Nucleon and Nucleon-Roper(?) Form Factors from Lattice QCD

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Jefferson Lab
Thomas Jefferson National Accelerator Facility

Hadron Electromagnetic Form Factors Workshop
ECT, Trento, Italy
May 13, 2008

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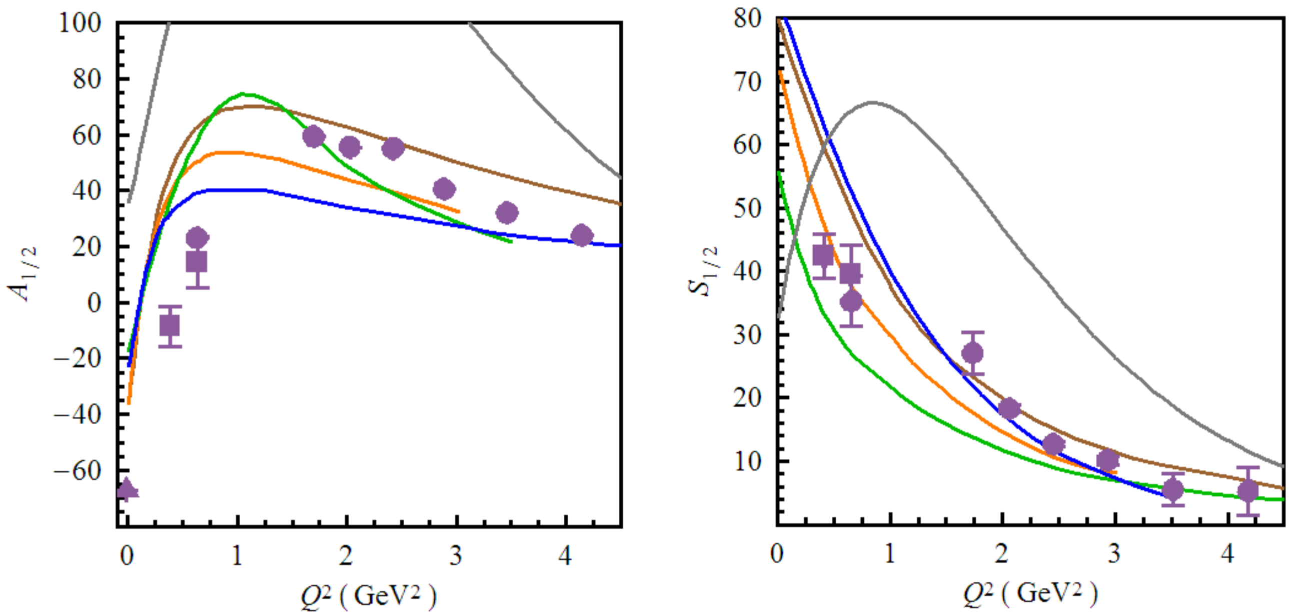
In collaboration with

Saul Cohen, Robert Edwards, and David Richards (JLab)

7n cluster @ JLab

N-P_{11} Form Factor

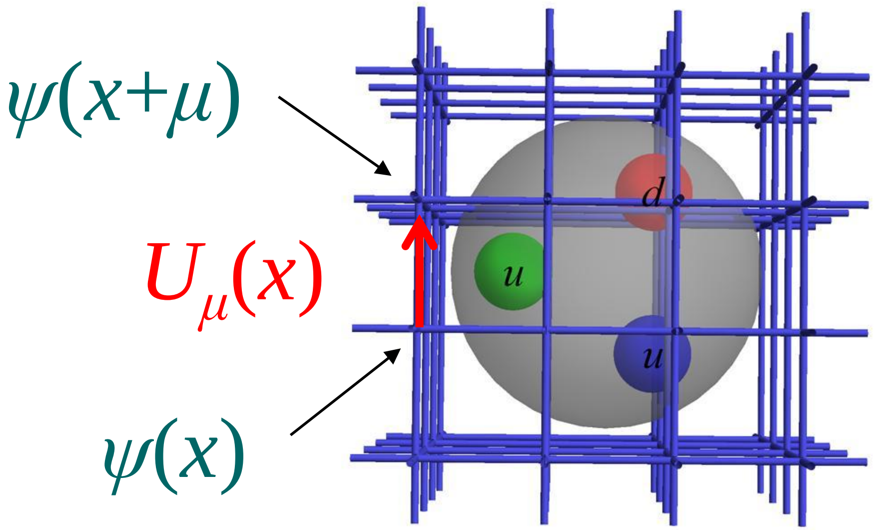
- Experiments at Jefferson Laboratory (**CLAS**), MIT-Bates, LEGS, Mainz, Bonn, GRAAL, and Spring-8
- *Helicity amplitudes* are measured (in 10^{-3} GeV$^{-1/2}$ units)
- Many models disagree (a selection are shown below)



- In this work, we will consider the case when the Roper has overlap with the first radial excited state of the nucleon

Lattice QCD

- Lattice QCD is a discrete version of continuum QCD theory



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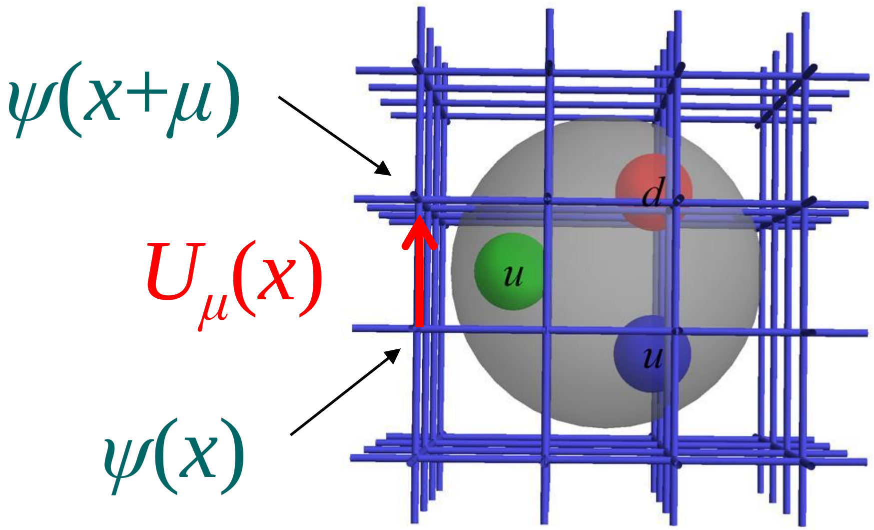


- Physical observables are calculated from the path integral

$$\langle 0|O(\overline{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int [dA][d\overline{\psi}][d\psi]\, O(\overline{\psi},\psi,A) e^{i\int d^4x\, \mathcal{L}^{\mathrm{QCD}}(\overline{\psi},\psi,A)}$$

- Use Monte Carlo integration combined with the "importance sampling" technique to calculate the path integral.
- Take $a \to 0$ and $V \to \infty$ in the continuum limit

Lattice QCD

- Lattice QCD is computationally intensive

$$\text{Cost} \approx \left(\frac{L}{\text{fm}}\right)^5 L_s \left(\frac{\text{MeV}}{M_\pi}\right)\left(\frac{\text{fm}}{a}\right)^6 \left(C_0 + C_1\left(\frac{\text{fm}}{a}\right)\left(\frac{\text{MeV}}{M_K}\right)^2 + C_2\left(\frac{a}{\text{fm}}\right)^2\left(\frac{\text{MeV}}{M_\pi}\right)^2\right)$$

