

# (e,e'p) and Nuclear Structure

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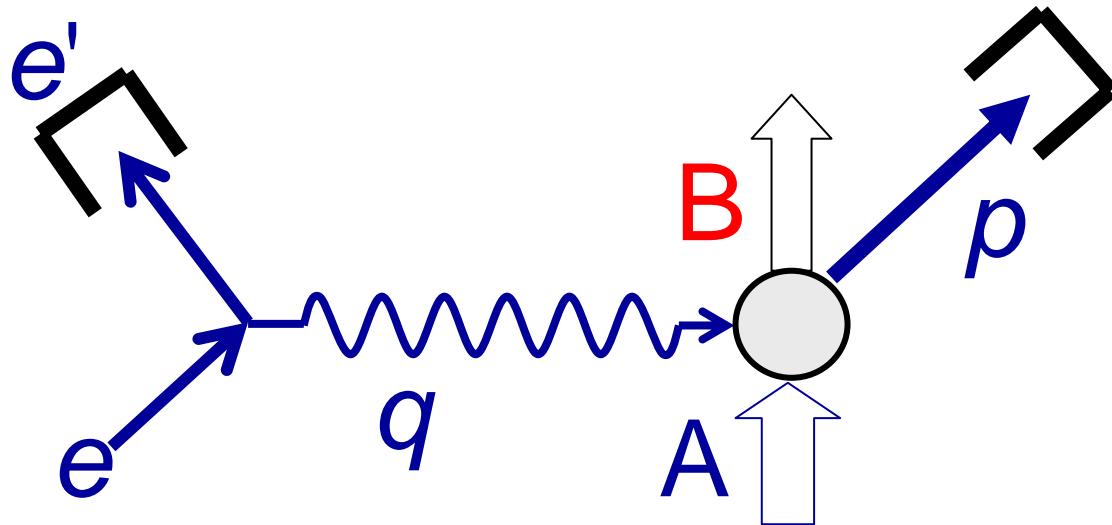
**E. Voutier**

**L. Weinstein**

# Outline

- Introduction
- Background
  - Experimental
  - Theoretical
- Nuclear Structure
- Medium-modified nucleons
  - Cross sections
  - Polarization transfer
- Studies of the reaction mechanism
- Few-body nuclei
  - The deuteron
  - $^{3,4}\text{He}$

A(e,e'p)B

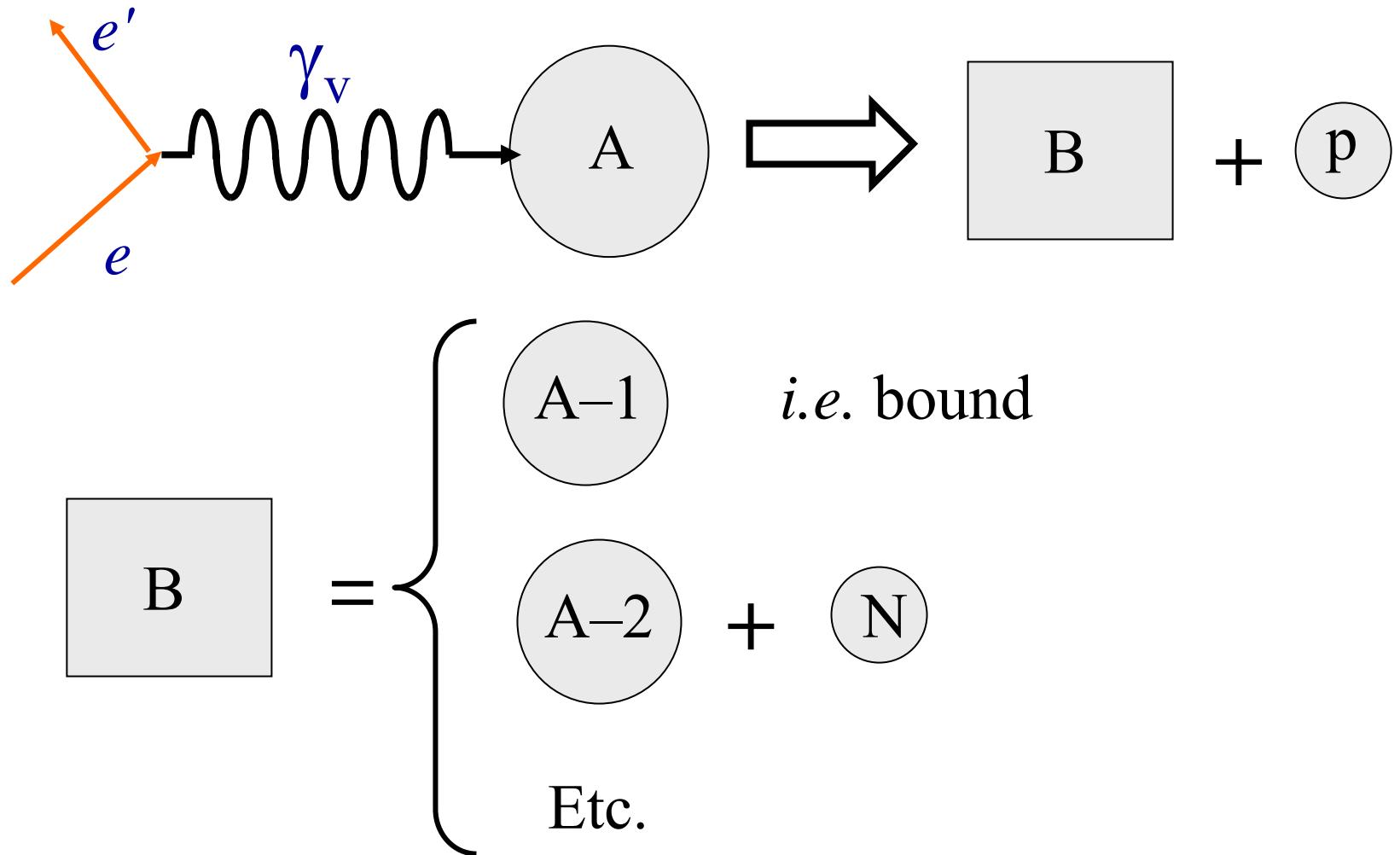


Known:  $e$  and  $A$

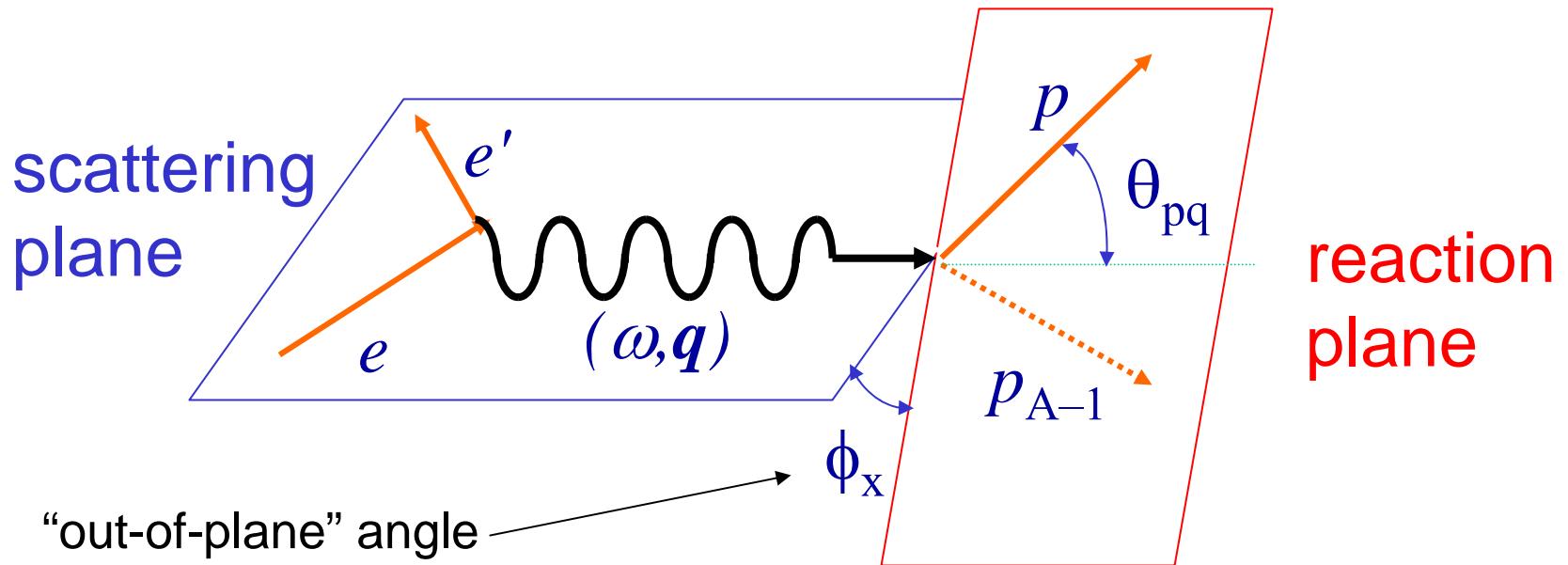
Detect:  $e'$  and  $p$

Infer:  $p_m = q - p = p_B$

# (e,e'p) - Schematically



# Kinematics



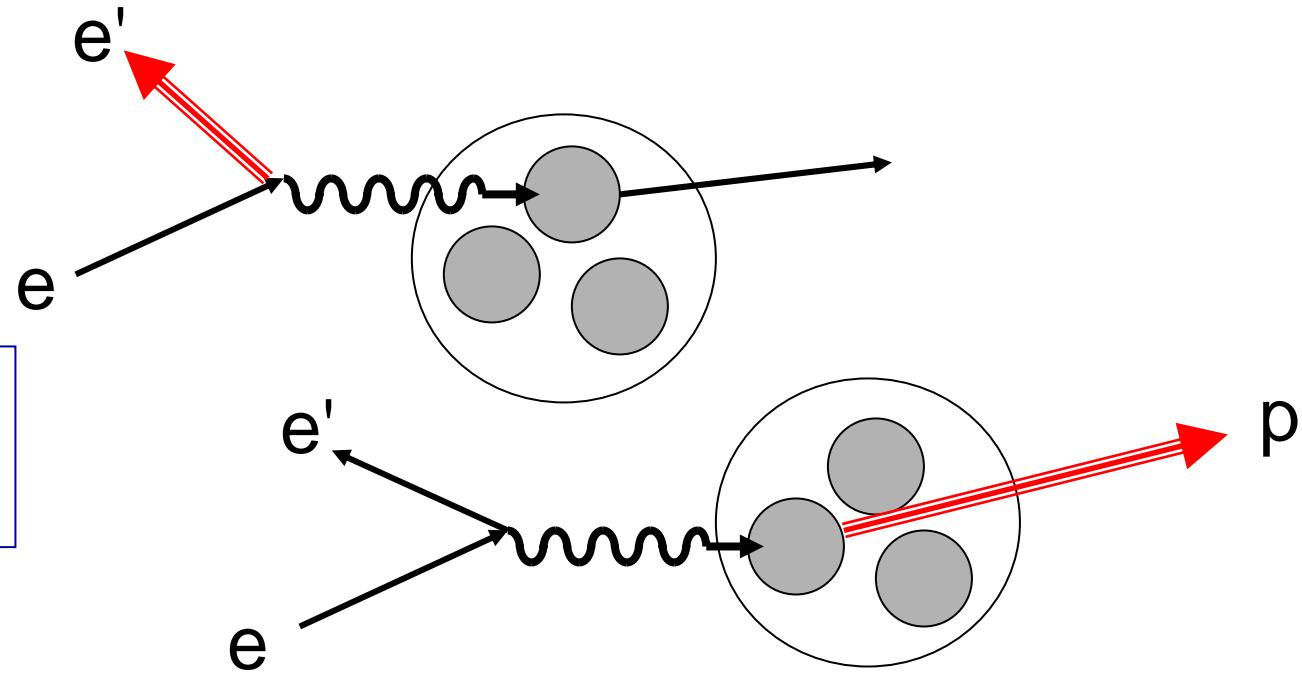
In ERL<sub>e</sub>:  $Q^2 \equiv -q_\mu q^\mu = \mathbf{q}^2 - \omega^2 = 4ee' \sin^2\theta/2$

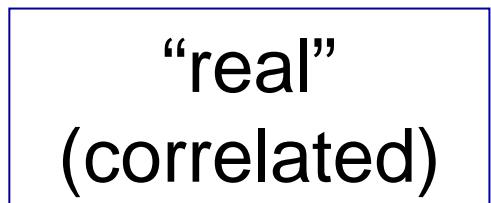
Missing momentum:  $\mathbf{p}_m = \mathbf{q} - \mathbf{p} = \mathbf{p}_{A-1}$

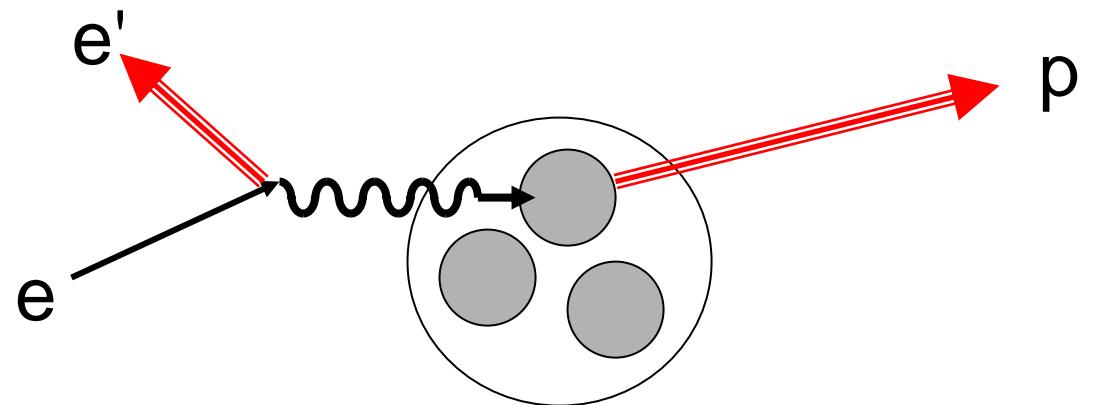
Missing mass:  $\varepsilon_m = \omega - T_p - T_{A-1}$

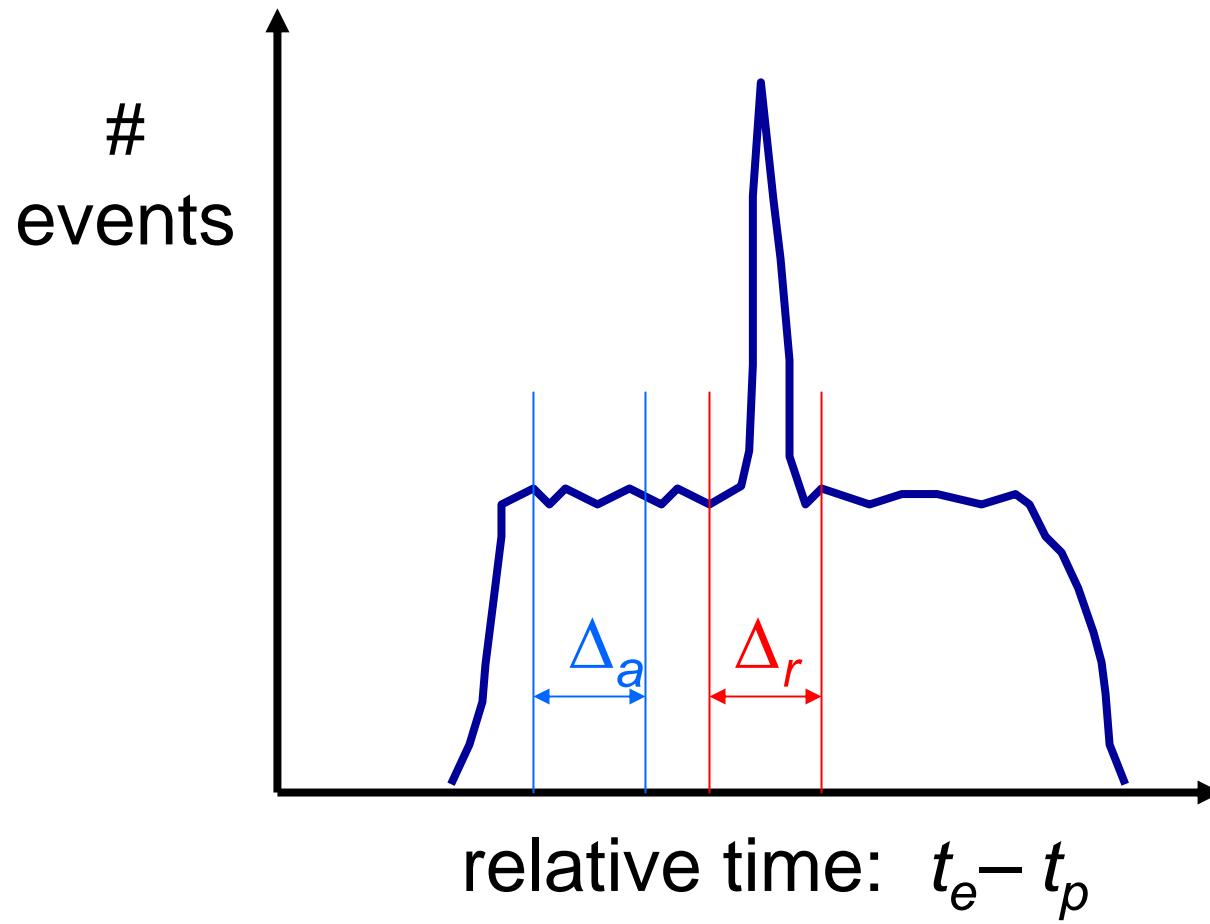
Some (Very Few)  
Experimental Details ...

  
Detected



 "real"  
(correlated)





$$C(x) = [C(x) \cap \text{Real}] - \frac{\Delta_r}{\Delta_a} \times [C(x) \cap \text{Accidental}]$$

$$\begin{aligned}\text{Accidentals Rate} &= R_e \times R_p \times \Delta\tau/\text{DF} \\ &\propto I^2 \Delta\tau/\text{DF}\end{aligned}$$

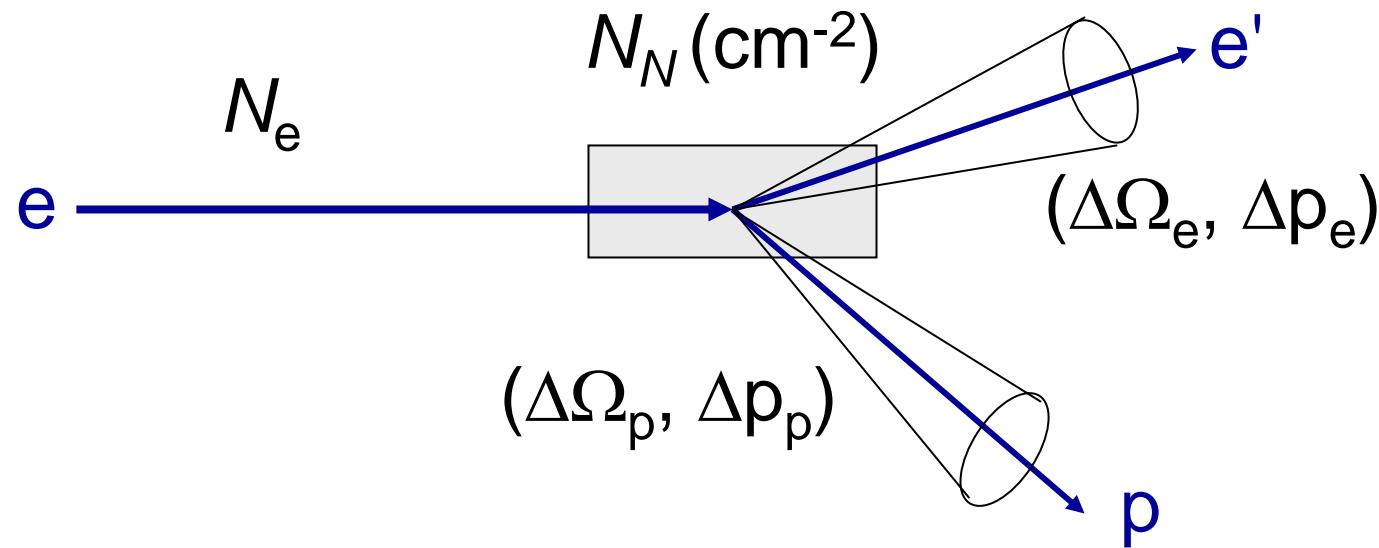
$$\begin{aligned}\text{Reals Rate} &= R_{eep} \\ &\propto I\end{aligned}$$

$$S:N = \text{Reals/Accidentals} \propto \text{DF } / (\Delta\tau * I)$$

**Compromise:**

Optimize  $S:N$  and  $R_{eep}$

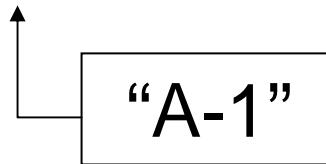
# Extracting the cross section



$$\left\langle \frac{d^6\sigma}{d\Omega_e d\Omega_p dp_e dp_p} \right\rangle = \frac{\text{Counts}}{N_e N_N \Delta\Omega_e \Delta\Omega_p \Delta p_e \Delta p_p}$$

Some Theory ...

