp = \vec{I}_{\perp(1)}$$

$I^\mu_{(1)}$ = one-body approx.

$$C^- = I^+_{(1)} \longrightarrow$$ to ensure gauge invariance of I^μ

- one has: $I^+ = I^+_{(1)}, \quad I^{\mu\neq+} \neq I^{\mu\neq+}_{(1)}$

Pseudoscalar (PS) mesons: $\langle P' | I^{\nu} | P \rangle = F_{PS}\left(Q^2\right)\left(P + P'\right)^{\nu}$

- Breit frame at $q^+ \neq 0$: $P^{\pm} = \sqrt{M_{PS}^2 + Q^2/4} \mp Q/2, \quad \vec{P}_{\perp} = 0$

- from $\nu = +$: $F_{PS} = e_1\, H_1\left(Q^2/M_{PS}^2\right) + e_2\, H_2\left(Q^2/M_{PS}^2\right)$

$$H_1(\alpha) = \frac{1-\alpha}{1-\frac{\alpha}{2}} \int d\xi\, d\vec{k}_{\perp} \sqrt{A(\xi, k_{\perp})\, A(\xi', k_{\perp})}\, \frac{w_{PS}(k)\, w_{PS}(k')}{4\pi}\, \frac{m(\xi)\, m(\xi') + k_{\perp}^2}{\sqrt{m^2(\xi) + k_{\perp}^2}\, \sqrt{m^2(\xi') + k_{\perp}^2}}$$

$$\begin{cases} \xi' = \alpha + (1-\alpha)\,\xi \\ \alpha = \dfrac{q^+}{P'^+} = \sqrt{\dfrac{Q^4}{4M_{PS}^4} + \dfrac{Q^2}{M_{PS}^2}} - \dfrac{Q^2}{2M_{PS}^2} \end{cases} \qquad \begin{cases} A(\xi, k_{\perp}) = \dfrac{M_0}{4\xi(1-\xi)} \left[1 - \dfrac{\left(m_1^2 - m_2^2\right)^2}{M_0^4}\right] \\ \mathrm{m}(\xi) = m_1(1-\xi) + m_2\,\xi \end{cases}$$

$w_{PS}(k)$ = canonical w.f. $\qquad H_1(k) \xrightarrow[\xi \to (1-\xi)]{m_1 \leftrightarrow m_2} H_2(k)$

within the LF at $q^+ \neq 0$ the form factors depend on Q^2 / M^2

light-front at $q^+ = 0$

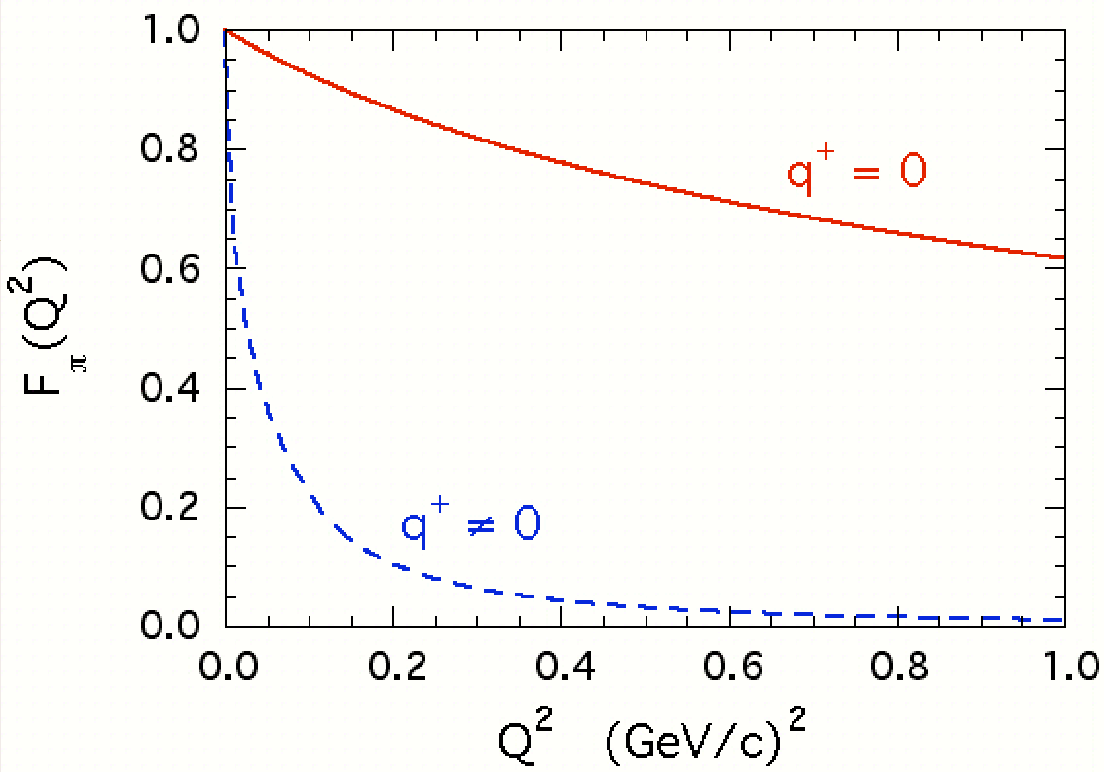
- standard Breit frame: $\begin{cases} q^+ = q^- = 0, \quad \vec{q}_\perp \neq \vec{0} \\ P^+ = \sqrt{M_{PS}^2 + Q^2/4}\,, \quad \vec{P}_\perp = -\vec{q}_\perp/2 \end{cases}$ $\qquad Q^2 = |\vec{q}_\perp|^2$

- from $\mu = +$: $\quad F_{PS}(Q^2) = e_1\, J_1(Q^2) + e_2\, J_2(Q^2)$

$$J_1(Q^2) = \int d\xi\, d\vec{k}_\perp \sqrt{A(\xi, k_\perp)\, A(\xi, k'_\perp)}\; \frac{w_{PS}(k)\, w_{PS}(k')}{4\xi}\; \frac{m^2(\xi) + \vec{k}_\perp \bullet \vec{k}'_\perp}{\sqrt{m^2(\xi) + k_\perp^2}\,\sqrt{m^2(\xi) + k'^2_\perp}}$$

$$\vec{k}'_\perp = \vec{k}_\perp + (1-\xi)\,\vec{q}_\perp \qquad\qquad J_1(Q^2) \xrightarrow[\vec{k}'_\perp = \vec{k}_\perp - \xi \vec{q}_\perp]{\xi \to 1-\xi} J_2(Q^2)$$

within the LF at $q^+ = 0$ the form factors depend on Q^2



[Simula ('02)]

$w_{\pi}(k)$ from OGE model

point-like CQ's

- since at $q^+ \neq 0$ f.f. depends on Q^2 / M^2, one has $r_{ch} \sim 1 / M$ [π: $r_{ch} \sim 5$ fm]

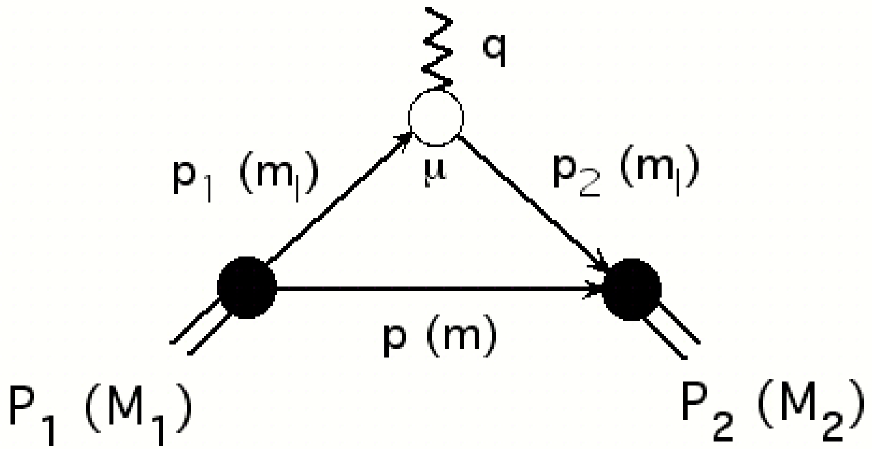
this is at variance with the phenomenology of light hadrons

$$r_{ch}^{\pi} = 0.660 \pm 0.024\ fm, \quad r_{ch}^{K} = 0.58 \pm 0.04\ fm, \quad r_{ch}^{p} = 0.883 \pm 0.014\ fm$$

$$M_K / M_\pi \simeq 3.5, \quad M_p / M_\pi \simeq 6.7$$

What is the origin of such a discrepancy ?

<u>Feynman triangle diagram</u>



spin-0 two-fermion systems

$$I^{\mu} = \sum_j e_j \Gamma^{\mu}$$

PS → PS transition

Frankfurt & Strikman ('79)
Lepage & Brodsky ('80)
Chung et al. ('88)
Sawicki ('92)
Frederico & Miller ('92)
Demchuk et al. ('96)
De Melo et al. ('99)
Melikhov & Simula ('02)
Frederico et al. ('02)
…

PS → PS transition:

$$I_F^{\mu}(P_1, P_2) = \frac{i}{(2\pi)^4} \int d^4 p \, \Lambda_1(p_1, P_1) \, \Lambda_2(p_2, P_2)$$

$$\cdot Tr\left[\frac{-\not p + m}{p^2 - m^2 + i\varepsilon} \gamma^5 \frac{\not p_2 + m_I}{p_2^2 - m_I^2 + i\varepsilon} \Gamma^{\mu} \frac{\not p_1 + m_I}{p_1^2 - m_I^2 + i\varepsilon} \gamma^5 \right]$$

Λ(p, P) = regularizing function (to be connected to a bound-state w.f.)