Norman Christ, LAT2007

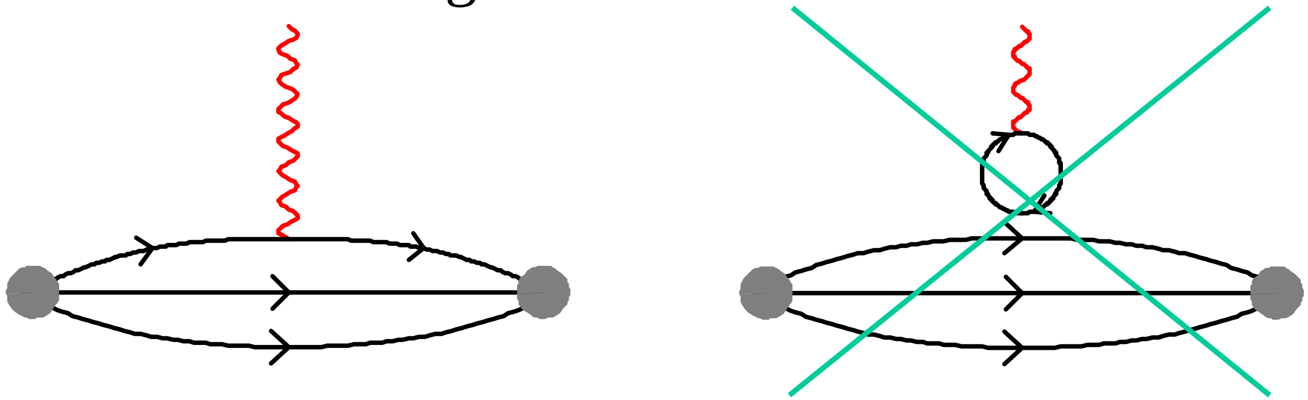
- Current major US 2+1-flavor gauge ensemble generation:
  - MILC: staggered, $a \sim 0.06$ fm, $L \sim 3$ fm, $M_\pi \sim 250$ MeV
  - RBC+UKQCD: DWF, $a \sim 0.09$ fm, $L \sim 3$fm, $M_\pi < 300$ MeV
- Chiral domain-wall fermions (DWF) at large volume (6 fm) at physical pion mass <u>may</u> be expected in 2011
- But for now….
  need a pion mass extrapolation $M_\pi \to (M_\pi)_{\text{phys}}$
  (use lattice/continuum chiral perturbation theory, if available)
- Soon be able to verify XPT directly in the chiral regime

Green Functions

- Three-point function with interpolation operator J

$$C_{\mathrm{3pt}}^{\Gamma,\mathcal{O}}\left(\vec{p},t,\tau\right)=\sum_{\alpha,\beta}\Gamma^{\alpha,\beta}\langle J_\beta\left(\vec{p},t\right)\mathcal{O}(\tau)\overline{J}_\alpha\left(\vec{p},0\right)\rangle$$

- Two contraction categories:



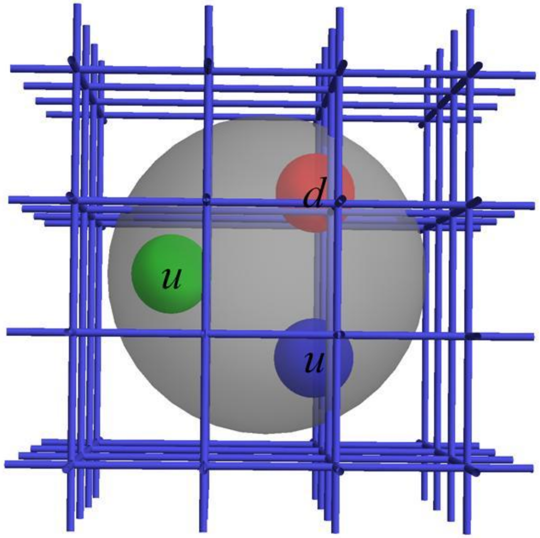
- We use only the "connected" construction for this work
- Ongoing investigation into "disconnected" contribution
- Euclidean space: signal falls exponentially with time

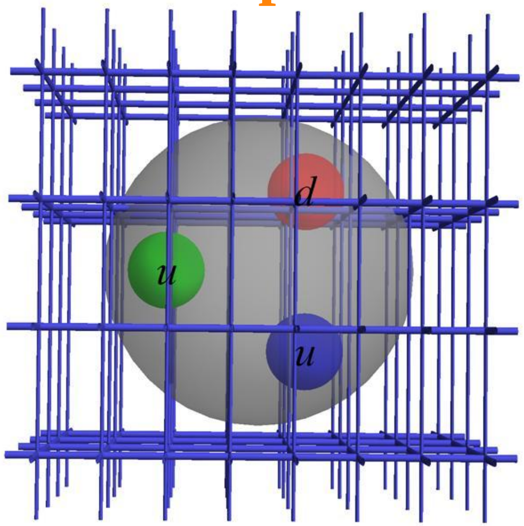
  dominated by ground state at large enough time

Lattice Setup

- Solution: increase resolution

Anisotropic lattice





Lattice Setup

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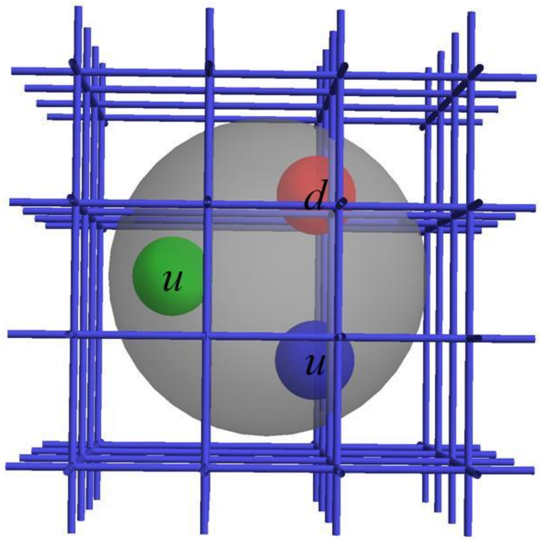
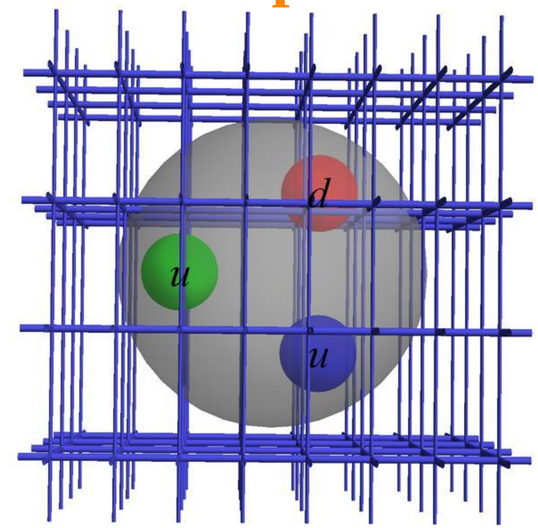
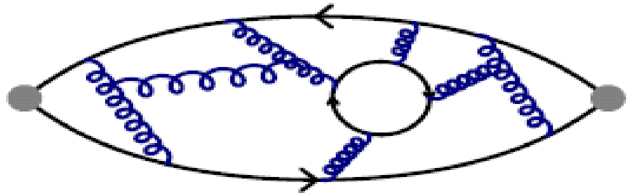
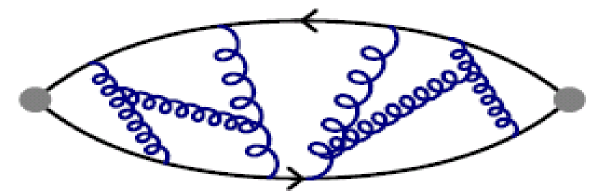




- "Quenched" for exploratory study





Lattice Setup

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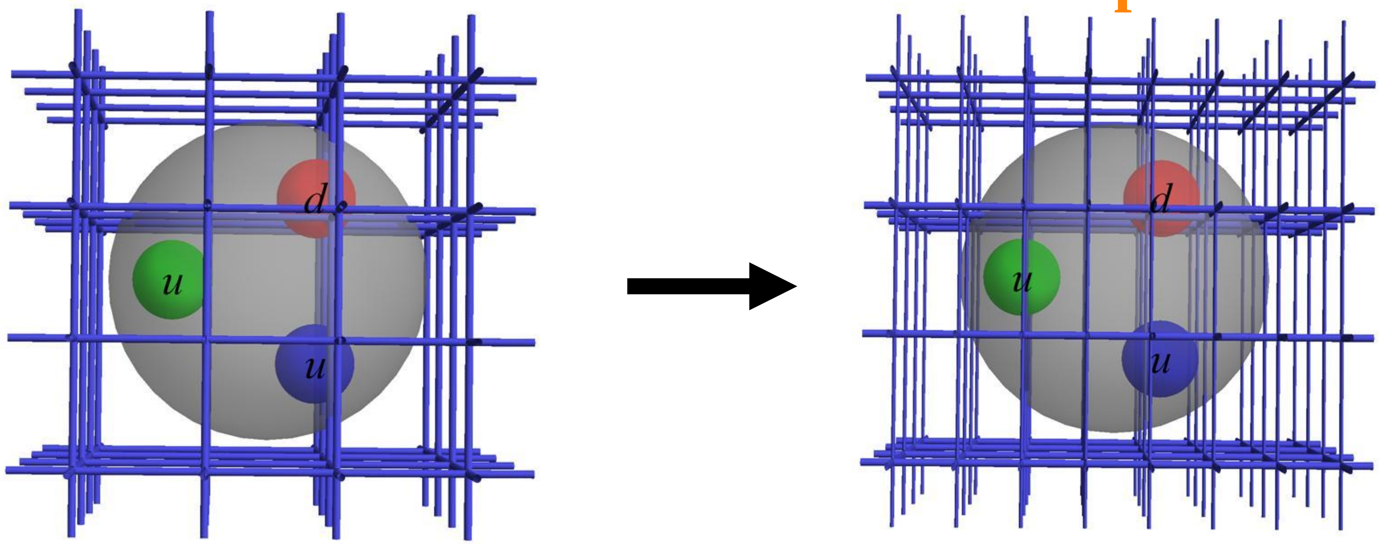