# Cross Section for A(e,e'p)B in OPEA



$$d\sigma_{\text{lab}} = \frac{1}{\beta} \frac{m_e}{e} \sum_{if} \left| M_{fi} \right|^2 \left[ \frac{m_e}{e'} \frac{d^3 k'}{(2\pi)^3} \right] \left[ \frac{m}{E} \frac{d^3 p}{(2\pi)^3} \right]$$
$$\times (2\pi)^4 \delta^4(P + P_{A-1} - Q - P_A)$$

where

$$M_{fi} = \frac{4\pi\alpha}{Q^2} \left\langle k' \lambda' \left| j_\mu \right| k \lambda \right\rangle \left\langle Bp \left| J^\mu \right| A \right\rangle$$

**Current-Current Interaction**

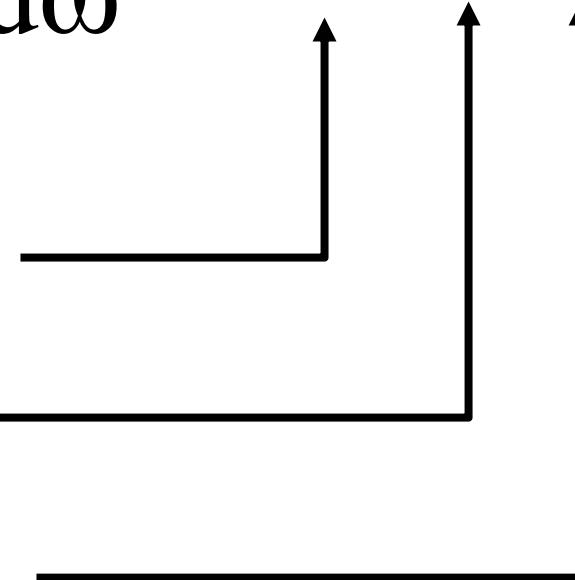
# Square of Matrix Element

$$\sum_{if} \left| M_{fi} \right|^2 = \left( \frac{4\pi\alpha}{Q^2} \right)^2 \underbrace{\sum_{if} \left\langle k' \lambda' | j_\mu | k \lambda \right\rangle^* \left\langle k' \lambda' | j_\nu | k \lambda \right\rangle}_{\mathcal{W}^{\mu\nu}} \\ \times \underbrace{\sum_{if} \left\langle Bp | J^\mu | A \right\rangle^* \left\langle Bp | J^\nu | A \right\rangle}_{\eta_{\mu\nu}}$$

# Cross Section in terms of Tensors

$$\frac{d^6\sigma}{d\Omega_e d\Omega_p dp d\omega} = \sigma_M \eta_{\mu\nu} W^{\mu\nu}$$

Mott cross section



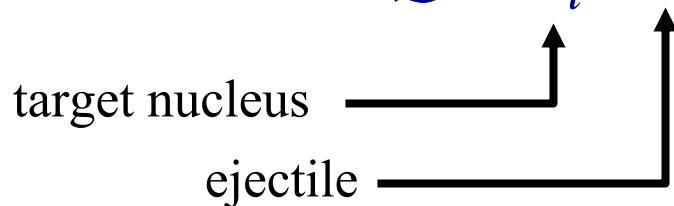
Electron tensor

Nuclear tensor

# Consider Unpolarized Case

Lorentz Vectors/Scalars

3 indep. momenta:  $Q, P_i, P$  ( $P_{A-1} = Q + P_i - P$ )



6 indep. scalars:  $\cancel{P_i^2}, \cancel{P^2}, Q^2, Q \cdot P_i, Q \cdot P, P \cdot P_i$

$$\begin{array}{c} \uparrow \\ \text{---} \\ \uparrow \end{array} = M_A^2 = m^2$$

# Nuclear Response Tensor

$$\begin{aligned} W^{\mu\nu} = & X_1 g_{\mu\nu} + X_2 q^\mu q^\nu + X_3 p_i^\mu p_i^\nu \\ & + X_4 p^\mu p^\nu + X_5 q^\mu p_i^\nu + X_6 p_i^\mu q^\nu \\ & + X_7 q^\mu p^\nu + X_8 p^\mu q^\nu + X_9 p^\mu p_i^\nu \\ & + X_{10} p_i^\mu p^\nu \\ & + (\text{PV terms like } \epsilon_{\mu\nu\rho\sigma} q_\rho p_\sigma) \end{aligned}$$

$X_i$  are the response functions

# Impose Current Conservation

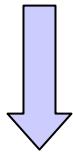
$$S^\nu \equiv q_\mu W^{\mu\nu} = 0$$

$$T^\mu \equiv q_\nu W^{\mu\nu} = 0$$

Then  $q_\nu S^\nu = 0$ ,  $p_\nu S^\nu = 0$ ,  $p_{i\nu} S^\nu = 0$

$q_\mu T^\mu = 0$ ,  $p_\mu T^\mu = 0$ ,  $p_{i\mu} T^\mu = 0$

Get 6 equations in 10 unknowns



4 independent response functions

# Putting it all together ...

$$\left( \frac{d^6\sigma}{d\Omega_e d\Omega_p dp d\omega} \right)_{LAB} = \frac{pE}{(2\pi)^3} \sigma_M [v_L R_L + v_T R_T + v_{LT} R_{LT} \cos \varphi_x + v_{TT} R_{TT} \cos 2\varphi_x ]$$

with

$$\sigma_M = \frac{\alpha^2 \cos^2 \theta/2}{4e^2 \sin^4 \theta/2}$$

$$v_L = \left( \frac{Q^2}{q^2} \right)^2 \quad v_T = \frac{Q^2}{2q^2} + \tan^2 \theta/2$$

$$v_{TT} = \frac{Q^2}{2q^2} \quad v_{LT} = \frac{Q^2}{q^2} \sqrt{\frac{Q^2}{q^2} + \tan^2 \theta/2}$$

# The Response Functions

Use spherical basis with z-axis along  $\mathbf{q}$ :

Nuclear 4-current

$$\left\{ \begin{array}{l} J_{fi}^0 \equiv J_{fi}^z = \frac{\omega}{q} \rho_{fi} \\ J_{fi}^{\pm 1} \equiv \mp \frac{1}{\sqrt{2}} (J_{fi}^x \pm i J_{fi}^y) \end{array} \right.$$

$$R_L = |\rho_{fi}(\vec{q})|^2 = \left( \frac{\vec{q}}{\omega} \right)^2 |J_{fi}^0(\vec{q})|^2$$

$$R_T = |J_{fi}^{+1}(\vec{q})|^2 + |J_{fi}^{-1}(\vec{q})|^2$$

$$R_{TT} = 2 \operatorname{Re} \{ J_{fi}^{+1}(\vec{q}) J_{fi}^{-1}(\vec{q}) \}$$

$$R_{LT} = -2 \operatorname{Re} \{ \rho_{fi}(\vec{q}) (J_{fi}^{+1}(\vec{q}) - J_{fi}^{-1}(\vec{q})) \}$$

# Response functions depend on scalar quantities

In lab:

$$\left. \begin{array}{l} Q \bullet P_i = \omega M_A \\ P \bullet P_i = E M_A \\ Q \bullet P = \omega E - q p \cos \theta_{pq} \end{array} \right\}$$

Can choose:

$$Q^2, \omega, \varepsilon_m, p_m$$

Note: no  $\phi_x$  dependence in response functions

# Including electron and recoil proton polarizations

$$\left( \frac{d^6\sigma}{d\Omega_e d\Omega_p dp d\omega} \right)_{LAB} = \frac{pE}{(2\pi)^3} \sigma_M \{ v_L (R_L + R_L^n S_n) + v_T (R_T + R_T^n S_n) \right. \\
 \left. + v_{LT} [(R_{LT} + R_{LT}^n S_n) \cos \varphi_x + (R_{LT}^l S_l + R_{LT}^t S_t) \sin \varphi_x] \right. \\
 \left. + v_{TT} [(R_{TT} + R_{TT}^n S_n) \cos 2\varphi_x + (R_{TT}^l S_l + R_{TT}^t S_t) \sin 2\varphi_x] \right. \\
 \left. + h v_{LT'} [(R_{LT'} + R_{LT'}^n S_n) \sin \varphi_x + (R_{LT'}^l S_l + R_{LT'}^t S_t) \cos \varphi_x] \right. \\
 \left. + h v_{TT'} (R_{TT'}^l S_l + R_{TT'}^t S_t) \right\}$$

with  $v_{LT'} = \frac{Q^2}{q^2} \tan \theta/2$        $v_{TT'} = \tan \theta/2 \sqrt{\frac{Q^2}{q^2} + \tan^2 \theta/2}$

and other  $v$ 's defined as before

# Extracting Response Functions

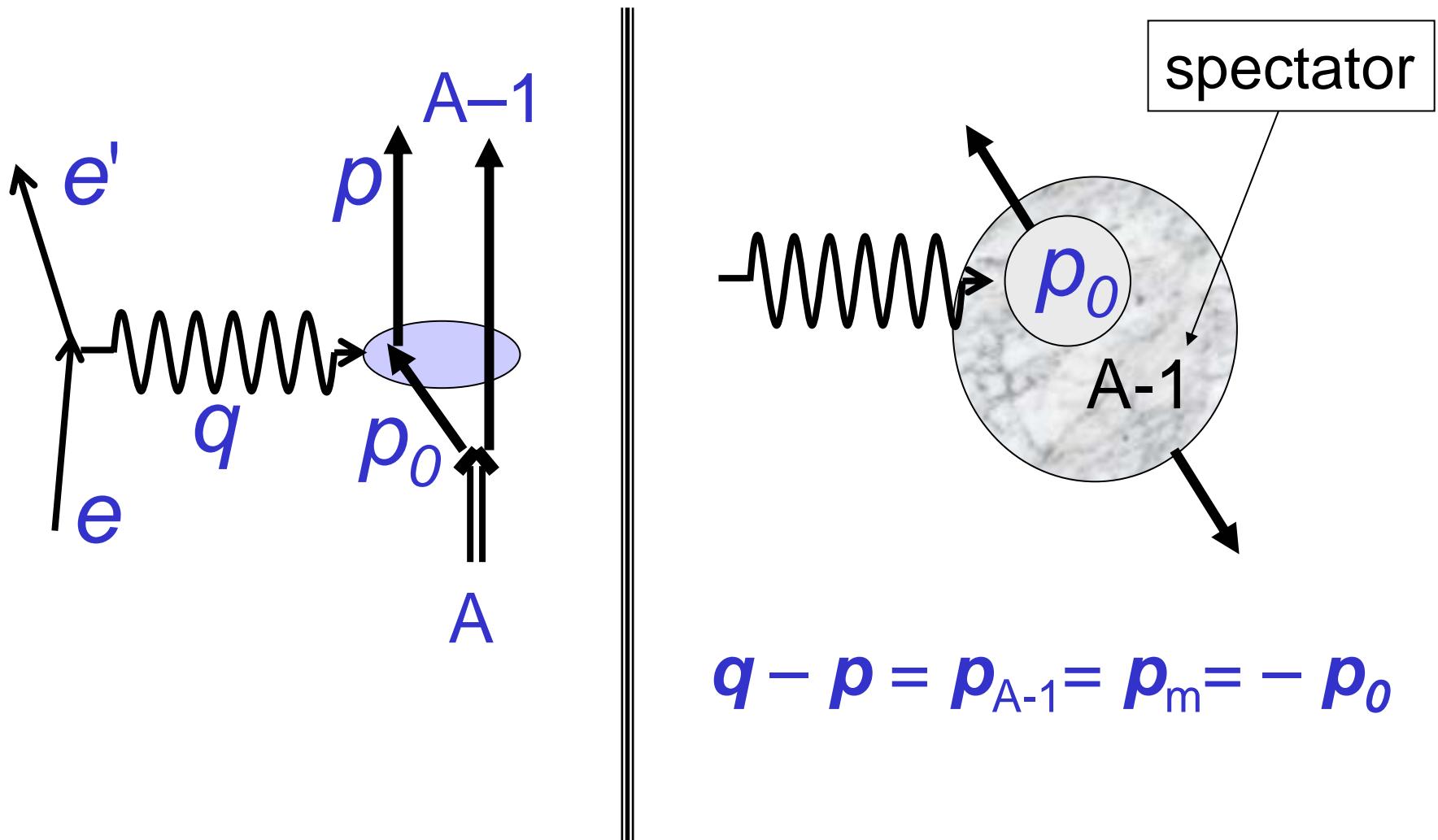
For instance:  $R_{LT}$  and  $A_\phi$  ( $=A_{LT}$ )

$$\sigma_{eep} = K\sigma_M [v_L R_L + v_T R_T + v_{LT} R_{LT} \cos \varphi_x + v_{TT} R_{TT} \cos 2\varphi_x]$$

$$R_{LT} = \frac{\sigma_{eep}(\varphi_x = 0) - \sigma_{eep}(\varphi_x = \pi)}{2K\sigma_M v_{LT}}$$

$$A_\phi = \frac{\sigma_{eep}(\varphi_x = 0) - \sigma_{eep}(\varphi_x = \pi)}{\sigma_{eep}(\varphi_x = 0) + \sigma_{eep}(\varphi_x = \pi)}$$

# Plane Wave Impulse Approximation (PWIA)



# The Spectral Function

In nonrelativistic PWIA:

$$\frac{d^6\sigma}{d\omega d\Omega_e dp d\Omega_p} = K \boxed{\sigma_{ep} S(p_m, \varepsilon_m)}$$

e-p cross section

nuclear spectral function

For bound state of recoil system:

$$\rightarrow \frac{d^5\sigma}{d\omega d\Omega_e d\Omega_p} = K' \sigma_{ep} \boxed{|\Phi(p_m)|^2}$$

proton momentum distribution

# The Spectral Function, cont'd.

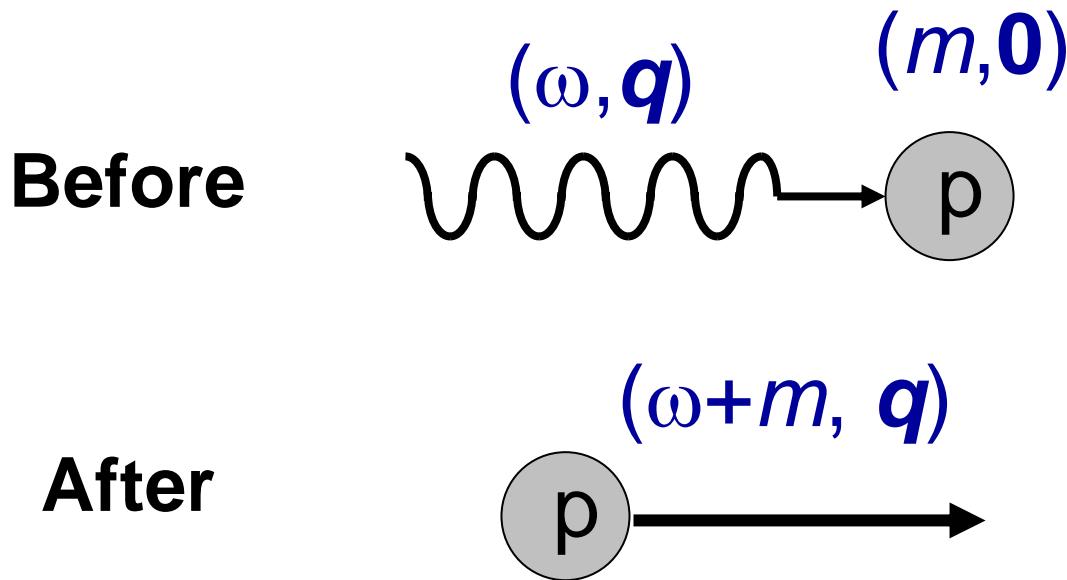
$$S(\vec{p}_0, E_0) = \sum_f \left| \langle B_f | a(\vec{p}_0) | A \rangle \right|^2 \delta(E_0 - \varepsilon_m)$$

where  $\vec{p}_0 = -\vec{p}_m$  = initial momentum

$E_0 = E - \omega$  = initial energy

Note:  $S$  is not an observable!

# Elastic Scattering from a Proton at Rest



Proton is on-shell  $\Rightarrow$   $(\omega + m)^2 - \mathbf{q}^2 = m^2$

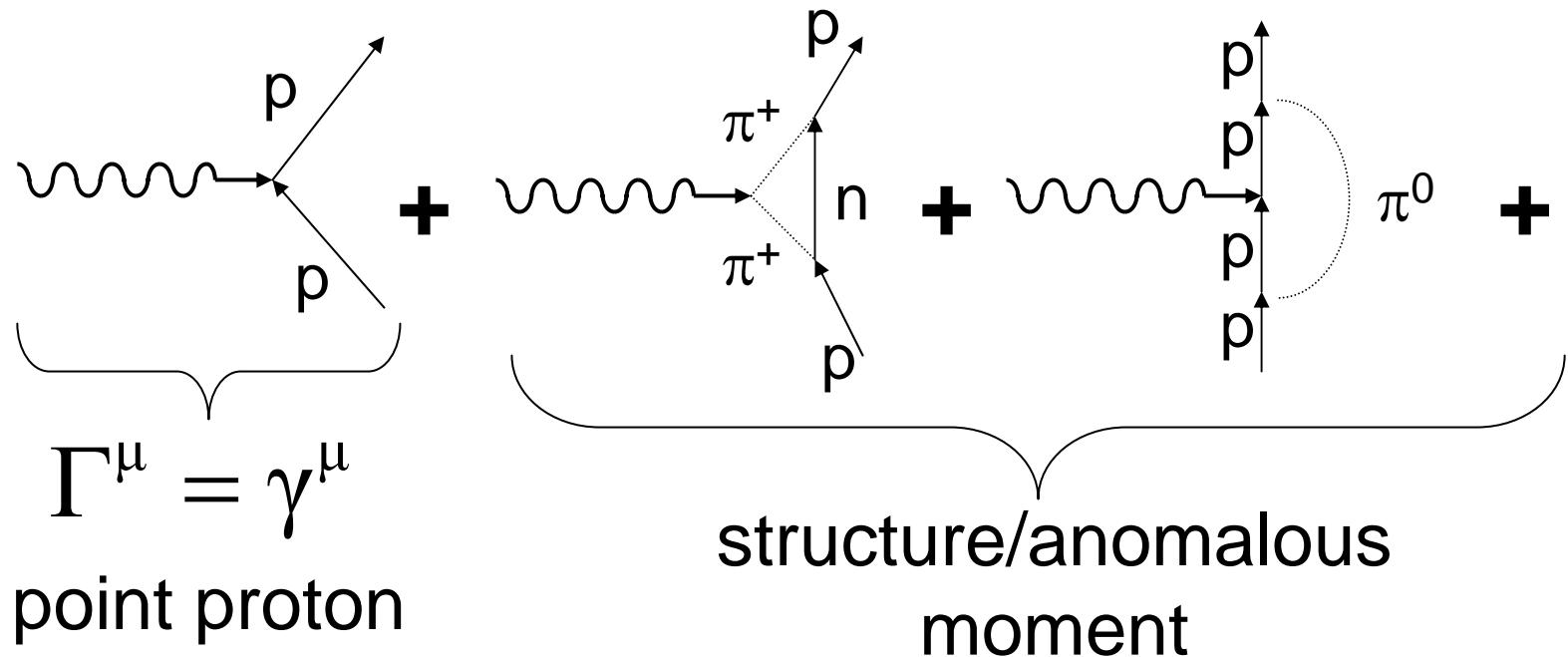
$$\omega^2 + 2m\omega + m^2 - \mathbf{q}^2 = m^2$$

$$\omega = Q^2 / 2m$$

## Scattering from a Proton , cont'd.

$$\left\langle p, s_f \middle| J^\mu \middle| p - q, s_i \right\rangle = \bar{U}_f \Gamma^\mu U_i$$

Vertex fcn



## Scattering from a Proton , cont'd.

Vertex fcn:

$$\Gamma^\mu = \gamma^\mu F_1(Q^2) + i\sigma^{\mu\nu} \frac{q_\nu}{2m} \kappa F_2(Q^2)$$

Dirac FF       Pauli FF 

Sachs FF's

$$\left\{ \begin{array}{l} G_E(Q^2) = F_1(Q^2) - \tau \kappa F_2(Q^2) \\ G_M(Q^2) = F_1(Q^2) + \kappa F_2(Q^2) \end{array} \right.$$

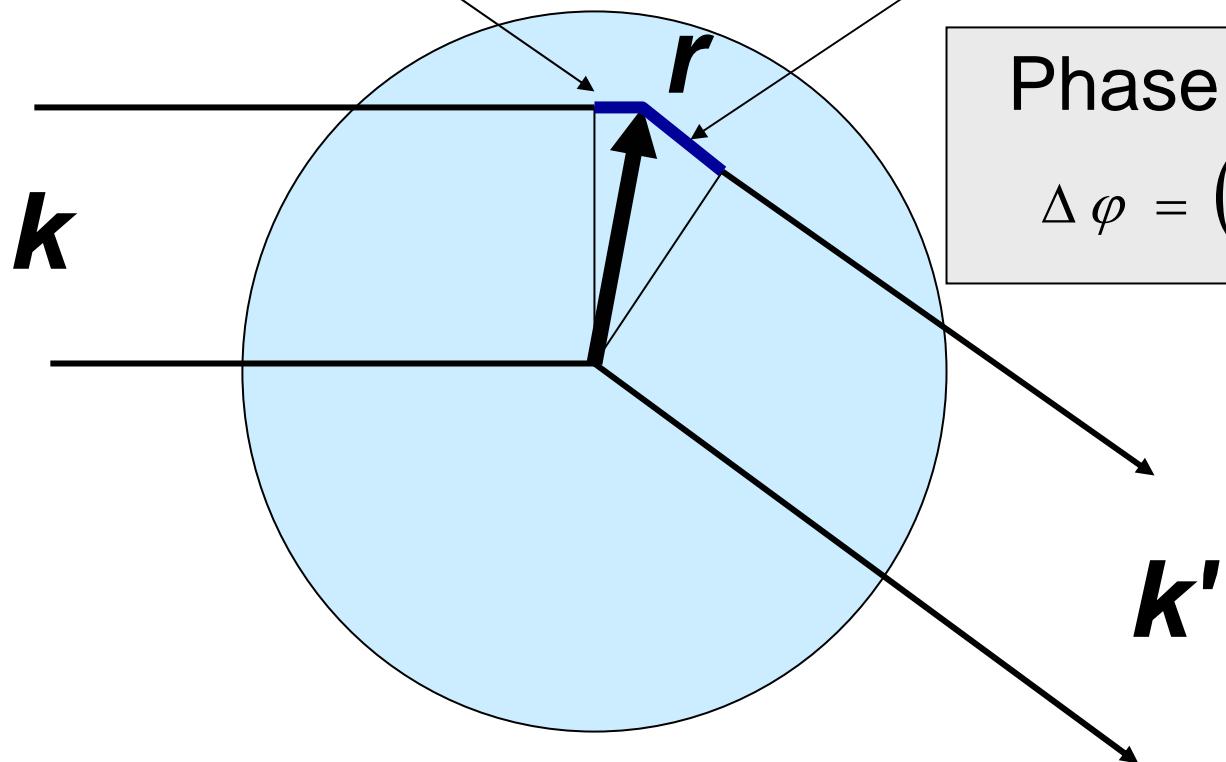
with  $\tau = \frac{Q^2}{4m^2}$

$G_E$  and  $G_M$  are the Fourier transforms of the charge and magnetization densities in the Breit frame.

# Form Factor

$$\Delta\varphi_1 = \vec{k} \cdot \vec{r}$$

$$\Delta\varphi_2 = -\vec{k}' \cdot \vec{r}$$



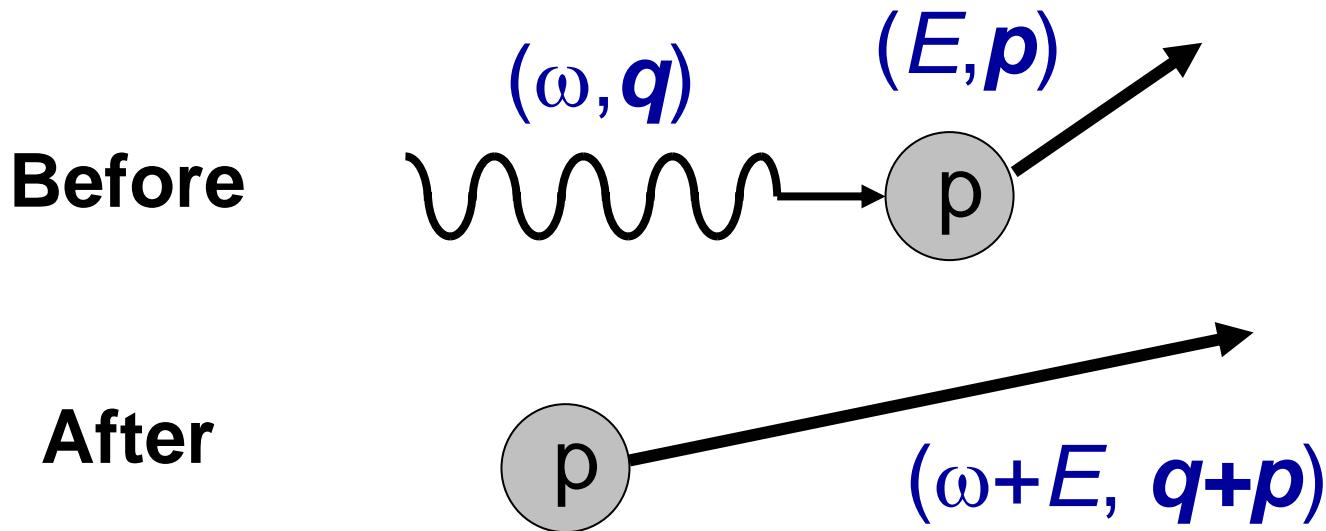
Amplitude at  $\mathbf{q}$ :  $F(\mathbf{q}) = \int d\vec{r} A(\vec{r}) e^{i\vec{q} \cdot \vec{r}}$

## Cross section for $ep$ elastic

$$\frac{d\sigma}{d\Omega} = f_{rec} \sigma_M \left[ \frac{G_E^2 + \tau G_M^2}{1 + \tau} + 2\tau \tan^2 \frac{\theta}{2} G_M^2 \right]$$

However,  $(e, e'p)$  on a **nucleus** involves scattering from **moving** protons, *i.e.* Fermi motion.

# Elastic Scattering from a Moving Proton



$$(\omega + E)^2 - (\mathbf{q} + \mathbf{p})^2 = m^2$$

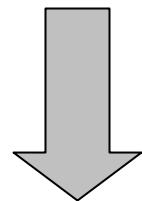
$$\omega^2 + 2E\omega + E^2 - \mathbf{q}^2 - 2\mathbf{p} \cdot \mathbf{q} - \mathbf{p}^2 = m^2$$

$$Q^2 = 2E\omega - 2\mathbf{p} \cdot \mathbf{q}$$

$$\omega (E/m) = (Q^2/2m) + \mathbf{p} \cdot \mathbf{q}/m$$

## Cross section for ep elastic scattering off moving protons

Follow same procedure as for unpolarized ( $e,e'p$ ) from nucleus



We get same form for cross section, with 4 response functions ...