Γ^{μ} = generic Dirac matrix $\left[\Gamma^{\mu} = \gamma^{\mu} \quad \text{or} \quad \Gamma^{\mu} = i\sigma^{\mu\nu} q_{\nu} \right]$

Note: $p_i + p = P_i, \quad P_i^2 = M_i^2 \;\Rightarrow\; p^2 \neq m^2$ and $p_i^2 \neq m_I^2 \qquad (i = 1, 2)$

within Hamiltonian formalisms (instant, point and front forms) constituents are always on-mass shell

connection with the triangle diagram is non-trivial

using light-front variables:

$$I_F^{\mu}(P_1,P_2) = \frac{i}{2(2\pi)^4}\int dp^- dp^+ d\vec{p}_\perp \; Tr\left\{(\not p + m)(\not P_2 - \not p + m_I)\Gamma^{\mu}(\not P_1 - \not p + m_I)\right\}$$

$$\cdot \frac{\Lambda_1 \Lambda_2}{p^+\left(P_1^+ - p^+\right)\left(P_2^+ - p^+\right)} \frac{1}{\left[p^- - p^-_{sp}\right]\left[p^- - p^-_{Z_1}\right]\left[p^- - p^-_{Z_2}\right]}$$

- three poles in p⁻ (assume no poles in Λ):

$$p^-_{sp} = \frac{m^2 + p_\perp^2}{p^+} - \frac{i\varepsilon}{p^+}$$

spectator pole

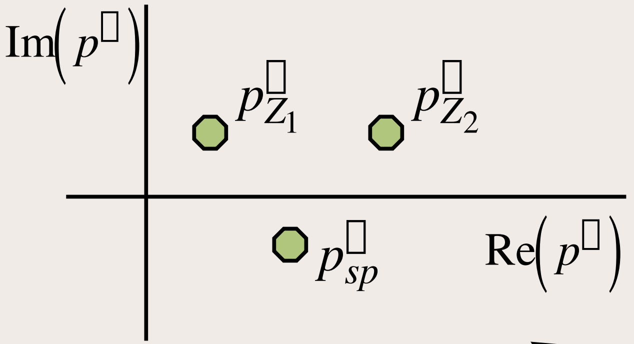
$$p^-_{Z_1} = P_1^- - \frac{m_I^2 + \left(\vec{P}_{1\perp} - \vec{p}_\perp\right)^2}{P_1^+ - p^+} + \frac{i\varepsilon}{P_1^+ - p^+}$$

$$p^-_{Z_2} = P_2^- - \frac{m_I^2 + \left(\vec{P}_{2\perp} - \vec{p}_\perp\right)^2}{P_2^+ - p^+} + \frac{i\varepsilon}{P_2^+ - p^+}$$

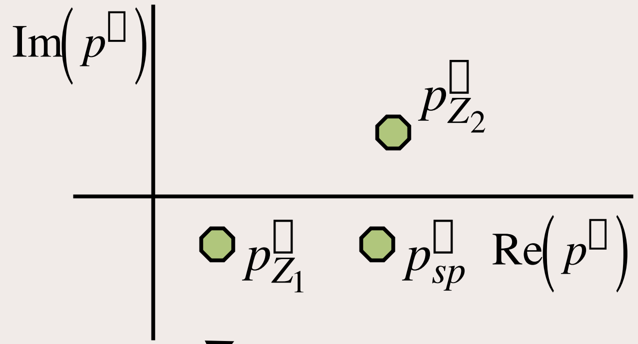
Z-graph poles

- consider: $P_1^+ > 0, \quad P_2^+ = P_1^+ + q^+ > P_1^+ \quad \Rightarrow \quad 0 \le p^+ \le P_2^+$

$0 \le p^+ \le P_1^+$

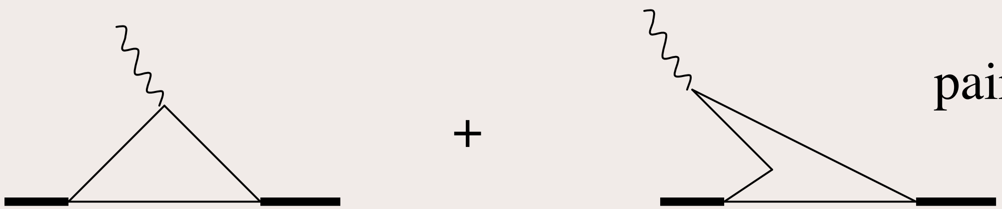


$P_1^+ < p^+ \le P_2^+$



$$I_F^-(P_1, P_2) = I_{sp}^-(P_1, P_2) + I_Z^-(P_1, P_2)$$

spectator + Z-graph contributions



pair creation

spectator term: spectator particle on-mass shell $\left(p^2 = m^2\right)$

$$I^{\mu}_{sp}(P_1,P_2) = \frac{N}{2(2\pi)^4}\int_0^{P_1^+} dp^+ \int d^2\vec{p}_\perp \, Tr\left\{(\not{p}+m)(\not{P}_2 - \not{p} + m_I)\gamma^{\mu}(\not{P}_1 - \not{p} + m_I)\right\}_{p^-=p^-_{sp}}$$

$$\cdot \frac{\chi_1 \chi_2}{p^+\left(P_1^+ - p^+\right)\left(P_2^+ - p^+\right)} \frac{1}{\left[p^-_{sp} - p^-_{Z_1}\right]\left[p^-_{sp} - p^-_{Z_2}\right]}$$

Z-graph term: active particle on-mass shell in the final state $\left(p_2^2 = m_I^2\right)$

$$I^{\mu}_{Z}(P_1,P_2) = \frac{-N}{2(2\pi)^4}\int_{P_1^+}^{P_1^+ + q^+} dp^+ \int d^2\vec{p}_\perp \, Tr\left\{(\not{p}+m)(\not{P}_2 - \not{p} + m_I)\gamma^{\mu}(\not{P}_1 - \not{p} + m_I)\right\}_{p^-=p^-_{Z_2}}$$

$$\cdot \frac{\chi_1 \chi_2}{p^+\left(P_1^+ - p^+\right)\left(P_2^+ - p^+\right)} \frac{1}{\left[p^-_{Z_2} - p^-_{sp}\right]\left[p^-_{Z_2} - p^-_{Z_1}\right]}$$

Note: when $q^+ \to 0$, then $I_Z^{\mu=+,\perp}(P_1,P_2) \to 0$

- no poles in Λ(p, P) $\Rightarrow$ ψ independent on p$^-$

- let us introduce the null four-vector ω which defines the direction normal to the null plane: $\omega = \left(\omega^0, \vec{\omega}\right) = (1, \hat{n})$ $\Rightarrow$ $\omega^2 = 0$

ω is along the "-" direction

- define the following new on-mass-shell momenta:

$$\tilde{p}_i = p_i - \omega \frac{p_i^2 - m_I^2}{2\,\omega \bullet p_i} \qquad \Rightarrow \qquad \tilde{p}_i^2 = m_I^2$$

$$\tilde{p} = p - \omega \frac{p^2 - m^2}{2\,\omega \bullet p} \qquad \Rightarrow \qquad \tilde{p}^2 = m^2$$

$\tilde{p}^{\mu}$ coincides with p^{μ} for $\mu = +, \perp$, while differs only for $\mu = -$

LF momenta (on-mass-shell) Feynman momenta (off-mass-shell) $\Rightarrow$ $\psi_i = \psi_i(p, P_i, \omega)$

new covariant structures depending on □ [Karmanov&Smirnov ('96)]

$$I_F^{\square} = I_F^{\square}\left(P_1, P_2, \square\right)$$

consider $q^+ = 0$: $\quad I_F^{\square}\left(P_1, P_2, \square\right) \;\square\; \underset{q^+=0}{\square\square\square} \;\square\; I_{sp}^{\square}\left(P_1, P_2, \square\right)$

Z-graph suppressed !

- covariant decomposition of the spectator term

$$I_{sp}^{\square}\left(P_1, P_2, \square\right) = \left(P_1 + P_2\right)^{\square} F_{PS}\left(q^2\right) + \frac{\square^{\square}}{\square \bullet P_1} B_{PS}\left(q^2\right)$$

note: only in case of the full current one must have $B_{PS} = 0$

- since □ is along the "-" direction, F_{PS} can be extracted using □ = + or □ = □

$$F_{PS}\left(q^2\right) = \frac{1}{2P_1^+} I_{sp}^+\left(P_1, P_2\right)$$

the form factors $F_{PS}(q^2)$ coincide with the LF result at $q^+ = 0$:

- instantaneous propagation: $(\not{p}_1 + m_I)\gamma^+(\not{\tilde{p}}_1 + m_I) = \gamma^+ \frac{p_i^- - \tilde{p}_i^-}{2}$

  killed in I^+_{sp} by $\gamma^+\gamma^+ = 0$

- connection with potential model w.f.'s:

$$\square_i(p, P_i, \square) = \square_i(M_{i0}) = \frac{\square}{\sqrt{2}}\left(M_{i0}^2 - M_i^2\right)\frac{\sqrt{M_{i0}\left[1 - \left(m_I^2 - m^2\right)^2 / M_{i0}^4\right]}}{\sqrt{M_{i0}^2 - (m_I - m)^2}}\, w_i(k)$$

$\sqrt{(\tilde{p} + \tilde{p}_i)^2}$

canonical w.f.

