



Nucleon Excited States-Theoretical Issues

- Overview: why study nucleon and meson excited states?
- Four QCD-based models of hadron structure
- How experiments can help resolve theoretical issues: salient examples
- Urgently needed theoretical developments



Why study nucleon and meson excited states?

1. Uniqueness: bound states of strongly-interacting, relativistic confined systems
2. Identification of important effective degrees of freedom in low-energy QCD
3. Potential discovery of entirely new forms of matter: glueballs, hybrids

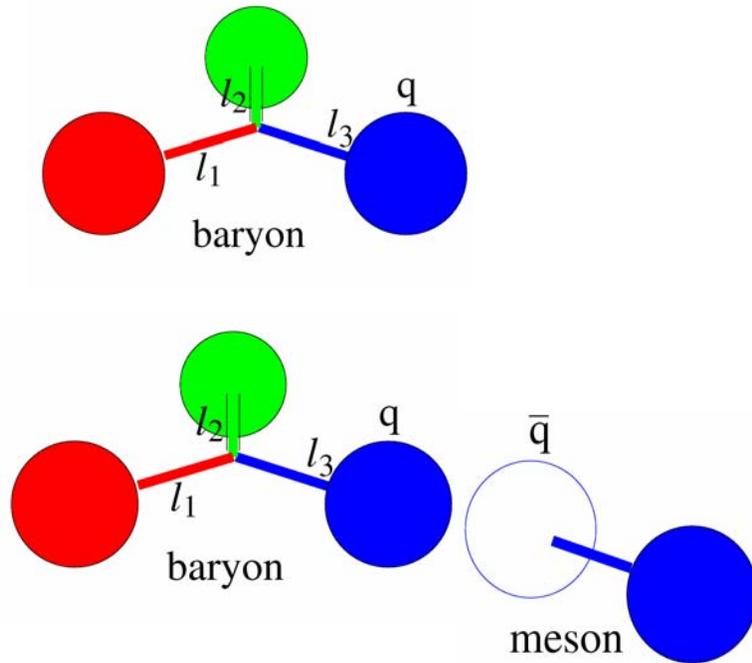


Uniqueness

- Unique?
- Nucleons interact strongly in nuclei...
 - Can isolate relevant low-energy d.f. (nucleons)
 - Can directly probe two-body potential in experiment
 - Few body systems of most A exist to test model N-N, N-N-N,... potentials
 - Can systematically expand around non-relativistic limit
 - Heavy effective degrees of freedom
 - Relatively large states



Uniqueness...



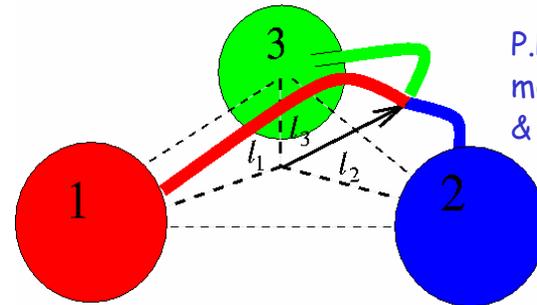
- Elementary d.f. are confined
 - Can only indirectly infer low-energy interaction
- Only qqq , $q\bar{q}$, $(qqqq\bar{q}')$ exist as bound states
- Not non-relativistic systems (unless all quarks heavy)



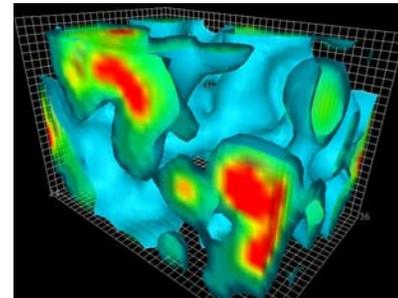
Effective degrees of freedom

Low-energy QCD:

- Constituent quarks (CQs), confined by flux tubes?
- Confined CQs, elementary meson fields?
- Confined CQs, gas of instantons?



P. Page, S.C. Flux-tube model of baryons & hybrids

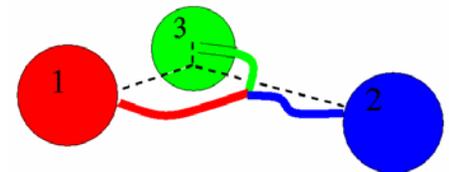
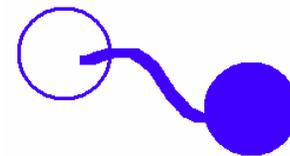
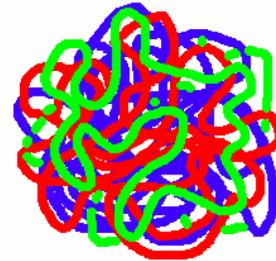