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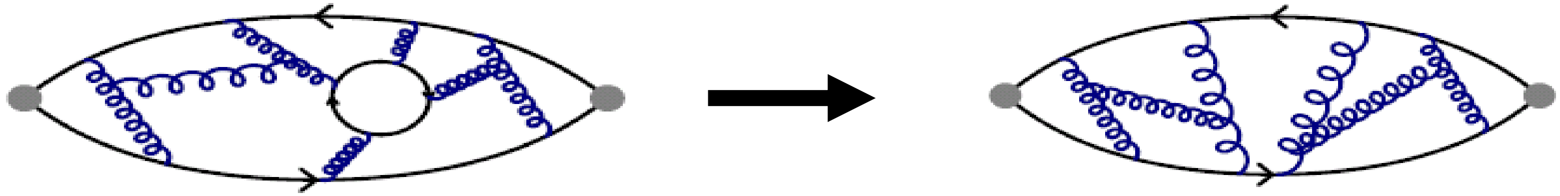
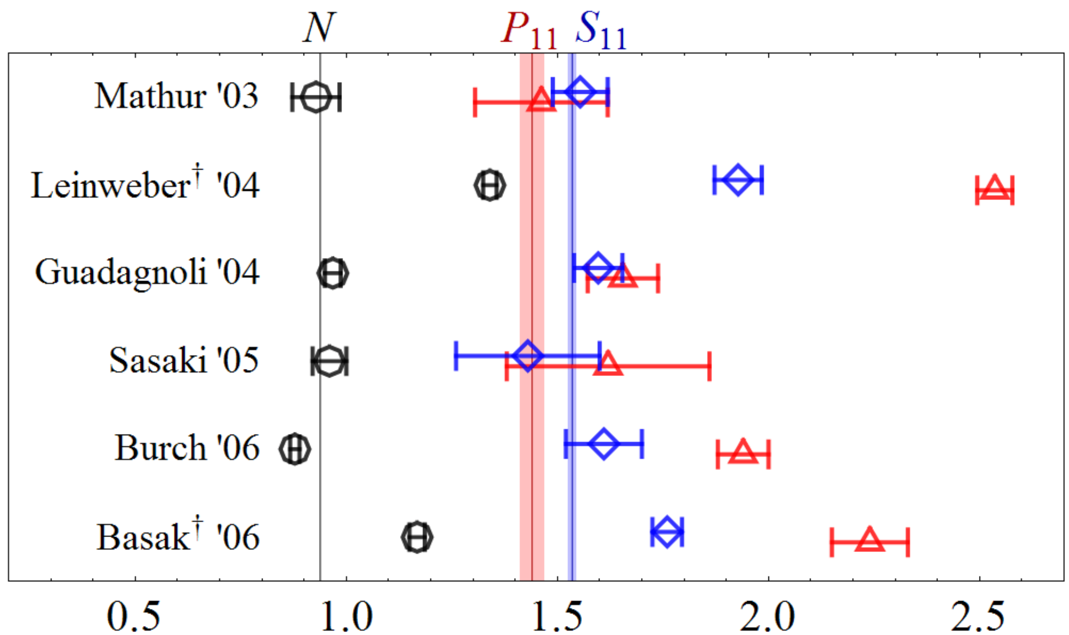


- $16^3 \times 64$ **an**isotropic lattice, $\xi = 3$
- Wilson gauge action + clover fermion action
- $a_t^{-1} \approx 6$ GeV and $a_s \approx 0.125$ fm ($L < 2$ fm)
- $m_\pi \approx 720$ (480 and 1100) MeV

Roper Resonance on the Lattice

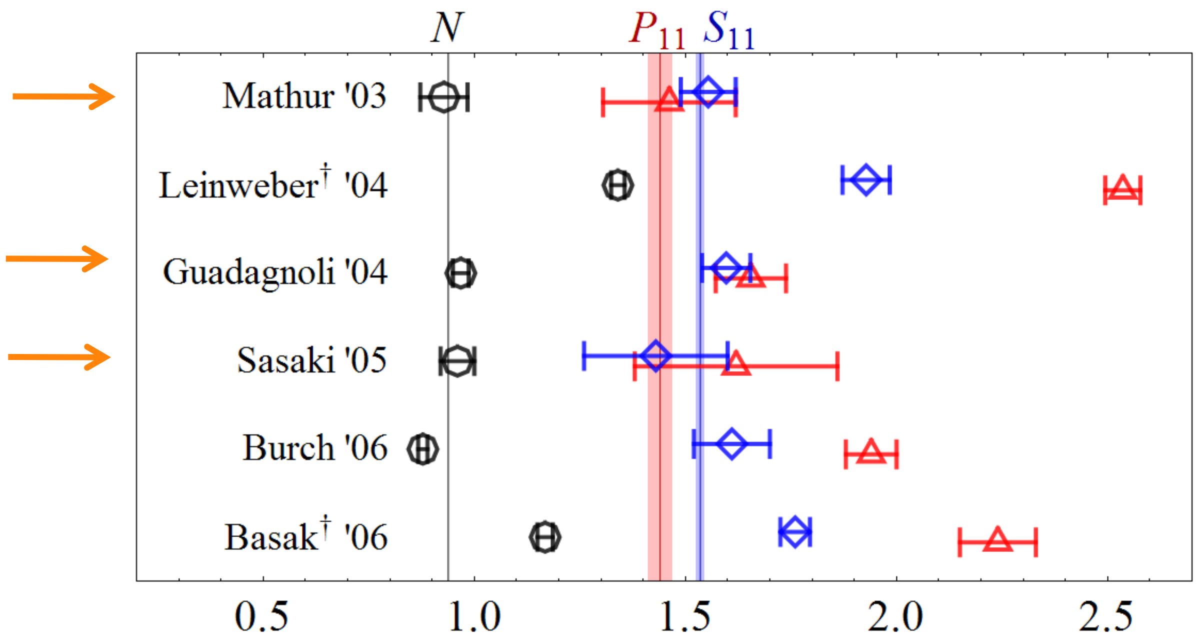
- Mostly done in "quenched" approx. ***N*, *P_{11}*, *S_{11}* spectrum**



Group	N_f	S_f	a_t^{-1} (GeV)	M_π (GeV)	L (fm)	Method	Extrapolation
Basak et al. [12]	0	Wilson	6.05	0.49	2.35	VM	N/A
Burch et al. [11]	0	CIDO	1.68,1.35	0.35–1.1	2.4	VM	$a + bm_\pi^2$
Sasaki et al. [9]	0	Wilson	2.1	0.61–1.22	1.5,3.0	MEM	$\sqrt{a + bm_\pi^2}$
Guadagnoli et al. [7]	0	Clover [13]	2.55	0.51–1.08	1.85	SBBM	$a + bm_\pi^2 + cm_\pi^4$
Leinweber et al. [8]	0	FLIC	1.6	0.50–0.91	2.0	VM	N/A
Mathur et al. [6]	0	Overlap [14]	1.0	0.18–0.87	2.4,3.2	CCF	$a + bm_\pi + cm_\pi^2$

