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Relativistic (Constituent) Quark Models

Silvano Simula

Istituto Nazionale di Fisica Nucleare (INFN)
Sezione Roma III

A Short Introduction

two of the main sources of information on hadron structure

<u>Mass Spectroscopy</u>

hadrons are bound states of
two and three quarks

[Gell-Mann, Zweig, Morpurgo, …]

Constituent Quarks

<u>Deep Inelastic Scattering</u>

probe the short-distance structure

parton model

[Bjorken, Feynmann, ..]

Current Quarks and Gluons

Quantum ChromoDynamics (QCD)

Mass Spectroscopy	Deep Inelastic Scattering
non-perturbative QCD regime	perturbative QCD regime
CQs as quasi-particles: dressing of valence quarks with gluons and quark-antiquark pairs	current quarks and gluons are the fundamental degrees of freedom
⇓	⇓
constituents with structure	point-like constituents

Outline of the lectures

1. Hadron masses and constituent quarks
2. Relativistic Hamiltonian formalisms
3. Hadron form factors in the light-front CQ model
4. Deep Inelastic Scattering: current vs. constituent quarks
5. Structure functions at low momentum transfer

Lecture 1

Hadron Mass Spectroscopy

and

Constituent Quarks

present status of CQ potential models: highlights and shadows

relativistic effects and the baryon spin-orbit puzzle

<u>Eightfold way</u>: hadrons are bound states of two and three *"quarks"* with approximate $SU_f(3)$ symmetry [Gell-Mann, Zweig, Morpurgo, …]

$\Omega^{-}(sss)$ $\Rightarrow$ a new quantum number: the color

after the advent of QCD *"quarks"* are Constituent Quarks (CQ's), viewed as effective degrees of freedom:

- # gluon and $q\bar{q}$ pairs degrees of freedom are frozen
- # their effects are hidden in the CQ mass (and size) and in the effective interaction among CQ's

CQ's have the same spin, color and flavor of the QCD quarks, but their masses are phenomenological parms to be fitted to hadron mass spectra

typical values:

$$m_U \simeq m_D \simeq 0.20 \div 0.35\,GeV,\; m_S \simeq 0.4 \div 0.5\,GeV$$
$$m_C \simeq 1.5\,GeV,\; m_B \simeq 5\,GeV$$

rest-frame baryon wave functions: $|\Psi_B\rangle \equiv |\Psi_{3q}(\vec{p}_1, \vec{p}_2, \vec{p}_3)\rangle$

$[\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0]$

rest-frame Schrödinger-type equation:

$$\hat{M}\,|\Psi_{3q}\rangle = \left(\hat{M}_0 + \hat{V}\right)|\Psi_{3q}\rangle = M_B\,|\Psi_{3q}\rangle \qquad (*)$$

$$\hat{M}_0 = \sum_{i=1}^{3} \sqrt{m_i^2 + |\vec{p}_i|^2}$$ = free-mass operator

$\hat{V}$ = interaction term

- Poincaré covariance is not manifest, but in principle Eq. (*) is relativistic

the first success of the CQ model has been the reproduction of the mass spectra of heavy quarkonia

(non-relativistic $Q\bar{Q}$ systems)

Cornell-type (or funnell) potential:

$$\hat{V} = V_0 + \sum_{i<j} \left(-\frac{3}{4} b\, r_{ij} + \frac{\alpha_s(r_{ij})}{r_{ij}} \right) F_i \cdot F_j$$

$= -4/3$ *mesons*

$= -2/3$ *baryons*

$r_{ij} \equiv |\vec{r}_i - \vec{r}_j|$

α_s = *running strong coupling*

$$F_i = \begin{cases} \lambda_i/2 & \textit{for quarks} \\ -\lambda_i^*/2 & \textit{for antiquarks} \end{cases} \rightarrow \; SU(3)\ \textit{color matrices}$$

one-gluon exchange (OGE) at short distances [De Rujula, Georgi, Isgur, Karl…]