D. Leinweber et al.
QCD vacuum action density

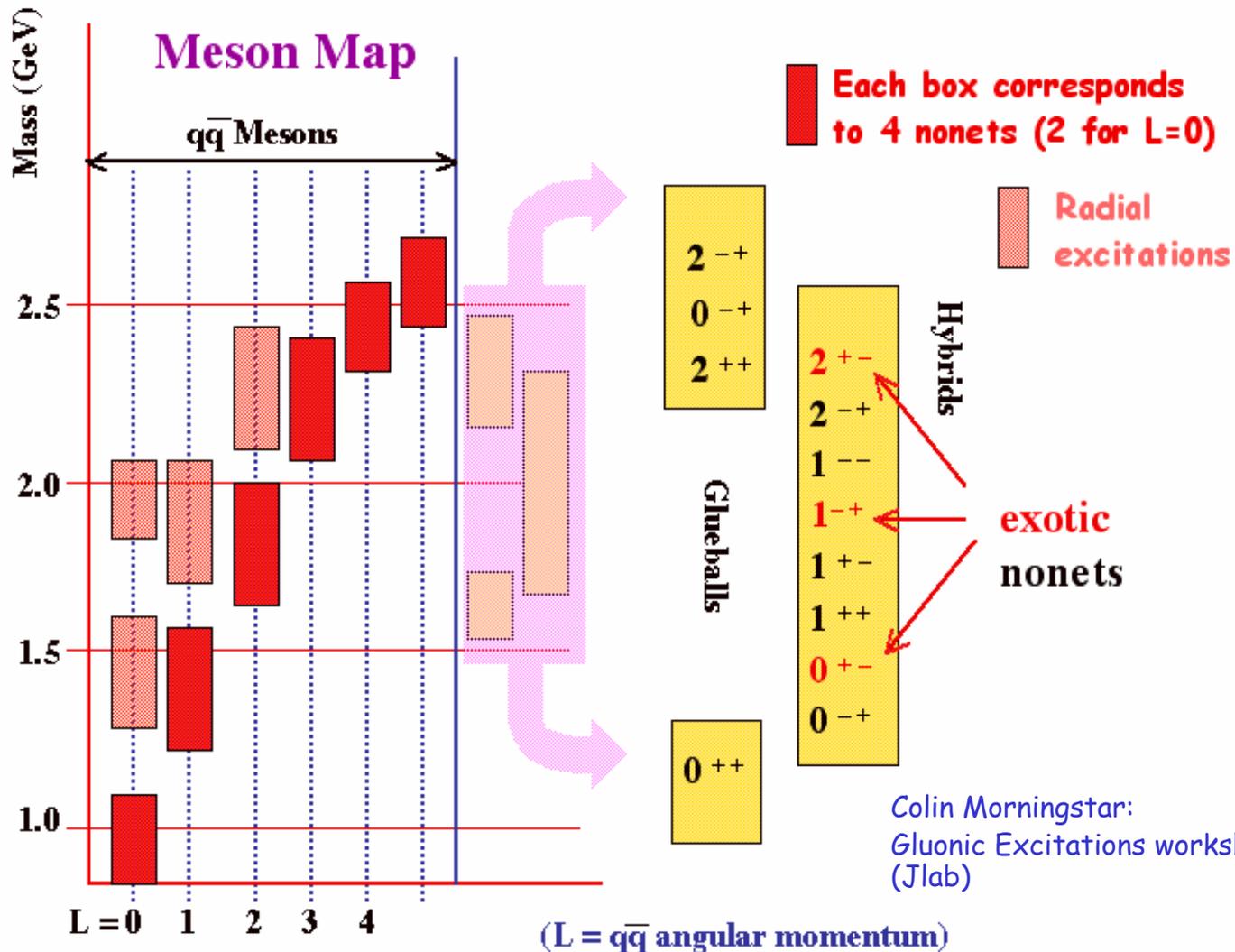


Entirely new forms of matter

- Gauge-field configurations provide confining potential
 - States of pure glue exist
 - Exotic states not light
 - Others mix with $q\bar{q}$
 - Glue may not be in ground state
 - Hybrid mesons: exotic quantum numbers
 - Hybrid baryons: no exotics, mix with qqq



Glueballs and hybrid mesons



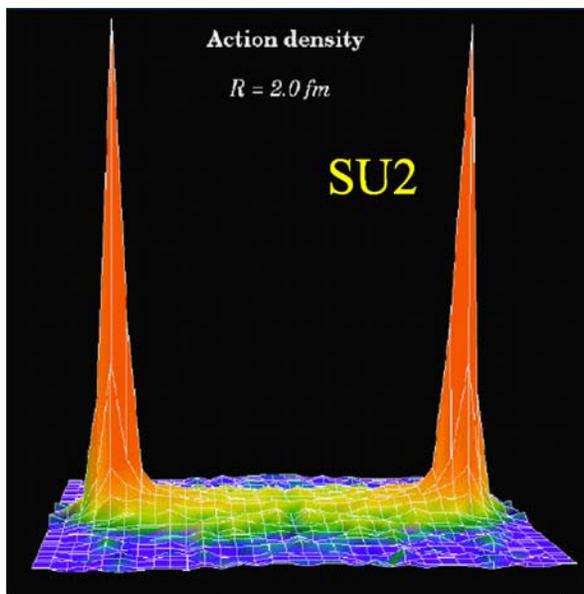
QCD-based models of hadron structure

- Why do we need models?
- Can solve for certain quantities in QCD using lattice gauge theory
 - Masses of lightest few states with given quantum numbers (especially pure glue)
 - Hadronic matrix elements of electro-weak operators
 - Especially heavy-quark hadrons: f_B, f_D, \dots
 - Heavy-quark potentials