Roper Resonance on the Lattice

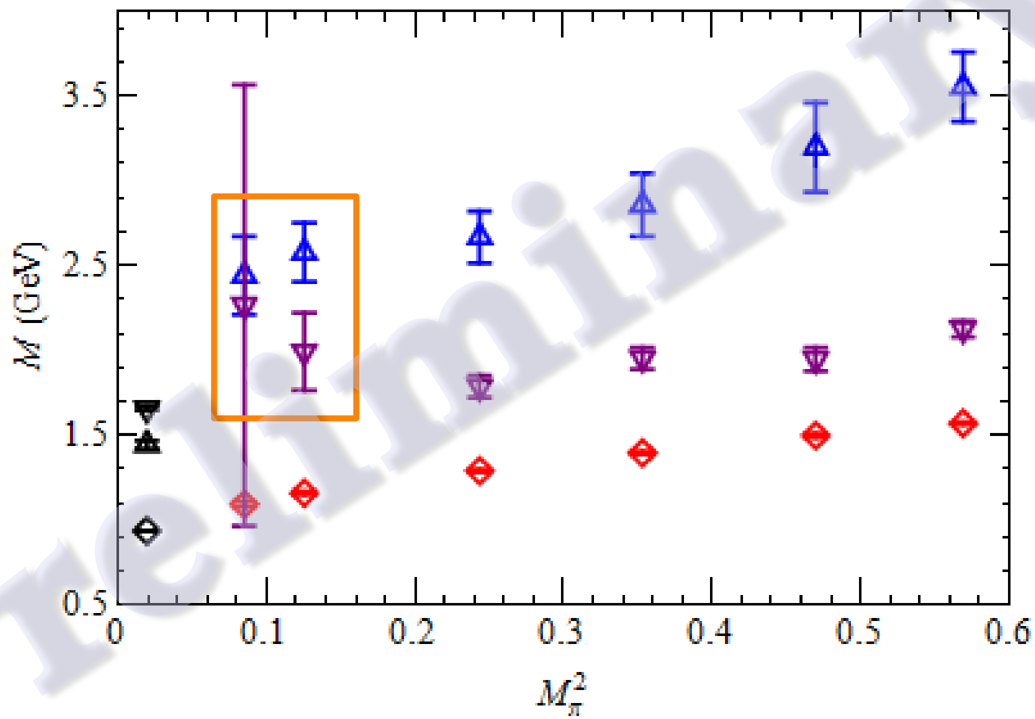
- Mostly done in "quenched" approx. N, P_{11}, S_{11} spectrum



- Questions
  - Differences in analysis approach? (Need a unified analysis on all data!)
  - Contamination by 2nd and higher excited states?
  - How is the chiral extrapolation is performed?
    Does it really work or coincidence?
  - Other systematics? Finite-volume effect, theory cut-off, N_f , etc.

Roper in Full QCD

- N_f = 2+1 mixed action (DWF+ asqtad) calculation (L ~ 2.5 fm)
- Symbols: J^P
  - ◇ $1/2^+$ ***N***
  - ▽ $1/2^-$ $\boldsymbol{S_{11}}$
  - △ $1/2^+$ $\boldsymbol{P_{11}}$



- Finite-volume effects starting at 350 MeV pion.?
- Prove or disprove Roper as the first radial excited state of nucleon?

Roper in Full QCD

- $N_f = 2+1$ mixed action (DWF+ asqtad) calculation ($L \sim 2.5$ fm)
- Symbols: J^P
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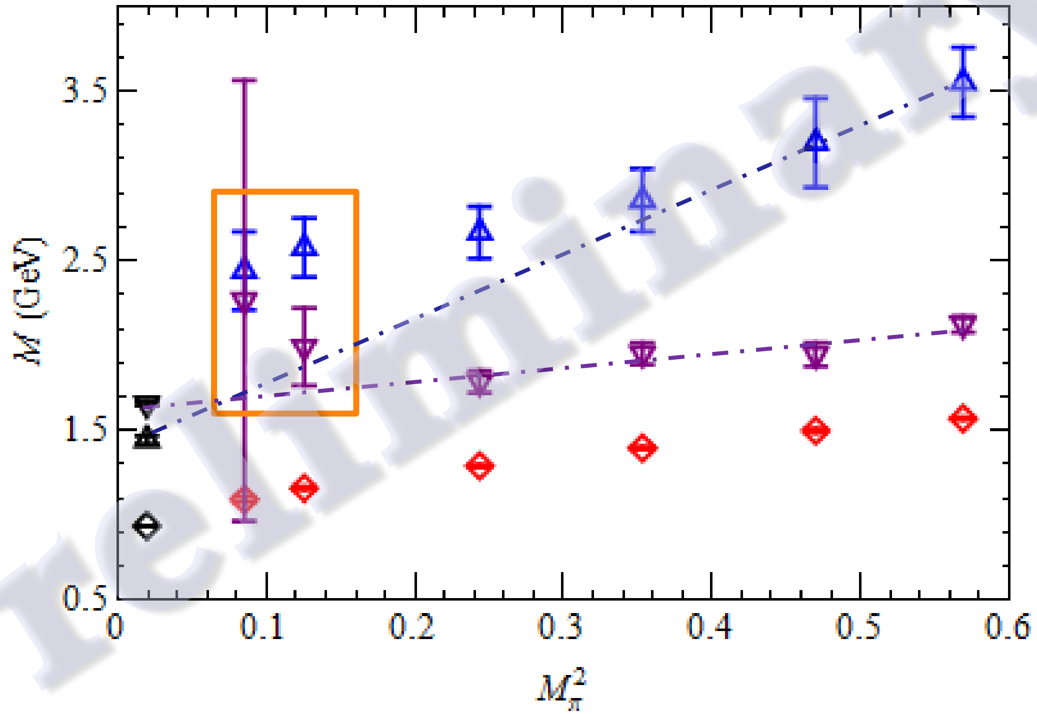


- Finite-volume effects starting at 350 MeV pion.?
- Prove or disprove Roper as the first radial excited state of nucleon?
- Not a crazy possibility (see the hand-drawn extrapolation lines)
- Stay tuned on future $N_f = 2+1$ lattice calculations

Form Factors

- The form factors are buried in the amplitudes

$$\Gamma^{(3),T}_{\mu,AB}(t_i,t,t_f,\vec{p}_i,\vec{p}_f)$$
$$= a^3 \sum_n \sum_{n'} \frac{1}{Z_j} \frac{Z_{n',B}(p_f) Z_{n,A}(p_i)}{4E'_n(\vec{p}_f)E_n(\vec{p}_i)} e^{-(t_f-t)E'_n(\vec{p}_f)} e^{-(t-t_i)E_n(\vec{p}_i)}$$
$$\times \sum_{s,s'} T_{\alpha\beta} u_{n'}(\vec{p}_f,s')_\beta \langle N_{n'}(\vec{p}_f,s')\,|j_\mu(0)|\,N_n(\vec{p}_i,s)\rangle \overline{u}_n(\vec{p}_i,s)_\alpha$$

- Nucleon form factor ($n = n' = 0$)

$$\langle N\,|V_\mu|\,N\rangle(q) = \overline{u}_N(p')\left[\gamma_\mu F_1(q^2) + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{2m}\right] u_N(p) e^{-iq\cdot x}$$

  - Multiple works done using lattice QCD in the past
    - easy cross-checks
  - Clear of excited-state contamination
    - better systematics

Form Factors

- The form factors are buried in the amplitudes

$$\Gamma^{(3),T}_{\mu,AB}(t_i,t,t_f,\vec{p}_i,\vec{p}_f)$$
$$= a^3\sum_n\sum_{n'}\frac{1}{Z_j}\frac{Z_{n',B}(p_f)Z_{n,A}(p_i)}{4E'_n(\vec{p}_f)E_n(\vec{p}_i)}e^{-(t_f-t)E'_n(\vec{p}_f)}e^{-(t-t_i)E_n(\vec{p}_i)}$$
$$\times\sum_{s,s'}T_{\alpha\beta}u_{n'}(\vec{p}_f,s')_\beta\langle N_{n'}(\vec{p}_f,s')\left|j_\mu(0)\right|N_n(\vec{p}_i,s)\rangle\overline{u}_n(\vec{p}_i,s)_\alpha$$

- Nucleon form factor ($n = n' = 0$)

$$\langle N\left|V_\mu\right|N\rangle(q) = \overline{u}_N(p')\left[\gamma_\mu F_1(q^2)+\sigma_{\mu\nu}q_\nu\frac{F_2(q^2)}{2m}\right]u_N(p)e^{-iq\cdot x}$$

