Baryon masses and axial couplings in the combined $1/N_c$ and Chiral expansions

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Based on arXiv:1210.2364 with José Goity

Non-perturbative regime of QCD

QCD fundamental theory of strong interactions

$$\mathcal{L}_{QCD} = \sum_f \bar{q}_f \left(iD_\mu \gamma^\mu - m_f \right) q_f - \frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu}$$

Hadrons and Nuclear physics

⇒ Solve QCD on a lattice

Brute force ab-initio simulations using the QCD lagrangian

⇒ Exploit available symmetries of QCD applicable to our particular system

Chiral symmetry, Heavy quark symmetry, large N_c limit, ...

Do these two methods dismiss or complement each other ?

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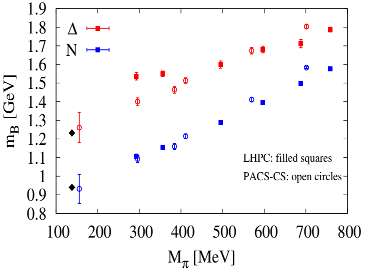
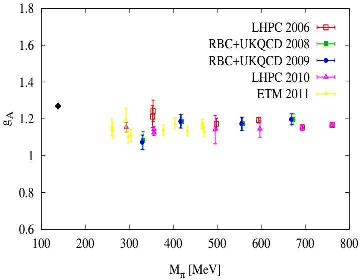
Complement indeed

Hadron structure from lattice QCD

BMW, LHPC, PACS-CS, ETMC, RBC, UKQCD, QCDSF, ...





... and a lot more ... magnetic moments, (transition) form factors, electromagnetic polarizabilities, etc.

Chiral & Large N_c limits of QCD

Chiral Symmetry: $SU_L(N_f) \times SU_R(N_f)$ for $m_q \to 0$

- Spontaneous Chiral Symmetry breaking: Goldstone bosons (π, K, η).
- Scale separation: mass of Goldstone bosons $\ll$ vector mesons ~ 1 GeV.
- EFT: Chiral Perturbation Theory, low-energy expansion in m_π / Λ_{QCD}
- Successful description of hadron properties in the low-energy region.

Large N_c limit: $SU(N_c)$, $N_c \to \infty$, $\lambda = g^2 N_c$ =const

- t'Hooft: planar diagrams $\mathcal{O}(N_c)$, non-planar diagrams suppressed $1/N_c$.
- Mesons are light, $m = \mathcal{O}(N_c^0)$, and stable, $\Gamma = \mathcal{O}(1/N_c)$.
- Witten: baryons color singlets with N_c valence quarks.
- Baryons are heavy particle $M = \mathcal{O}(N_c)$ with size independent of N_c.
- Meson-baryon coupling $\frac{g_A}{F_\pi} \partial_i \pi^a G^{ia} = \mathcal{O}(\sqrt{N_c})$.
- Ordering of all QCD effects in powers of $1/N_c$.

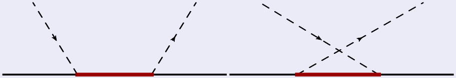
The role of the Δ: spin-flavor symmetry

Gervais & Sakita (1984), Dashen & Manohar (1993)

From Witten's counting rules:

Pion-nucleon scattering amplitude $\sim \mathcal{O}(1)$ ... but pion-nucleon vertex $\sim \mathcal{O}(\sqrt{N_c})$

Cancellation between diagrams: consistency conditions



$$\propto \frac{N_c^2 g_A^2}{F_\pi^2}\left[X^{ia}, X^{jb}\right] = O(1)$$

Spin-Flavor $SU(2N_f)$ symmetry $\{S^i, T^a, G^{ia}\}$

GS baryon = spin-flavor symmetric multiplet **B** of $SU(2N_f)$ with S=I

$$\left[S^i, T^a\right] = 0$$

$$\left[S^i, S^j\right] = i\epsilon^{ijk} S^k \,,\ \left[T^a, T^b\right] = i\epsilon^{abc} T^c$$

$$\left[S^i, G^{ja}\right] = i\epsilon^{ijk} G^{ka} \,,\ \left[T^a, G^{ib}\right] = i\epsilon^{abc} G^{ic}$$

$$\left[G^{ia}, G^{jb}\right] = \frac{i}{2N_F}\delta^{ab}\epsilon^{ijk} S^k + \frac{1}{4}\delta^{ij} f^{abc} T^c + \frac{i}{2}\epsilon^{ijk} d^{abc} G^{kc} \,.$$

$$S = I = \frac{1}{2}, \frac{3}{2}, \cdots, \frac{N_c}{2}$$

$$\mathbf{B} = \begin{pmatrix} N \\ \Delta_{3/2} \\ \vdots \\ \Delta_{N_c/2} \end{pmatrix} \to \mathbf{B} = \begin{pmatrix} N \\ \Delta \end{pmatrix}$$