consider $q^+ \neq 0$: again $\dfrac{I_F^+(P_1, P_2)}{(P_1 + P_2)^+}$ provides the form factor $F_{PS}(q^2)$

But now: $I_F^+(P_1, P_2) = I_{sp}^+(P_1, P_2) + I_Z^+(P_1, P_2) \neq I_{sp}^+(P_1, P_2)$

only $\dfrac{I_{sp}^+(P_1, P_2)}{(P_1 + P_2)^+}$ coincides with the LF result at $q^+ \neq 0$

the origin of the discrepancy between the LF approaches at $q^+ = 0$ and $q^+ \neq 0$ is the pair creation process (Z-graph)

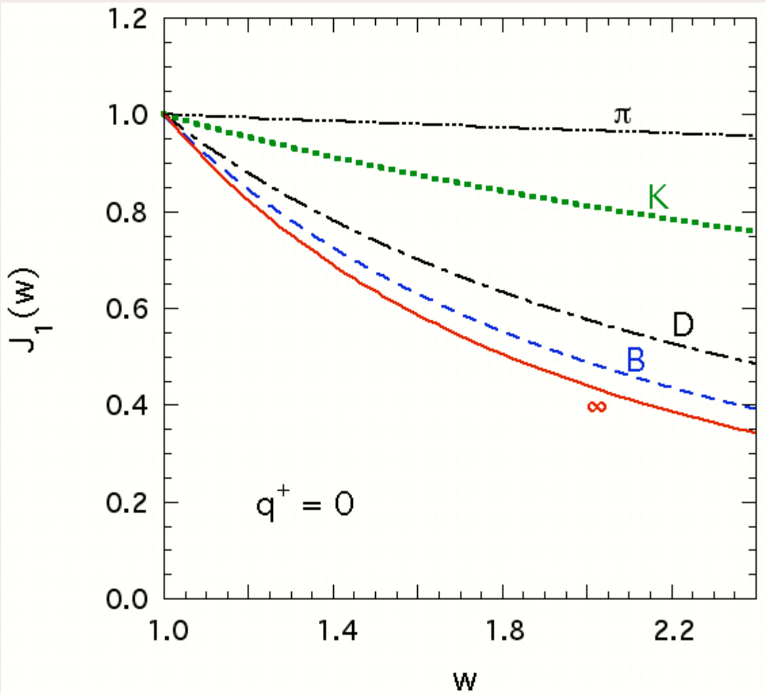
a many-body current !!!

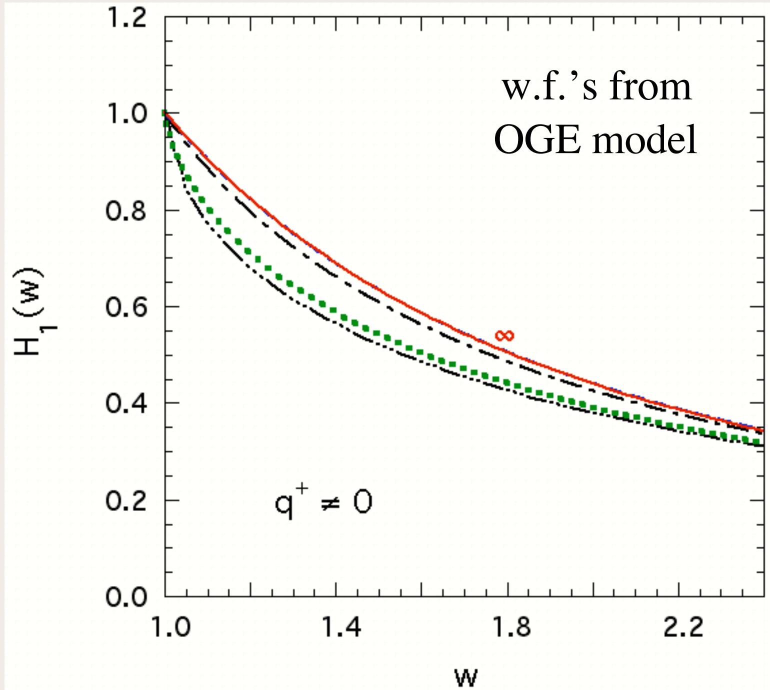
LF formalism
- $q^+ = 0$: Z-graph suppressed $\Rightarrow$ minimize many-body currents
- $q^+ \neq 0$: Z-graph active $\Rightarrow$ maximize many-body currents

pair creation process is sensitive to the mass of the active particle

$$I_Z^{\square}(P_1, P_2) \; \square \; \square_{m_I \square} \; \square \; \square \; 0$$

f.f.'s calculated at $q^+ = 0$ and $q^+ \neq 0$ should coincide in the heavy-quark limit



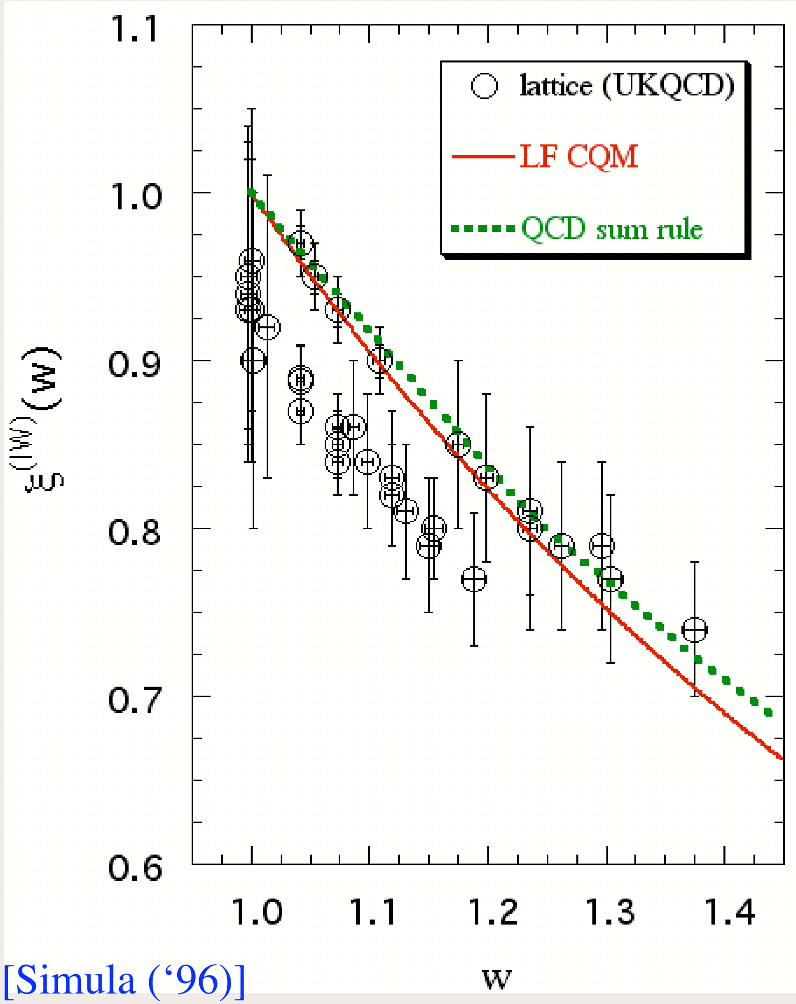


[Simula ('02)]

$$w = P_1 \bullet P_2 / M_{PS}^2 = 1 + Q^2 / 2M_{PS}^2$$

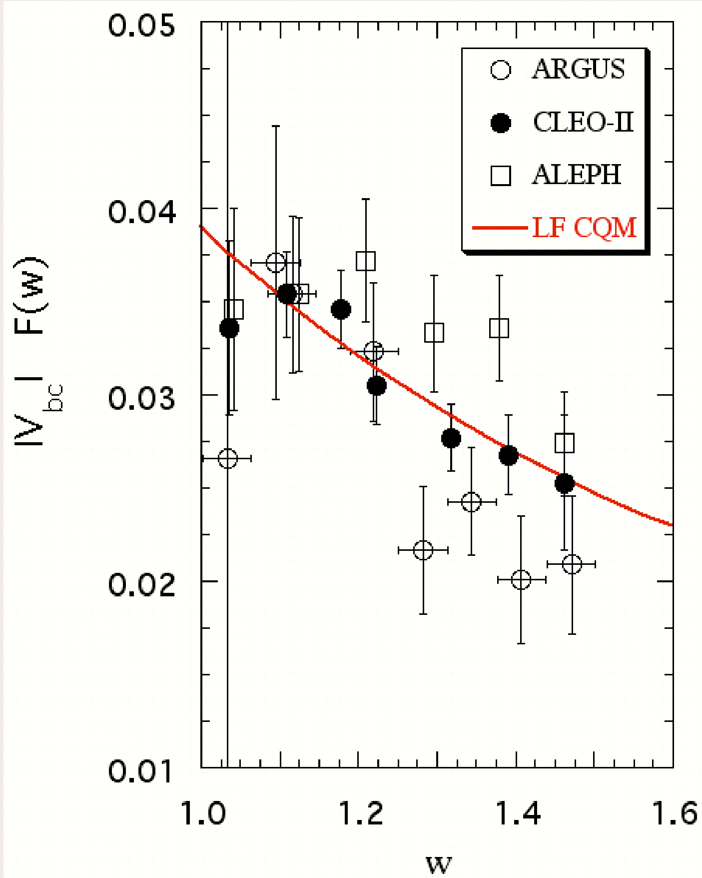
$$\lim_{m_1 \to \infty} J_1(w) = \lim_{m_1 \to \infty} H_1(w) \equiv \xi^{(IW)}(w) = \text{Isgur–Wise function}$$

heavy-quark symmetry of QCD [Isgur&Wise ('89)] $\quad \xi^{(IW)}(w=1) = 1$ in QCD