- Nucleon-Roper form factor ($n = 0$, $n' = 1$ or $n = 1$, $n' = 0$)

$$\langle N_2\left|V_\mu\right|N_1\rangle_\mu(q) = \overline{u}_{N_2}(p')\left[F_1(q^2)\left(\gamma_\mu-\frac{q_\mu}{q^2}\not{q}\right)+\sigma_{\mu\nu}q_\nu\frac{F_2(q^2)}{M_{N_1}+M_{N_2}}\right]u_{N_1}(p)e^{-iq\cdot x}$$

- Need as better input from two-point correlators as possible

Variational Method

- Generalized eigenvalue problem:

[*C. Michael, Nucl. Phys. B 259, 58 (1985)*]
[*M. Lüscher and U. Wolff, Nucl. Phys. B 339, 222 (1990)*]

  - Construct the matrix

$$C_{i\ j}(t) = \langle 0 \mid \mathcal{O}_i(t)^\dagger \mathcal{O}_j(0) \mid 0\rangle$$

  - Solve for the generalized eigensystem of

$$C(t_0)^{-1/2}C(t)C(t_0)^{-1/2}v = \lambda(t,t_0)v$$

with eigenvalues

$$\lambda_n(t,t_0) = e^{-(t-t_0)E_n}(1 + \mathcal{O}(e^{-|\delta E|(t-t_0)}))$$

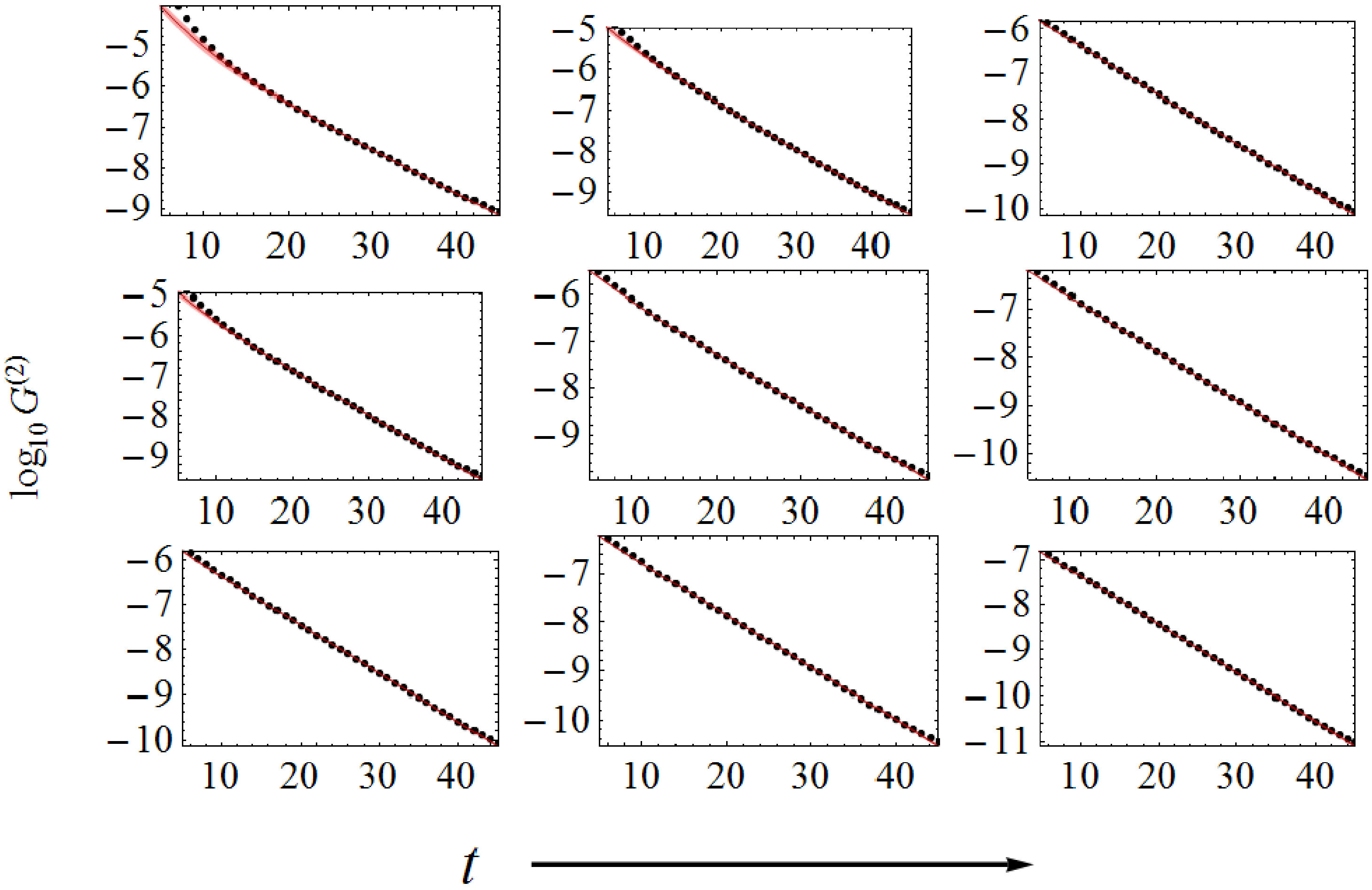
Now the original correlator matrix can be approximated by

$$C_{ij} = \sum_{n=1}^{r}(C(t_0)^{1/2}v_n^*)_i(v_nC(t_0)^{1/2})_j\lambda_n(t,t_0) = \sum_n \frac{E_n+m}{2E_n}Z_{i,n}Z_{j,n}e^{-E_nt}$$

  - Three smearings (*i*,*j*) are chosen for this work
    - 2nd excited state is contaminated by remaining states