# Response functions for ep elastic scattering off moving protons

$$R_L = \left[ \frac{(E_0 + E)}{2m} \right]^2 W_1 - \frac{\vec{q}^2}{4m^2} W_2$$

$$R_T = 2\tau W_2 + \frac{\vec{p}^2 \sin^2 \theta_{pq}}{m^2} W_1$$

$$R_{LT} = - \frac{(E_0 + E) |\vec{p}| \sin \theta_{pq}}{m^2} W_1$$

$$R_{TT} = \frac{\vec{p}^2 \sin^2 \theta_{pq}}{m^2} W_1$$

with

$$W_1 = F_1^2 + \tau(\kappa F_2)^2 \quad W_2 = (F_1 + \kappa F_2)^2$$

# Quasielastic Scattering

For  $E \approx m$ :

$$\omega \approx (Q^2/2m) + \mathbf{p} \cdot \mathbf{q} / m$$

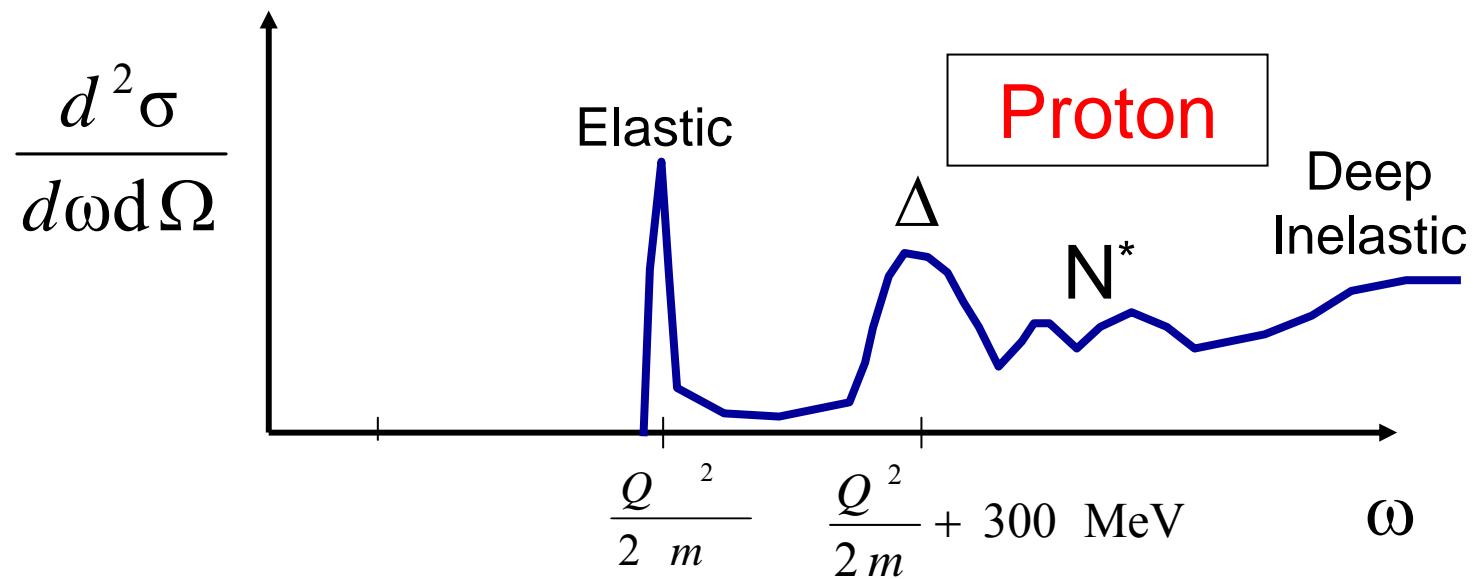
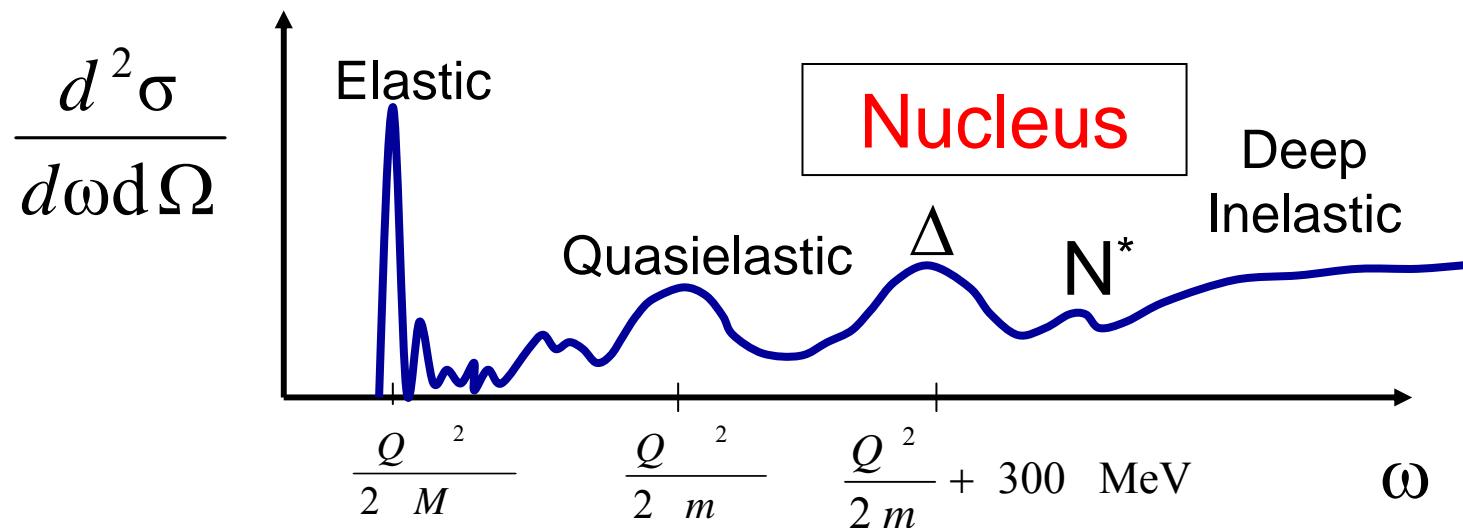
If we “quasielastically” scatter from nucleons within nucleus:

Expect peak at:

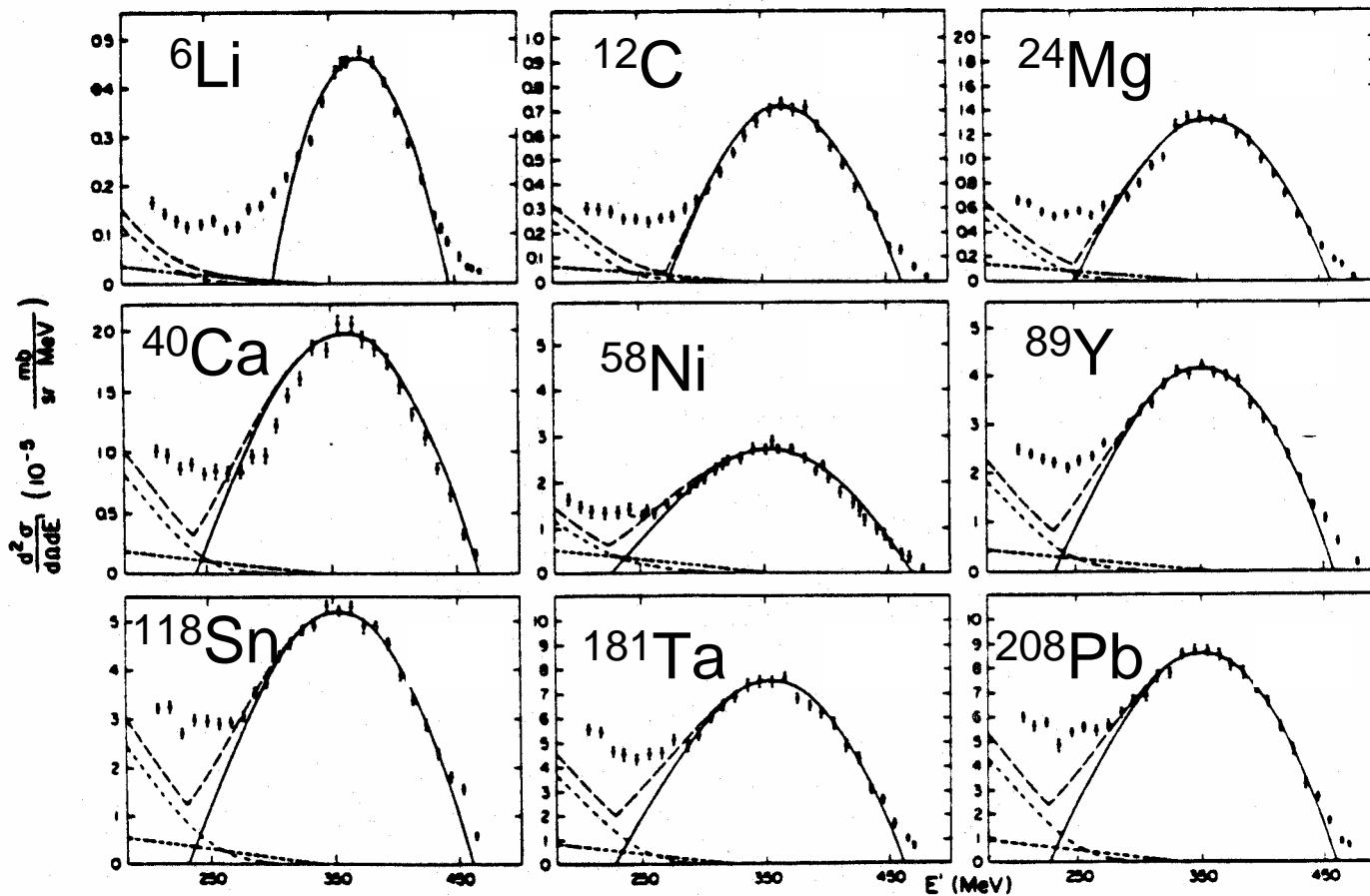
$$\omega \approx (Q^2/2m)$$

Broadened by Fermi motion:  $\mathbf{p} \cdot \mathbf{q} / m$

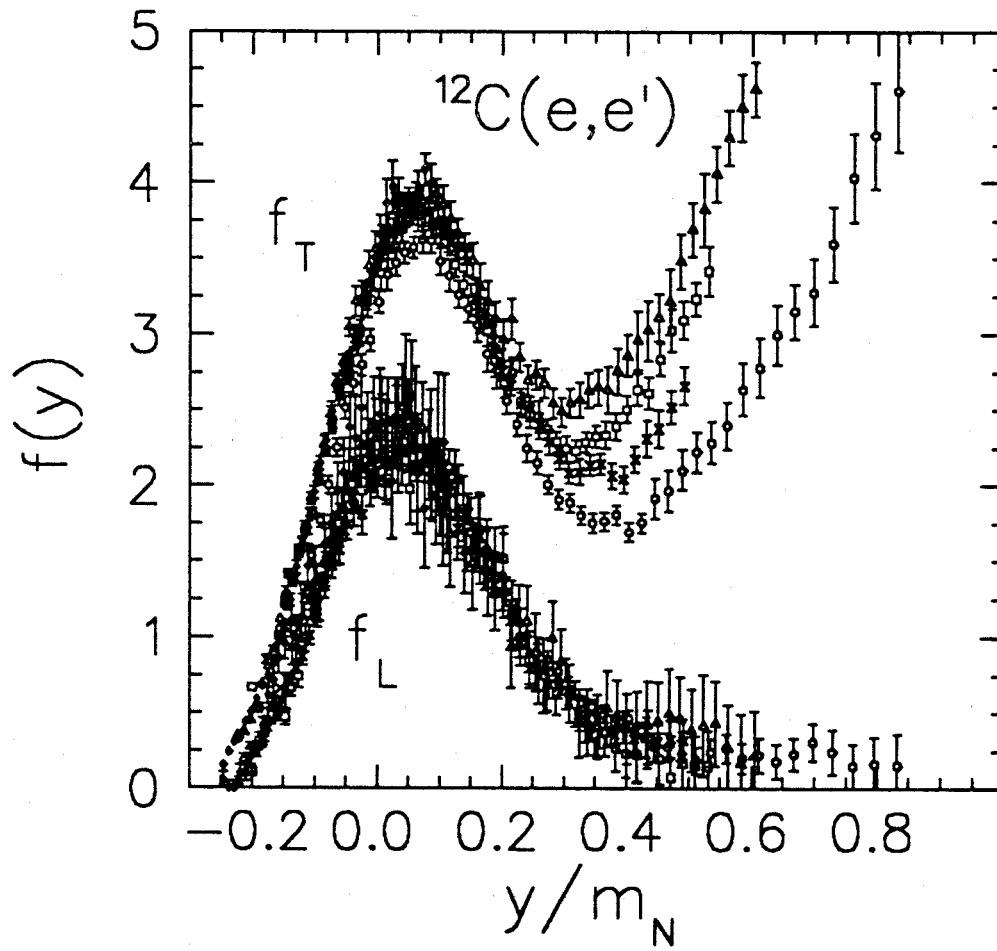
# Electron Scattering at Fixed $Q^2$



# Quasielastic Electron Scattering



R.R. Whitney *et al.*, Phys. Rev. C **9**, 2230 (1974).



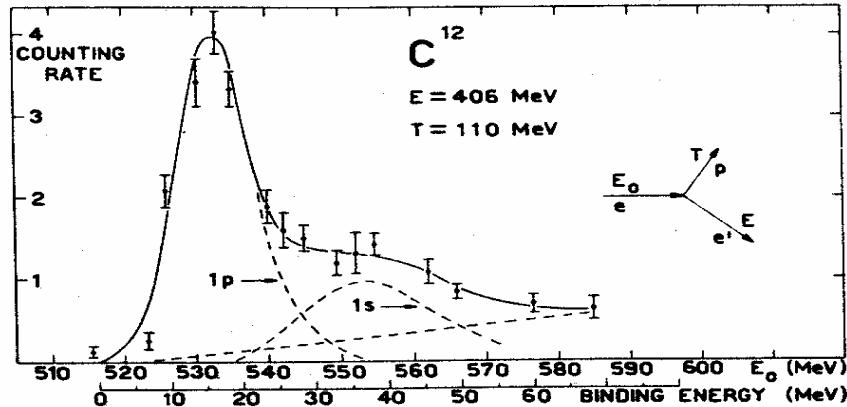
Data: P. Barreau *et al.*, Nucl. Phys. **A402**, 515 (1983).

$y$ -scaling analysis: J.M. Finn, R.W. Lourie and B.H. Cottman,  
Phys. Rev. C **29**, 2230 (1984).

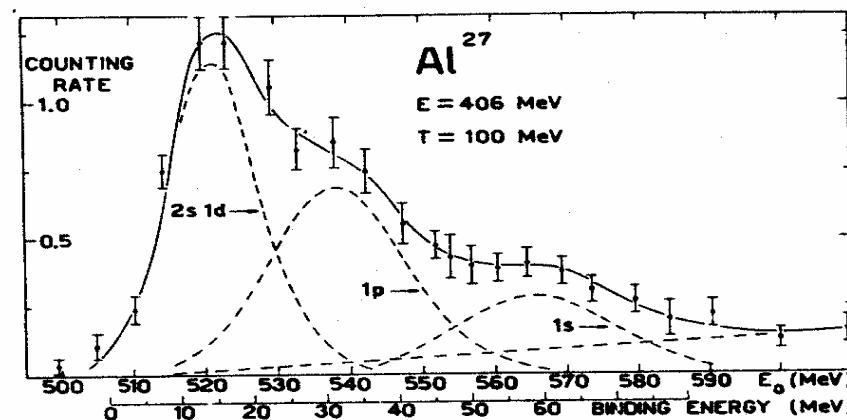
# Nuclear Structure

# First, a bit of history: The first ( $e, e'p$ ) measurement

$^{12}\text{C}(e, e'p)$



$^{27}\text{Al}(e, e'p)$



Frascati  
Synchrotron,  
Italy

U. Amaldi, Jr. *et al.*,  
Phys. Rev. Lett. 13,  
341 (1964).

FIG. 2. Electron-proton coincidence counting rate per  $10^{11}$  equivalent quanta at 550 MeV as a function of the incident energy. The dashed lines indicate the contributions of the various shells and the background as explained in the text.

## (e,e'p) advantages over (p,2p)

- Electron interaction relatively weak: OPEA is reasonably accurate.
- Nucleus is very transparent to electrons: Can probe deeply bound orbits.

However: ejected proton is strongly interacting. The “cleanness” of the electron probe is somewhat sacrificed.

FSI must be taken into account.

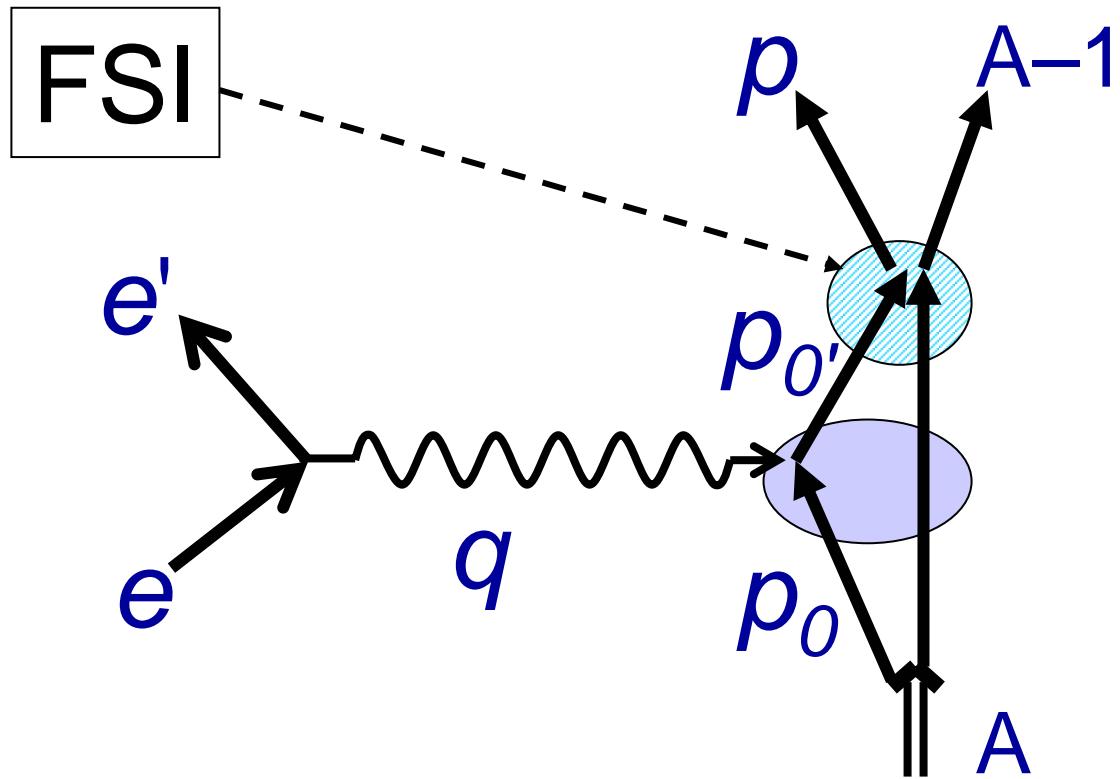
Recall, in nonrelativistic PWIA:

$$\frac{d^6\sigma}{d\omega d\Omega_e dp d\Omega_p} = K \sigma_{ep} S(p_m, \epsilon_m)$$

where  $\mathbf{q} - \mathbf{p} = \mathbf{p}_m = -\mathbf{p}_o$

FSI destroys simple connection  
between the measured  $\mathbf{p}_m$  and the  
proton initial momentum (not an  
observable).

# Final State Interactions (FSI)



$$\vec{q} - \vec{p} = \vec{p}_{A-1} \neq \vec{p}_0$$

# Distorted Wave Impulse Approximation (DWIA)

Treat outgoing proton distorted waves in presence of potential produced by residual nucleus (*optical potential*).

$$\frac{d^6\sigma}{d\omega d\Omega_e dp d\Omega_p} = K \sigma_{ep} [S^D(p_m, \varepsilon_m, p)]$$

“Distorted” spectral function



Optical potential is constrained by proton elastic scattering data.

Problems with this approach:

- Residual nucleus contains hole state, unlike the target in p+A scattering.
- Proton scattering data is surface dominated, whereas ejected protons in (e,e'p) are produced within entire nuclear volume.

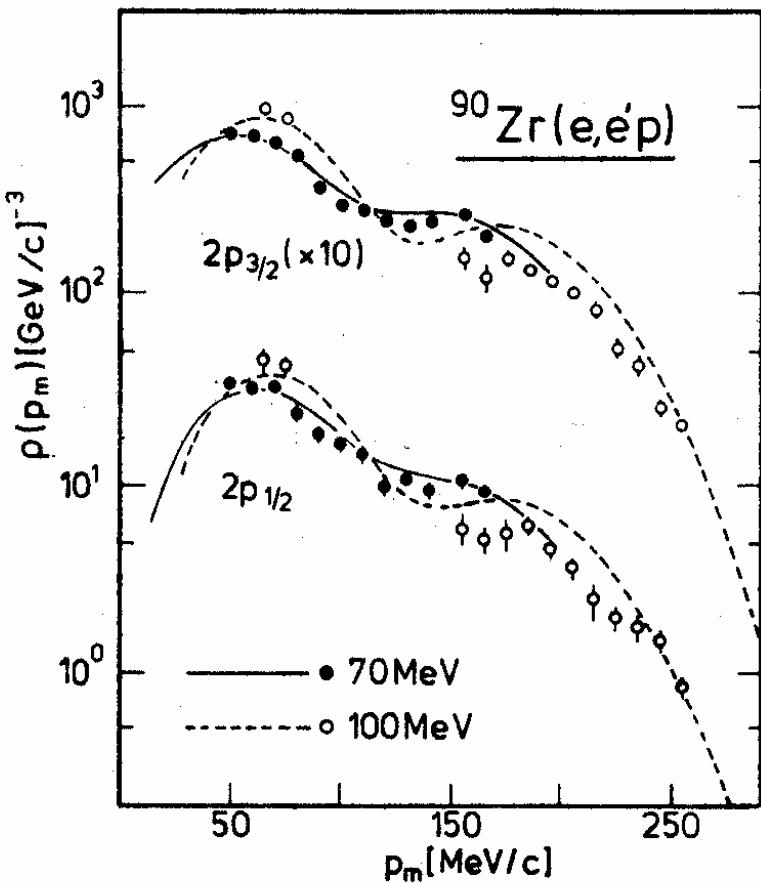


Fig. 1. Experimental momentum distributions for the  $2p_{3/2}$  and  $2p_{1/2}$  transitions in the  ${}^{90}\text{Zr}(e,e'p){}^{89}\text{Y}$  reaction. The curves correspond to DWIA calculations for the two proton energies (set I in table 1).

100 MeV data  
is significantly  
overestimated  
by DWIA near  
2<sup>nd</sup> maximum.

NIKHEF-K  
Amsterdam

J.W.A. den Herder, *et al.*, Phys. Lett. B 184, 11 (1987).