[Simula ('96)]

$B \to D^* \ell \nu_\ell$



spin-1 two-fermion systems

\# solution of the angular condition problem at $q^+ = 0$

- first step: introduce the amplitude tensor $T^{\mu,\alpha\beta}$

$$\langle P', s' | I^{\mu} | P, s \rangle = e_{\alpha}^{*}(P', s')\, T^{\mu,\alpha\beta}\, e_{\beta}(P, s)$$

e(P, s) = polarization four-vectors:
$$\begin{cases} e(P, s=0) = \dfrac{1}{M_V}\left(\dfrac{-M_V^2 + P_\perp^2}{P^+}, P^+, \vec{P}_\perp\right) \\ e(P, s=\pm 1) = \left(\dfrac{2}{P^+}\vec{\varepsilon}_\perp(\pm 1)\cdot\vec{P}_\perp, 0, \vec{\varepsilon}_\perp(\pm 1)\right) \end{cases}$$

$$\vec{\varepsilon}_\perp(\pm 1) = \mp\frac{1}{\sqrt{2}}(1, \pm i)$$

properties:
$$\begin{cases} e(P, s)\bullet P = 0 \\ \displaystyle\sum_{s=0,\pm 1} e_{\alpha}^{*}(P, s)\, e_{\beta}(P, s) = -g_{\alpha\beta} + \frac{P_{\alpha}P_{\beta}}{M_V^2} \end{cases}$$

- second step: take into account the orientation of the null plane (ω):

$$T^{+,\mu\nu} = J^{+,\mu\nu} + B^{+,\mu\nu}(\omega)$$

- ω-independent structures: seven form factors

$$J^{+,\mu\nu} = -(P+P')^{+}\left\{F_1(Q^2)\left[g^{\mu\nu} - \frac{P'^{\mu}P^{\nu}}{M_V^2} - \frac{P'^{\nu}P^{\mu}}{M_V^2} + \frac{P'^{\mu}P^{\nu}}{M_V^2}\,\frac{P\bullet P'}{M_V^2}\right] + \frac{F_2(Q^2)}{2M_V^2}\left[q^{\mu} - \frac{P'\bullet q}{M_V^2}P'^{\mu}\right]\left[q^{\nu} - \frac{P\bullet q}{M_V^2}P^{\nu}\right]\right\}$$

$$+F_3(Q^2)\left\{\left[g^{+\mu} - \frac{P'^{+}P'^{\mu}}{M_V^2}\right]\left[q^{\nu} - \frac{P\bullet q}{M_V^2}P^{\nu}\right] - \left[g^{+\nu} - \frac{P^{+}P^{\nu}}{M_V^2}\right]\left[q^{\mu} - \frac{P'\bullet q}{M_V^2}P'^{\mu}\right]\right\}$$

$$-(P+P')^{+}\left\{H_1(Q^2)\frac{P^{\mu}P'^{\nu}}{M_V^2} + \frac{H_2(Q^2)}{2M_V^2}\left(q^{\mu}P'^{\nu} - q^{\nu}P^{\mu}\right)\right\} + H_3(Q^2)\left[g^{+\mu}P'^{\nu} + g^{+\nu}P^{\mu}\right]$$

$$+H_4(Q^2)\,q^{+}\,\frac{q^{\mu}P'^{\nu} + q^{\nu}P^{\mu}}{M_V^2}$$

$$e^{*}_{\mu}(P',s')\,J^{+,\mu\nu}\,e_{\nu}(P,s) \propto -(P+P')^{+}\left[F_1(Q^2)\,e^{*}(P',s')\bullet e(P,s) + F_2(Q^2)\,e^{*}(P',s')\bullet q\;e(P,s)\bullet q/2M_V^2\right]$$
$$+F_3(Q^2)\left[e^{*+}(P',s')\,e(P,s)\bullet q - e^{+}(P,s)\,e^{*}(P',s')\bullet q\right]$$

- the form factors H_j (j=1, .., 4) describe the loss of transversity with respect to external momenta and do not contribute to the matrix element of the current

- ω-dependent structures: eight spurious form factors

$$B^{\mu\alpha\beta}(\omega) = \frac{1}{(\omega\cdot P)}\omega^{\mu}\left[B_1(Q^2)g^{\alpha\beta} + B_2(Q^2)\frac{q^{\alpha}q^{\beta}}{M^2} + B_3(Q^2)\frac{\omega^{\alpha}\omega^{\beta}}{(\omega\cdot P)^2}\right.$$

$$\left. + B_4(Q^2)\frac{q^{\alpha}\omega^{\beta} - q^{\beta}\omega^{\alpha}}{(\omega\cdot P)}\right] + (P+P')^{\mu}\left[B_5(Q^2)\frac{\omega^{\alpha}\omega^{\beta}}{(\omega\cdot P)^2}\right.$$

$$\left. + B_6(Q^2)\frac{q^{\alpha}\omega^{\beta} - q^{\beta}\omega^{\alpha}}{(\omega\cdot P)}\right] + B_7(Q^2)\frac{g^{\mu\alpha}\omega^{\beta} + g^{\mu\beta}\omega^{\alpha}}{(\omega\cdot P)} + B_8(Q^2)q^{\mu}\frac{q^{\alpha}\omega^{\beta} + q^{\beta}\omega^{\alpha}}{(\omega\cdot P)}$$

- (B_5+ B_7) describes the violation of the angular condition:

$$\Delta(Q^2) = (1+2\eta)I^{+}_{11}(Q^2) + I^{+}_{1-1}(Q^2) - \sqrt{8\eta}\,I^{+}_{10}(Q^2) - I^{+}_{00}(Q^2) = -B_5(Q^2) - B_7(Q^2)$$

- B_8 describes the possible loss of gauge invariance of the given approximation of the e.m. current operator

- Third step: choose the components with all the indexes different from "-"

$$B^{\mu,\alpha\beta}=0 \quad \text{if} \quad \mu,\alpha,\beta \neq - \quad \Rightarrow \quad \text{no spurious terms}$$

explicit solution in the standard Breit frame where [Melikhov&Simula ('02)]

$$q=\left(q^-,q^+,q_x,q_y\right)=\left(0,0,Q,0\right) \quad \text{and} \quad \vec{P}_\perp+\vec{P}'_\perp=0$$

$$\begin{cases} F_1\left(Q^2\right)=\dfrac{J^{+,yy}}{2P^+} \\ F_2\left(Q^2\right)=\dfrac{1}{2\eta}\,\dfrac{J^{+,yy}-J^{+,xx}}{2P^+}+\dfrac{1}{2(1+\eta)}\,\dfrac{J^{+,++}}{2P^+}-\dfrac{1}{\sqrt{\eta(1+\eta)}}\,\dfrac{J^{+,x+}}{2P^+} \\ F_3\left(Q^2\right)=\dfrac{J^{y,xy}}{Q}-\sqrt{\dfrac{1+\eta}{\eta}}\,\dfrac{J^{y,+y}}{Q} \end{cases}$$

unique solution, no ambiguity in the extraction of form factors

use of μ = + and α = y [transverse to $\vec{q}_\perp$ in the transverse plane (x,y)]

note that $e^*_\alpha(P',s')\,T^{\mu,\alpha\beta}\,e_\beta(P,s)$ may involve indexes α and β equal to "-"