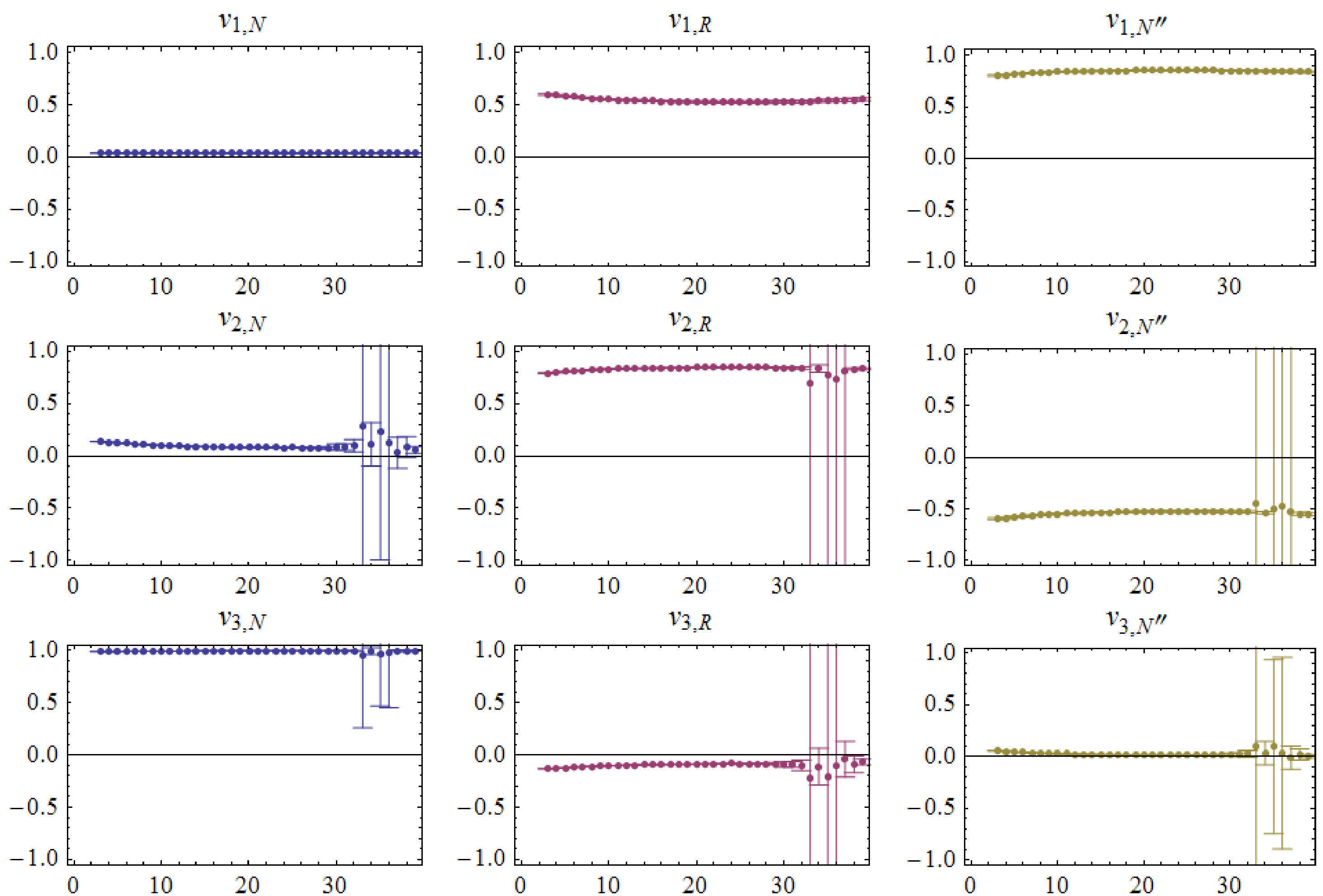
Variational Method

- Reconstruct two-point correlators from Z and λ



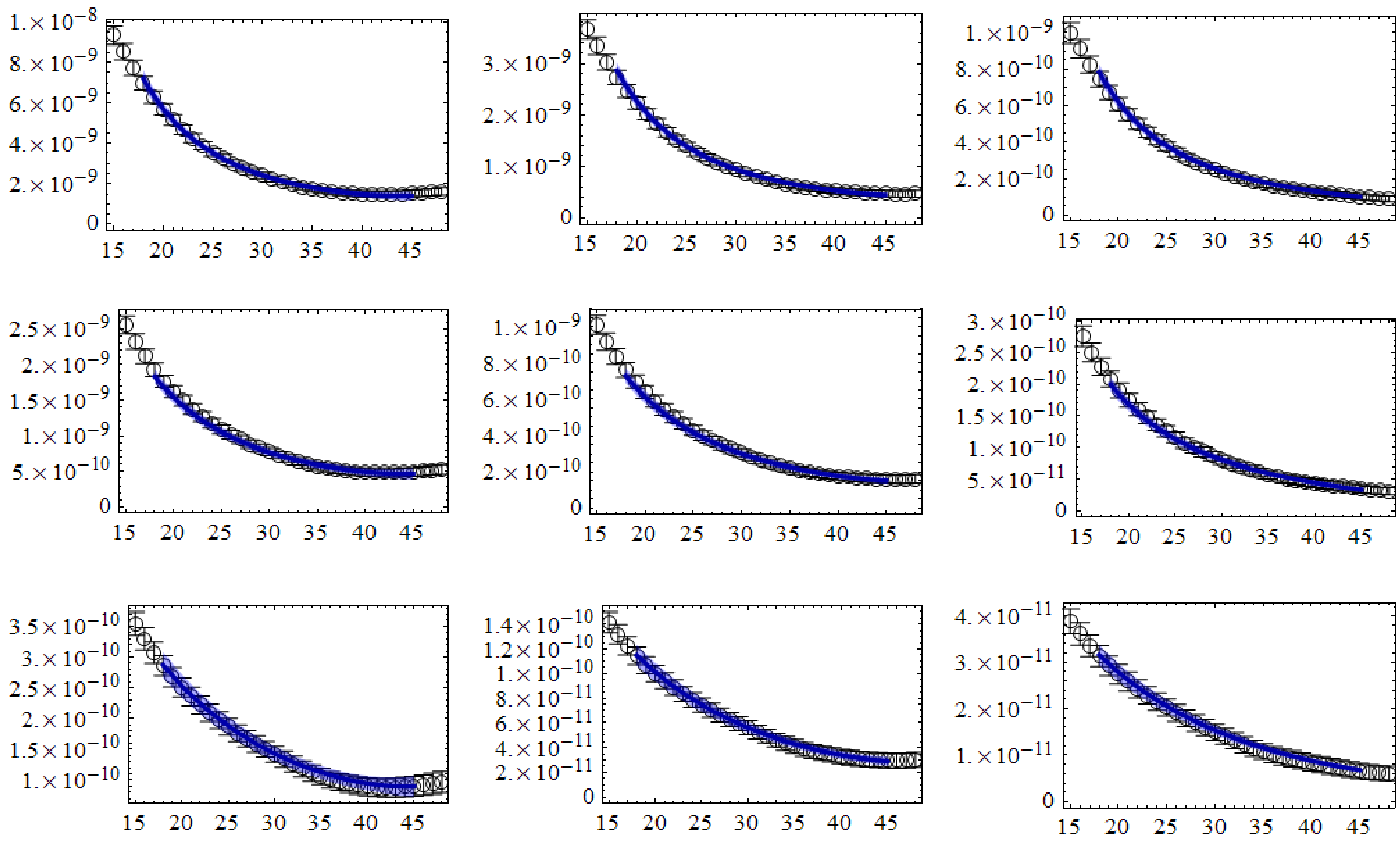
Variational Method

- Eigenvectors (at $p = 0$) show overlap of smearings with states



Three-Point Fitting

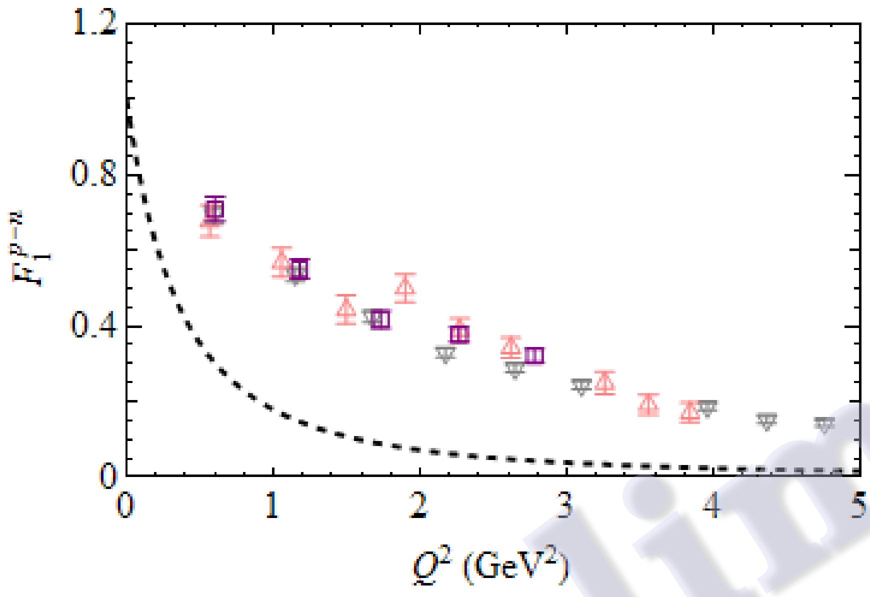
- Example: $P_f = \{0,0,0\}$, $P_i = \{0,1,1\}$, V_4