+

linear confinement at large separations (as suggested by lattice QCD and Regge behavior of excited meson masses)

details of the mass spectra requires spin-dependent components

T-matrix element for $q\bar{q}$ scattering: [see Lucha, Schoberl, Gromes: PR 200 (1991)]

$$T_{fi} = \frac{1}{(2\pi)^6} \frac{1}{\sqrt{E_{p_1} E_{p_2} E_{q_1} E_{q_2}}} \bar{u}(q_1)\Gamma u(p_1) K(k^2) \bar{v}(p_2)\Gamma v(q_2)$$

$$k = p_1 - q_1 = p_2 - q_2$$

$\Gamma \otimes \Gamma$ *structure*				
scalar	1	1		V_S
pseudoscalar	γ_5	γ_5		0
vector	γ_μ	γ^μ	$\xrightarrow{\textit{non relativistic reduction}}$	V_V
axial-vector	$\gamma_\mu\gamma_5$	$\gamma^\mu\gamma_5$		$V_A\left(\vec{S}_1 \bullet \vec{S}_2\right)$
tensor	$\sigma_{\mu\nu}$	$\sigma^{\mu\nu}$		$V_T\left(\vec{S}_1 \bullet \vec{S}_2\right)$

OGE model up to terms of order 1/m^2

exchange of a vector massless boson

$$\hat{V}^{(spin)} = -\sum_{i<j} F_i \bullet F_j \frac{\alpha_s(r_{ij})}{r_{ij}} \left\{ \frac{8\pi}{3}\delta(\vec{r}_{ij})\, \vec{S}_i \bullet \vec{S}_j \right.$$

spin-spin contact term

$$+ \frac{1}{r_{ij}^3}\left[3\vec{S}_i \bullet \vec{r}_{ij}\, \vec{S}_j \bullet \vec{r}_{ij} - \vec{S}_i \bullet \vec{S}_j\right]$$

tensor term

$$\left. + \frac{m_i m_j}{r_{ij}^3}\left[\frac{\vec{r}_{ij} \times \vec{p}_i \bullet \vec{S}_i}{2m_i^2} - \frac{\vec{r}_{ij} \times \vec{p}_j \bullet \vec{S}_j}{2m_j^2} + \frac{\vec{r}_{ij} \times \vec{p}_i \bullet \vec{S}_j}{m_i m_j} - \frac{\vec{r}_{ij} \times \vec{p}_j \bullet \vec{S}_i}{m_i m_j} \right] \right\}$$

2- and 3-body
spin-orbit terms

$\delta(\vec{r}_{ij}) \longrightarrow$ Gaussian smearing (may be related to relativistic effects and/or to the CQ size)

contact term is short-ranged, which agrees with the small splitting between <^3P_J> and $^1P_1$ multiplets in the meson sector