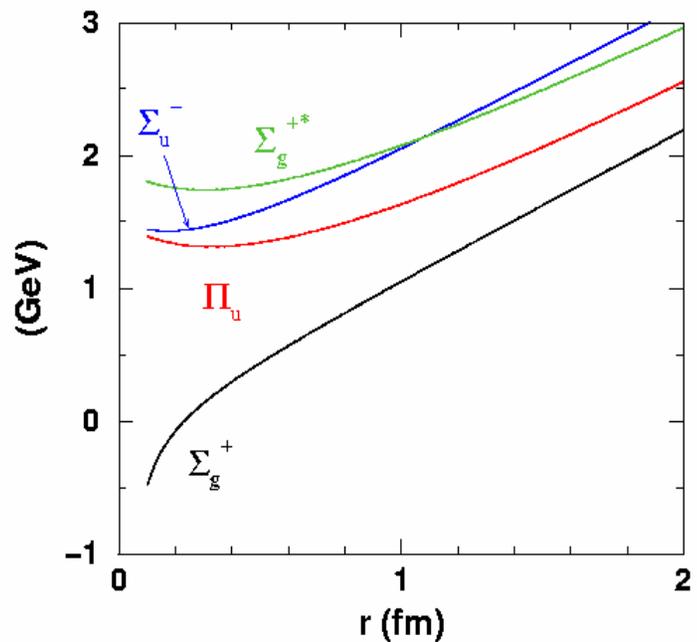


Why do we need models?

- Heavy quark potentials in mesons
 - Action density has flux-tube at large r
 - Potentials deviate from flux-tube expectations at small r



Bali et al.



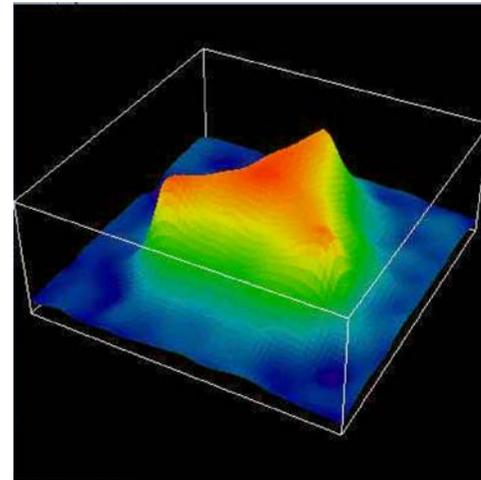
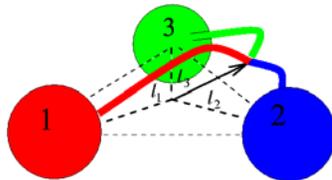
Juge, Kuti, & Morningstar



Heavy quark potentials in baryons

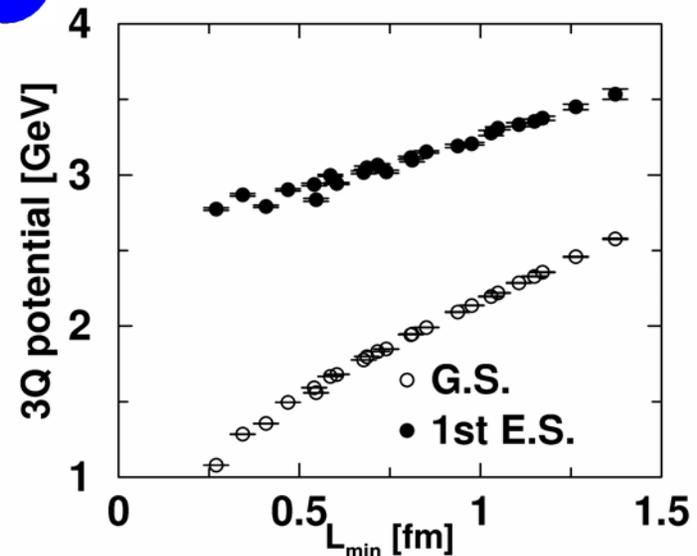
- Abelian action distribution of gluons and light quarks nr. QQQ

Ichie, Bornyakov,
Struer & Schierholz



Takahashi & Suganuma:

- Calculate $L_{\min} = l_1 + l_2 + l_3$
- plot ground and first excited state energies of glue (V_B and V_{H1}) vs. L_{\min}



Why do we need models...?

- Description of full spectrum requires models based on QCD
 - Quarks are confined
 - pair-wise linear confinement
 - string potential L_{\min} , scale set by meson string tension
 - Spin and flavor-dependent "hyperfine" interactions are present between quarks
 - Models differ in mechanism for short-distance, spin-dependent interactions
 - Different pictures of the important physics!



Gluon-exchange models

- Emphasize:
 - Connection to heavy-quark limit
 - Universality of meson and baryon physics
- Quarks exchange gluons at short distance
 - color-magnetic hyperfine interactions
 - e.g. DeRujula, Georgi, Glashow (ground states)

$$M = \sum_{i=1}^3 m_i + \frac{2\alpha_s}{3} \frac{8\pi}{3} \langle \delta^3(\mathbf{r}) \rangle \sum_{i<j=1}^3 \frac{\mathbf{S}_i \cdot \mathbf{S}_j}{m_i m_j}$$



Gluon-exchange models...