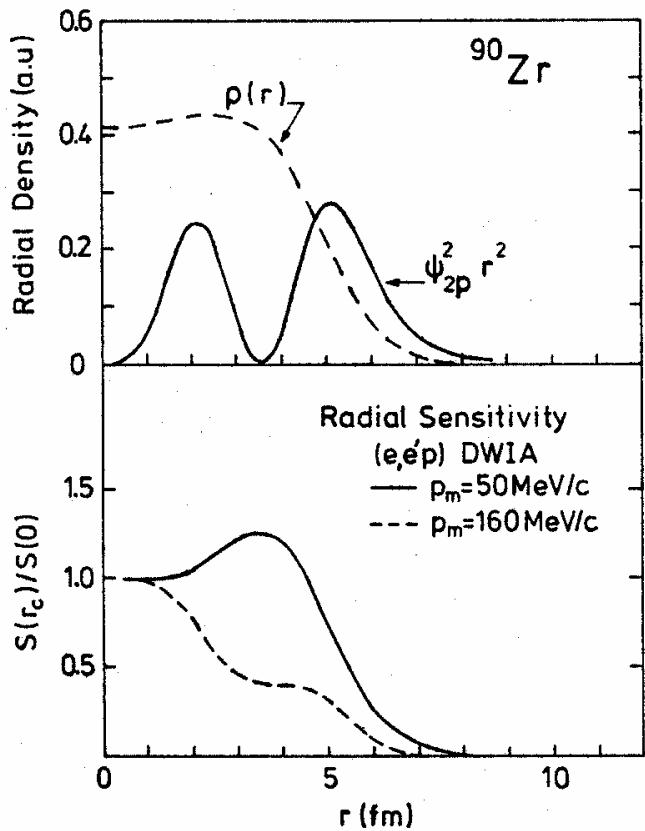
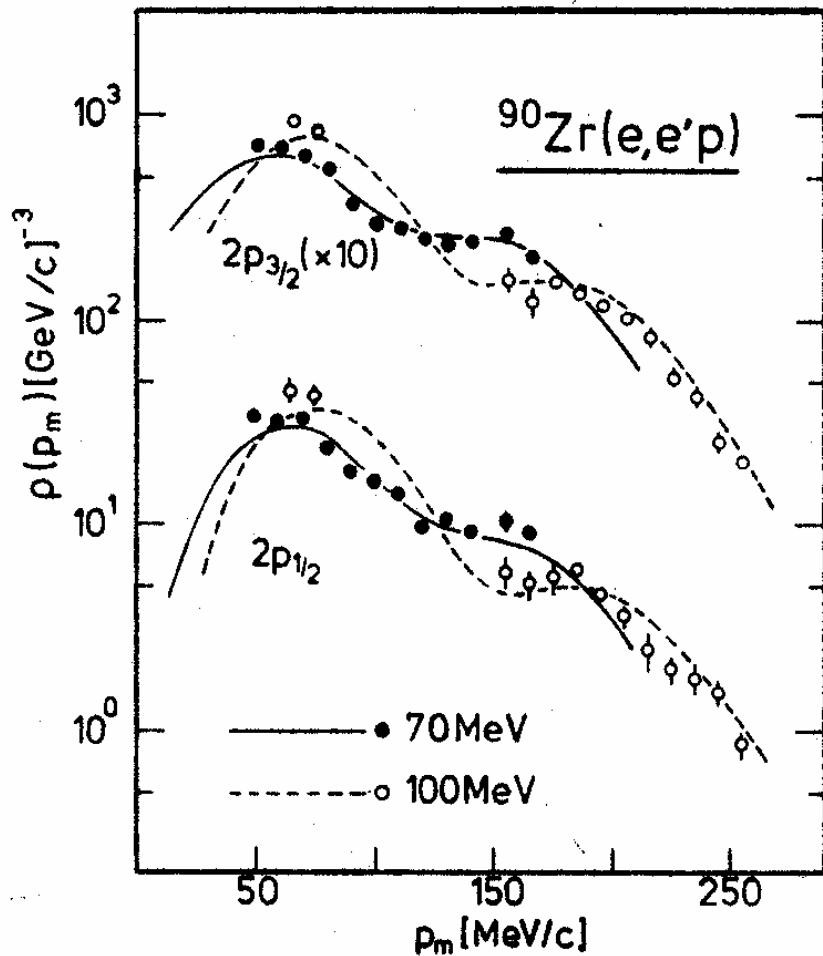


Fig. 2. Radial sensitivity,  $S(r_c)$ , as a function of the lower integration limit  $r_c$  (see text). For reference the radial dependence of the charge density ( $\rho$ ) and of the 2p wave functions ( $\psi_{2p}$ ) are indicated as well (not to scale).

$$\rho_a^{\text{th}}(p_m, \vec{p}) = S_a \left| \int_{r_c=0}^{\infty} \chi^{(-)*}(\vec{r}_p, \vec{p}) \times \exp(i\vec{q} \cdot \vec{r}_p) \psi_a(\vec{r}_p) d\vec{r}_p \right|^2$$

At  $p_m \approx 160$  MeV/c,  
wf is probed in  
nuclear interior.

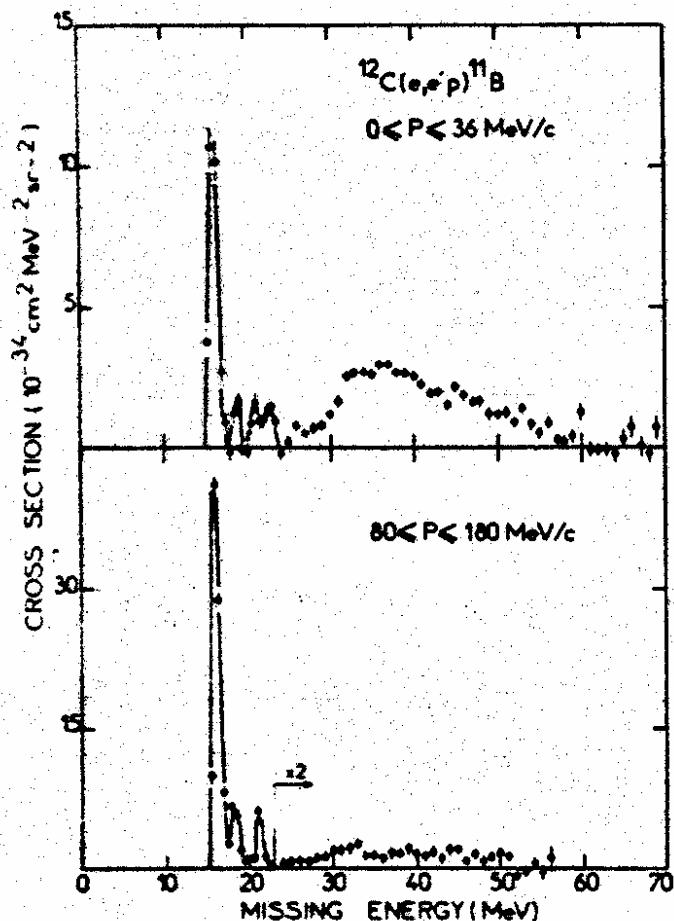


Adjusting optical potential renders good agreement while maintaining agreement with p+A elastic.

Fig. 3. Same as fig. 1, but for the modified optical potential (set II in table 1).

J.W.A. den Herder, et al., Phys. Lett. B 184, 11 (1987).

Saclay  
Linac,  
France



$^{12}\text{C}(\text{e}, \text{e}'\text{p})^{11}\text{B}$

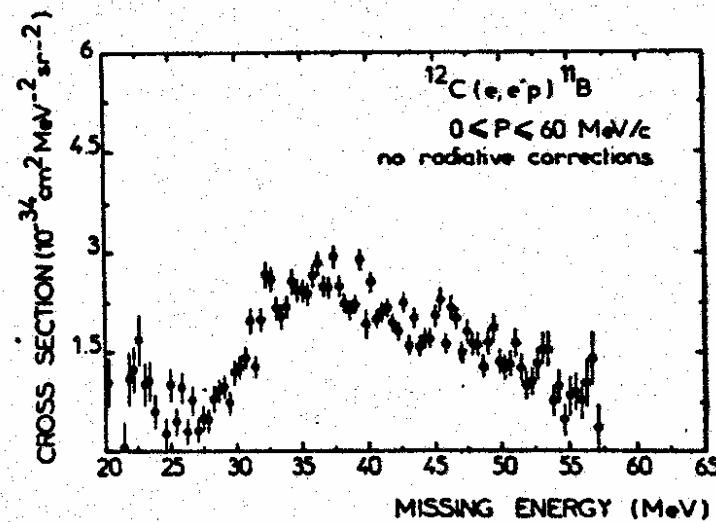
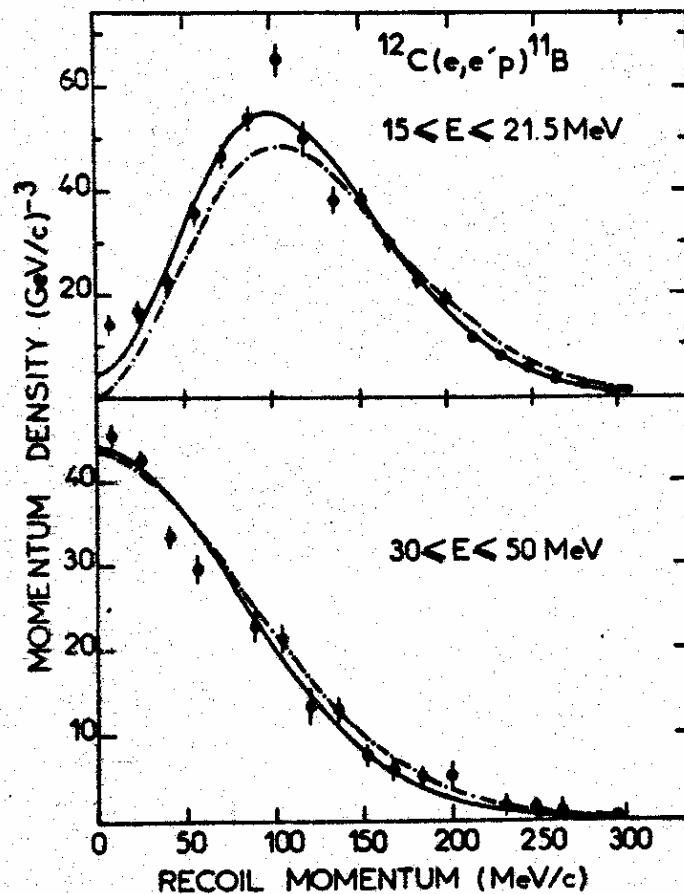


Fig. 9. Missing energy spectra from  $^{12}\text{C}(\text{e}, \text{e}'\text{p})$ , (a)  $0 \leq P \leq 36 \text{ MeV}/c$ , (b)  $80 \leq P \leq 180 \text{ MeV}/c$  and (c)  $0 \leq P \leq 60 \text{ MeV}/c$  for  $20 \leq E \leq 60 \text{ MeV}$ .

J. Mougey et al., Nucl. Phys. **A262**, 461 (1976).

p-shell  
 $l=1$

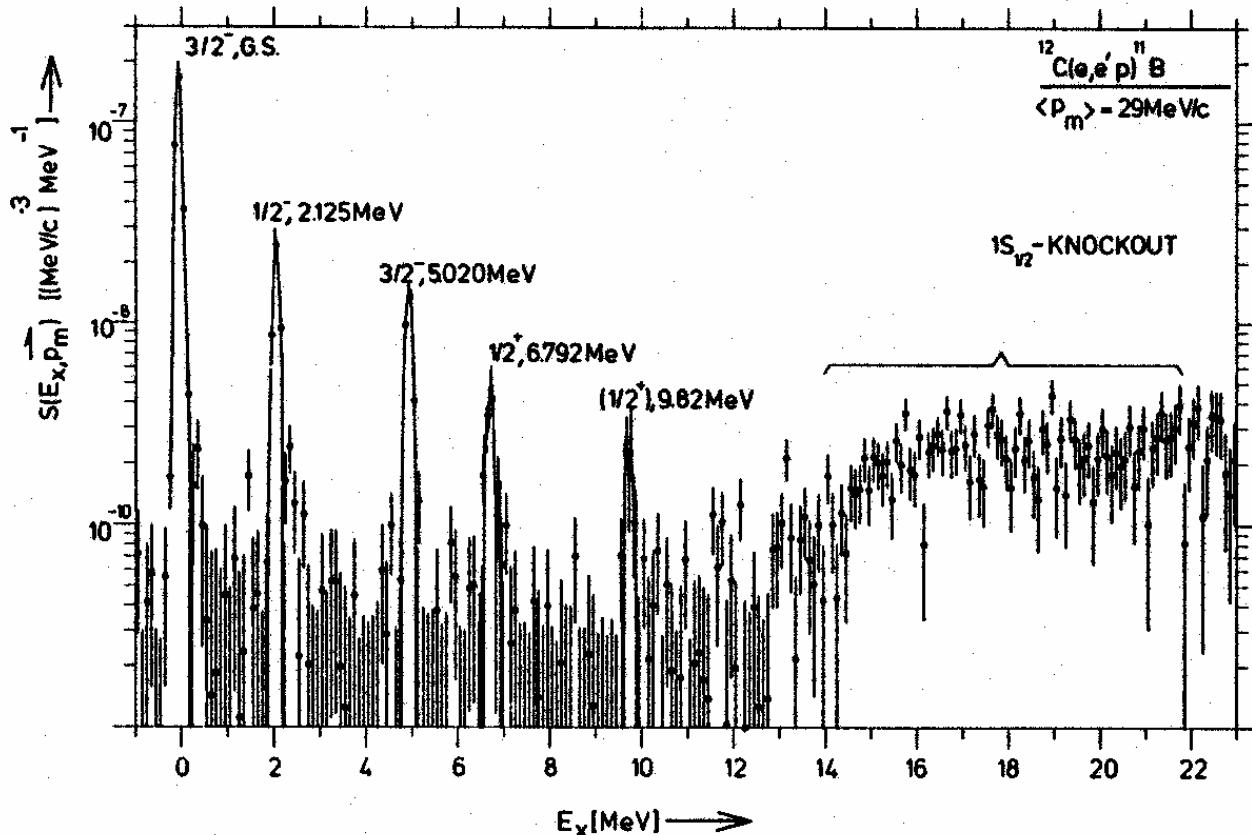
s-shell  
 $l=0$



$^{12}\text{C}(\text{e},\text{e}'\text{p})^{11}\text{B}$

Saclay  
Linac,  
France

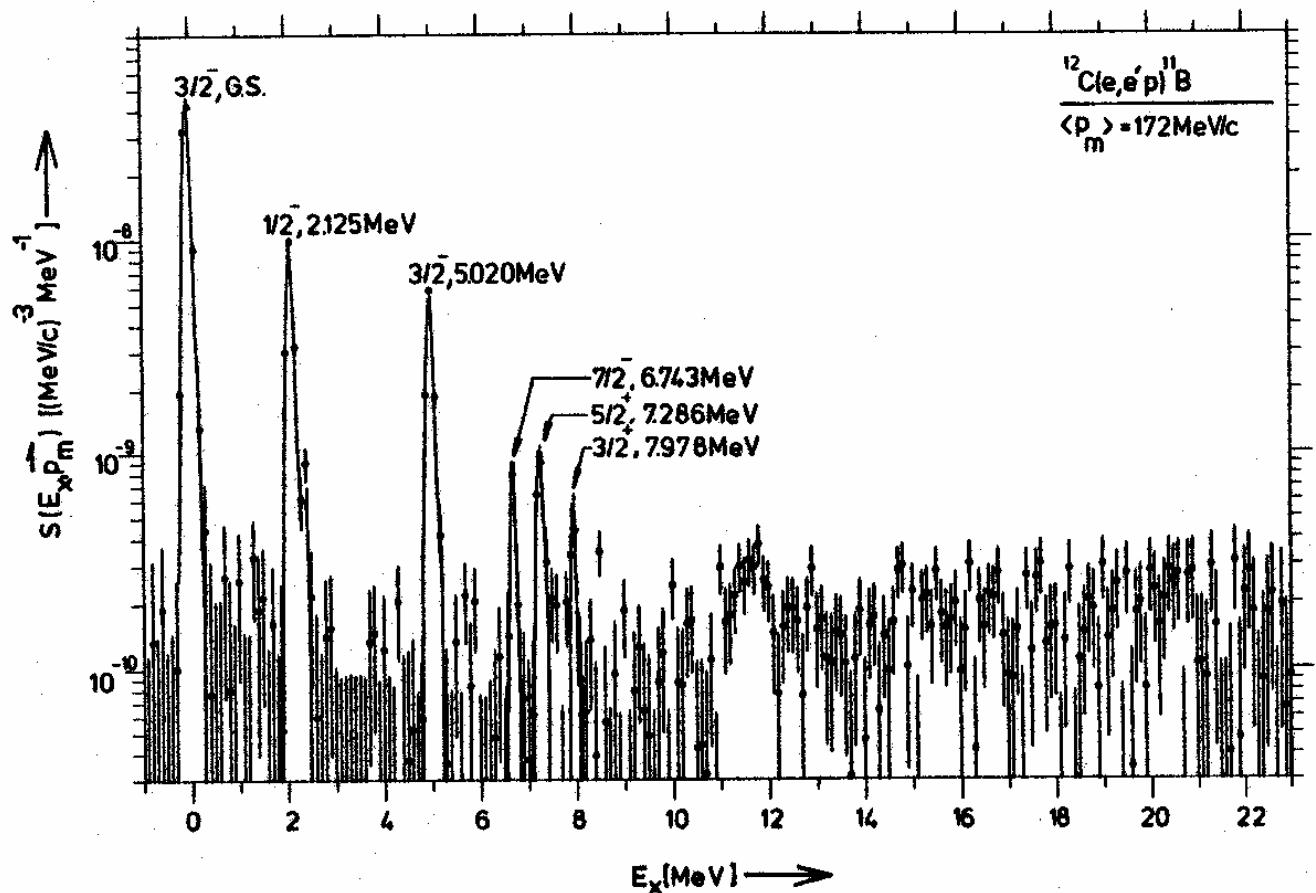
Fig. 10. Momentum distribution from  $^{12}\text{C}(\text{e}, \text{e}'\text{p})$ ; (a)  $15 \leq E \leq 21.5$  MeV and (b)  $30 \leq E \leq 50$  MeV. The solid and dashed lines represent DWIA and PWIA calculations respectively, with normalization obtained by a fit to the data.



NIKHEF-K  
Amsterdam

Fig. 1. Excitation-energy spectrum of the reaction  $^{12}\text{C}(\text{e},\text{e}'\text{p})^{11}\text{B}$  at a central value of the missing momentum  $p_m = 29 \text{ MeV}/c$ . The spectrum has been sorted in 100 keV bins.

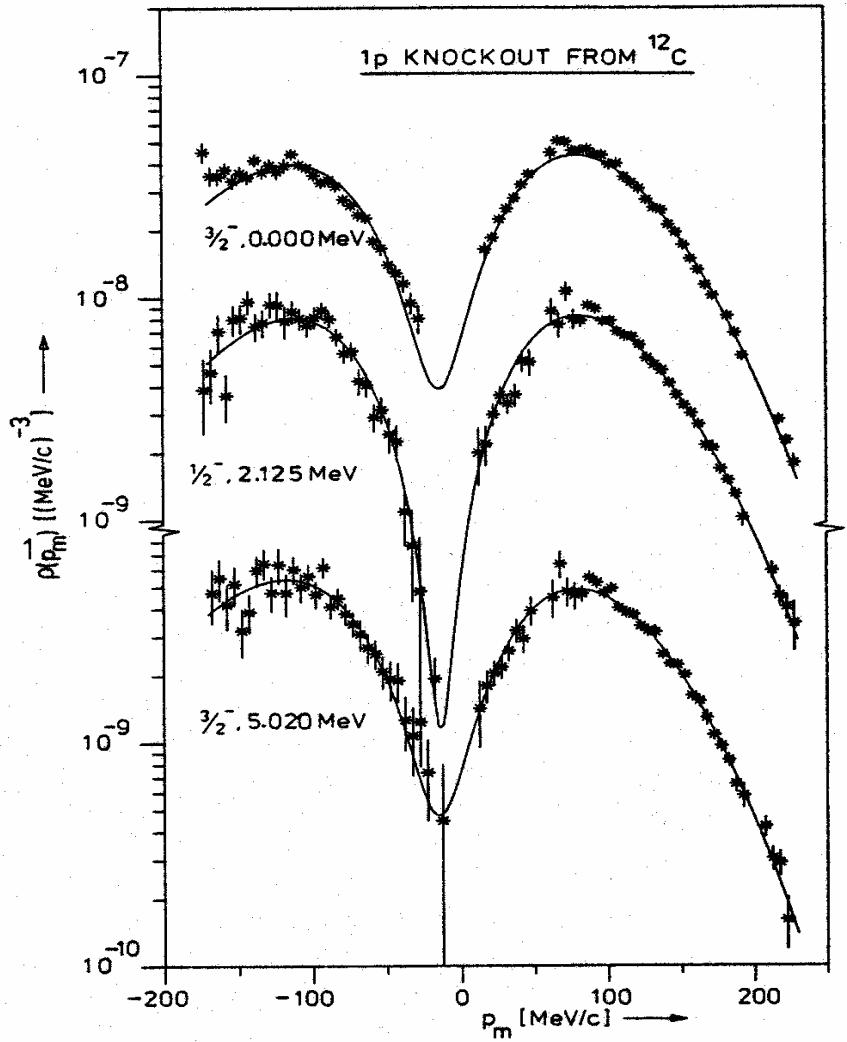
G. van der Steenhoven *et al.*, Nucl. Phys. **A484**, 445 (1988).



NIKHEF-K  
Amsterdam

Fig. 2. Same as fig. 1 but for  $p_m = 172 \text{ MeV}/c$ .

G. van der Steenhoven *et al.*, Nucl. Phys. **A484**, 445 (1988).



$^{12}\text{C}(\text{e},\text{e}'\text{p})^{11}\text{B}$

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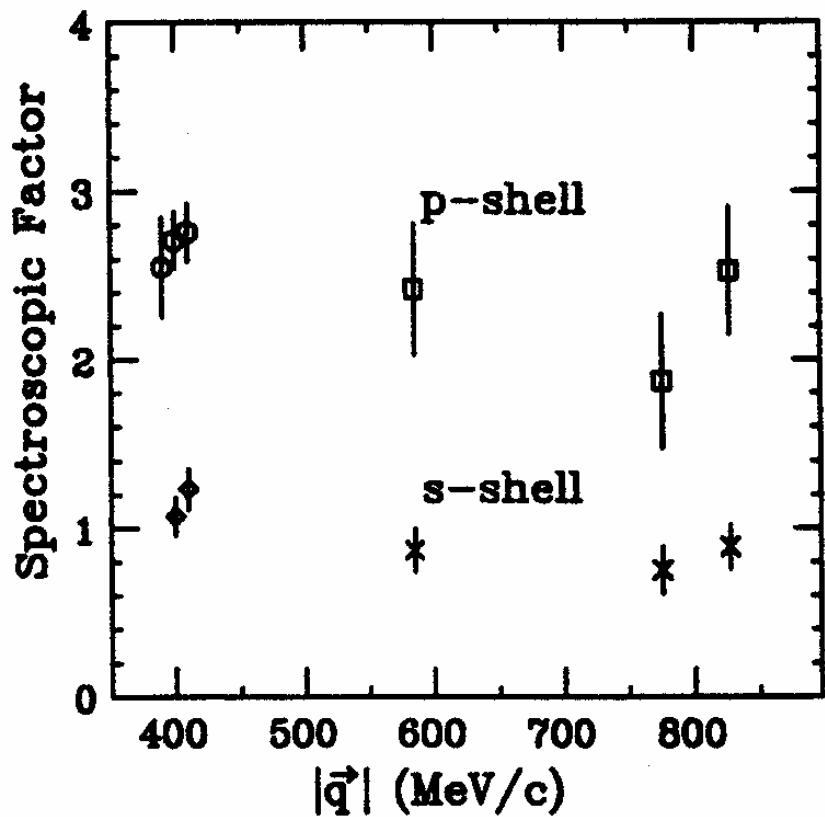
DWIA  
calculations fit  
data reasonably  
well.

Missing strength  
observed  
however.

Fig. 11. Momentum distributions for 1p knockout from  $^{12}\text{C}$  leading to the  $\frac{3}{2}^-$  ground state, the  $\frac{1}{2}^-$  state at 2.125 MeV and the  $\frac{3}{2}^-$  state at 5.020 MeV in  $^{11}\text{B}$ . The curves represent DWIA calculations employing the MCO potential. The fitted parameters are listed in table 6 (after a correction for the omitted couplings using table 5).