Outline

- Combined $1/N_c$ & Chiral expansions ($SU(2)$ case)
- Baryon mass and axial current
- Analysis of lattice QCD results
- Summary and Outlook

HBChPT $[SU_L(N_f) \times SU_R(N_f)]$ $\oplus$ $1/N_c$ Expansion $[SU(2N_f)]$

ξ-expansion: $1/N_c = \mathcal{O}(\xi) = \mathcal{O}(p)$

$\mathcal{O}(\xi)$ lagrangian

$$\mathcal{L}_{\mathbf{B}}^{(1)} = \mathbf{B}^\dagger \left(iD_0 + \mathring{g}_A u_{ia} G^{ia} - \frac{C_{HF}}{N_c} \hat{S}^2 - \frac{c_1}{2} N_c \, \chi_+ \right) \mathbf{B},$$

chiral building blocks

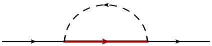
$$u = \exp(i\pi^a \tau^a / 2F_\pi) \quad , \quad u_\mu = u^\dagger (i\partial_\mu + r_\mu) u - u(i\partial_\mu + l_\mu) u^\dagger,$$

$$D_\mu \mathbf{B} = \partial_\mu \mathbf{B} - i\Gamma_\mu^a \hat{I}^a \mathbf{B} \quad , \quad \Gamma_\mu = \frac{1}{2}(u^\dagger (i\partial_\mu + r_\mu) u + u(i\partial_\mu + l_\mu) u^\dagger),$$

$$\chi = 2B_0(s + ip) \quad , \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u,$$

$$(L, R) : \mathbf{B} = h(L, R, u)\mathbf{B} \quad , \quad (L, R) : u_\mu = h(L, R, u) u_\mu h^\dagger (L, R, u).$$

Baryon mass to 1-loop



$$m_{\mathbf{B}}(S) = N_c m_0 + \frac{C_{HF}}{N_c} S(S+1) + c_1 N_c M_\pi^2 + \delta m_{\mathbf{B}}^{1-loop+CT}(S),$$

$$\delta\Sigma_{(1-loop)} = i\,\frac{\mathring{g}_A^2}{F_\pi^2}\,\frac{1}{d-1}\,\sum_n G^{ia}\mathcal{P}_n G^{ia}\; I_{(1-loop)}(\delta m_n - p^0, M_\pi)\,,$$

$$\mathcal{L}_\Sigma^{CT} = \mathbf{B}^\dagger \left\{ \frac{m_1(N_c)}{N_c} + \frac{C_{HF1}(N_c)}{N_c^2}\hat{S}^2 + \frac{C_{HF2}(N_c)}{N_c^3}\hat{S}^4 + \mu_1(N_c)\chi_+ + \frac{\mu_2(N_c)}{N_c}\chi_+\hat{S}^2 \right.$$
$$\left. + \left(\frac{w_1(N_c)}{N_c} + \frac{w_2(N_c)}{N_c}\hat{S}^2 + \frac{w_3(N_c)}{N_c^3}\hat{S}^4 + (z_1(N_c)N_c + \frac{z_2(N_c)}{N_c}\hat{S}^2)\chi_+ \right)(iD_0 - \delta m) \right\} \mathbf{B}$$

LECs are of the form $X(N_c) = X_0 + X_1/N_c + \cdots$ and $X = X(\mu) + \gamma_X \lambda_\epsilon$.

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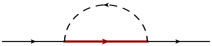

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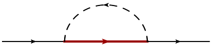

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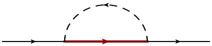

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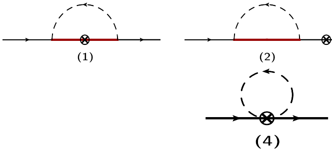
Axial current



$\sim \mathcal{O}(N_c^2)$

$\sim \mathcal{O}(N_c^0)$

- Axial current: $g_A^{BB'} = \mathring{g}_A + \langle \mathbf{B}' | \delta A^{ia}_{1-loop} | \mathbf{B} \rangle / \langle \mathbf{B}' | G^{ia} | \mathbf{B} \rangle$

$$\delta A^{ia}_{1-loop} = \delta A^{ia}_{1-loop}(1) + \delta A^{ia}_{1-loop}(2+3) + \delta A^{ia}_{1-loop}(4)$$

- Cancellation of the N_c-counting violating terms in the large N_c limit:

$$\delta A^{ia}_{1-loop}(1) + \delta A^{ia}_{1-loop}(2+3) \Big|_{N_c \to \infty} = -i \frac{\mathring{g}_A^3}{3F_\pi^2} \left\{ \frac{1}{2} [[G^{jb}, G^{ia}], G^{jb}] \frac{\partial}{\partial \mathfrak{p}^0} I_{1-loop}(\mathfrak{p}^0, M_\pi) + \cdots \right\}$$