- Predict presence of additional tensor interactions

$$H_{\text{hyp}}^{ij} = \frac{2\alpha_s}{3m_i m_j} \left\{ \frac{8\pi}{3} \mathbf{S}_i \cdot \mathbf{S}_j \delta^3(\mathbf{r}_{ij}) + \frac{1}{r_{ij}^3} \left[\frac{3(\mathbf{S}_i \cdot \mathbf{r}_{ij})(\mathbf{S}_j \cdot \mathbf{r}_{ij})}{r_{ij}^2} - \mathbf{S}_i \cdot \mathbf{S}_j \right] \right\}$$

- Tensor

- mixes states split by contact interaction
- D-waves in the nucleon and Δ

- Where are spin-orbit interactions?



One-boson exchange models

- Emphasize:
 - aspects of QCD at low momenta imposed by chiral symmetry
 - Goldstone-boson nature of π , K , η , ... fields
- Bosons exchanged between quarks

$$H_\chi \sim - \sum_{i < j} \frac{V(\mathbf{r}_{ij})}{m_i m_j} \lambda_i^F \cdot \lambda_j^F \sigma_i \cdot \sigma_j$$

- No spin-orbit from OBE (confinement?)



Instanton-based model

- Another flavor-dependent possibility: instanton-induced interactions
- Present if qq in S-wave, $l=0$, $S=0$ state

$$\langle q^2; S, L, T | W | q^2; S, L, T \rangle = -4g \delta_{S,0} \delta_{L,0} \delta_{T,0} \mathcal{W}$$

- W is a contact interaction (has range λ)
 - causes no shifts in Δ^* masses
 - No tensor interaction, or spin-orbit forces
- Applied to excited states
 - Blask, Bohn, Huber, Metsch & Petry
 - solve Bethe-Salpeter equation



Instanton-induced interactions

- Quarks confined by linear q-q potential

$$V(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = A_3 + B_3 \sum_{i < j} |\mathbf{r}_i - \mathbf{r}_j|$$

- Relativistic treatment, so need to choose Dirac structure of potential

$$A_3 = a \frac{3}{4} \left[\mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} + \gamma^0 \otimes \gamma^0 \otimes \mathbf{1} + \gamma^0 \otimes \mathbf{1} \otimes \gamma^0 + \mathbf{1} \otimes \gamma^0 \otimes \gamma^0 \right]$$

$$B_3 = b \frac{1}{2} \left[-\mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} + \gamma^0 \otimes \gamma^0 \otimes \mathbf{1} + \gamma^0 \otimes \mathbf{1} \otimes \gamma^0 + \mathbf{1} \otimes \gamma^0 \otimes \gamma^0 \right]$$

- Form chosen to reduce spin-orbit effects
- Reproduces correct "Regge trajectories"



Dynamical approaches

- Are **all** (or many) excited baryon states dynamically generated?
 - States are poles in scattering matrix
 - Potentials chosen to reproduce low-energy scattering data (chiral dynamics)
 - Generate poles by iterating interaction based on potentials, coupled channels
 - E. Oset et al., M. Lutz, S. Krewald et al.



Important required developments

- Experiment + theoretical analysis:
 - Will ultimately sort out (or synthesize) these pictures
 - Steer lattice groups toward important quantities to calculate



Important required developments

1. Better determination of properties of states known to exist (say PDG 4^* , 3^*)
2. Verification/removal of poorly determined states, discovery of "missing" resonances
3. Evidence of overpopulation of states in some partial waves, decay signatures?
 - Hybrids
 - Possible N , Σ partners of $S=+1$ pentaquarks
4. Development of coupled-channel analysis (required for 1.-3.)



1. Properties of existing states

- E.g.: some models predict (tensor) mixing between S_{11} and D_{13} ($N^{*1/2}$ -, $N^{*3/2}$ -) states
 - Mixing angles differ in different approaches (no mixing at all with instantons)
- Theory: calculate mixing angles, effects on decays to:
 - $N\pi$, $N\eta$, ΛK , $N\pi\pi$ ($N\rho$, $\Delta\pi$,...)
- Experiment + analysis: find accurate partial widths



Mixing angles

- Physical states are admixtures of two possible L,S combinations

$$N(1535)1/2^- = \cos(\theta_S) N^2P1/2^- - \sin(\theta_S) N^4P1/2^-$$

$$N(1650)1/2^- = \sin(\theta_S) N^2P1/2^- + \cos(\theta_S) N^4P1/2^-$$

$$N(1520)3/2^- = \cos(\theta_D) N^2P3/2^- - \sin(\theta_D) N^4P3/2^-$$

$$N(1700)3/2^- = \sin(\theta_D) N^2P3/2^- + \cos(\theta_D) N^4P3/2^-$$

- Lattice QCD should also be able to determine θ_S and θ_D
 - enough time (CPU and elapsed!)
 - clever choice of correlators...



Properties of existing states...

- E.g. is the Roper resonance:
 - A qqq (radial) excitation?
 - Dynamically-generated bound state?
 - S. Krewald et al., iterated $N\sigma$ interaction
 - no elementary excitation needed to fit data!
 - Hybrid? Pentaquark?
 - Bag/flux-tube models: lightest hybrids include P_{11} ($N_{1/2^+}$) states at 1500/1900 MeV
 - Chiral-soliton picture anti-decuplet $N(1647)$
 - More than one of the above?