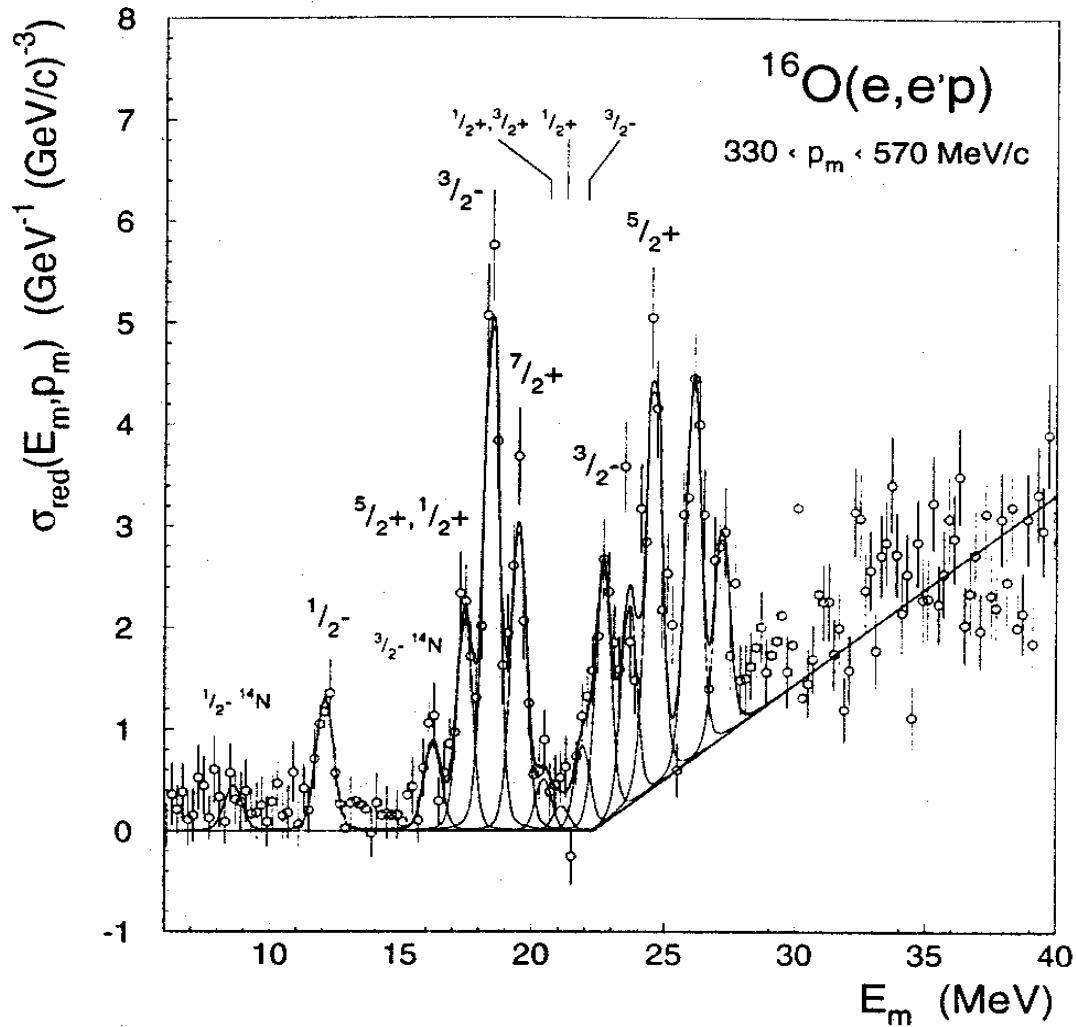
G. van der Steenhoven, et al., Nucl. Phys. A480, 547 (1988).

# $^{12}\text{C}(\text{e},\text{e}'\text{p})$



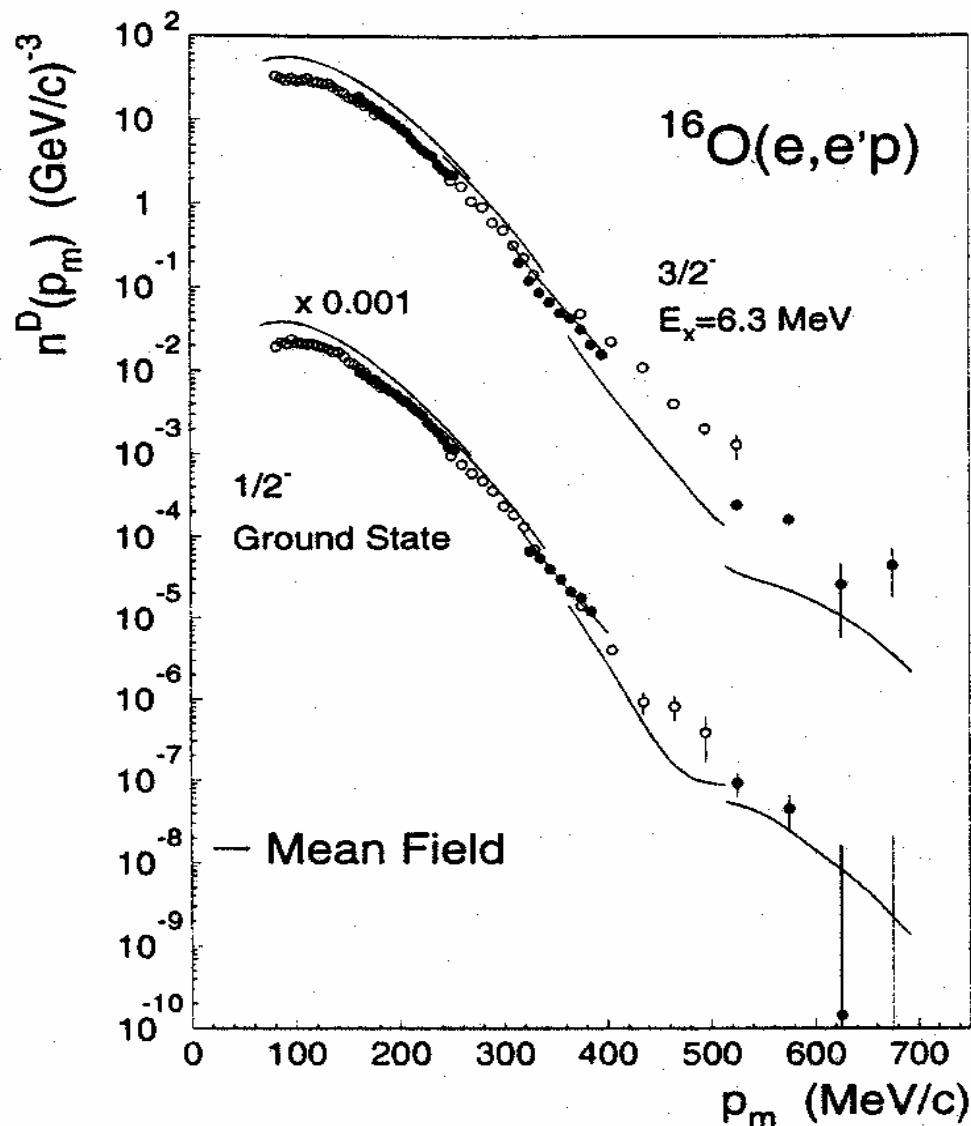
Bates  
Linear  
Accelerator

L.B. Weinstein *et al.*, Phys. Rev. Lett. **64**, 1646 (1990).



MAMI  
Mainz,  
Germany

K.I. Blomqvist *et al.*, Phys. Lett. B 344, 85 (1995).

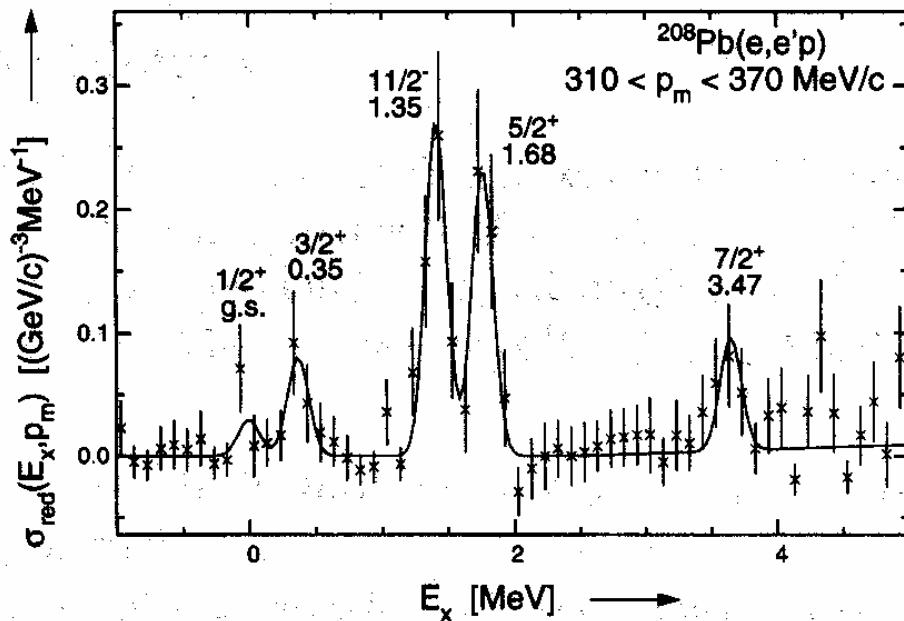


MAMI  
Mainz,  
Germany

Factorization violated.

DWIA calculations  
underpredict at high  $p_m$ .

Neglected MEC's &  
relativistic effects.  
Offshell effects  
uncertain at high  $p_m$ .



**208Pb(e,e'p)**

AmPS NIKHEF-K  
Amsterdam

FIG. 1. The reduced cross section of the reaction  $^{208}\text{Pb}(e, e'p)$  at an average missing momentum of 340 MeV/c, showing the knock out of valence protons to discrete states in  $^{207}\text{Tl}$ , labeled by their spin, parity, and excitation energy. The solid curve is the result of a fit to the spectrum.

I. Bobeldijk *et al.*, Phys. Rev. Lett. **73**, 2684 (1994).

$^{208}\text{Pb}(e, e'p)$

## AmPS NIKHEF-K Amsterdam

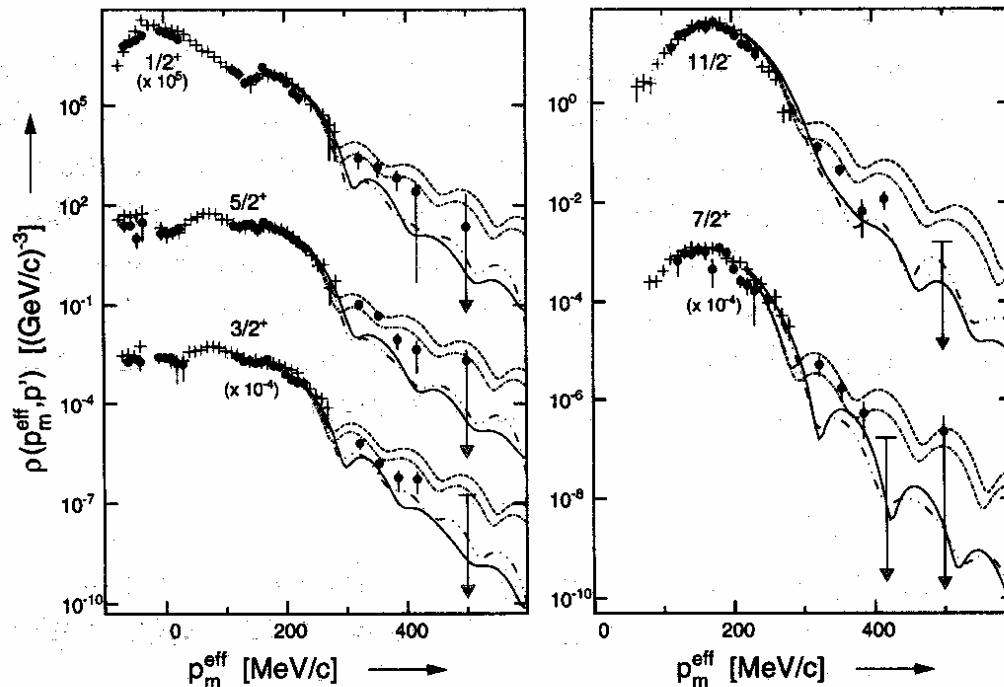


FIG. 2. Missing-momentum distributions for the transitions to the  $\frac{1}{2}^+$ ,  $\frac{3}{2}^+$ ,  $\frac{11}{2}^-$ ,  $\frac{5}{2}^+$ , and  $\frac{7}{2}^+$  states in the reaction  $^{208}\text{Pb}(e, e'p)$  at excitation energies of 0.00, 0.35, 1.35, 1.68, and 3.47 MeV, respectively. The present data are represented by solid circles, the plus marks have been measured by Quint [16]. The solid curves are knockout calculations in the distorted-wave impulse approximation. The calculations including correlations as proposed by Pandharipande [8], Ma and Wambach [10], and Mahaux and Sartor [12] are represented by dash-double-dotted, dashed, and dot-dashed curves, respectively.

Long-range correlations important.

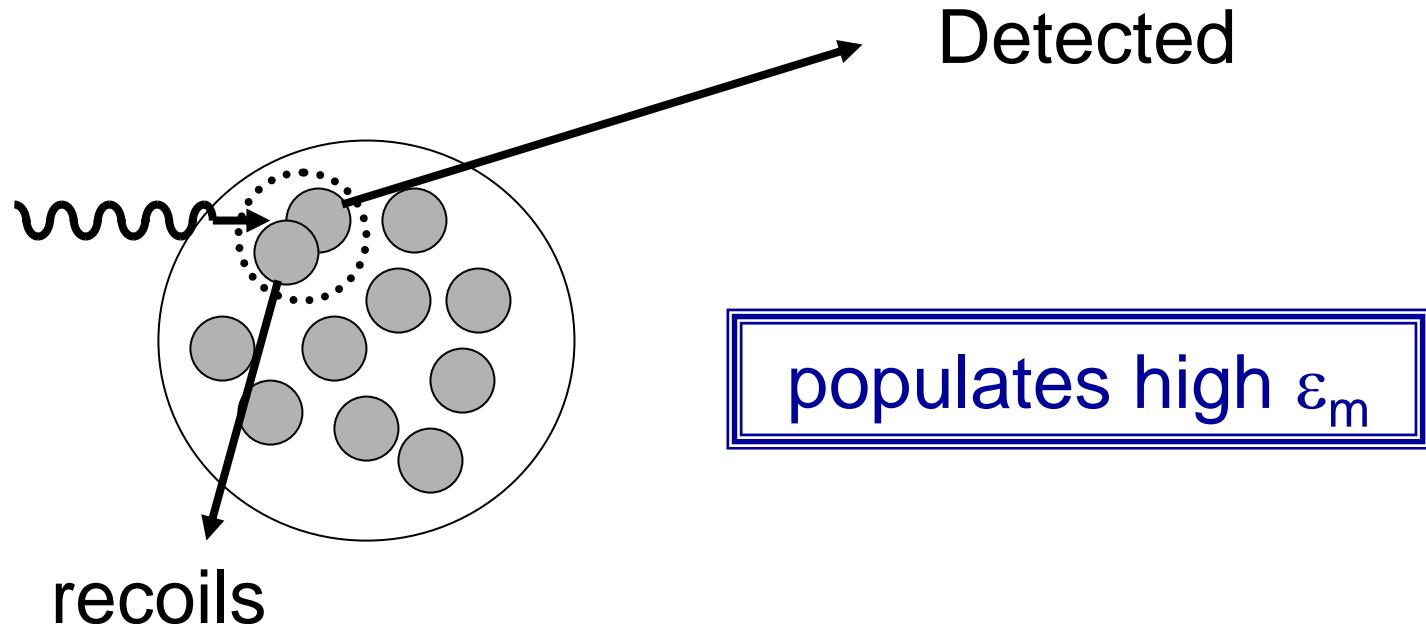
SRC and TC less so, but expected to grow with  $\varepsilon_m$ .

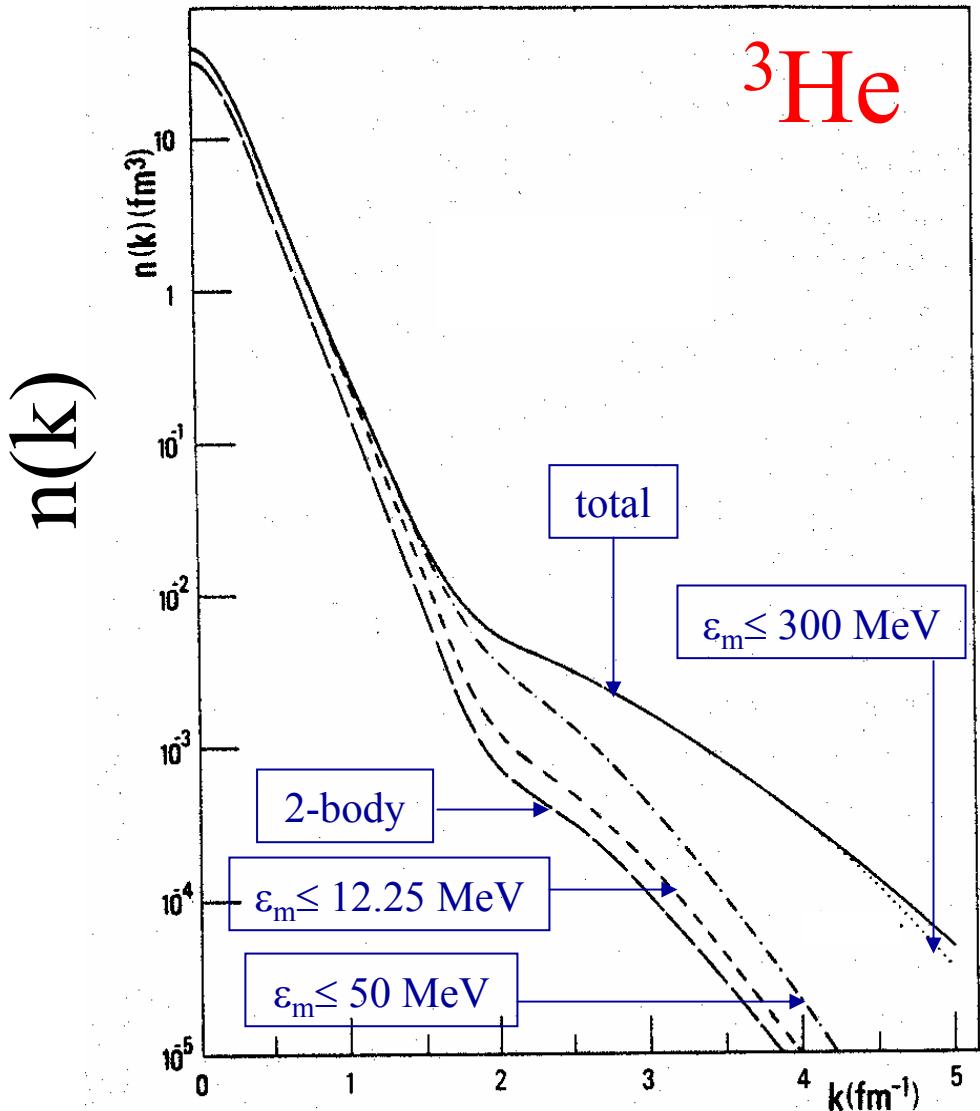
## Some of the lessons learned:

- $(e,e'p)$  sensitive probe of single-particle orbits.
- Proton distortions (FSI) must be accounted for to reproduce shape of spectral function. Energy dependence of FSI breaks factorization.
- Missing strength in valence orbits, even after accounting for FSI
- At high  $P_m$  significant discrepancies found relative to calculations.

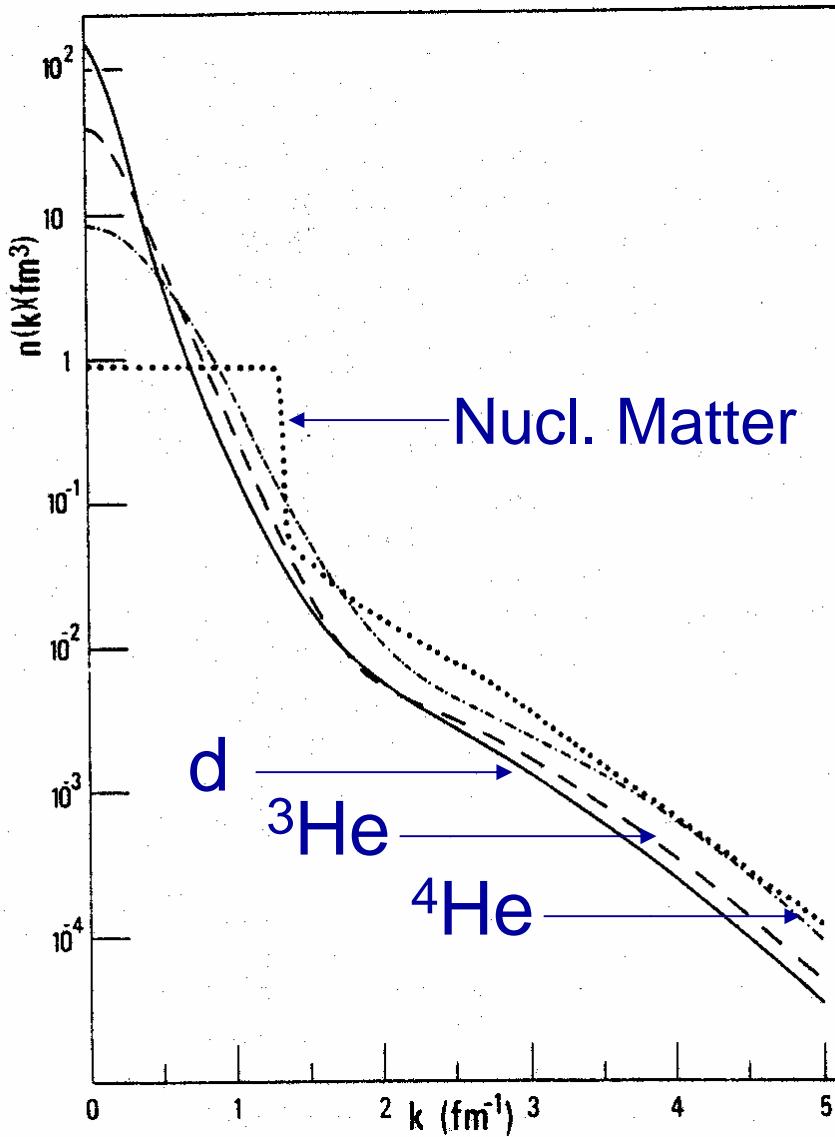
# Where does the “missing” strength go?

One possibility:





SRC dominate high  $k (=p_m)$  and are related to large values of  $\varepsilon_m$ .



Similar shapes for  
few-body nuclei  
and nuclear matter  
at high  $k (=p_m)$ .

C. Ciofi degli Atti, E. Pace and G. Salmè,  
Phys. Lett. **141B**, 14 (1984).

# Medium-Modified Nucleons

# Searching for Medium Effects on the Nucleon ...

In parallel kinematics:

$$\frac{d^6\sigma}{d\Omega_e d\Omega_p dp d\omega} = \frac{pE}{(2\pi)^3} \sigma_M [v_L R_L + v_T R_T]$$

Can write  $ep$  elastic cross section as:

$$\frac{d\sigma}{d\Omega} = f_{rec} \sigma_M [v_L k_L G_E^2 + v_T k_T G_M^2]$$

with  $k_L = \frac{|\vec{q}|^2}{Q^2}$  and  $k_T = \frac{Q^2}{2m^2}$

Relate  $R_T/R_L$  to in-medium proton FF's

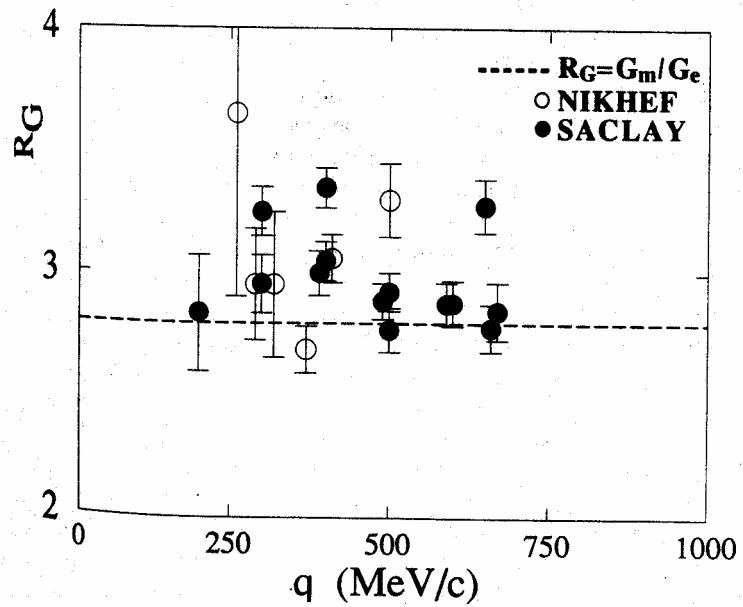
$$R_G \equiv \frac{m|\vec{q}|}{Q^2} \sqrt{\frac{2R_T}{R_L}} \rightarrow \frac{\tilde{G}_M}{\tilde{G}_E}$$

↑  
PWIA

This relies on (unrealistic) model assumptions!