S-wave mesons: $\pi(^1S_0) - \rho(^3S_1) \approx 600\ MeV$

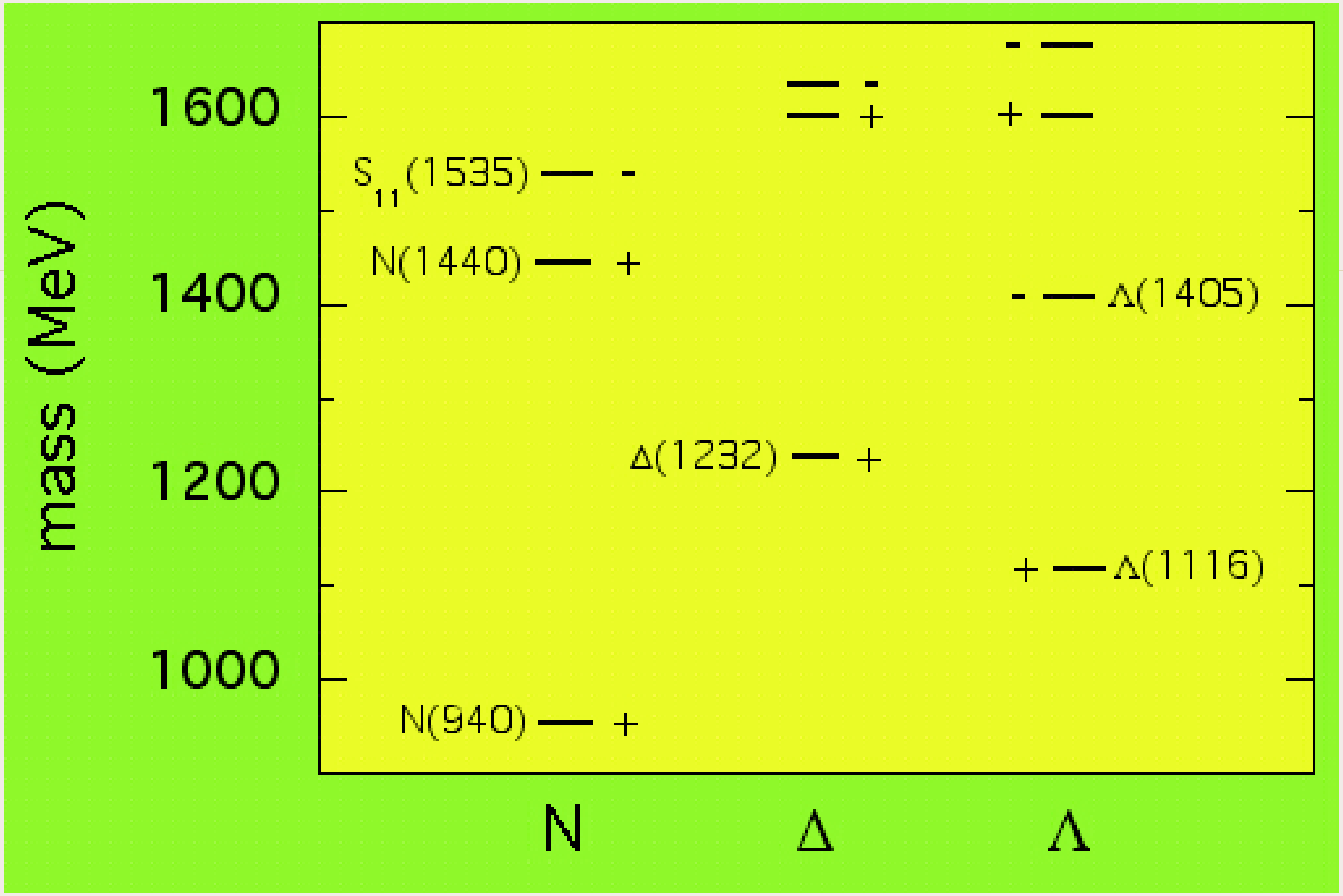
P-wave mesons: $b_1(^1P_1) - a_1(^3P_1) \approx 30\ MeV$

further spin-orbit term: Thomas-Fermi precession of the confining term

$$\hat{V}^{(TF)} = \sum_{i<j} F_i \bullet F_j \frac{3b}{4} \frac{1}{r_{ij}} \left[\frac{\vec{r}_{ij} \times \vec{p}_i \bullet \vec{S}_i}{2m_i^2} - \frac{\vec{r}_{ij} \times \vec{p}_j \bullet \vec{S}_j}{2m_j^2} \right]$$

- TF term gives rise to relevant matrix elements

- OGE spin-orbit term is opposite in sign to the TF term



Low-lying masses of N, □ and □ states from PDG

parity ordering

spin-orbit splitting: $S_{11}(1535)$ - $D_{13}(1520)$

OGE model works ~ well both in heavy- and light-quark sectors as well as in mesons and in baryons

BUT

1) it still relies on the non-relativistic limit + corrections up to $1/m^2$, while $\langle p \rangle > m \sim \Lambda_{QCD}$:

need of a *relativized* model

2) the strength of the spin-orbit force in baryon is significantly lower than the one expected from non-relativistic CQ models:

baryon spin-orbit puzzle:

$$M[S_{11}(1535)] - M[D_{13}(1520)] \sim 15 \text{ MeV}$$

the flaws of the non-relativistic OGE model were corrected in:

Godfrey and Isgur: PR D32 (1985) for $q\,\bar{q}'$ systems

Capstick and Isgur: PR D34 (1986) for q q'q'' systems

phenomenological approach for including relativistic effects:

momentum-dependent suppression of the interaction strength

$$\hat{V}^{(rel.)} = \left(\frac{m_i m_j}{E_i E_j}\right)^{\frac{1}{2}+\epsilon} \hat{V}^{(non-rel.)}$$