Roper resonance

- Photo-couplings incompatible with (OGE) qqq interpretation
 - Accurate determinations of photo-couplings (in coupled-channel analysis) required
- EM form factor from $e-N$
 - Should fall off rapidly if state is predominantly a baryon-meson effect
- Focus on P_{11} partial wave (also other states)
- Lattice:
 - Roper heavy in quenched calculations, lighter (threshold?) as pion mass is lowered: more development needed!



2. Missing and 1^* , 2^* states

- Why bother finding new states or confirming/removing old ones?
 - E.g.: current debate about chiral-symmetry restoration in spectrum
 - Prediction of pairing of +ve/-ve parity states with same J higher in spectrum
 - 1^* states $N_{1/2^+}(2100)$ & $N_{1/2^-}(2090)$ identified as doublet (Cohen and Glzman)
 - PDG: $S_{11}(2090)$ "any structure above 1800 MeV"
 1. 1885 +/- 30 MeV vs. 1928 +/- 59 (43 MeV)
 2. 2125 +/- 75 vs. 2180 +/- 80 (55 MeV)
 3. 2050 +/- 20 vs. 1880 +/- 20 (-165 MeV)



Missing resonances

- Symmetric (qqq) potential models:
 - Agree on **number** of excited states of a given character
 - Disagree on their **place** in spectrum, especially at higher energy
 - many positive (and doubly-excited negative) parity states not seen in analyses of data: "missing" resonances
- Largest differences in predictions for (formation &) decay-channel couplings
 - Model proponents **must** calculate baryon-meson (all open channels) and photo- couplings



Missing resonances

- Finding several missing (+ve parity) resonances:
 - Would verify symmetric qqq correct picture
- PDG states established in analyses of $N\pi$ elastic scattering
 - States which couple weakly to $N\pi$ will be "missing"
 - **Evidence** for them should show up in other ($N\pi\pi$, ΔK ,...) final states, excited with EM probes from nucleon targets (make N^* or Δ^*)
 - Their existence will be **established** in multi-channel analyses of several final states



3. Unconventional states

- All baryon J^P quantum numbers possible with qqq : no exotic hybrids
 - Light hybrid baryon states (flux-tube):
 - $S_{qqq}=1/2$ states: $N1/2^+$, $N3/2^+$ at $\sim 1870 \pm 100$ MeV
 - $S_{qqq}=3/2$ states: $D1/2^+$, $D3/2^+$, $D5/2^+$ approx. 2075 ± 100 MeV
 - Theory needs to examine decays
 - Easily identified decay signatures?
 - Electromagnetic couplings? (Burkert and Li)



Unconventional states...

- Partners N, Σ of Θ^+ with $J^P=1/2^+$
 - will mix with conventional states
 - May have significant hidden strangeness: decays?
- Because of mixing, discovery may require overpopulation of states
 - Another important reason to carefully study P_{11} ($P_{13}, P_{31}, P_{33}, F_{35}$) partial wave!



4. Development of coupled-channel analysis

- Grand challenge for hadron structure physics:
 - Extraction of model-independent information about overlapping, broad resonances from EM-production and hadron scattering data



Analysis of N^* (and meson) data

Masses, widths, decay branches,
photocouplings, EM form factors

from

Partial wave data in many (all open)
channels; multipoles in γN

from

Scattering data



Analysis of N^* (and meson) data...

- Necessary ingredients?
- Coupled-channel unitarity
 - E.g. K-matrix approach: (D.M. Manley, KSU)

$$T = \frac{1 + iK}{1 - iK}, \quad K = K^\dagger$$

- K contains resonance information, background terms
- CMB (Cutkosky; Vrana, Dytman and Lee) model:
 - all channels re-scatter into all others via loops
- Effective Lagrangians: T. Sato and T.-S. H. Lee; GWU group: C. Bennhold, H. Haberzettl; Mainz group: L. Tiator, D. Drechsel,...



Analysis of N^* (and meson) data...

- Fitting ambiguities can be lessened by imposing necessary analytic structure of amplitudes
 - Resonances appear as poles
 - Thresholds cause branch cuts, amplitudes on various sheets related
 - Analytic structure can be made compatible with unitarity (CMB model)



Analysis of N^* (and meson) data...

- Theory must provide:

- Strong form factors: e.g. $N(1535)$ to $N\eta$ as a function of decay momentum (for loops)

$$\pi N \rightarrow N(1535) \rightarrow N(-k)\eta(k) \rightarrow N(1535) \rightarrow \pi N$$

- Open threshold causes cusp in $N\pi$ elastic scattering amplitude
 - Amplitude is integral, involves form factor: not an observable!