spurious terms present in the matrix elements of the e.m. current covariant LF

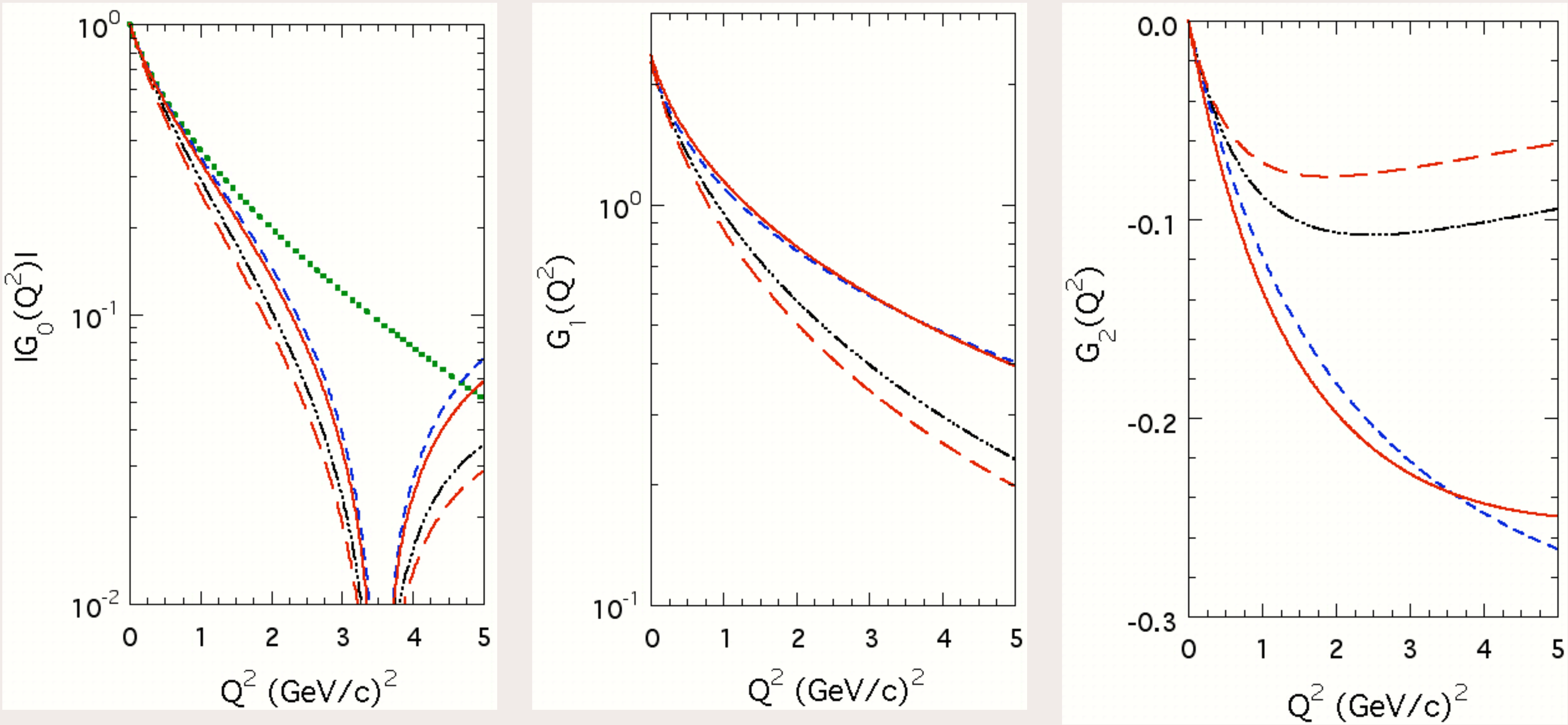
BH

FFS

CCKP

GK

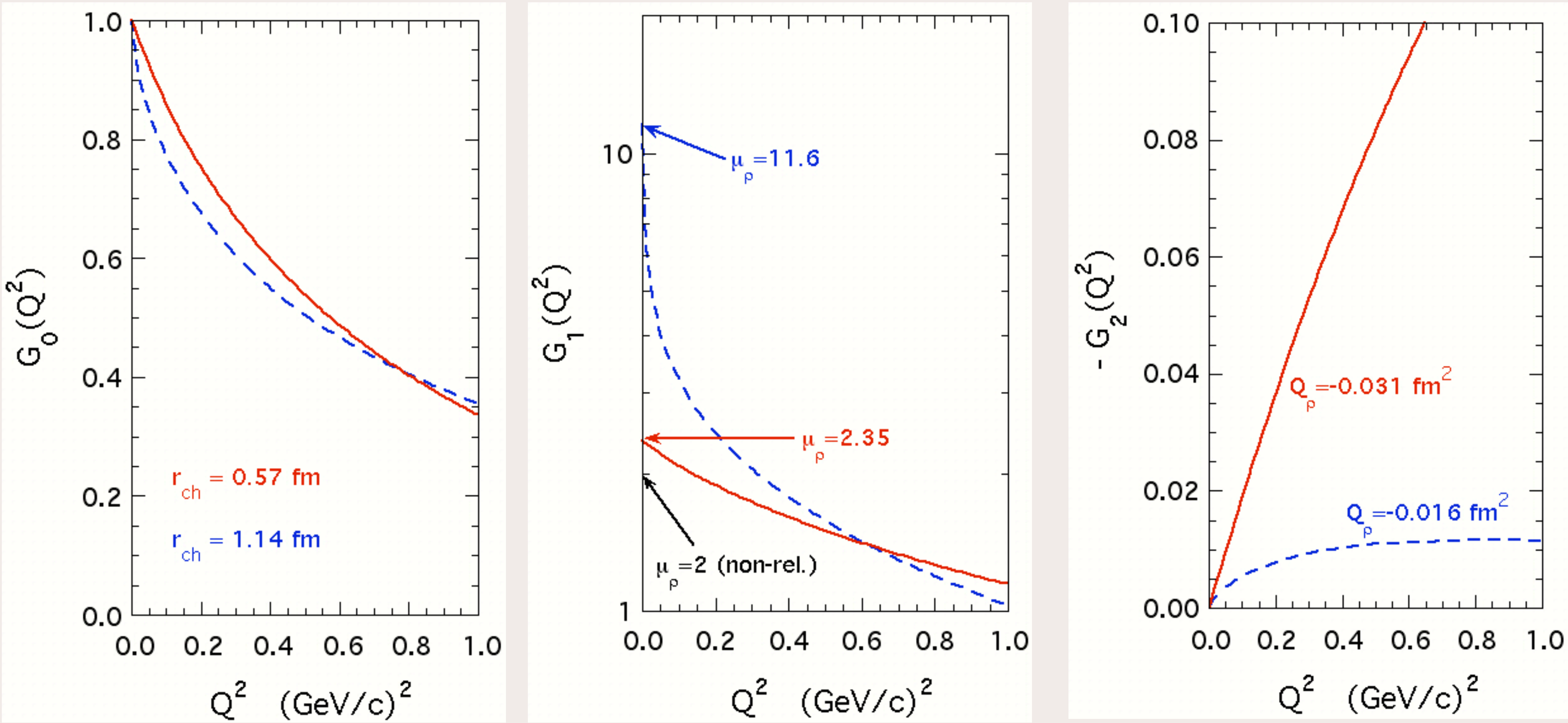
□-meson w.f. from OGE model



Note: GK prescription close to the covariant result

again comparison between $q^+ = 0$ and $q^+ \neq 0$:

w.f. from OGE model



Z-graph relevant for all ρ-meson form factors [$M_\rho = 0.77$ GeV]

N - N* transition: LF at $q^+ = 0$ [Cardarelli&Simula ('00), Simula ('01)]

$$\langle p^* \nu' | I^\mu | p \nu \rangle = I^\mu(\nu'\nu) = \bar{u}(p^* \nu') \left[J^\mu + \frac{B^\mu(\omega)}{\omega \bullet P} \right] u(p \nu) \qquad \left(J^P = \frac{1}{2}^+, T = \frac{1}{2} \right)$$

- ω-independent structures: two form factors

$$J^\mu = F_1(Q^2) \left[\gamma^\mu + q^\mu \frac{M^* - M}{Q^2} \right] + F_2(Q^2) \frac{i\sigma^{\mu\nu} q_\nu}{M^* + M}$$

- ω-dependent structures: five (spurious) form factors

$$B^\mu(\omega) = B_1(Q^2) \left[\not{\omega} - \frac{2\omega \cdot P}{(M^* + M)(1+\eta)} \right] \left[P^\mu + q^\mu \frac{M^{*2} - M^2}{2Q^2} \right]$$

$$+ B_2(Q^2) \omega^\mu + B_3(Q^2) \frac{\not{\omega}}{\omega \cdot P} \omega^\mu + (M^* - M) B_4(Q^2) \frac{\not{\omega}}{\omega \cdot P} q^\mu$$

$$+ (M^* - M) B_5(Q^2) i\sigma^{\mu\nu} \omega_\nu$$

Note: $q_\mu J^\mu = 0$ and $q_\mu B^\mu \propto M^* - M$

standard Breit frame: $q = \left(q^{-}, q^{+}, q_x, q_y\right) = (0, 0, Q, 0)$ and $\vec{P}_{\perp} + \vec{P}'_{\perp} = -\vec{q}_{\perp} \dfrac{M^2 - M^{*2}}{Q^2}$

$$I^{+}(\lambda'\lambda) = 2P^{+}\left\{\left[F_1\left(Q^2\right) + \frac{\tau B_1\left(Q^2\right)}{1+\tau}\right]\delta_{\lambda'\lambda} - \left[F_2\left(Q^2\right) + \frac{B_1\left(Q^2\right)}{1+\tau}\right]\frac{iQ\left(\sigma_y\right)_{\lambda'\lambda}}{M^* + M}\right\}$$

$$I^{y}(\lambda'\lambda) = \left[F_1\left(Q^2\right) + F_2\left(Q^2\right)\right] iQ\left(\sigma_z\right)_{\lambda'\lambda} + \left[F_1\left(Q^2\right) + F_2\left(Q^2\right) + B_5\left(Q^2\right)\right] i\left(M^* - M\right)\left(\sigma_x\right)_{\lambda'\lambda}$$

Sachs form factors

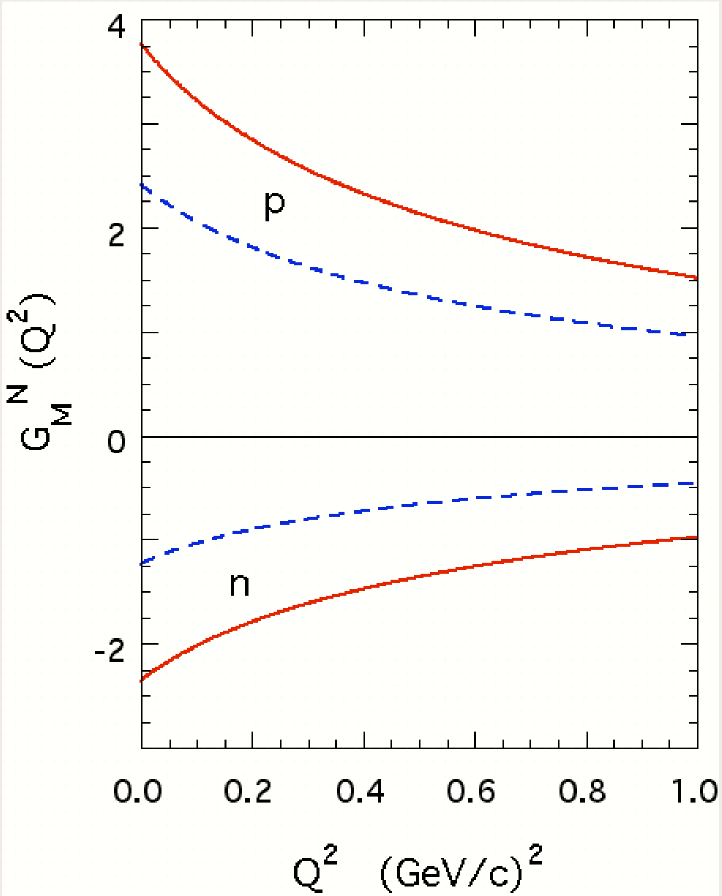
$$G_E\left(Q^2\right) = F_1\left(Q^2\right) - \tau F_2\left(Q^2\right) = \frac{1}{2} Tr\left[\frac{I^{+}}{2P^{+}}\right] - \frac{iQ}{2\left(M^* + M\right)} Tr\left[\frac{I^{+}}{2P^{+}}\sigma_y\right]$$