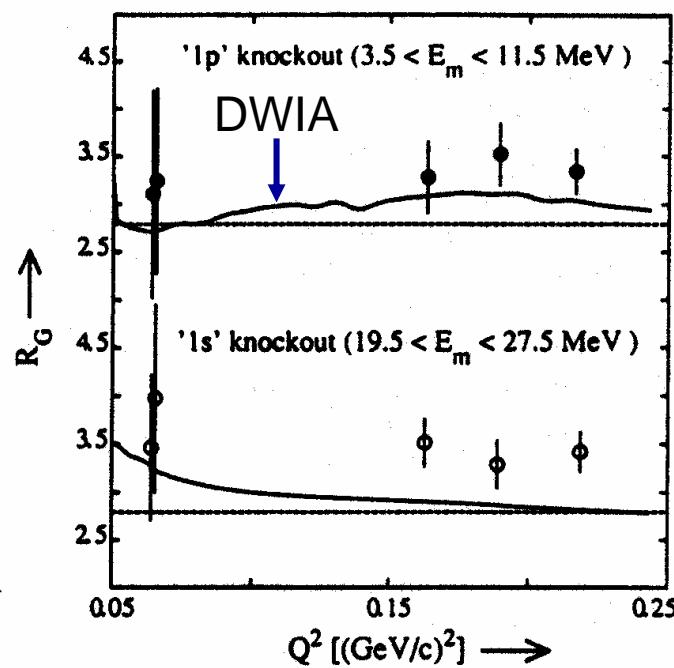
Nonetheless ...

## $^2\text{H}(\text{e},\text{e}'\text{p})\text{n}$



J.E. Ducret *et al.*,  
Phys. Rev. C **49**, 1783 (1994).

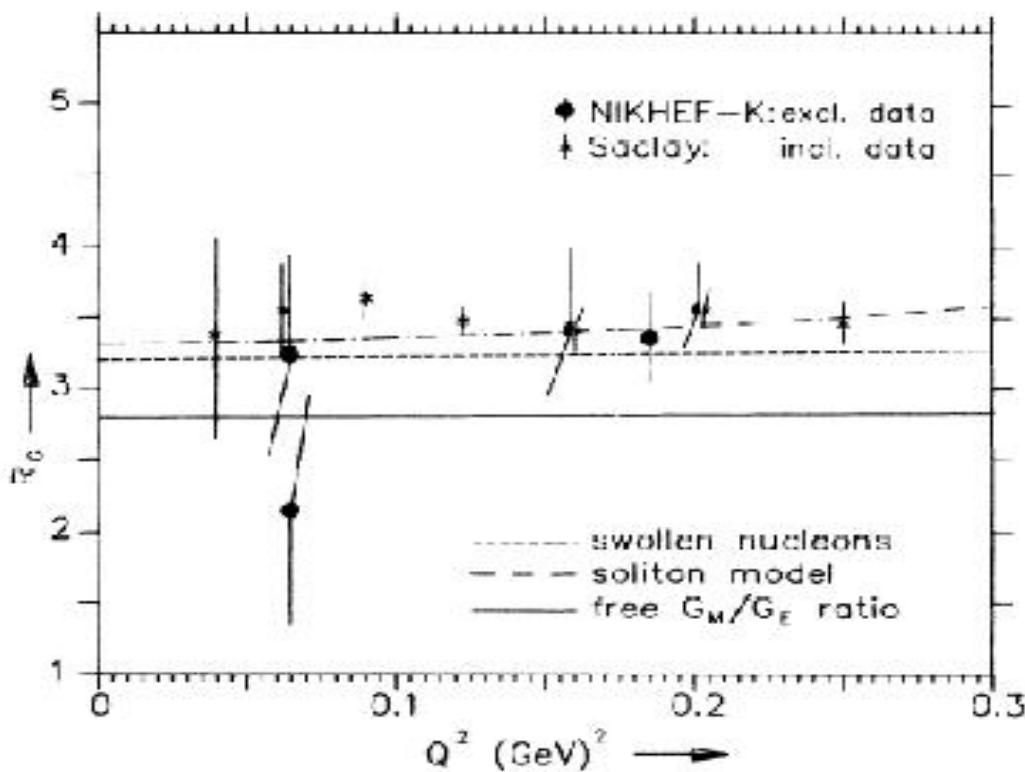
## $^6\text{Li}(\text{e},\text{e}'\text{p})$



J.B.J.M. Lanen *et al.*,  
Phys. Rev. Lett. **64**, 2250 (1990).

NIKHEF-K  
Amsterdam

## $^{12}\text{C}(\text{e},\text{e}'\text{p})$ and $^{12}\text{C}(\text{e},\text{e}')$



G. Van der Steenhoven *et al.*,  
Phys. Rev. Lett. **57**, 182 (1986)

JLab  
Hall C

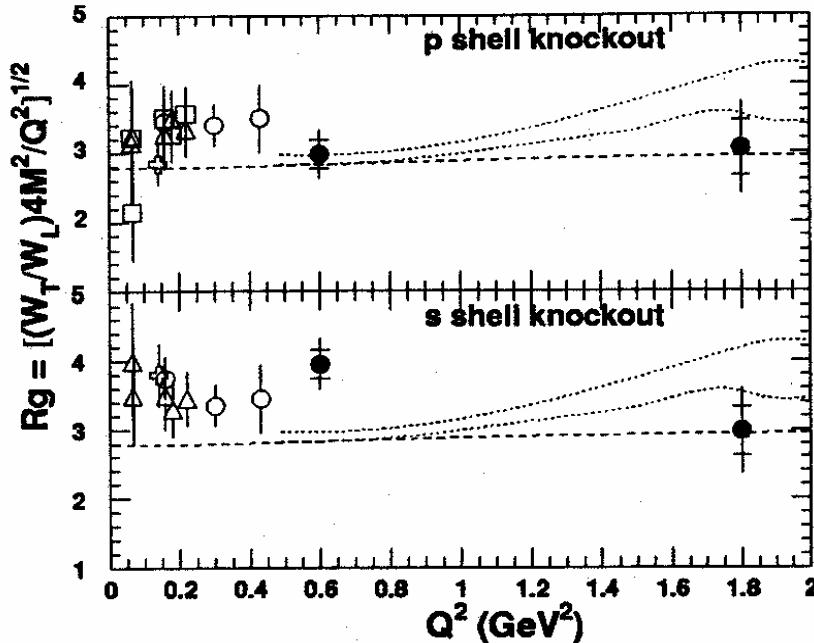
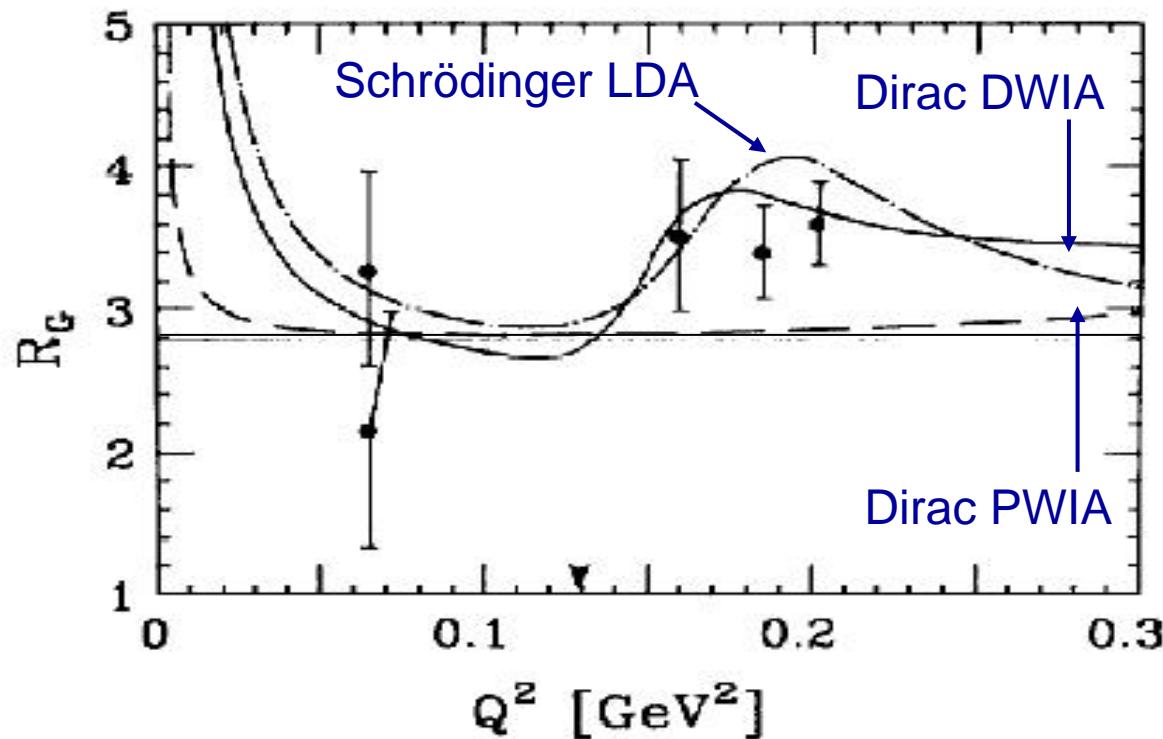


FIG. 3.  $R_G = \sqrt{W_T 4M_p^2 / W_L Q^2}$  for  $^{12}\text{C}$  (solid) from the measurements of this experiment with  $^6\text{Li}$  ( $p$  shell: open squares [3], open circles [25], and  $s$  shell: open triangles [3], open circles [25]) and  $^{12}\text{C}$  ( $p$  shell: open cross [1], open triangles [15], and  $s$  shell: open cross [1]). The top panel is for the  $p$  shell region and bottom panel is for the  $s$  shell region. The inner error bar represents that statistical error and the outer error bar includes the systematic error. The dashed line represents  $R_G$  for the free proton with the dipole electric and Ref. [18] magnetic form factor while the dotted lines represent the one sigma error band of the recent proton results of Ref. [27].

However, large FSI  
effects can mimic this  
behavior ...

# FSI calculations for $^{16}\text{O}$ $1\text{p}_{3/2}$

Data for  $^{12}\text{C}$   $1\text{p}_{3/2}$



T.D. Cohen, J.W. Van Orden, A. Picklesimer,  
Phys. Rev. Lett. **59**, 1267 (1987)

Another, less model-dependent,  
method ...

Polarization Transfer

# Proton Polarization and Form Factors

Free  $\vec{e} p$  scattering\*

$$I_0 P'_x = -2 \sqrt{\tau(1 + \tau)} G_E G_M \tan\left(\frac{\theta_e}{2}\right)$$

$$I_0 P'_z = \frac{e + e'}{m} \sqrt{\tau(1 + \tau)} G_M^2 \tan^2\left(\frac{\theta_e}{2}\right)$$

$$I_0 = G_E^2 + \tau G_M^2 \left[ 1 + 2(1 + \tau) \tan^2\left(\frac{\theta_e}{2}\right) \right]$$

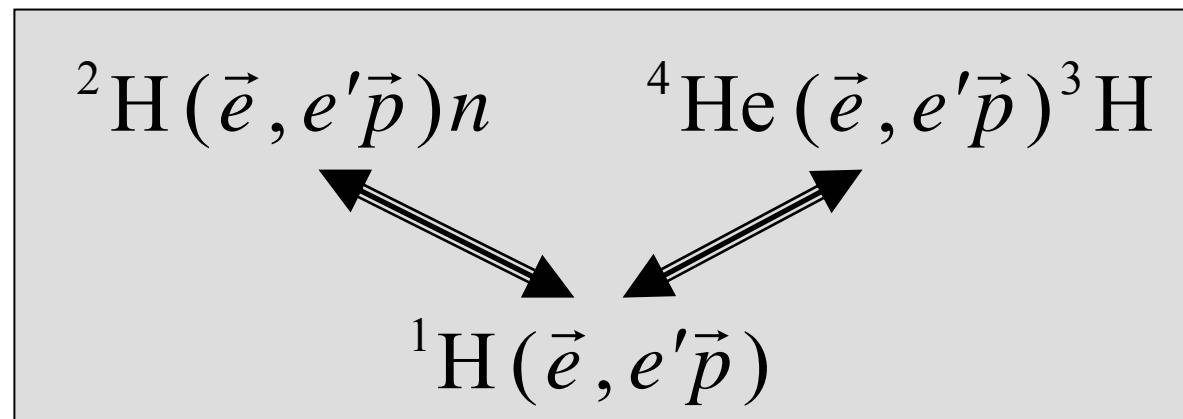
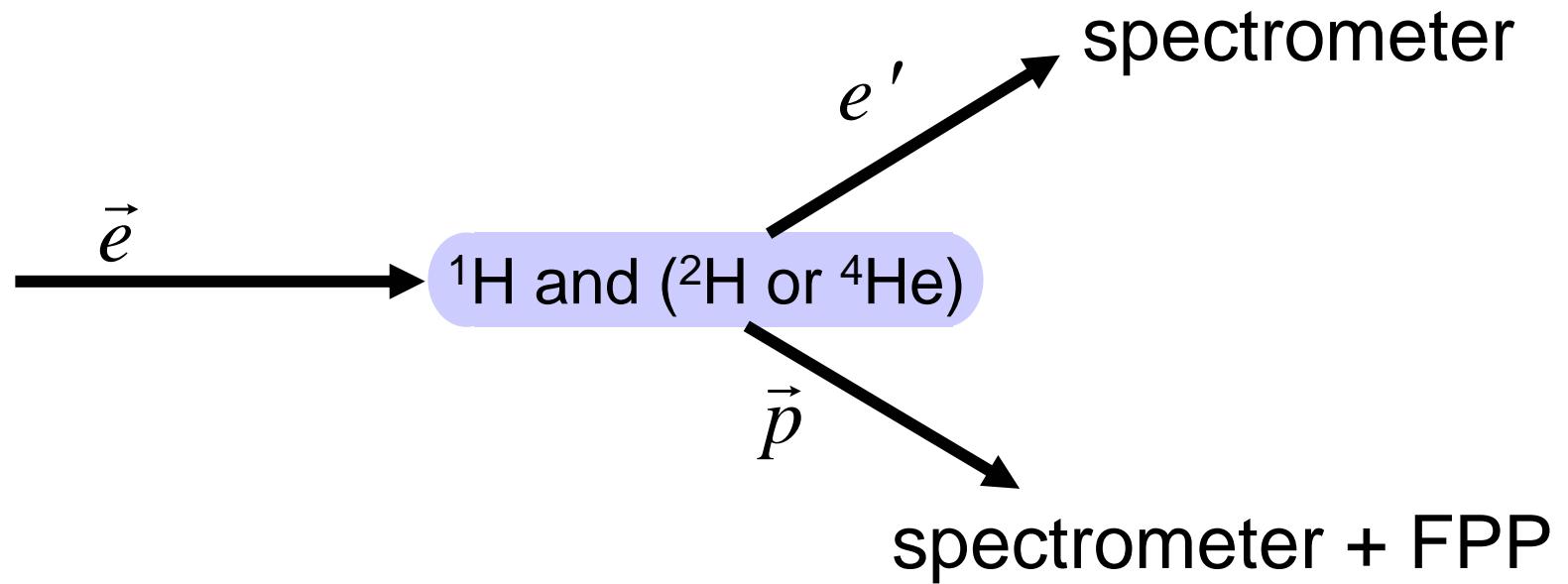
$$\frac{G_E}{G_M} = -\frac{P'_x}{P'_z} \cdot \frac{e + e'}{2m} \tan\left(\frac{\theta_e}{2}\right)$$

in nucleus  
model assumptions

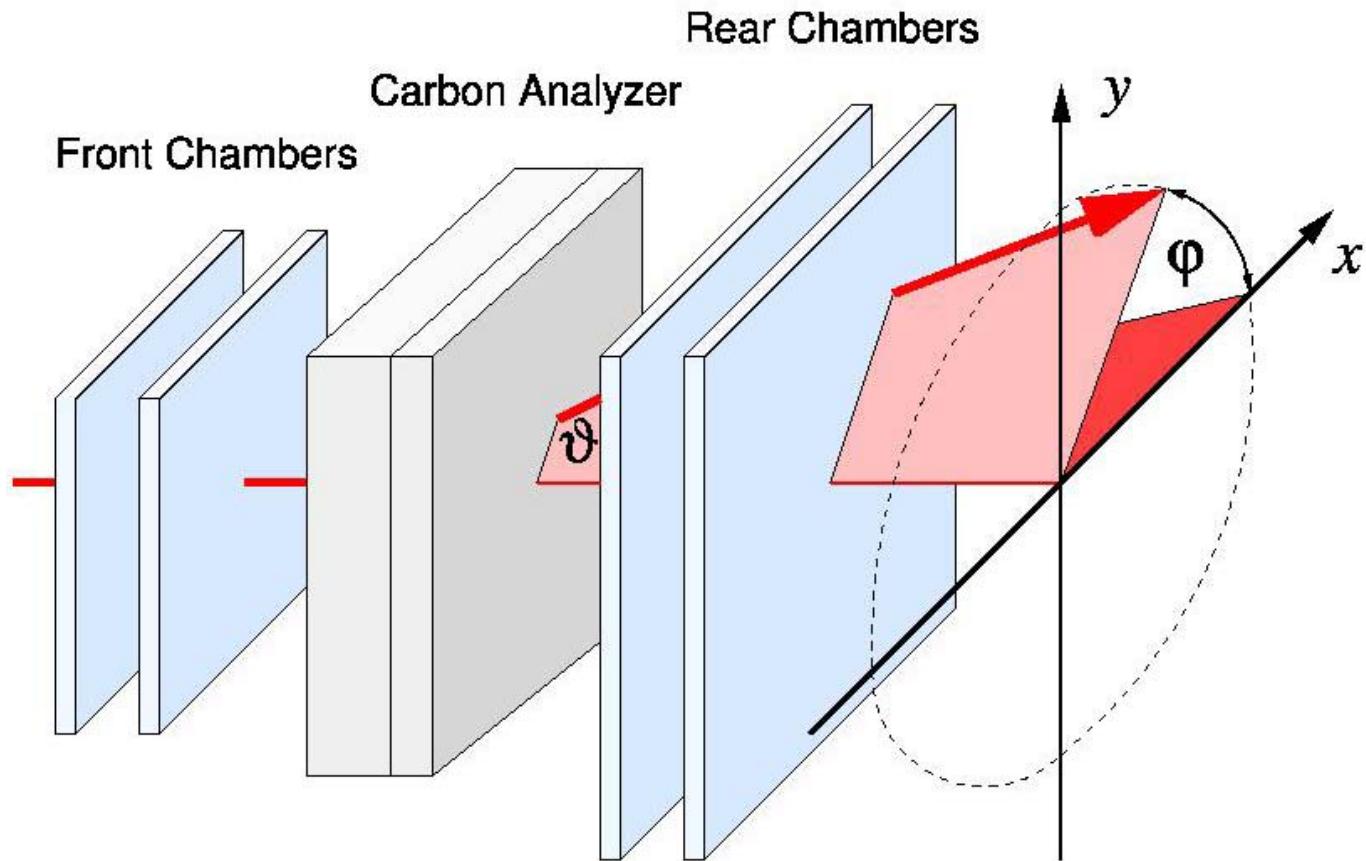
$$\frac{\tilde{G}_E}{\tilde{G}_M}$$

\* R. Arnold, C. Carlson and F. Gross, Phys. Rev. C **23**, 363 (1981).

# Polarization Transfer in Hall A



# Measuring the Proton Polarization: FPP



# Density Dependent Form Factors

Quark-Meson Coupling Model (QMC):

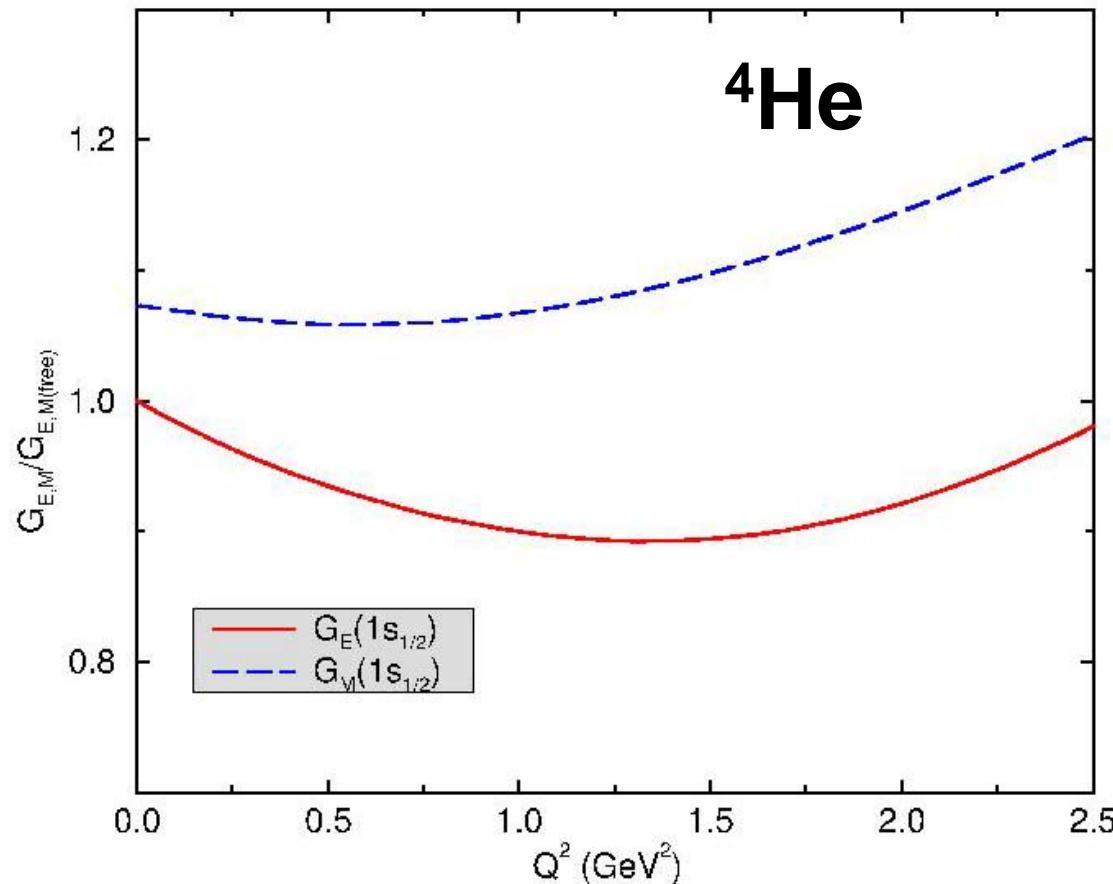
$$\overline{G}_\alpha(Q^2) = \frac{\int d^3r w_\alpha(r) G(Q^2, \rho_B(r))}{\int d^3r w_\alpha(r)}$$

For (e,e'p)

$$w_\alpha = \exp(i\vec{q} \cdot \vec{r}) \chi^{(-)}(\vec{p}', \vec{r})^* \phi_\alpha(r)$$

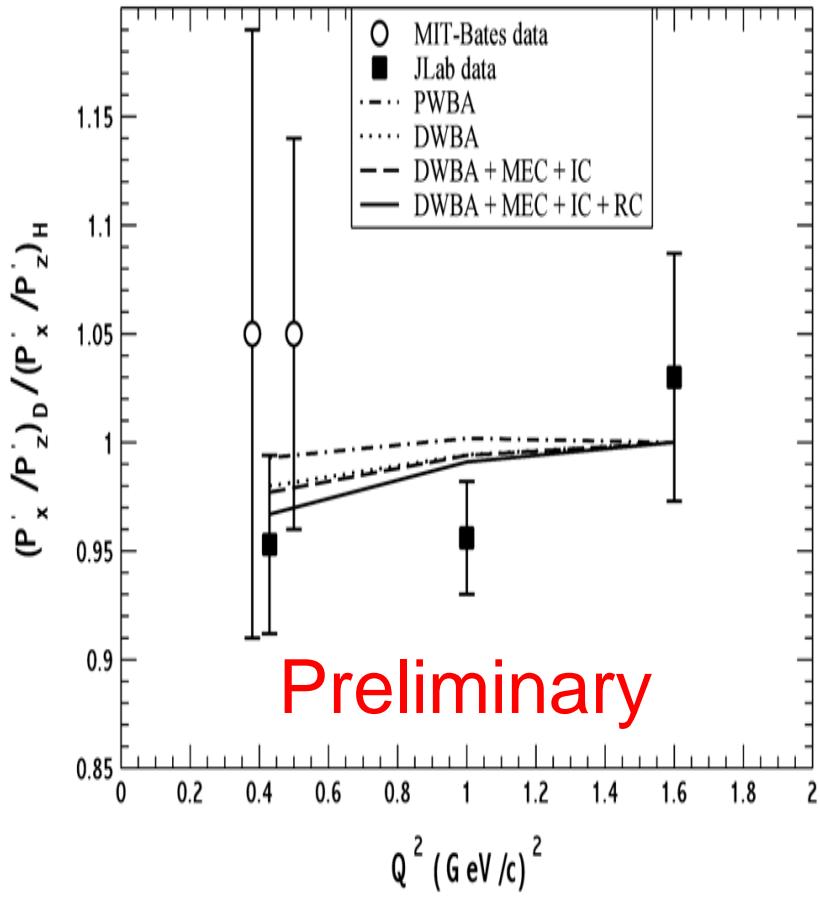
D.H. Lu, , A.W. Thomas, K. Tsushima, A.G. Williams, K. Saito, Phys. Lett. **B** 417, 217 (1998).