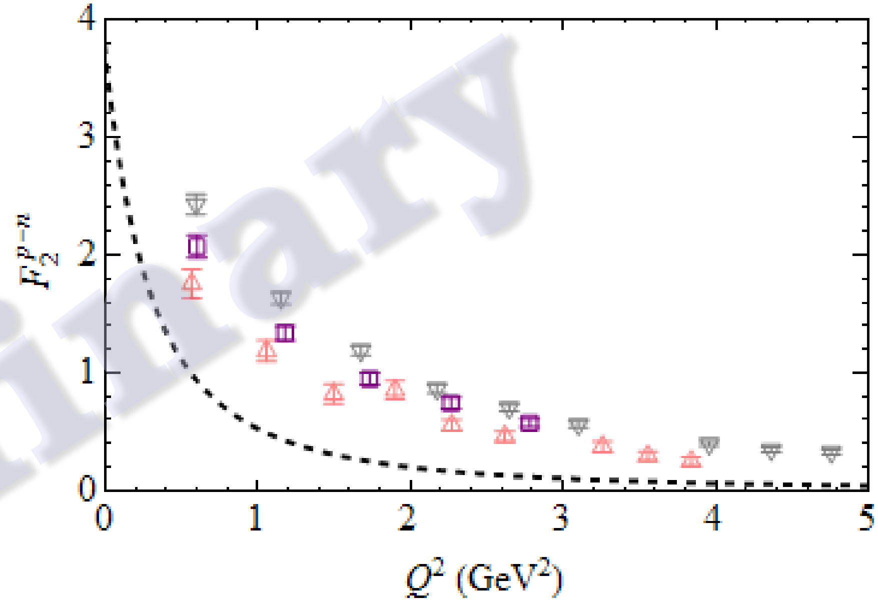


Nucleon Form Factors

- Pion masses around 480, 720 and 1100 MeV

Isovector F_1 **Isovector F_2**





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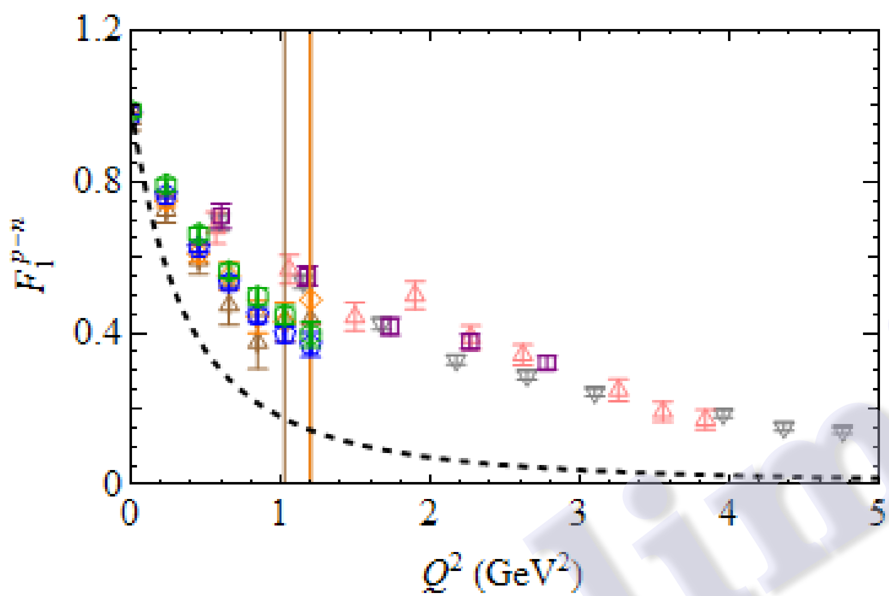
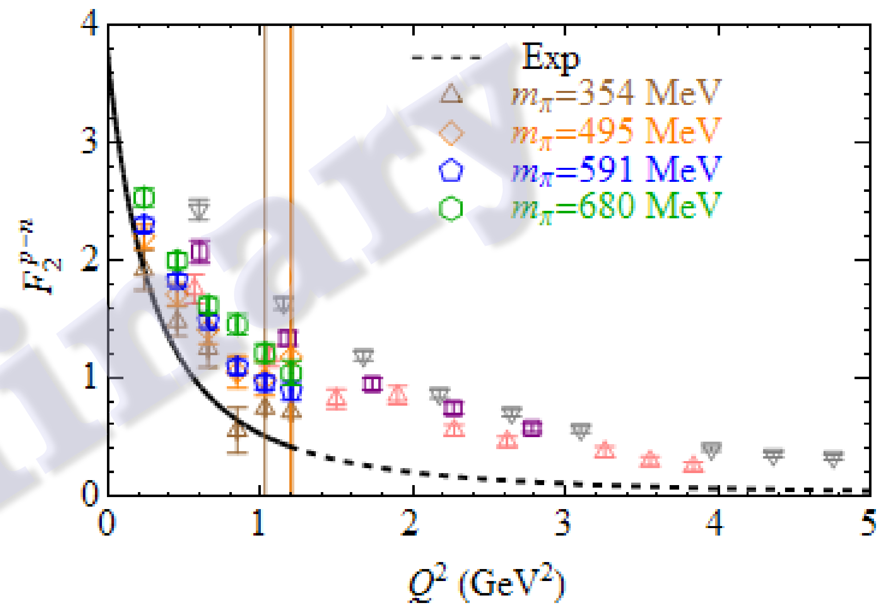




- Compare with $N_f = 2+1$ mixed action (DWF+ asqtad) calculation with "ratio" approach:

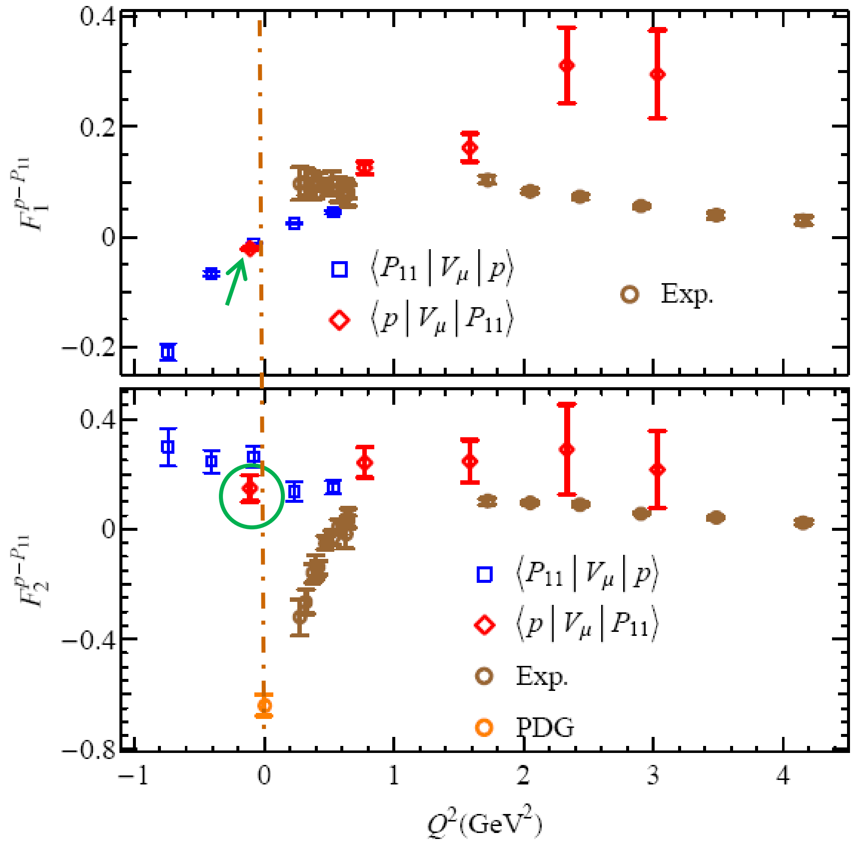
$$R_\mu = \frac{\Gamma^{(3),\mathcal{P}}_{\mu,GG}(t_i,t,t_f,\vec{p}_i,\vec{p}_f)}{\Gamma^{(2)}_{GG}(t_i,t_f,\vec{p}_f)} \times \left(\frac{\Gamma^{(2)}_{LG}(t,t_f,\vec{p}_i)\Gamma^{(2)}_{GG}(t_i,t,\vec{p}_f)\Gamma^{(2)}_{LG}(t_i,t_f,\vec{p}_f)}{\Gamma^{(2)}_{LG}(t,t_f,\vec{p}_f)\Gamma^{(2)}_{GG}(t_i,t,\vec{p}_i)\Gamma^{(2)}_{LG}(t_i,t_f,\vec{p}_i)}\right)^{\frac{1}{2}}$$