ϵ = parameter(s) to be fixed by the reproduction of the mass spectra

merit of the OGE model: *unified description* of meson and baryon mass spectra from light to heavy quarks

- same dynamical mechanism for the hyperfine interaction (exchange of one gluon)
- same values of the phenom. parms (13 in total in the uds sector), with few exceptions only

Ground-state baryon masses in MeV (uds sector)

baryon	$N\left(\frac{1}{2}^+\right)$	$□\left(\frac{3}{2}^+\right)$	$□\left(\frac{1}{2}^+\right)$	$□\left(\frac{1}{2}^+\right)$	$□\left(\frac{3}{2}^+\right)$	$□\left(\frac{1}{2}^+\right)$	$□\left(\frac{3}{2}^+\right)$	$□\left(\frac{3}{2}^+\right)$
OGE	960	1230	1115	1190	1370	1305	1505	1635
exp. (*PDG*)	939	1232	1116	1193	1384	1318	1532	1672

Main problems with OGE model

1) incorrect parity ordering:

	OGE	exp.
Roper $\left(\frac{1}{2}^{+}\right)$	1540	1440^{+30}_{-10}
S_{11} $\left(\frac{1}{2}^{-}\right)$	1460	1535^{+20}_{-15}

2) the "Λ(1405)" problem:

OGE	exp.
1550	1405^{+5}_{-5}

NK molecule ?

However, one should add also the problem of *spin-orbit splittings in baryons*

the baryon spin-orbit puzzle is (only partially) explained in the (relativized) OGE model via two mechanisms:

- partial compensation among the TF precession term [~ - 1/ r] and the OGE spin-orbit component [~ 1 / r^3]
- suppression of the interaction strength at high momenta

splitting	$D_{13} - N$	$D^*_{13} - N$	$S_{11} - N$	$S^*_{11} - N$	$S_{13} - N$
OGE	628	763	564	652	679
exp.	582^{+10}_{-5}	761^{+50}_{-50}	597^{+20}_{-15}	711^{+30}_{-10}	681^{+55}_{-5}

baryons : $\square^{(s.o.)}_{TF} = 0.30,$ $\square^{(s.o.)}_{OGE} = 0.0$

mesons : $\square^{(s.o.)}_{TF} = -0.055,$ $\square^{(s.o.)}_{OGE} = -0.035$

ad-hoc extra suppression in baryons !

alternative models for the CQ interaction

PS-meson exchange model [Robson, Glozmann, Riska, …]

instanton-inspired models [Diakonov, Petrov, Metsch…]

collective models [Iachello, …]

hypercentral quark model [Giannini, …]

…

PS-meson exchange model

- massless QCD: $SU(f)_L \quad SU(f)_R$ symmetry
- no multiplets in the mass spectra associated to chiral symmetry

$$\Downarrow$$

chiral symmetry is realized in the hidden Nambu-Goldstone mode

$$\Downarrow$$

existence of an octet of PS mesons representing the Goldstone bosons arising from the spontaneous breaking of the chiral symmetry

$$\hat{V}^{(GBE)} = \hat{V}^{(\mathrm{conf.})} + \hat{V}^{(\mathrm{octet})} + \hat{V}^{(\mathrm{singlet})}$$

structure of the octet and singlet terms:

original U(3) nonet

$$\hat{V}^{(\mathrm{octet})} = \sum_{i<j} \left[\sum_{a=1}^{3} \hat{V}_{\pi} \lambda_i^a \bullet \lambda_j^a + \sum_{a=4}^{7} \hat{V}_{K} \lambda_i^a \bullet \lambda_j^a + \hat{V}_{\eta} \lambda_i^a \bullet \lambda_j^a \right]$$

$$\hat{V}^{(\mathrm{singlet})} = \sum_{i<j} \frac{2}{3} \hat{V}_{\eta'}$$

λ^a = SU(3) flavor matrices (a=1, …, 8)