# Quark-Meson Coupling Model



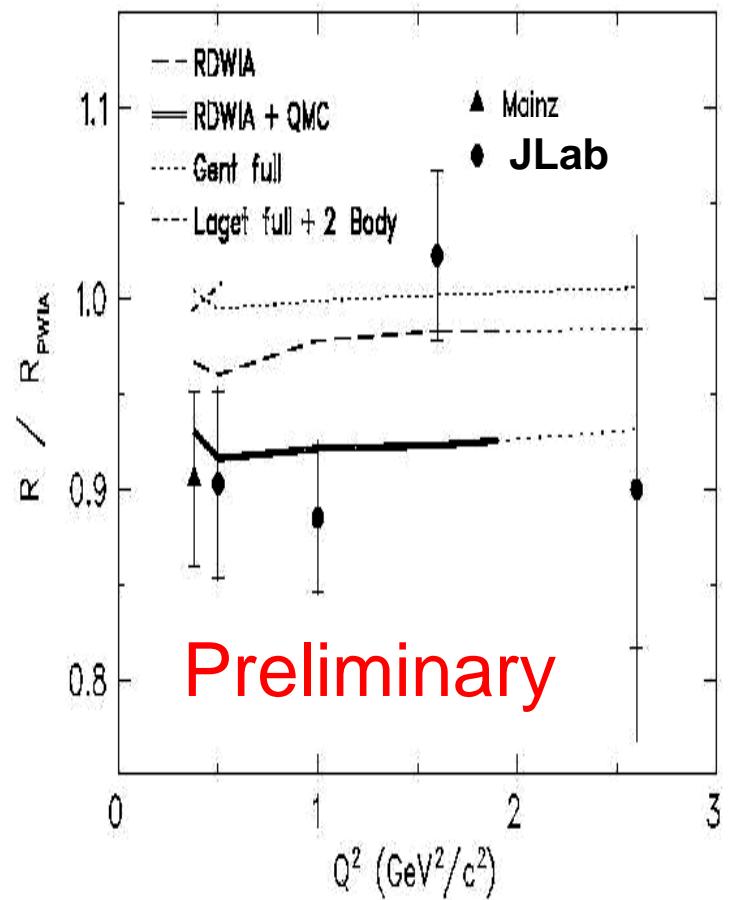
D.H. Lu, K. Tsushima, A.W. Thomas, A.G. Williams and K. Saito,  
Phys. Lett. **B417**, 217 (1998) and Phys. Rev. C **60**, 068201 (1999).

# $^2\text{H}(\vec{e}, e' \vec{p})n$



Calculations by Arenhövel

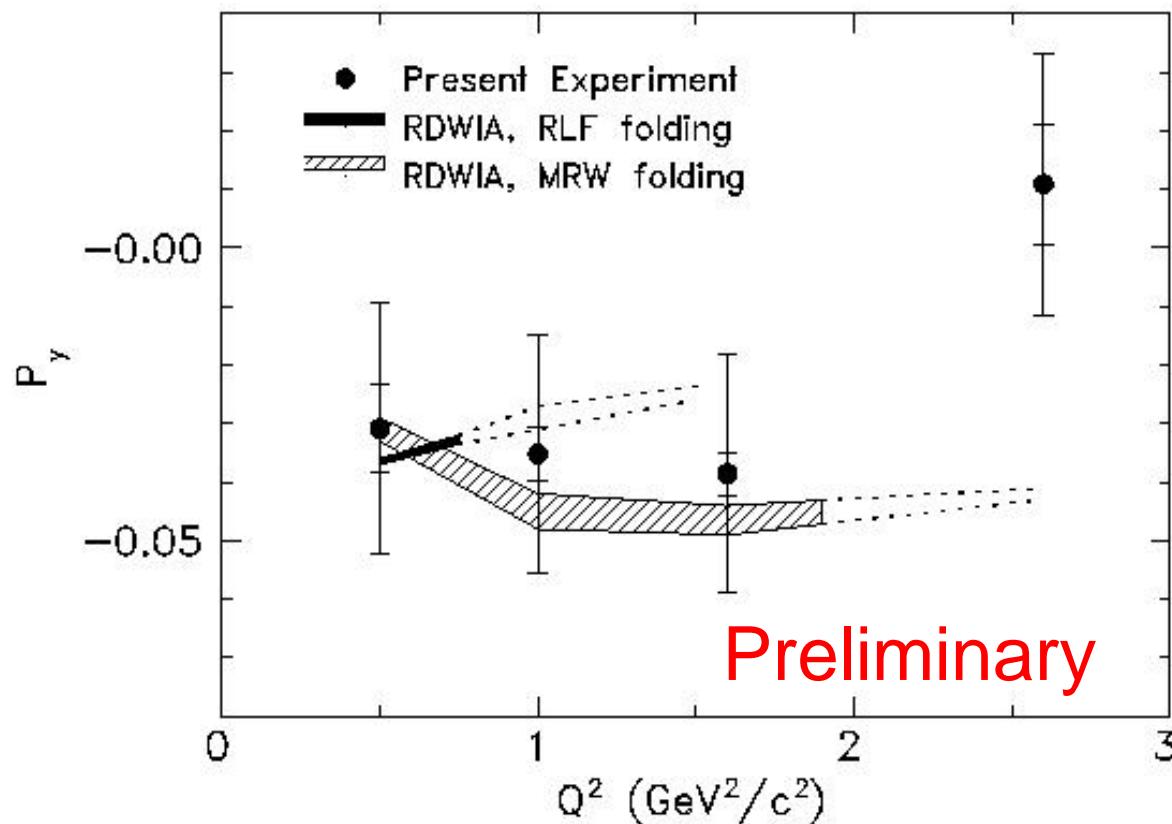
# $^4\text{He}(\vec{e}, e' \vec{p})^3\text{H}$



RDWIA calculations by Udias *et al.*

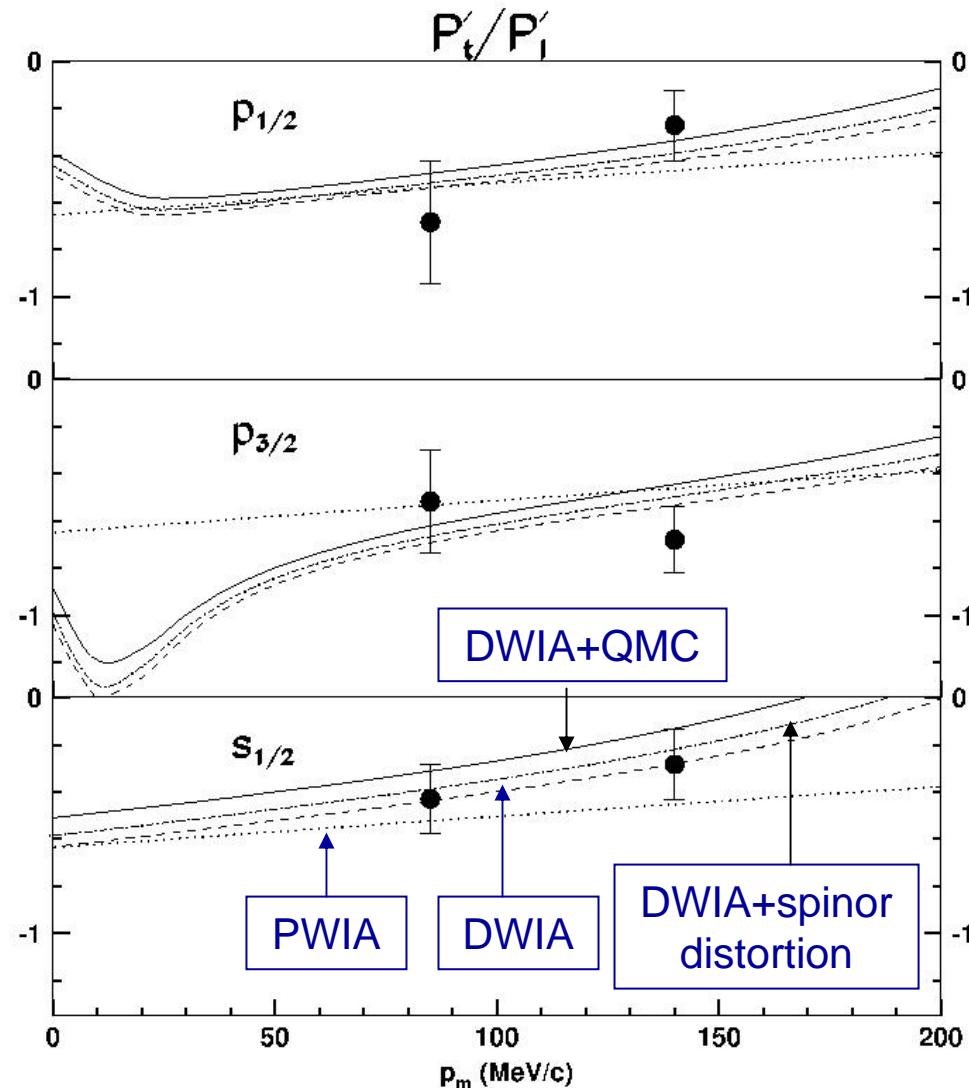
# Induced Polarization – ${}^4\text{He}$

JLab E93-049



$P_y=0$  in PWIA: test of FSI

$^{16}\text{O}(\vec{e}, e' \vec{p})^{15}\text{N}$  at  $Q^2 = 0.8 (\text{GeV}/c)^2$

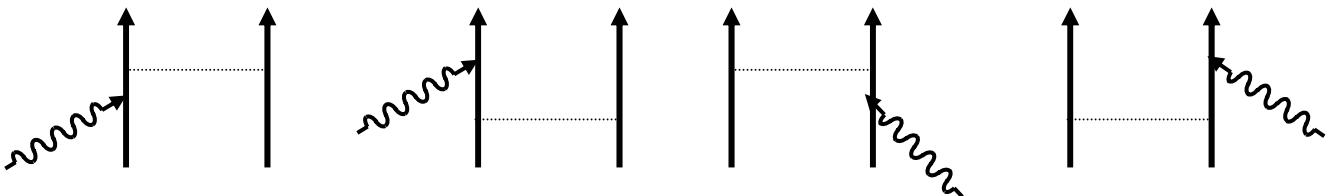


S. Malov *et al.*, Phys. Rev. C **62**, 057302 (2000).

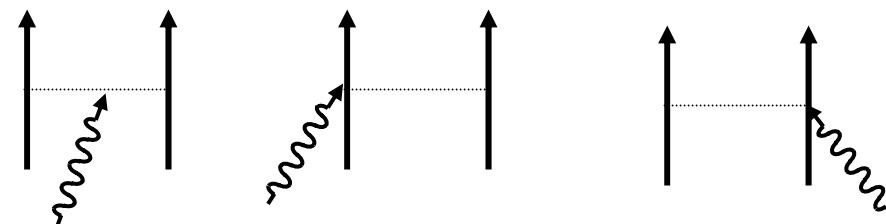
# Studies of the Reaction Mechanism

# Correlations and Interaction Currents

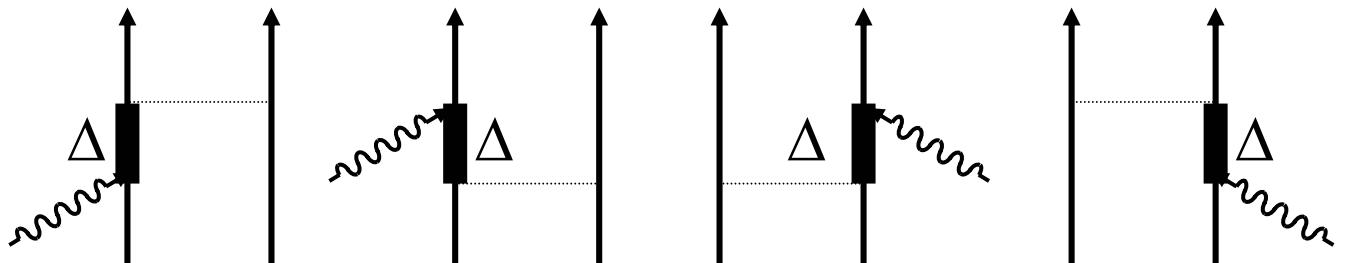
Correlations



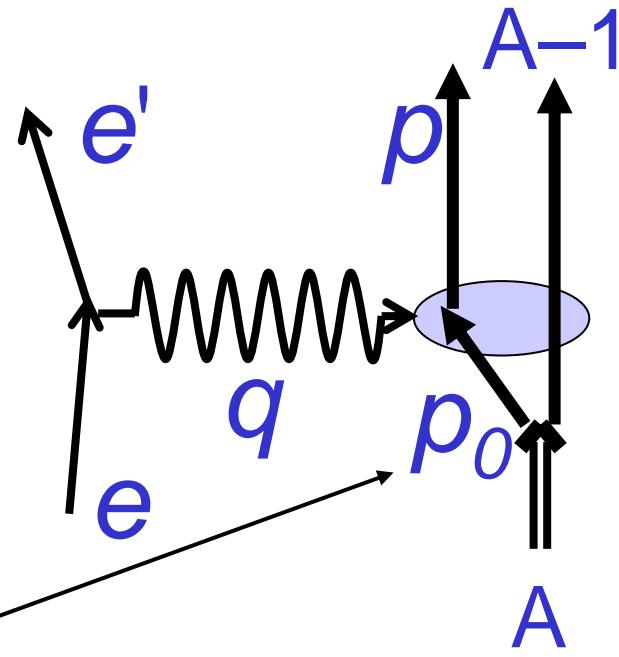
MEC's



IC's



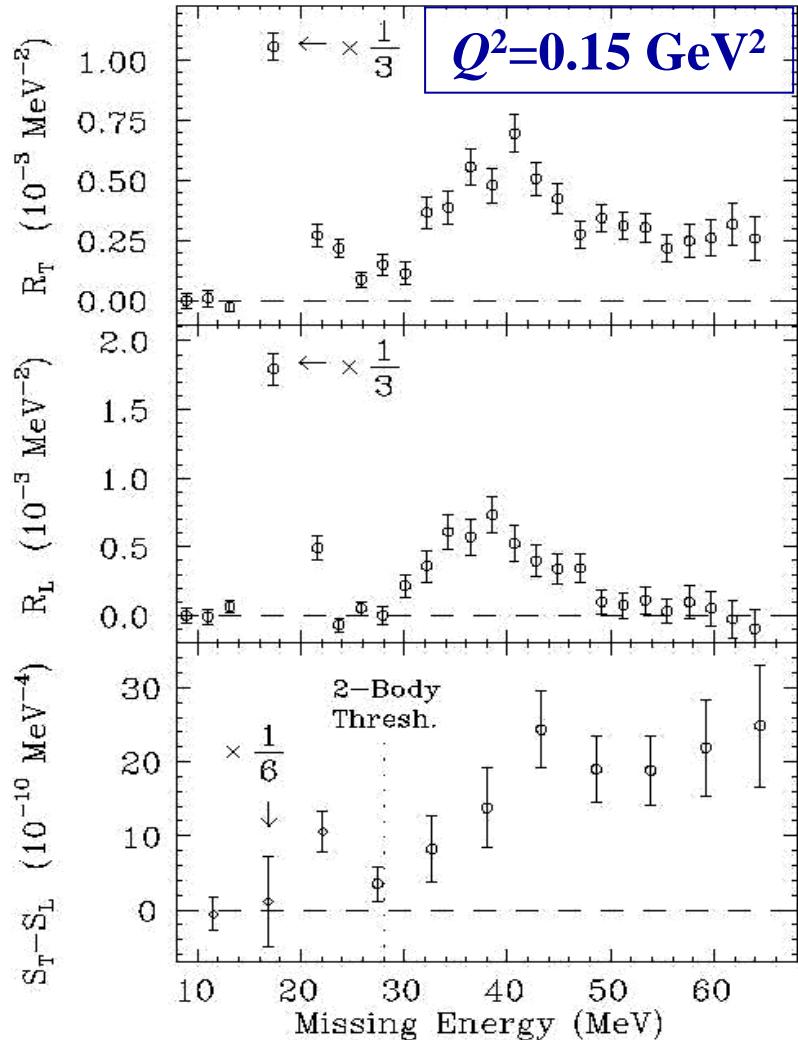
# Off-shell Effects



initial proton is bound

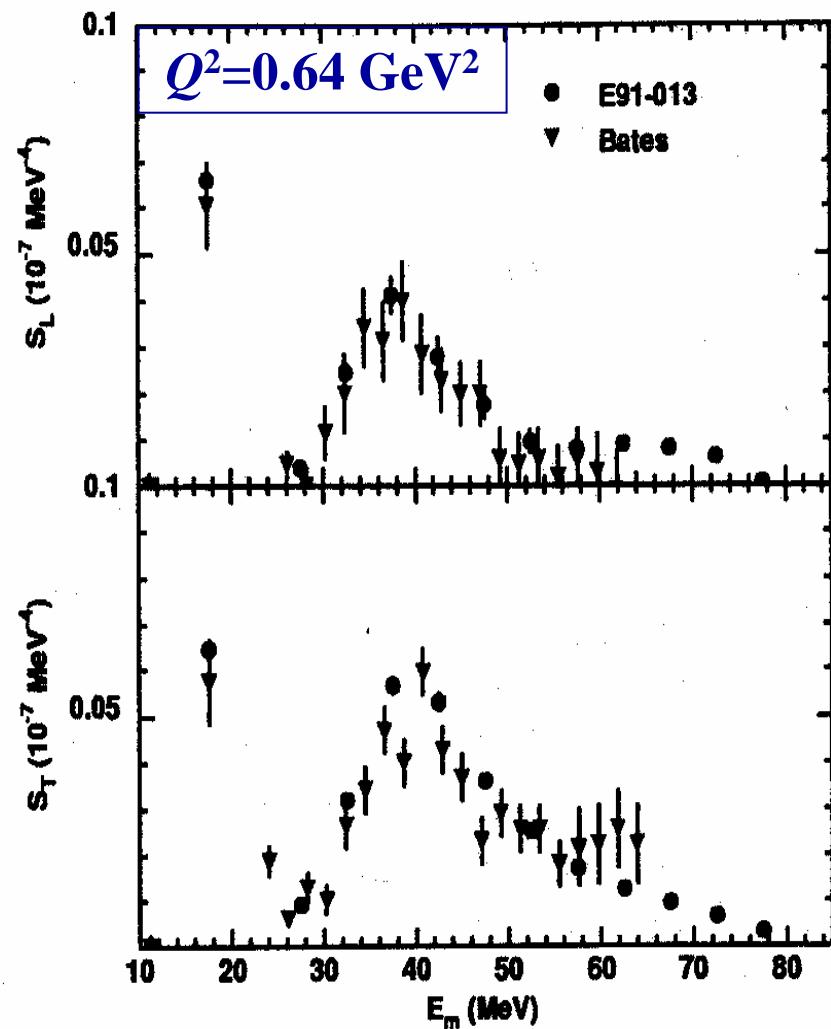
Vertex function is not well defined. The “Gordon identity” leads to alternative forms, equivalent only when proton is on-shell.

# $^{12}\text{C}(\text{e},\text{e}'\text{p})$ L/T Separations



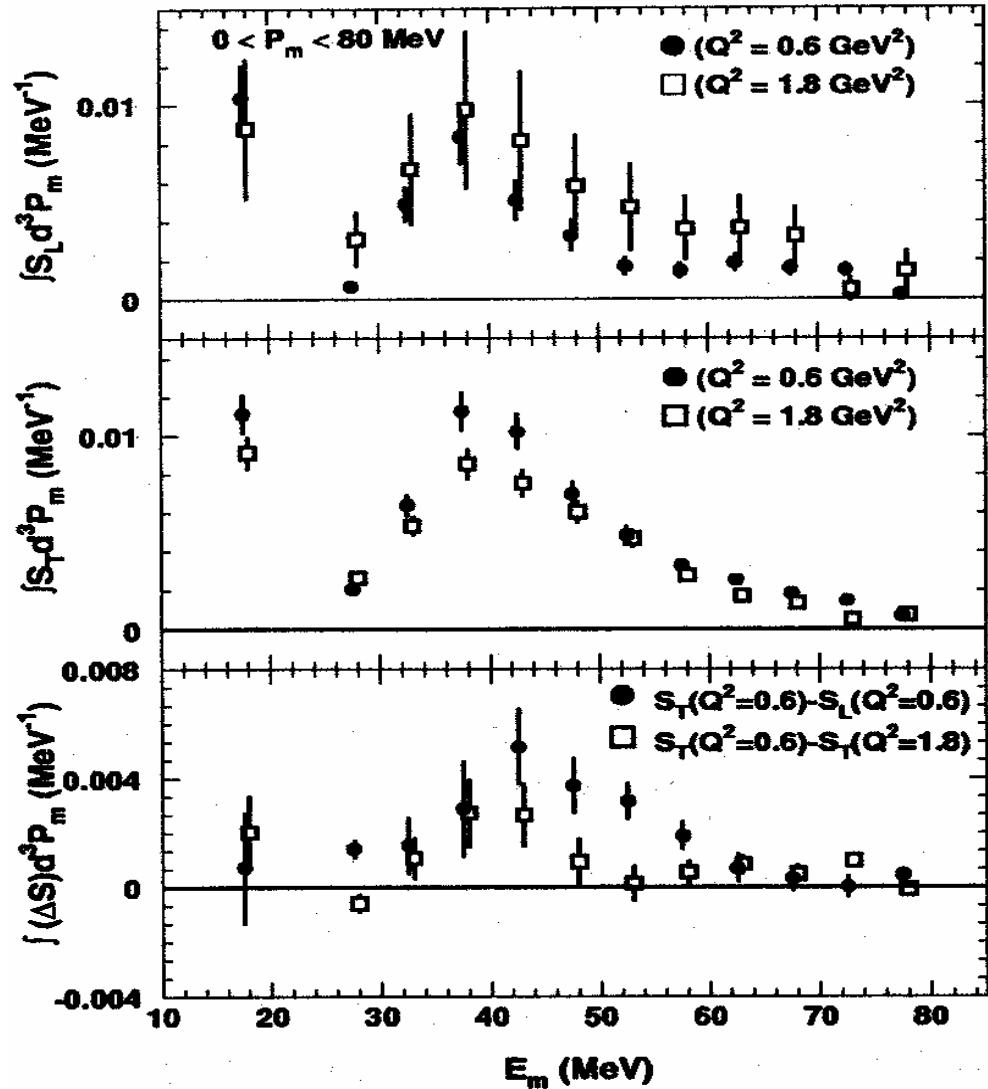
P.E. Ulmer *et al.*, Phys. Rev. Lett. **59**, 2259 (1987).