$$G_M\left(Q^2\right) = F_1\left(Q^2\right) + F_2\left(Q^2\right) = -\frac{i}{2Q} Tr\left\{I^{y}\sigma_z\right\}$$

use of μ = + for the charge form factor, and of μ = y for the magnetic form factor

effects of the loss of rotational covariance can be manifest also in systems with J < 1 using different components of the e.m. current

nucleon magnetic form factors:



point-like CQ's: $I^+(0) = \sum_j e_j \gamma^+$

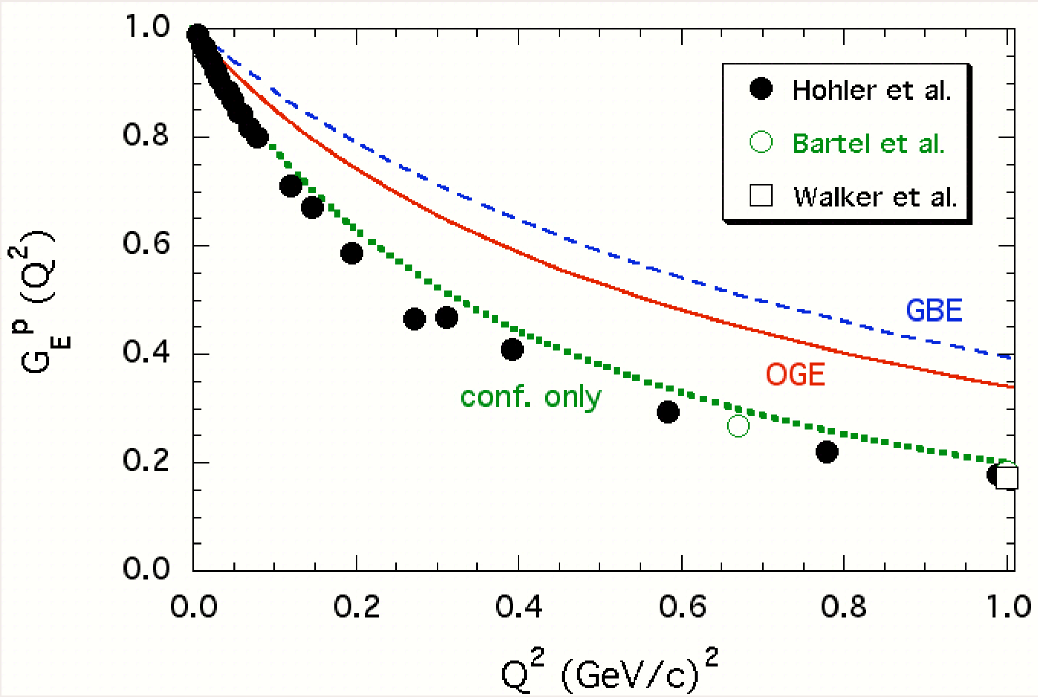
nucleon w.f. from OGE model

from I^+

$$G_M^N(Q^2) = \frac{1}{2}\,\mathrm{Tr}\left[I^+ \left(1 + \frac{2M}{Q}\, i\sigma_y \right) \right]$$

from I^y

$$G_M^N(Q^2) = -\frac{P^+}{Q}\,\mathrm{Tr}\left[I^y \; i\sigma_z \right]$$



point-like CQ's:

$$I^+(0) = \sum_j e_j \gamma^+$$

proton charge radius underestimated

↓

CQ size ?

CQ form factors: isovector (IV=U-D) and isoscalar (IS=U+D) f.f.'s

$$f_1^{IV(IS)}\left(Q^2\right) = \frac{A_1^{IV(IS)}}{1+B_1^{IV(IS)}Q^2} + \frac{1-A_1^{IV(IS)}}{\left(1+C_1^{IV(IS)}Q^2\right)^2}$$

$$f_2^{IV(IS)}\left(Q^2\right) = \kappa^{IV(IS)}\left[\frac{A_2^{IV(IS)}}{\left(1+B_2^{IV(IS)}Q^2\right)^2} + \frac{1-A_2^{IV(IS)}}{\left(1+C_2^{IV(IS)}Q^2\right)^3}\right]$$

k^{IV} and k^{IS} fixed by μ_p and μ_n

↓

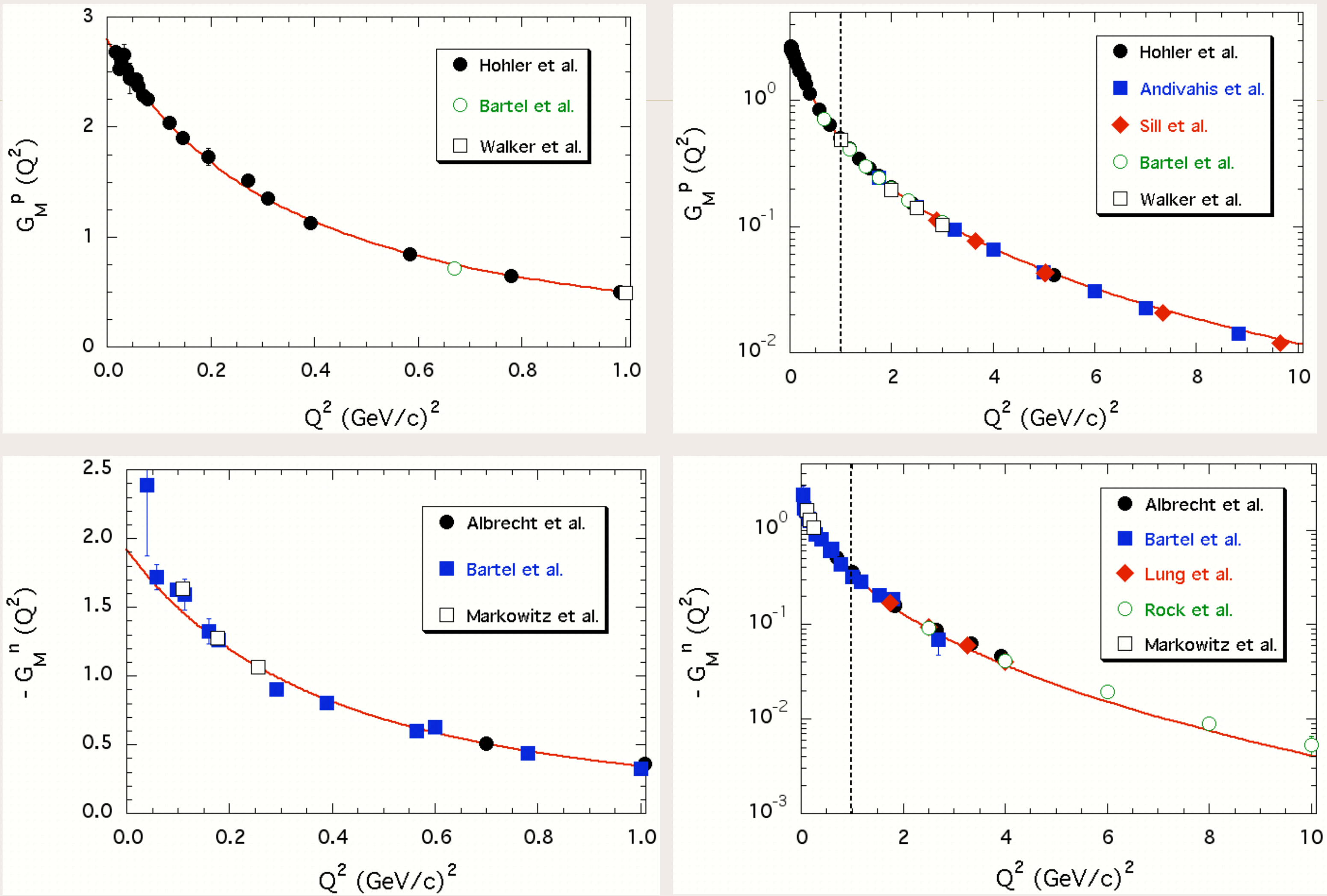
8 parms

CQ f.f. parms fixed by data below $Q^2 = 1\ (\mathrm{GeV}/c)^2 \sim$ (scale of □SB)^2