CQ-meson potential:

$$\hat{V}_M = \vec{S}_i \bullet \vec{S}_j \frac{g_{QQM}^2}{4\pi} \frac{1}{3 m_i m_j} \left\{ \mu_M^2 \frac{e^{-\mu_M r_{ij}}}{r_{ij}} - \Lambda_M^2 \frac{e^{-\Lambda_M r_{ij}}}{r_{ij}} \right\}$$

$$\hat{V}_{\pi} \neq \hat{V}_K \neq \hat{V}_{\eta} \neq \hat{V}_{\eta'}$$

mass terms

$U_A(1)$ anomaly

smearing function

$\Lambda_M = \Lambda_{\pi} \approx 1\,GeV$

$$\hat{V}^{(GBE)} = \hat{V}^{(\text{conf.})} + \hat{V}^{(\text{octet})} + \hat{V}^{(\text{singlet})}$$

- color dependent
- flavor independent

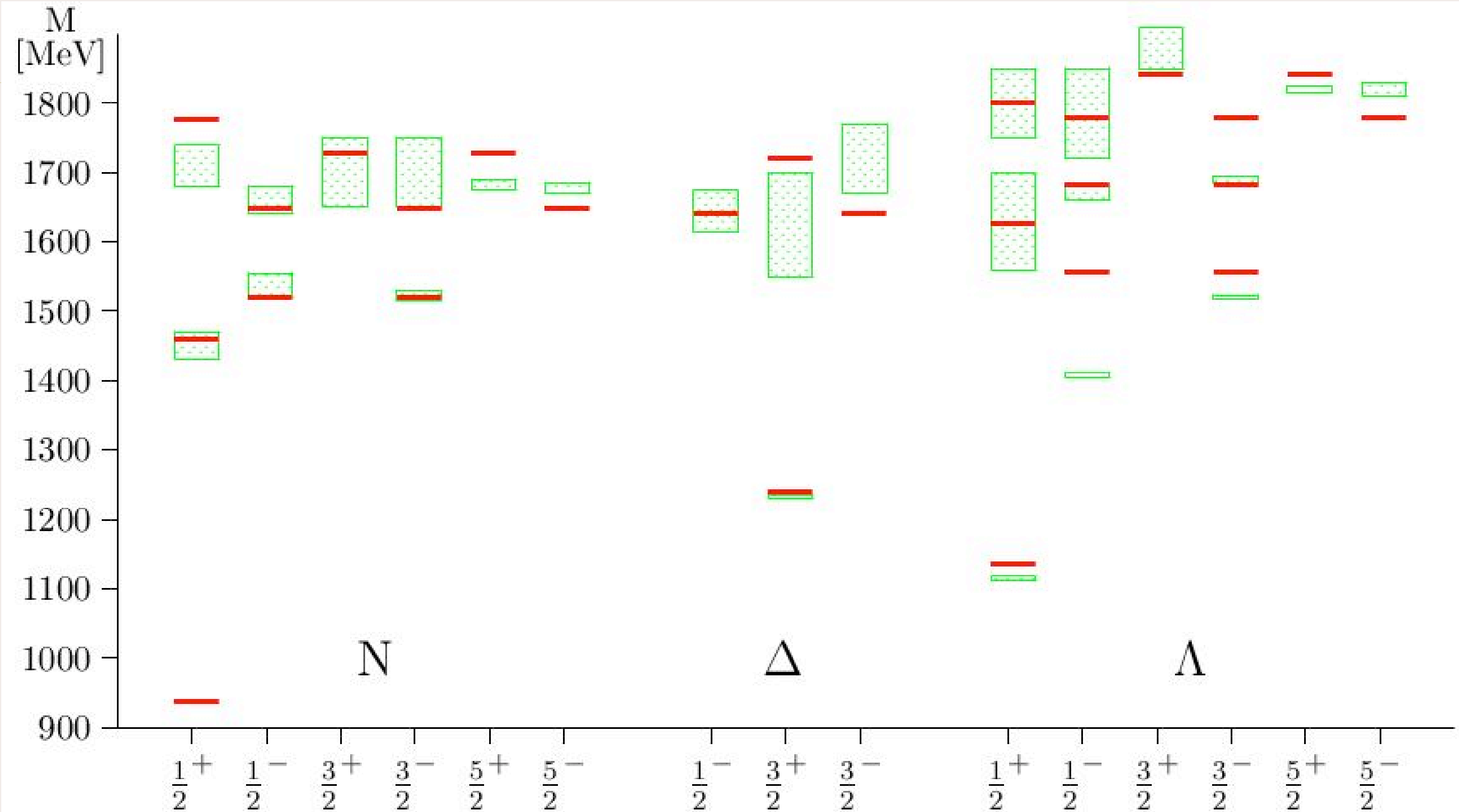
- color independent
- flavor dependent

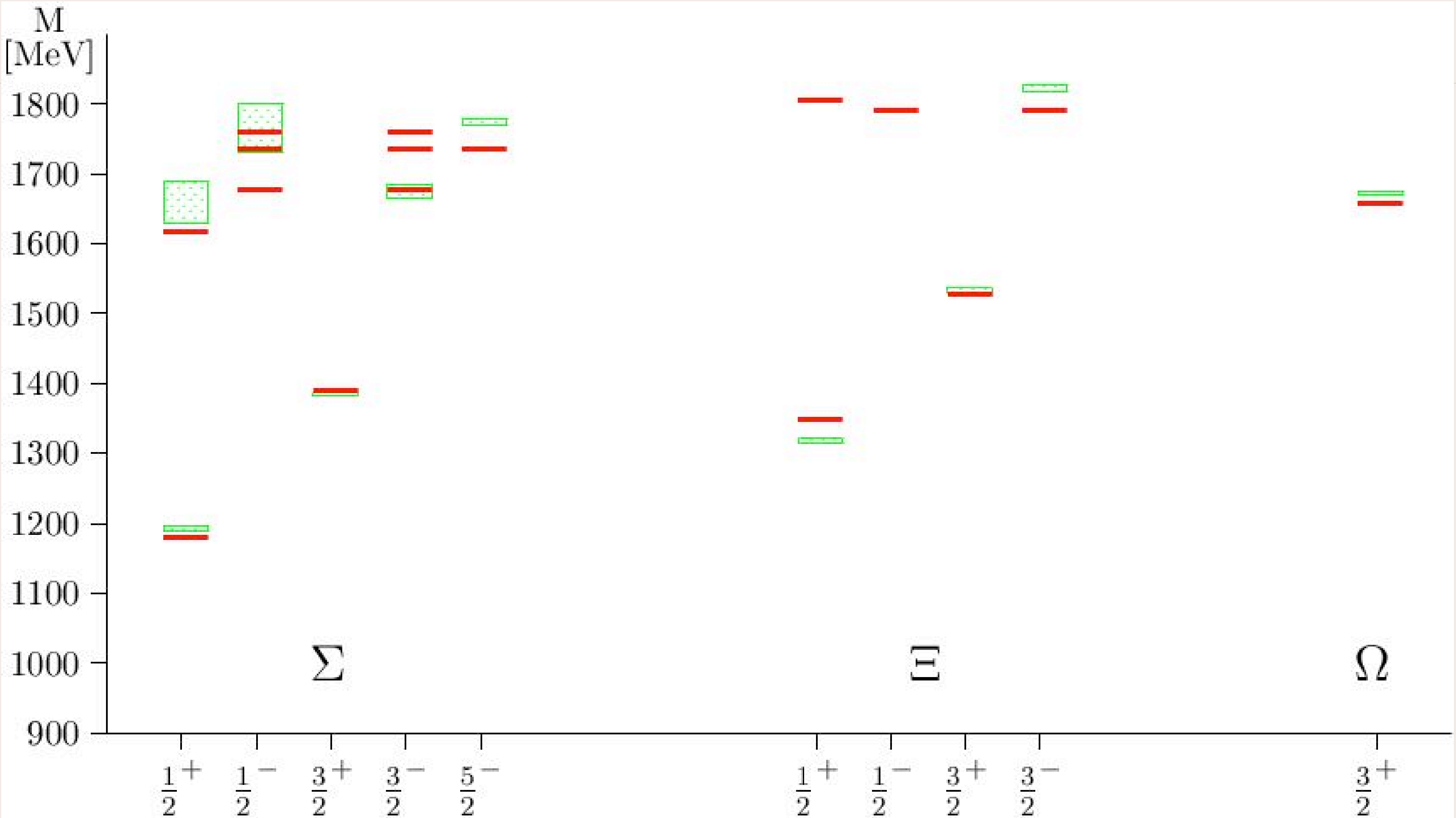
good reproduction of the baryon spectrum in the uds sector with few parms

correct ordering between negative- and positive-parity states [due to the flavor dependence of the GBE interaction]

figures

Graz group: NPA 663 (2000)