- Counter-term lagrangian

$$\mathcal{L}_A^{CT} = \mathbf{B}^\dagger u_{ia} \left(\frac{C_0^A}{N_c} G^{ia} + \frac{C_1^A}{4} \{\chi_+, G^{ia}\} + \frac{C_2^A}{N_c^2} \{\hat{S}^2, G^{ia}\} + \frac{C_3^A}{N_c} [\hat{S}^2, G^{ia}] + \frac{C_4^A}{N_c} S^i I^a \right) \mathbf{B}$$

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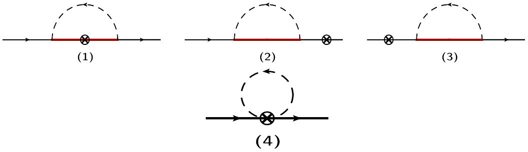

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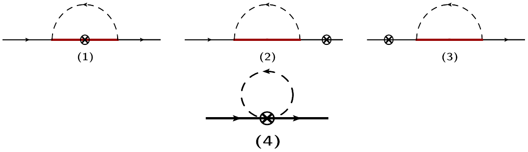

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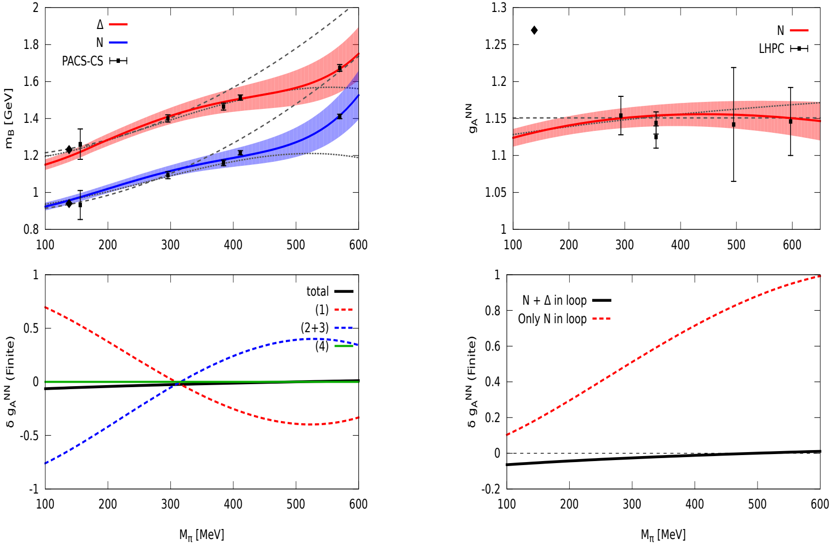
Analysis of LQCD results

to N^{2}LO, meaning $\mathcal{O}(\xi^3)$ in masses and $\mathcal{O}(\xi^2)$ in g_A



Analysis of LQCD results (some numbers)

- Fitted LECs: m_0, C_{HF}, $\mathring{g}_A$, c_1, μ_2, z_1 and C_1^A

Collab.	Order	m_N [MeV]	m_Δ [MeV]	g_A^{NN}	χ^2	χ^2/DOF
PACS-CS + LHP	LO	935(17)	1236(17)	1.15(1)	20.15	3.36
	NLO	957(14)	1222(14)	1.13(1)	28.80	3.20
	N^2LO	957(14)	1195(14)	1.131(12)	41.79	3.48
LHP + LHP	LO	928(14)	1306(15)	1.15(1)	59.69	9.95
	NLO	961(9)	1257(10)	1.13(1)	73.08	8.12
	N^2LO	886(10)	1180(12)	1.08(1)	147.97	12.33
PACS-CS + ETM	LO	936(17)	1236(17)	1.162(13)	21.01	2.10
	NLO	958(15)	1222(14)	1.139(11)	29.93	2.30
	N^2LO	957(14)	1192(14)	1.158(19)	42.89	2.68
LHP + ETM	LO	928(14)	1307(14)	1.162(12)	60.55	6.06
	NLO	960(9)	1256(10)	1.137(11)	74.06	5.69
	N^2LO	895(10)	1193(12)	1.055(12)	143.13	8.95

- Sigma terms evaluated at the physical point

$$\sigma^N = 65 \pm 8\mathrm{MeV}$$
$$\sigma^\Delta = 90 \pm 9\mathrm{MeV}$$

Summary

- Chiral Symmetry and Large N_c are fundamental features of QCD.
- It is very important to have a theoretical framework where both are consistently incorporated.
- This is possible thanks to the ξ-expansion where one links

$$1/N_c = \mathcal{O}(\xi) = \mathcal{O}(p)$$

- This expansion corresponds to the SSE with large N_c constraints.
- At N^{2}LO in the ξ-expansion we have a satisfactory description of LQCD results for N and Δ masses and g_A.
- A important cancellation at $N_c = 3$ between diagrams results into a mild quark mass dependence of g_A which is not manifest when the Δ is off.
- The physical g_A cannot be fitted along with LQCD results. A systematic analysis of finite size and volume effects needs to be done.