low Q^2 [< 1 $(\mathrm{GeV}/c)^2$] high Q^2

G_M^p

$-G_M^n$

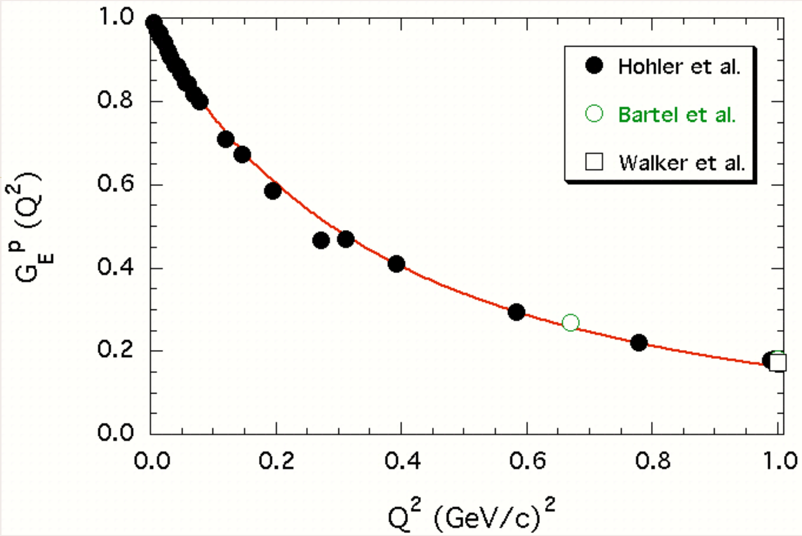


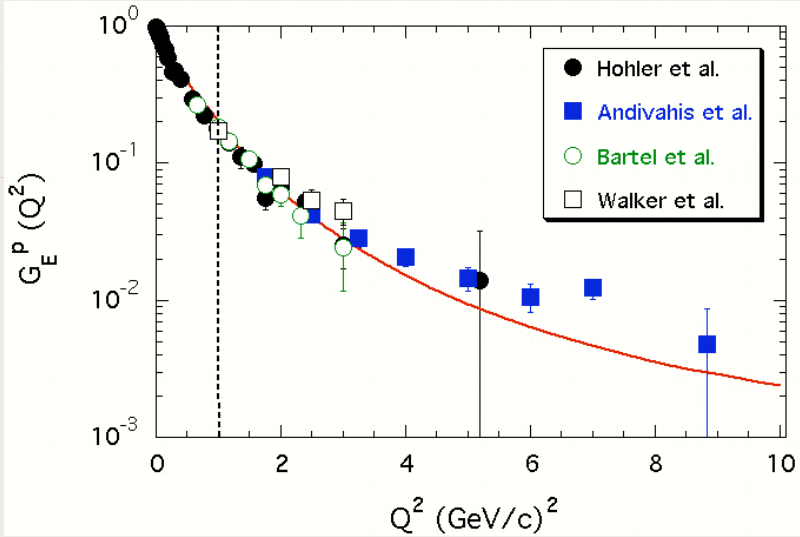
soft physics important at least up to $Q^2 \sim 10\ (\mathrm{GeV}/c)^2$

low Q^2 [< 1 $(GeV/c)^2$]

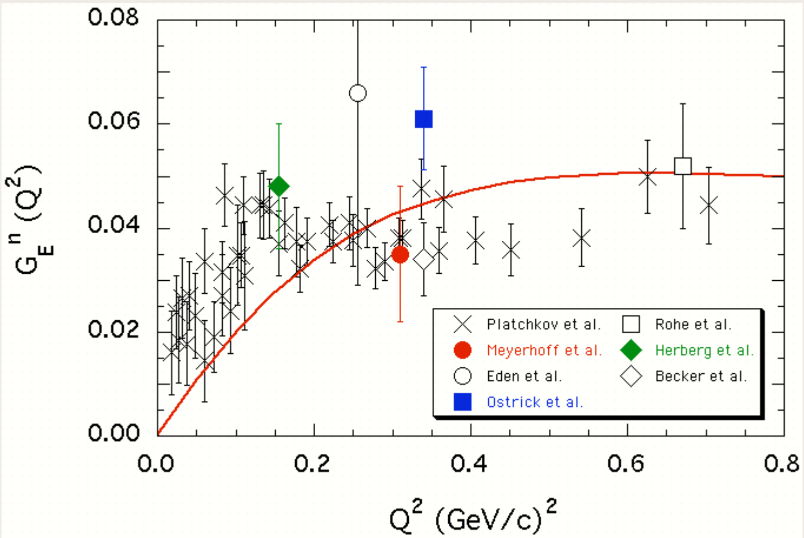
high Q^2

G_E^p





G_E^n



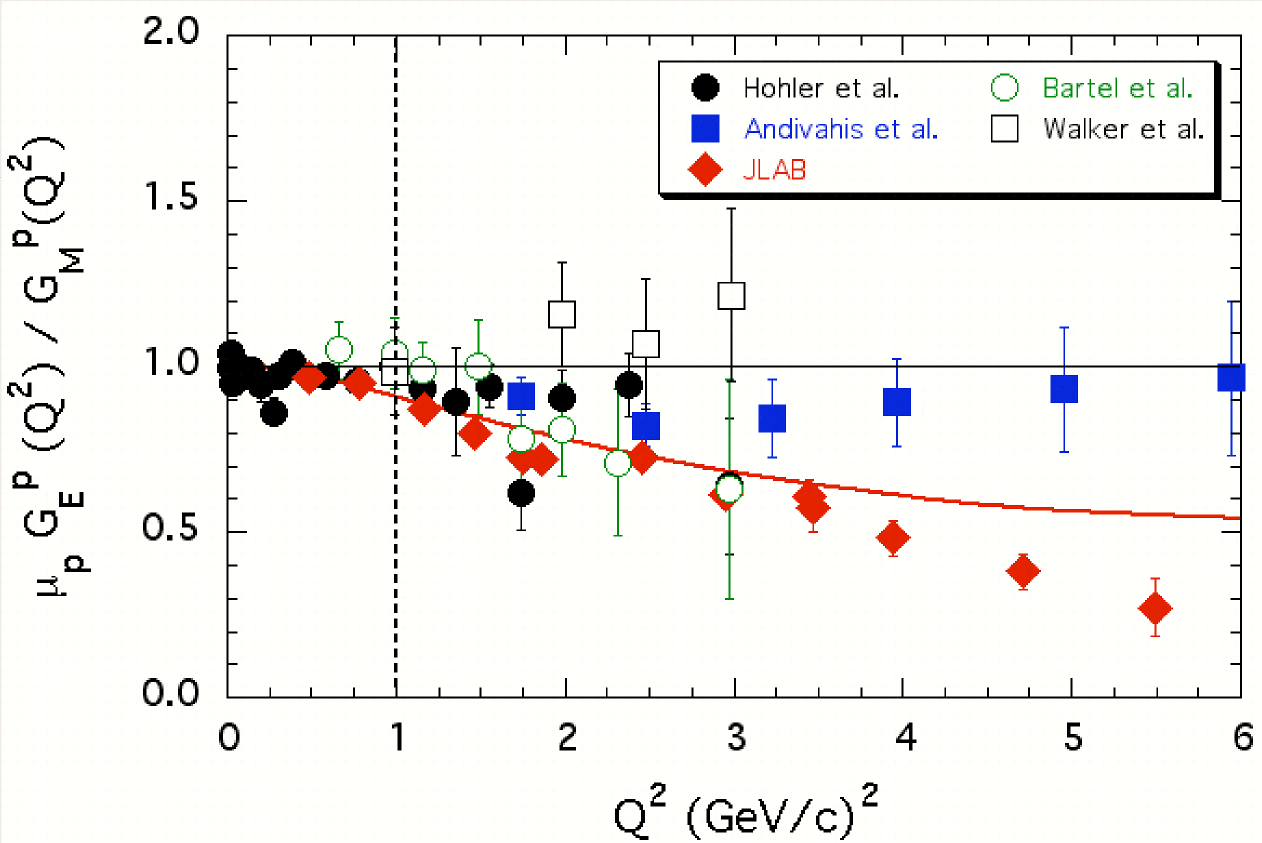
neutron charge radius
underestimated by 50%

many-body currents ?

CQ size: $r_{ch}^U = 0.43$ fm

$r_{ch}^D = 0.45$ fm

ratio of electric to magnetic proton f.f.'s [see update in Arrington ('03)]