M [MeV]
1800
1700
1600
1500
1400
1300
1200
1100
1000
900
Σ
Ξ
Ω
$\frac{1}{2}^+$ $\frac{1}{2}^-$ $\frac{3}{2}^+$ $\frac{3}{2}^-$ $\frac{5}{2}^-$
$\frac{1}{2}^+$ $\frac{1}{2}^-$ $\frac{3}{2}^+$ $\frac{3}{2}^-$
$\frac{3}{2}^+$

open problems

- the interaction is still non-relativistic, while $\langle p \rangle > m \sim \Lambda_{QCD}$
- tensor and spin-orbit components are neglected

PS meson exchange produce a tensor term, but it does not produce any spin-orbit term:

$$\hat{V}^{(GBE,\,tens.)} = \sum_{a=1}^{8} \lambda_i^a \bullet \lambda_j^a \, \frac{g_{QQM}^2}{4\pi} \, \frac{1}{3 m_i m_j} \left\{ \mu_M^2 \, \frac{e^{-\mu_M r_{ij}}}{r_{ij}} \left(1 + \frac{3}{\mu_M r_{ij}} + \frac{3}{\mu_M^2 r_{ij}^2} \right) - smearing\ term \right\} \left[3 \vec{S}_i \bullet \vec{r}_{ij} \, \vec{S}_j \bullet \vec{r}_{ij} - \vec{S}_i \bullet \vec{S}_j \right]$$

spin-orbit term due to confinement should be added:

$$\hat{V}^{(TF)} = -\sum_{i<j} \frac{b}{2 r_{ij}} \left(\frac{\vec{r}_{ij} \times \vec{p}_i \bullet \vec{S}_i}{2 m_i^2} - \frac{\vec{r}_{ij} \times \vec{p}_j \bullet \vec{S}_j}{2 m_j^2} \right)$$

same parms [from PRD 58 (‘98)]

splitting	*no tens.* *no s.o.*	*with tens.* *no s.o.*	*with tens.* *with s.o.*	exp. (PDG)
$\Delta - N$	295	217	252	294^{+2}_{-2}
$Roper - N$	528	568	557	502^{+30}_{-10}
$D_{13} - N$	596	651	555	582^{+10}_{-5}
$D^{*}_{13} - N$	698	698	796	761^{+50}_{-50}
$S_{11} - N$	596	604	665	597^{+20}_{-15}
$S^{*}_{11} - N$	698	850	1044	711^{+30}_{-10}
$S_{31} - N$	692	717	668	681^{+55}_{-5}
$F_{15} - N$	781	854	606	792^{+10}_{-5}

significant effects from PS tensor term as well as from the TF spin-orbit component

possible solutions: exchange of vector mesons (ρ, K^*, ω, ϕ)

$$\hat{V}^{(vector)} = \sum_{i<j}\left[\sum_{a=1}^{3}\hat{V}_{\rho}\lambda_i^a\cdot\lambda_j^a + \sum_{a=4}^{7}\hat{V}_{K^*}\lambda_i^a\cdot\lambda_j^a + \hat{V}_{\phi}\lambda_i^a\cdot\lambda_j^a\right] + \frac{2}{3}\hat{V}_{\omega}$$

central term

spin-spin term

$$\hat{V}_M = \sum_{i<j}\left\{V_M(r_{ij}) + \frac{2}{3m_im_j}\nabla^2 V_M(r_{ij})\,\vec{S}_i\cdot\vec{S}_j\right.$$

tensor term

$$+\frac{1}{3m_im_j}\left(\frac{1}{r_{ij}}\frac{\partial}{\partial r_{ij}}V_M(r_{ij}) - \frac{\partial^2}{\partial r_{ij}^2}V_M(r_{ij})\right)\left[3\vec{S}_i\cdot\vec{r}_{ij}\,\vec{S}_j\cdot\vec{r}_{ij} - \vec{S}_i\cdot\vec{S}_j\right]$$