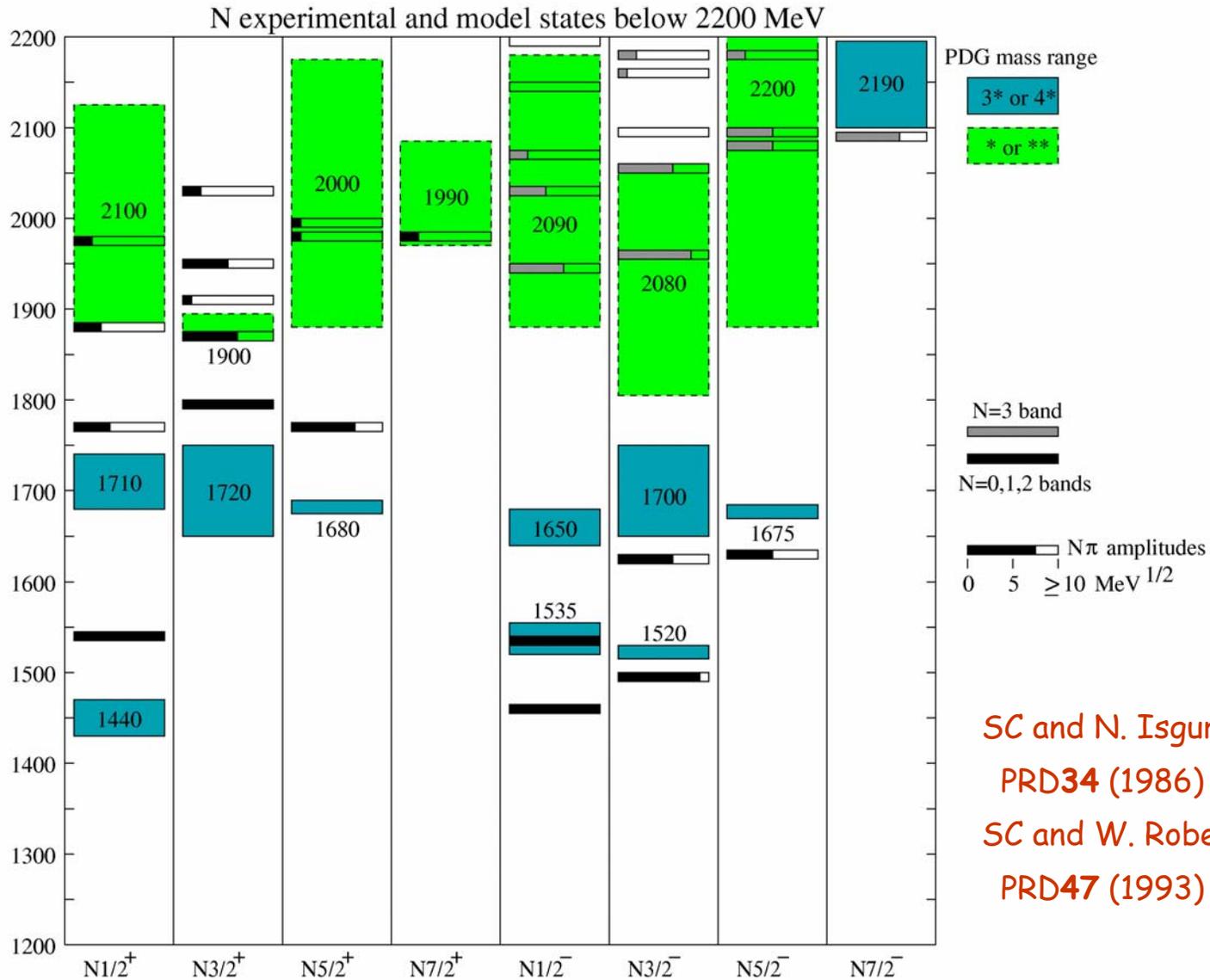
Theoretical ingredients...

- Theory must provide:
 - Technique for constraining background amplitudes
 - Based on physics of competing processes
e.g. t-channel (meson) exchange
 - Consistent with unitarity, analyticity, gauge invariance