— LF CQM

recoil polarization $p(\vec{e}, e' \vec{p})$

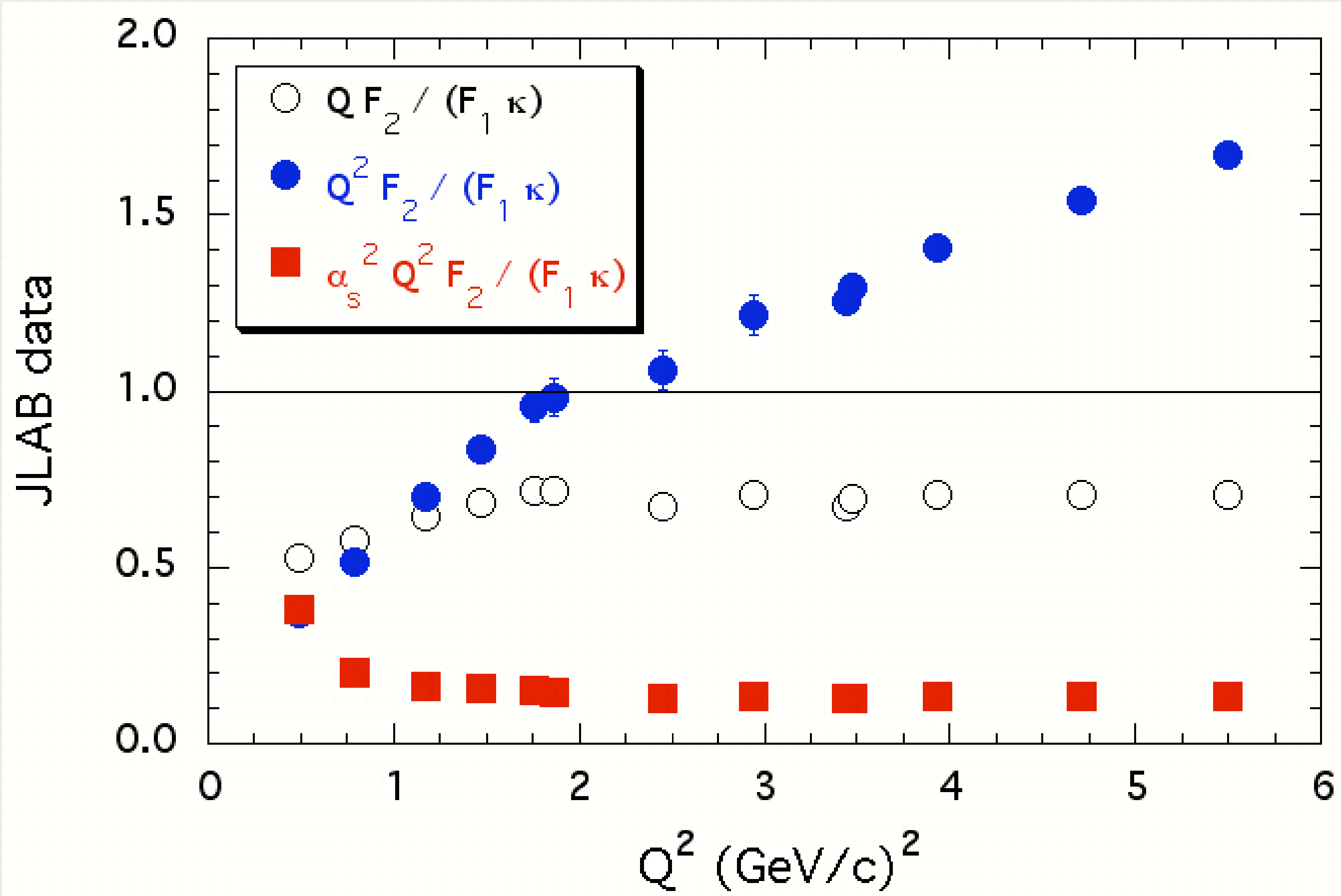
$$\frac{G_E}{G_M} = -\frac{P_t}{P_l}\,\frac{(E + E')\tan(\theta/2)}{2M}$$

$P_{l(t)}$ = long. (trans.) pol.

Rosenbluth separation $p(e, e' p)$

$$\frac{d\sigma}{d\Omega}\,\frac{\varepsilon\left(1+Q^2/4M^2\right)}{\sigma_{Mott}} = \frac{Q^2}{4M^2}G_M^2 + \varepsilon G_E^2$$

ε = long. photon pol.



JLAB data versus pQCD

pQCD (quark counting rules) : $Q^2 F_2(Q^2)/F_1(Q^2) \to$ const. no ! [Brodsky et al. ('73)]

pQCD (with Sudakov resum.) : $\alpha_s^2(Q^2)\, Q^2 F_2(Q^2)/F_1(Q^2) \to$ const. O.K. [Ji et al. ('03)]

relativistic CQ model : $Q F_2(Q^2)/F_1(Q^2) \sim$ const. O.K. [Miller ('02)]

nature of the Roper resonance [P_{11}(1440)]

- q^3 assignment: radial excitation of the nucleon

$\Downarrow$

same spin-flavor structure of the nucleon

- $q^3 G$ admixture [Barnes, Burkert, Carlson, Close, Li, Mukhopadhyay, …]

$q^3 G$ is orthogonal to the nucleon in the spin-flavor space

different Q^2-behavior of the form factors for q^3 or $q^3 G$ configurations

<u>helicity amplitudes</u>

$A_{1/2}(Q^2) \quad G_M(Q^2)$

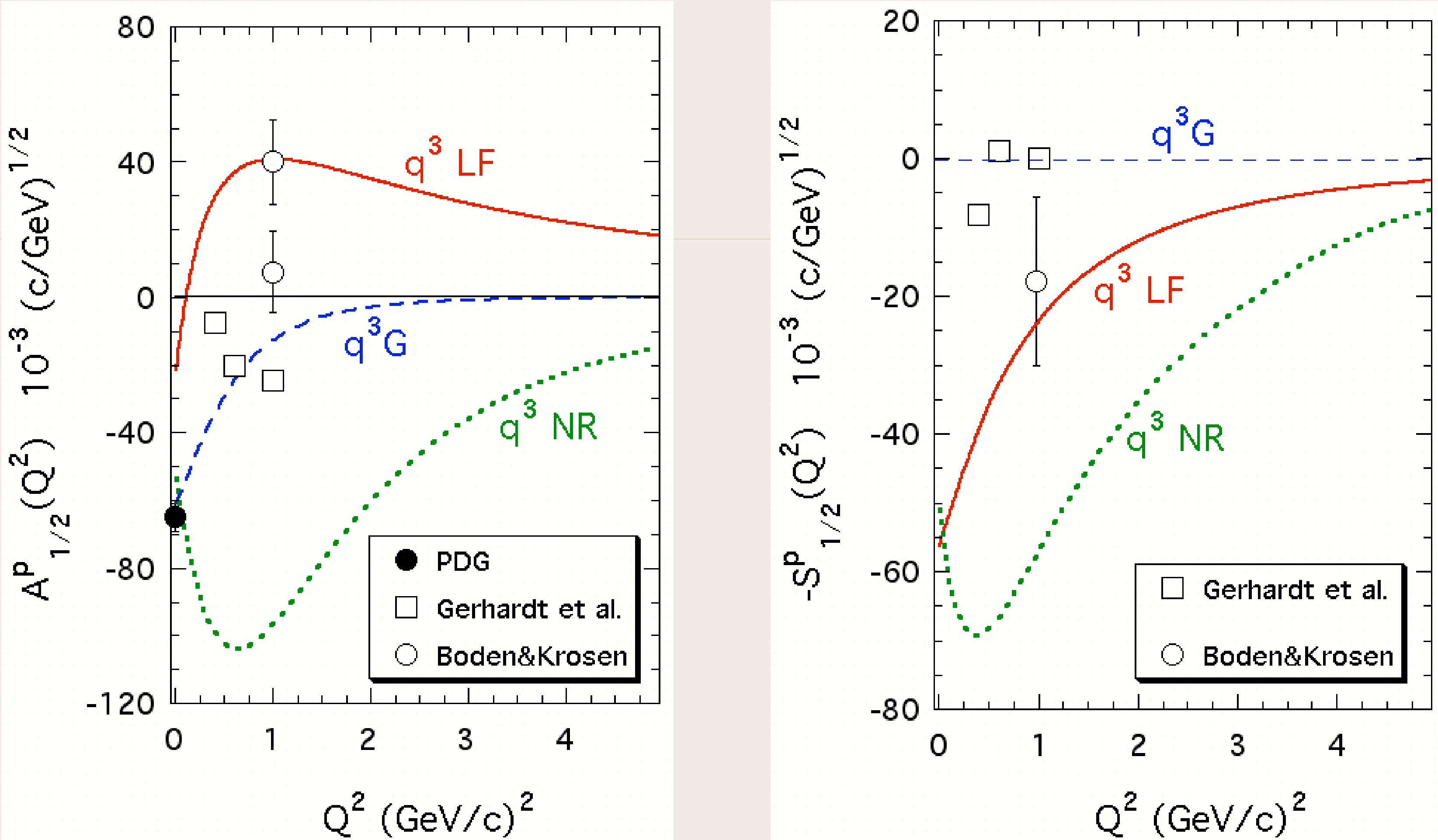
$S_{1/2}(Q^2) \quad G_E(Q^2)/Q^2$

<u>some JLAB experiments</u>:

91-002 (Burkert, Stoler, Taiuti)

93-036 (Chasteler, Minehart, Weller)

…



- relativistic effects reduce the helicity amplitudes and may change their sign

- the fastest fall-off is exhibited by the hybrid q^3G model

- relativistic quark model underestimates $A_{1/2}$ at the photon point ($Q^2 = 0$)

helicity amplitudes of many transition to nucleon resonances are being investigated systematically at JLAB

some JLAB experiments:

89-037 (Burkert, Minehart)	PC
89-039 (Dytman, Giovannetti)	PC
89-042 (Burkert, Minehart)	PC
91-002 (Burkert, Stoler, Taiuti)	PC
91-011 (Frullani, Kelly, Sarty)	C
93-006 (Burkert, Ripani)	PC
93-036 (Chasteler, Minehart, Weller)	C
94-014 (Stoler, Napolitano)	C
99-107 (Burkert, Minehart, Stoler, Taiuti)	C
01-002 (Frolov, Koubarowski, Stoler)	A

…

SUMMARY

the light-front formalisms at $q^+ = 0$ is presently the most suitable approach for developing a **relativistic CQ model**

- it allows the suppression of the pair-creation process and it matches the Feynman triangle diagram for a subset of amplitudes from which the form factors can be extracted

- the angular condition is solved and a one-body approximation for the e.m. current can be formulated free of spurious effects due to the orientation of the null plane

relativistic effects are important for the calculation of hadron e.m. form factors, particularly up to $Q^2 \sim 1$ (GeV/c)$^2 \sim$ (scale of □SB)^2

nucleon and pion data for $Q^2 < 1$ (GeV/c)^2 can be reproduced using the OGE w.f.'s by introducing a **CQ size** of 0.40 - 0.45 fm