$$+\frac{1}{2r_{ij}}\frac{\partial}{\partial r_{ij}}V_M(r_{ij})\left(\frac{\vec{r}_{ij}\times\vec{p}_i\cdot\vec{S}_i}{2m_i^2} - \frac{\vec{r}_{ij}\times\vec{p}_j\cdot\vec{S}_j}{2m_j^2}\right)$$

$$\left.+\frac{1}{m_im_jr_{ij}}\frac{\partial}{\partial r_{ij}}V_M(r_{ij})\left(\frac{\vec{r}_{ij}\times\vec{p}_i\cdot\vec{S}_j}{m_im_j} - \frac{\vec{r}_{ij}\times\vec{p}_j\cdot\vec{S}_i}{m_im_j}\right)\right\}$$

2- and 3-body spin-orbit terms

- smeared function: $V_M\left(r_{ij}\right) = \frac{g_{QQV}^2}{4\pi} \frac{1}{4m_i m_j} \left[\mu_M^2 \frac{e^{-\mu_M r_{ij}}}{r_{ij}} - \Lambda_M^2 \frac{e^{-\Lambda_M r_{ij}}}{r_{ij}} \right]$

- CQ masses: from Graz version

- meson masses: from PDG

- confinement term: $V_0 + \frac{1}{2} \sum_{i<j} b\, r_{ij}$ one parm: b

- PS meson exchange: $\begin{cases} g_{QQ\pi} = g_{QQ\eta} \\ g_{QQ\eta'}^2 / g_{QQ\pi}^2 = 1.34 \end{cases}$ one parm: $g_{QQ\pi}$

- vector meson exhange: $\begin{cases} g_{QQ\rho} = g_{QQ\omega} \\ g_{QQ\phi}^2 / g_{QQ\rho}^2 = 1.34 \end{cases}$ one parm: $g_{QQ\rho}$

- smearing functions: from Graz version

solution found:

	$b\left(GeV\ fm^{-1}\right)$	$g_{QQ\square}^{2}/4\square$	$g_{QQ\square}^{2}/4\square$
$PS\ only$	0.92	0.67	□□□□□
$PS + V$	0.78	0.51	0.12

PS + V

$splitting$	$no\ tens., no\ s.o.$	$with\ tens., with\ s.o.$	$\exp.(PDG)$
$\square\square N$	292	280	294^{+2}_{-2}
$Roper - N$	513	521	502^{+30}_{-10}
$D_{13} - N$	605	554	582^{+10}_{-5}
$D_{13}^{*} - N$	677	752	761^{+50}_{-50}
$S_{11} - N$	605	653	597^{+20}_{-15}
$S_{11}^{*} - N$	677	948	711^{+30}_{-10}
$S_{31} - N$	673	653	681^{+55}_{-5}
$F_{15} - N$	803	641	792^{+10}_{-5}

tensor terms from PS and V mesons almost compensate each other, but spin-orbit is too large ⟹ scalar meson exchange ? relativistic suppression ?

OGE model

- unified mechanisms for meson and baryons
- applicable to light- and heavy-quark sectors

- no effects from □SB (besides m_Q) [and also $U_A(1)$-breaking]
- incorrect parity ordering
- ad-hoc extra suppression of spin-orbit force in baryons

GBE model

- effects from □SB
- correct parity ordering

- applicable only to light baryons
- no relativized version
- ad-hoc absence of spin-orbit terms

relativistic effects are important for the CQ interaction

possible solution of the "*baryon spin-orbit puzzle*"

see Nathan Isgur: PRD 62 (2000) 014025 and 054026

problem with the nucleon size:

	$\langle r \rangle_p (fm)$	$\langle r^2 \rangle_n (fm^2)$
OGE model	0.32	-0.0083
GBE model	0.32	-0.0090
exp.	0.837 ± 0.011	-0.113 ± 0.005

Thus, we need to construct:

- relativistic CQ wave functions
- relativistic current operators

next two lectures