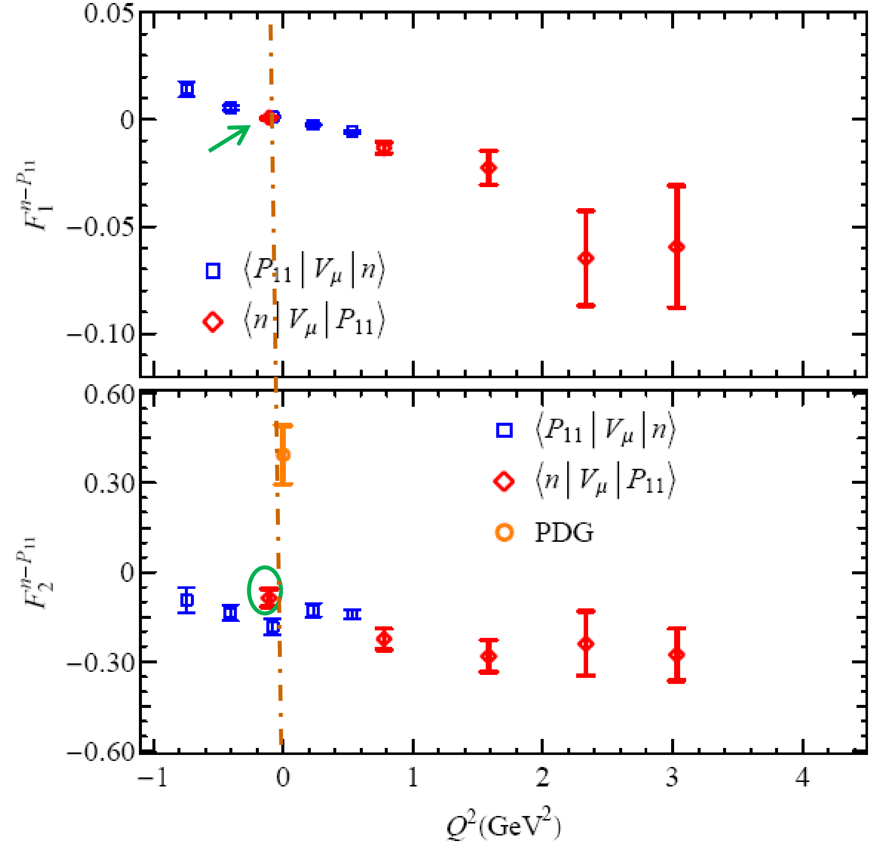
and SVD solutions

Nucleon-Roper Form Factors

- Completed exploratory study on quenched lattices *arXiv:0803.3020*

Proton-P_{11} **720 MeV Pion** **Neutron-P_{11}**



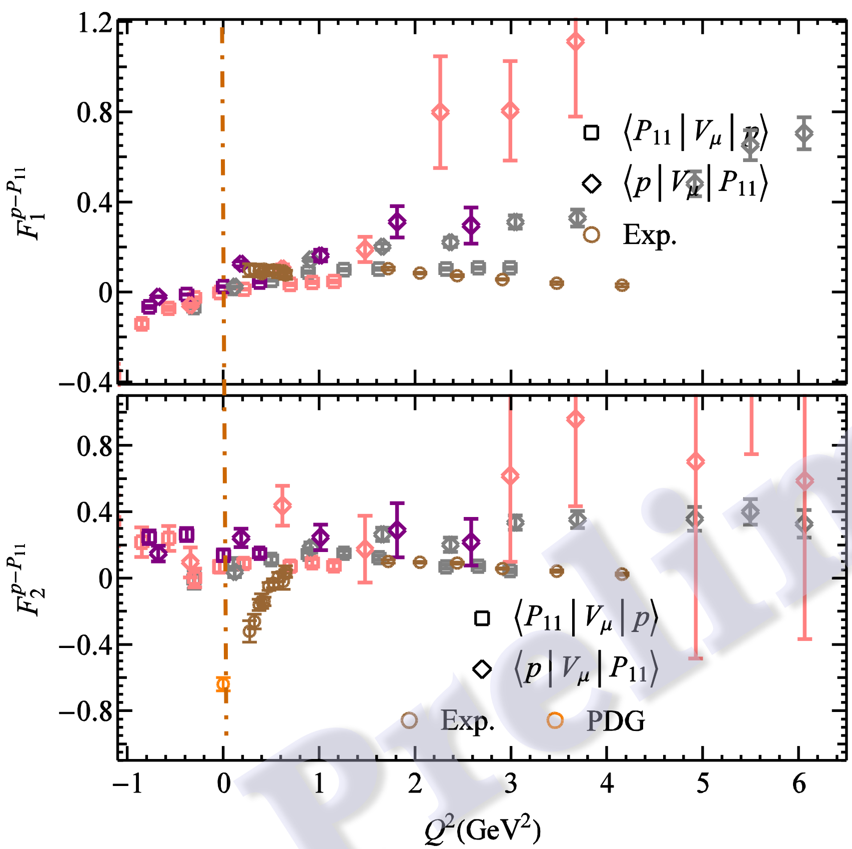


- Possible decaying state (circled above)
- 200 configurations give us reasonable signal
- Lower pion mass will shift the time-like region to space-like region

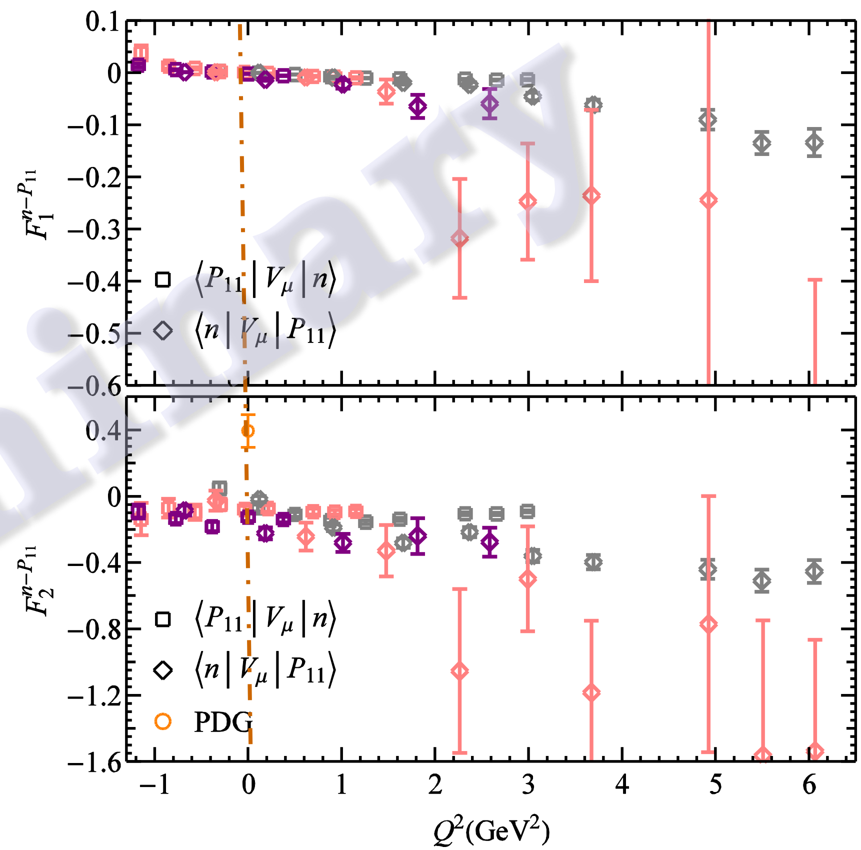
Nucleon-Roper Form Factors

- Add two more mass points at $m_\pi \sim$ **480** and **1100** MeV

Proton-P_{11}



Neutron-P_{11}



- Need to remove the points with potential decay kinematics
- May want to calculate on a second volume

Challenges for the Future

As one goes to lighter pion-mass regions…

- Need more statistics to get sufficient single-to-noise ratio

$$\frac{\text{Signal}}{\text{Noise}} = \frac{\langle J(t)J(0)\rangle}{\frac{1}{\sqrt{N}}\sqrt{\langle |J(t)J(0)|^2\rangle - \langle J(t)J(0)\rangle^2}}$$
$$\sim \frac{Ae^{-M_n t}}{\frac{1}{\sqrt{N}}\sqrt{Be^{-3m_\pi t} - Ce^{-2M_n t}}}$$
$$\sim \sqrt{N}De^{-(M_n - \frac{3}{2}m_\pi)t}$$

Challenges for the Future

As one goes to lighter pion-mass regions…

- Need more statistics to get sufficient single-to-noise ratio
- Decay channels open up…
  - Conservative approach: using multiple volumes to identify single- and multiple-particle states
- Further improvement:
  - Symmetry breaking on the lattice
    - construct more operators that would generate the same quantum numbers in $a = 0$ world
  - More data input
  - Better discrimination among different states

Summary

Lattice QCD calculations of N-P_{11} form factors…

- We demonstrate a method to determine N-N^* form factors
- Large Q^2 momentum N-N form factors
- Test case is in a small "quenched" box with large pion mass

Further along our roadmap…

- Starting full-QCD anisotropic lattice calculations this summer
- Search over low and larger Q^2 regions
- Other N-N^* form factors. The methodology developed can be applied to many other excited-nucleon form factors.