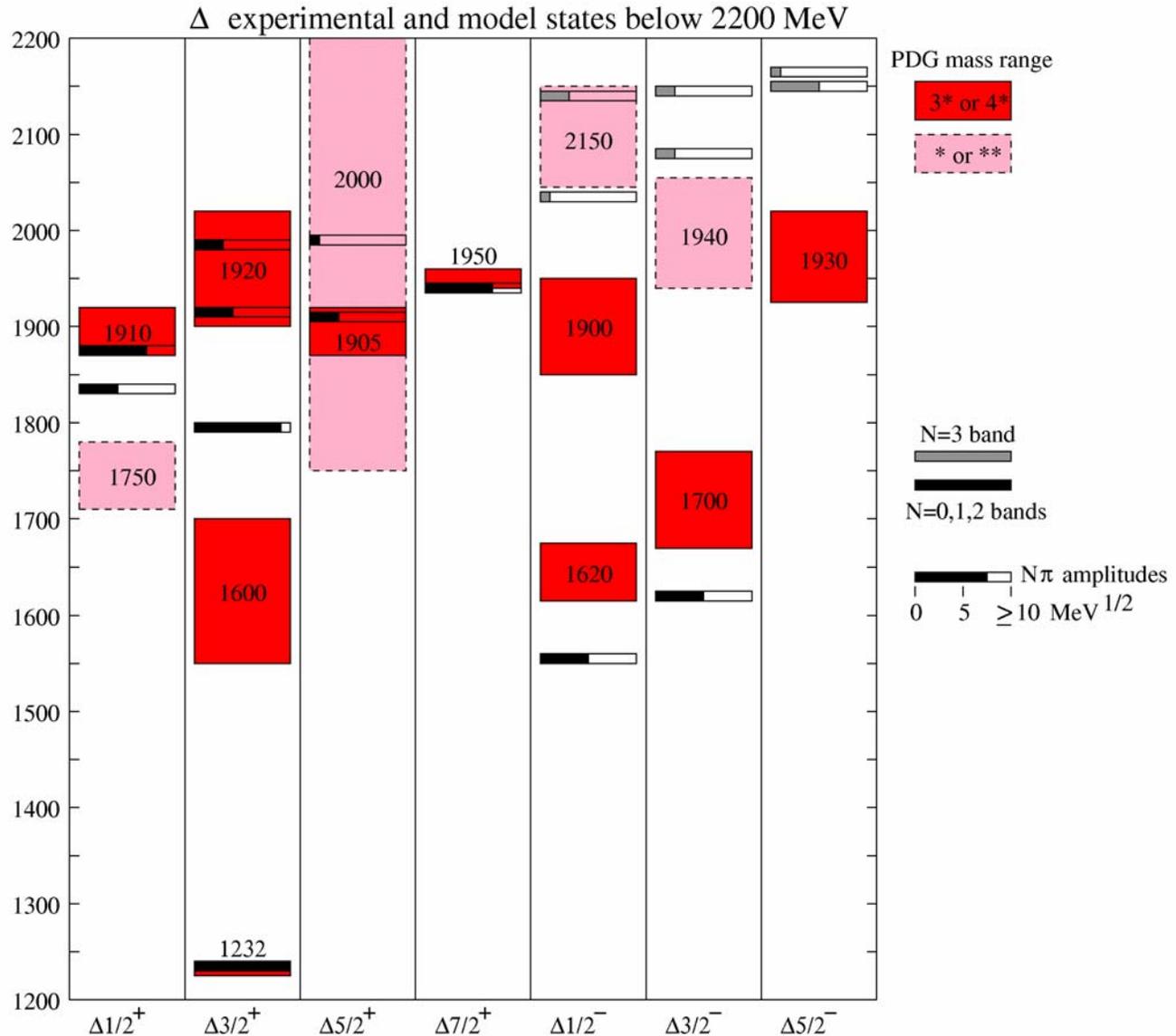




Nucleon model states and $N\pi$ couplings

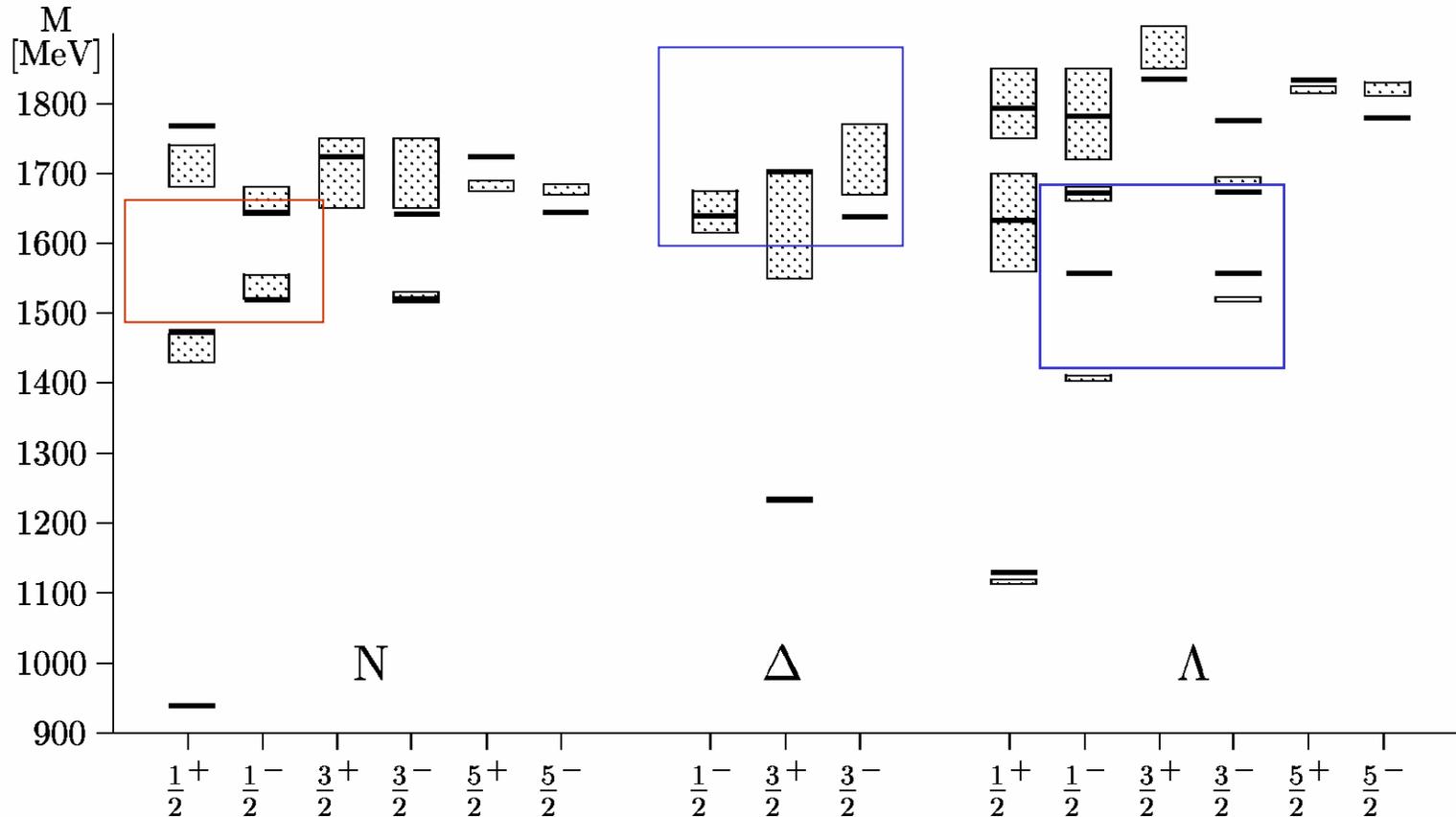


Δ model states and $N\pi$ couplings



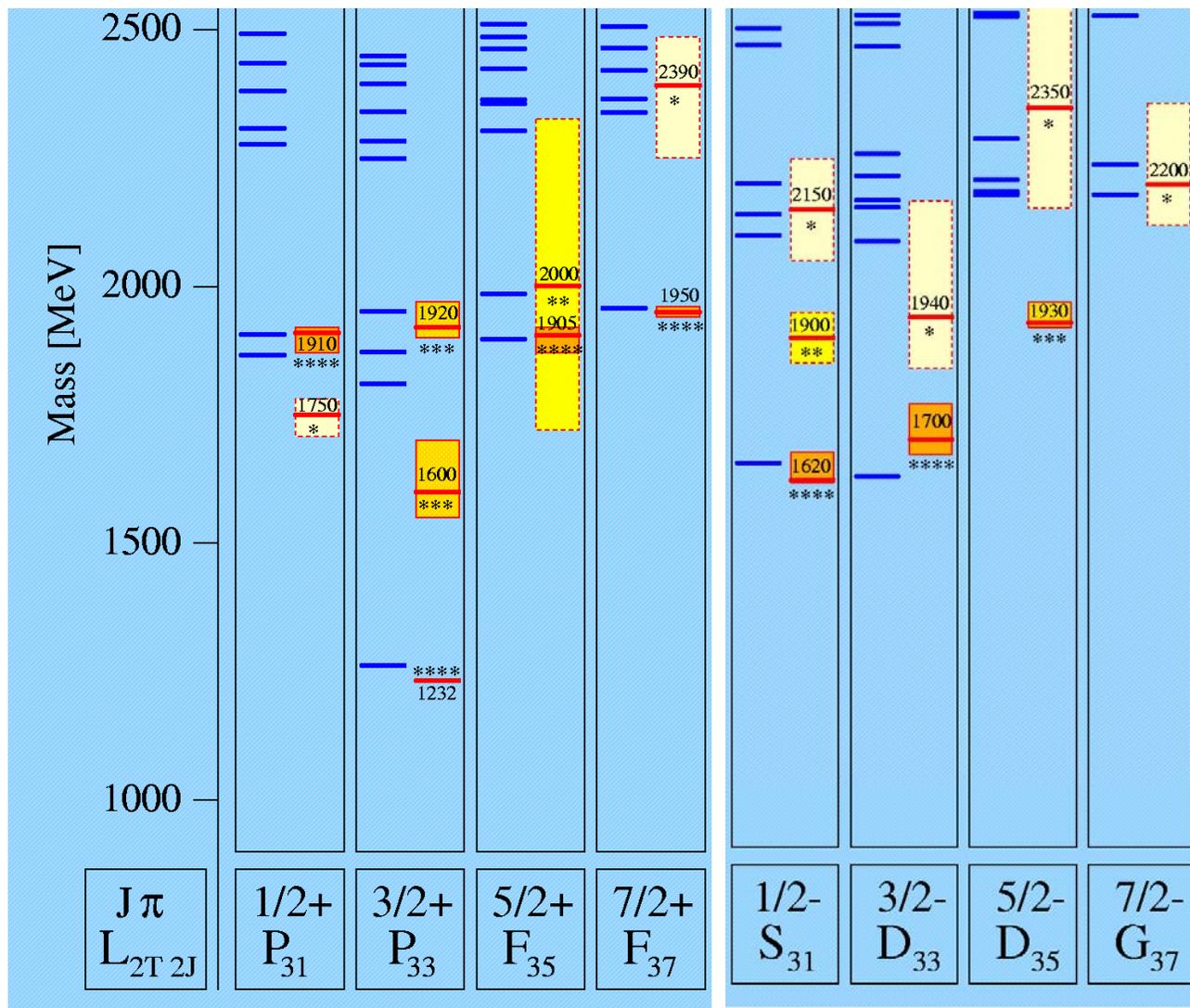
OBE spectrum...

- OBE Results for spectrum: *Glazman, Plessas, Theussl, Wagenbrunn, & Varga*



Instanton-induced interactions...

- spectrum of Δ^* only from confining potential
- Blask, Bohn, Huber, Metsch & Petry



N^* spectrum from 't Hooft's force