Bates Linear Accelerator



D. Dutta *et al.*, Phys. Rev. C **61**, 061602 (2000).

JLab Hall C

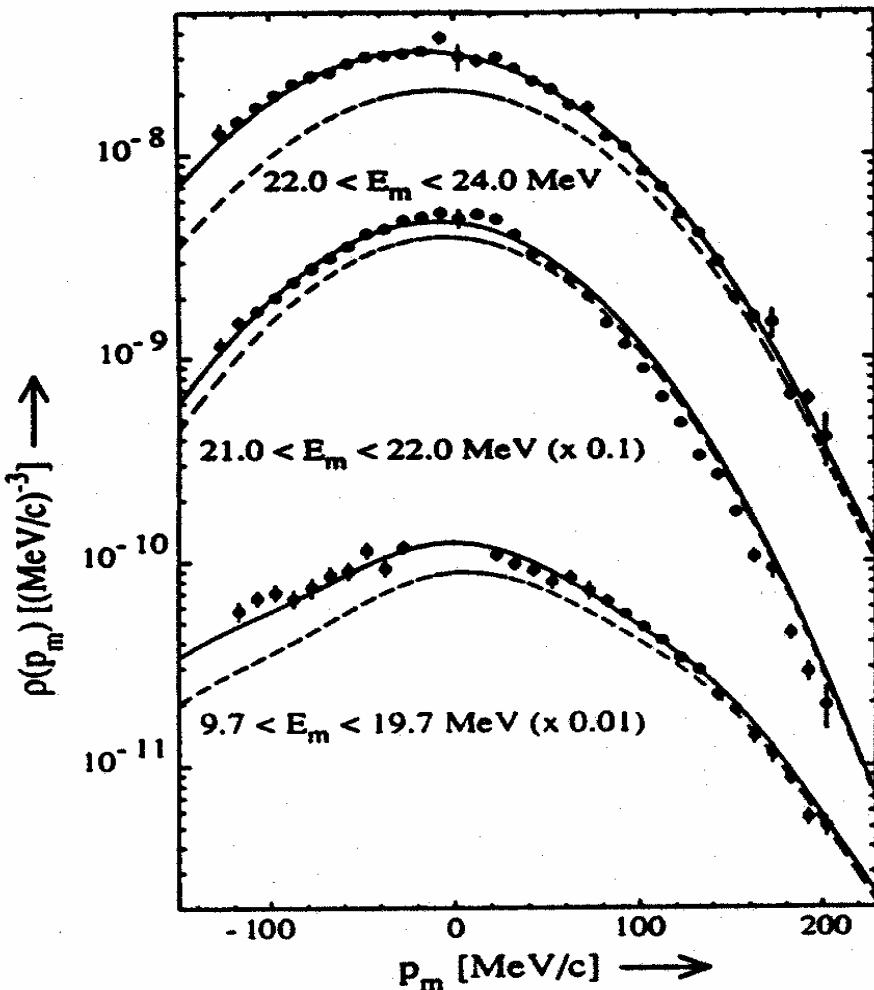


Excess transverse strength at high  $\varepsilon_m$ .  
Persists, though perhaps declines, at higher  $Q^2$ .

JLab Hall C

D. Dutta *et al.*, Phys. Rev. C 61, 061602 (2000).

# ${}^6\text{Li}(\text{e},\text{e}'\text{p})$ T/L Ratio



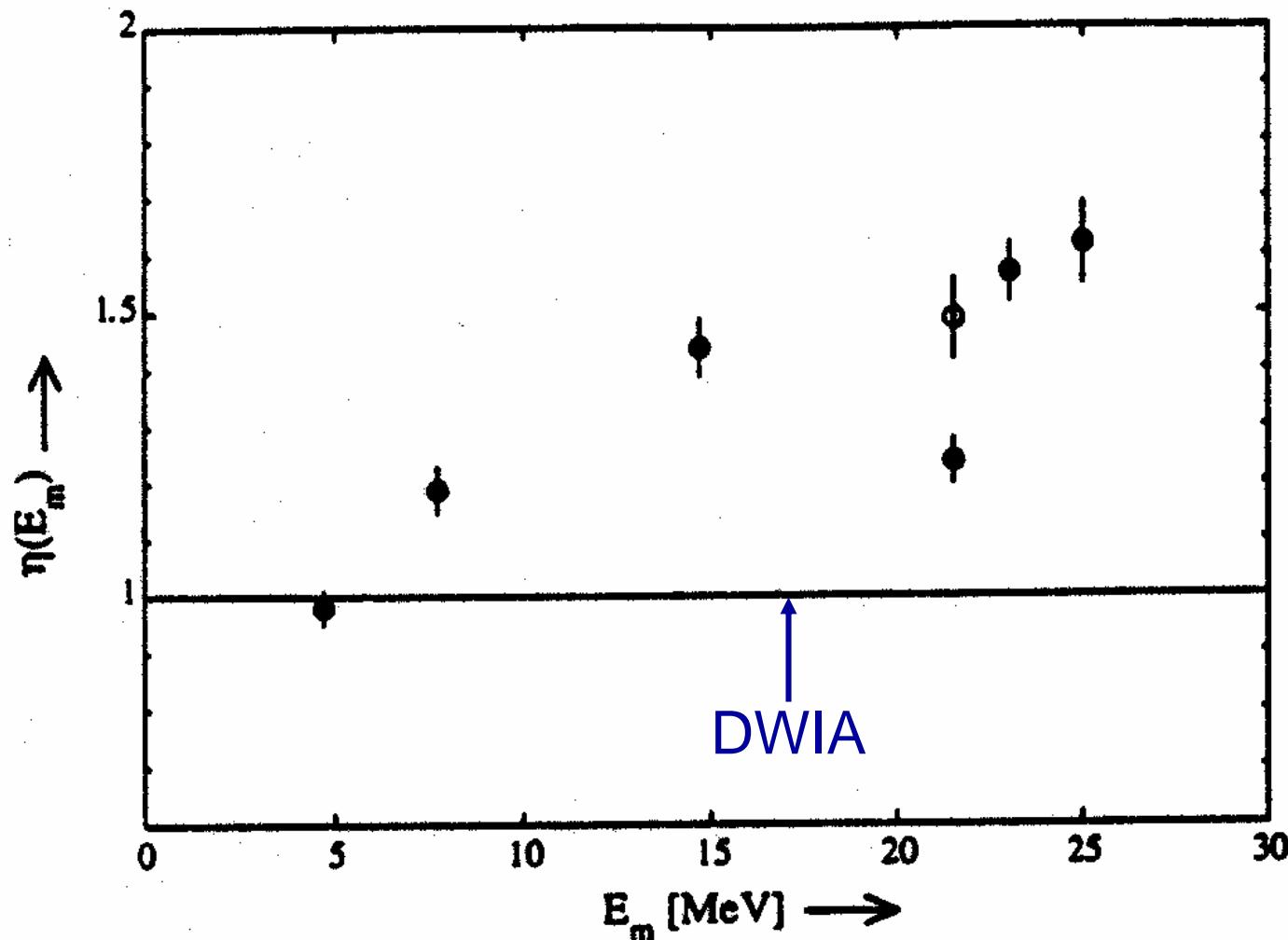
DWIA (dashed) fails to describe overall strength.

Scaling transverse amplitude in DWIA (solid) gives good agreement → deduce scale factor,  $\eta$ .

NIKHEF-K  
Amsterdam

J.B.J.M. Lanen *et al.*, Phys. Rev. Lett. **64**, 2250 (1990).

# ${}^6\text{Li}(\text{e},\text{e}'\text{p})$ T/L Ratio



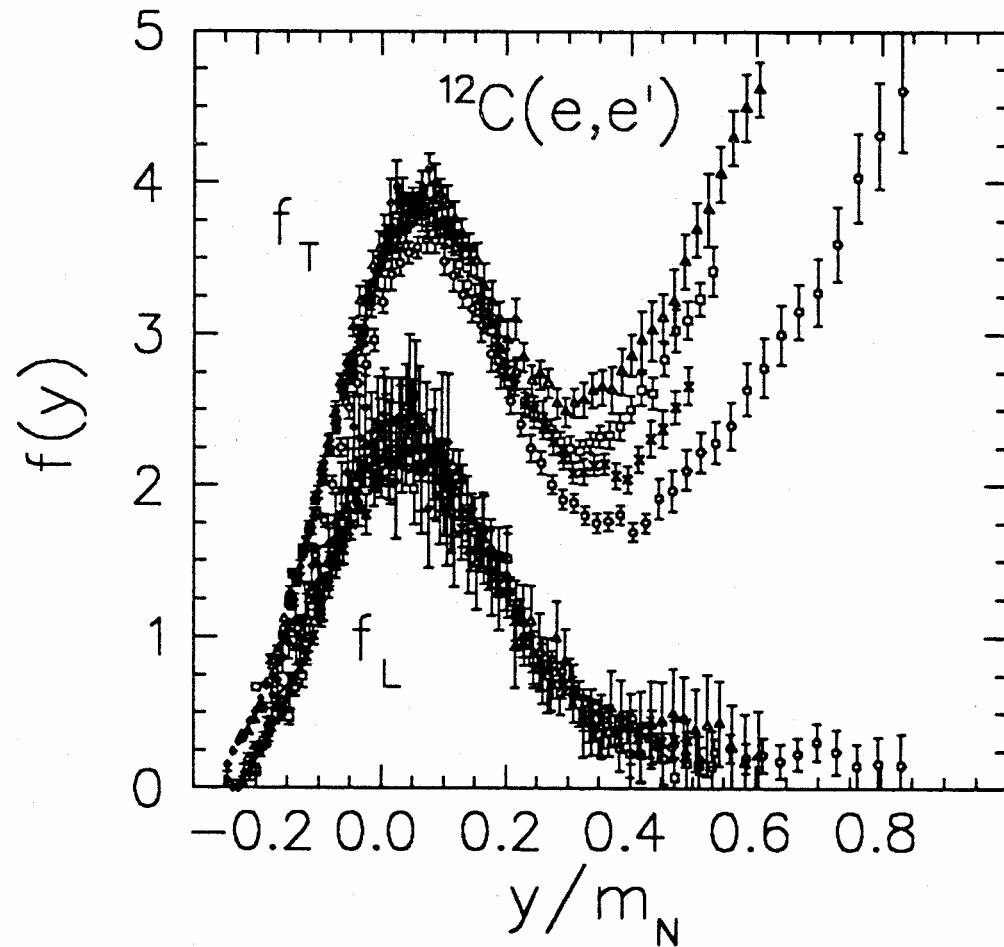
J.B.J.M. Lanen *et al.*, Phys. Rev. Lett. **64**, 2250 (1990).

## The L/T separations suggest

- Additional transverse reaction mechanism above 2-nucleon emission threshold.
- MEC's primarily transverse in character. Suggestive of two-body current.

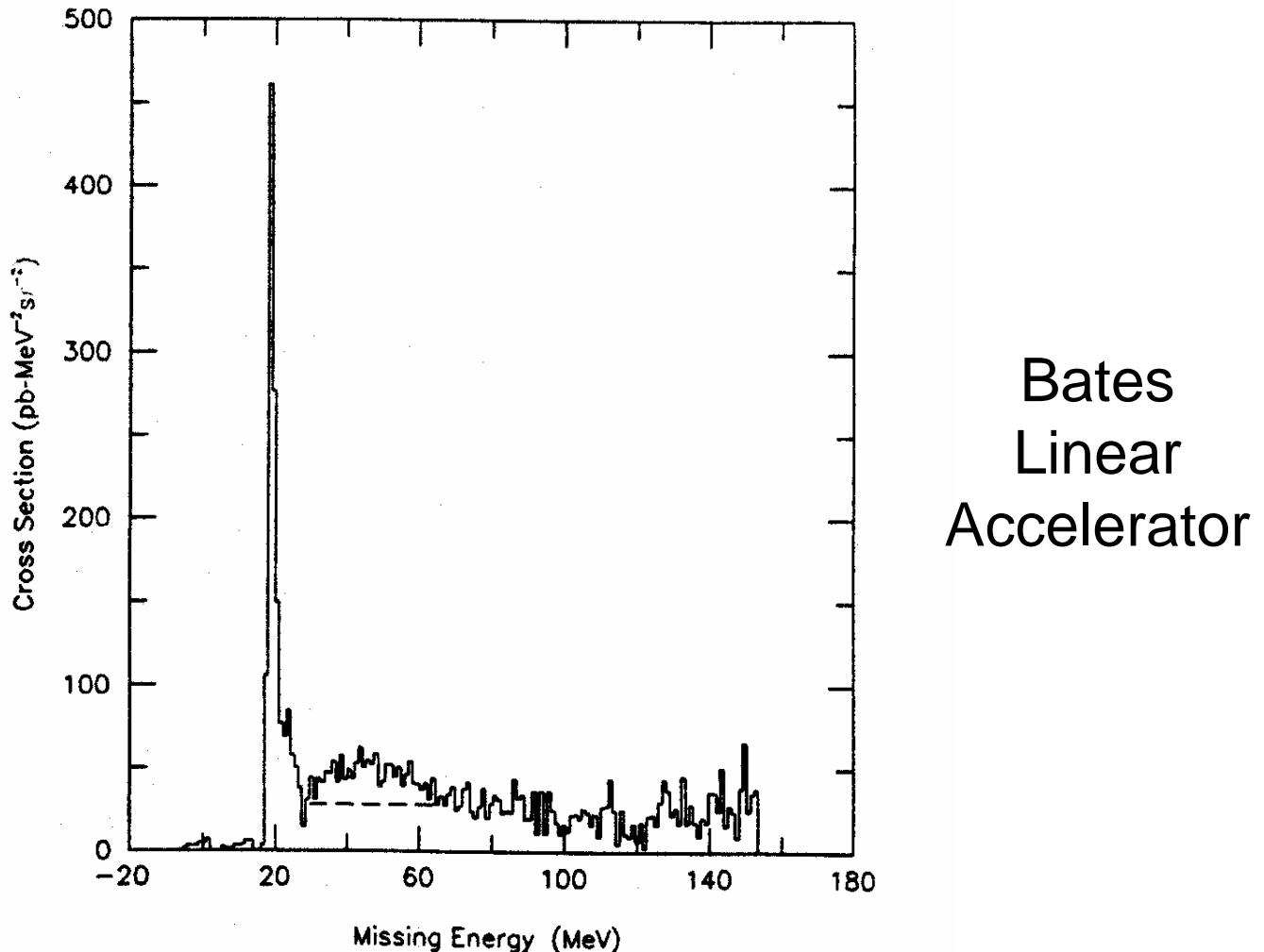
Reminiscent of ...

T/L anomaly  
in inclusive  
 $(e, e')$ :



J.M. Finn, R.W. Lourie and B.H. Cottman, Phys. Rev. C **29**, 2230 (1984).

# $^{12}\text{C}(\text{e},\text{e}'\text{p})$ in “Dip Region”



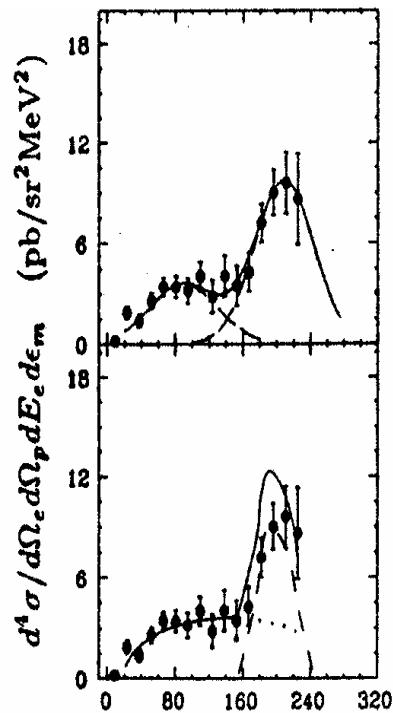
R.W. Lourie *et al.*, Phys. Rev. Lett. **56**, 2364 (1986).

Data from: Bates Linear Accelerator

# $^{12}\text{C}(\text{e},\text{e}'\text{p})$

“Delta”

Between dip and  $\Delta$

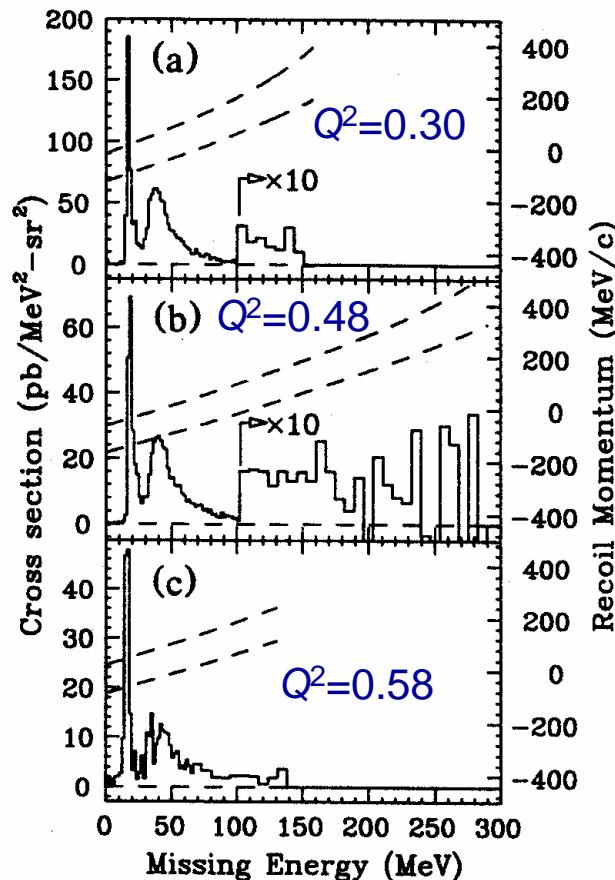


Missing Energy (MeV)

H. Baghaei *et al.*,  
Phys. Rev. C **39**, 177 (1989).

Bates Linear Accelerator

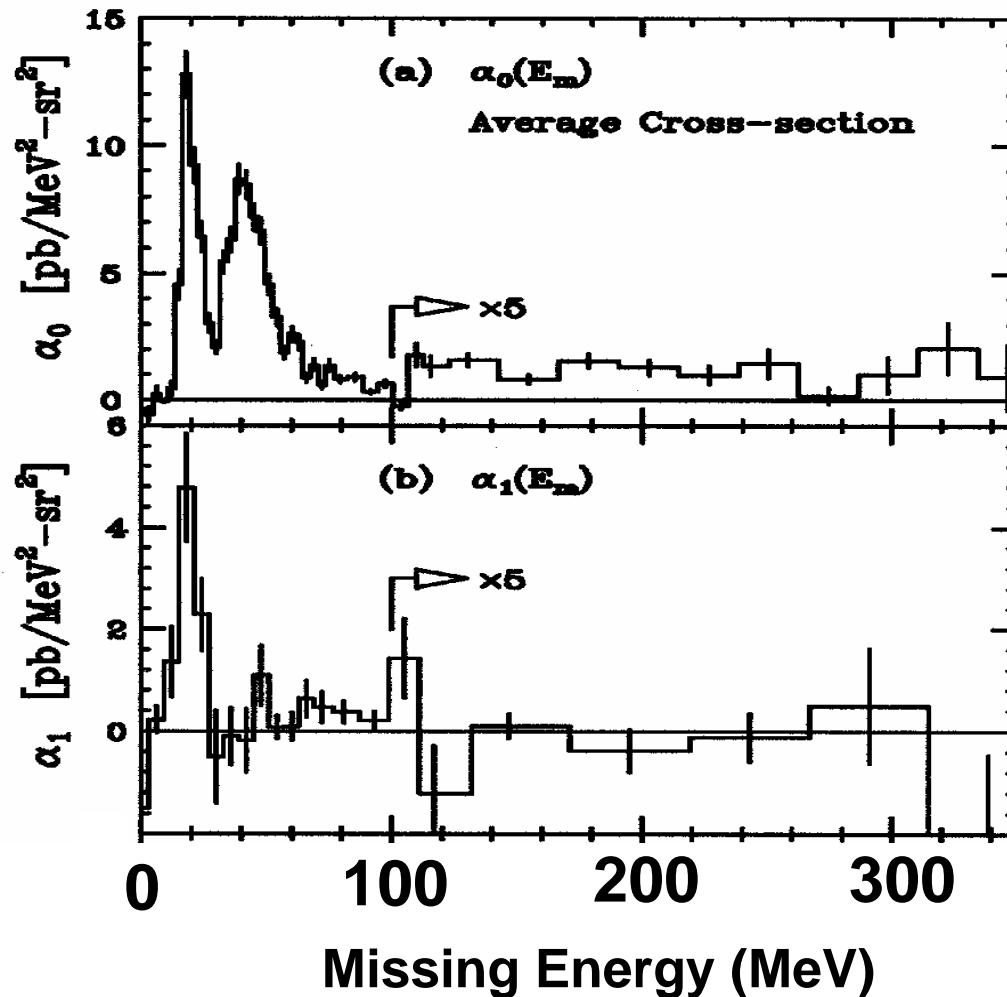
Quasielastic



L.B. Weinstein *et al.*,  
Phys. Rev. Lett. **64**, 1646 (1990).

Bates Linear Accelerator

# $^{12}\text{C}(\text{e},\text{e}'\text{p})$ $q=990 \text{ MeV}/c$ , $\omega=475 \text{ MeV}$



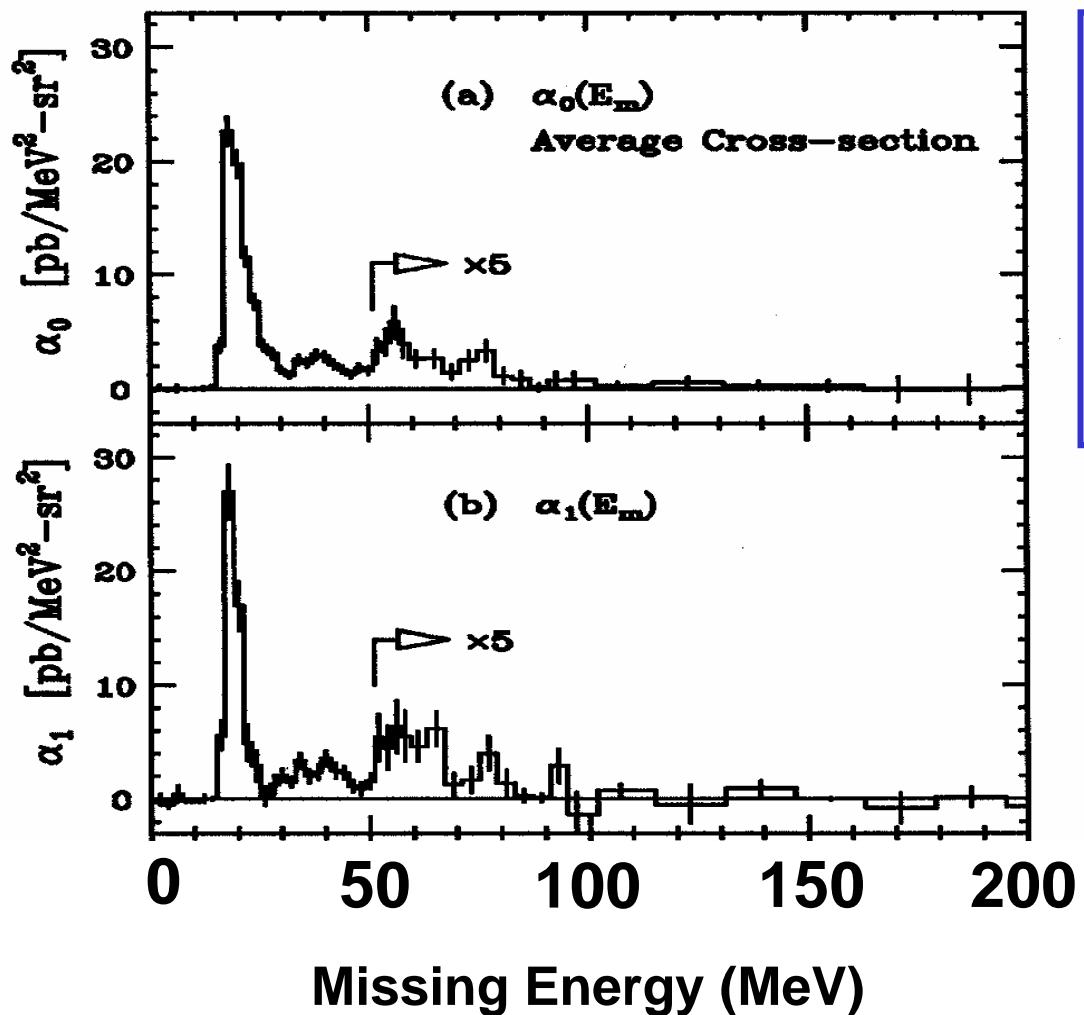
$$\frac{d^6\sigma}{d\Omega_e d\Omega_p d\omega d\varepsilon_m} = \sum_{l=0}^{l_{\max}} \alpha_l(\varepsilon_m) P_l\left(\frac{\omega - \omega_0}{\Delta\omega/2}\right)$$

For  $60 < \varepsilon_m < 100 \text{ MeV}$ ,  
continuum cross section  
increases strongly with  $\omega$ .  
Large continuum strength  
continues up to 300 MeV.

Bates Linear  
Accelerator

Figure adapted from J.H. Morrison *et al.*,  
Phys. Rev. C **59**, 221 (1999).

# $^{12}\text{C}(\text{e},\text{e}'\text{p})$ $q=970 \text{ MeV}/c$ , $\omega=330 \text{ MeV}$



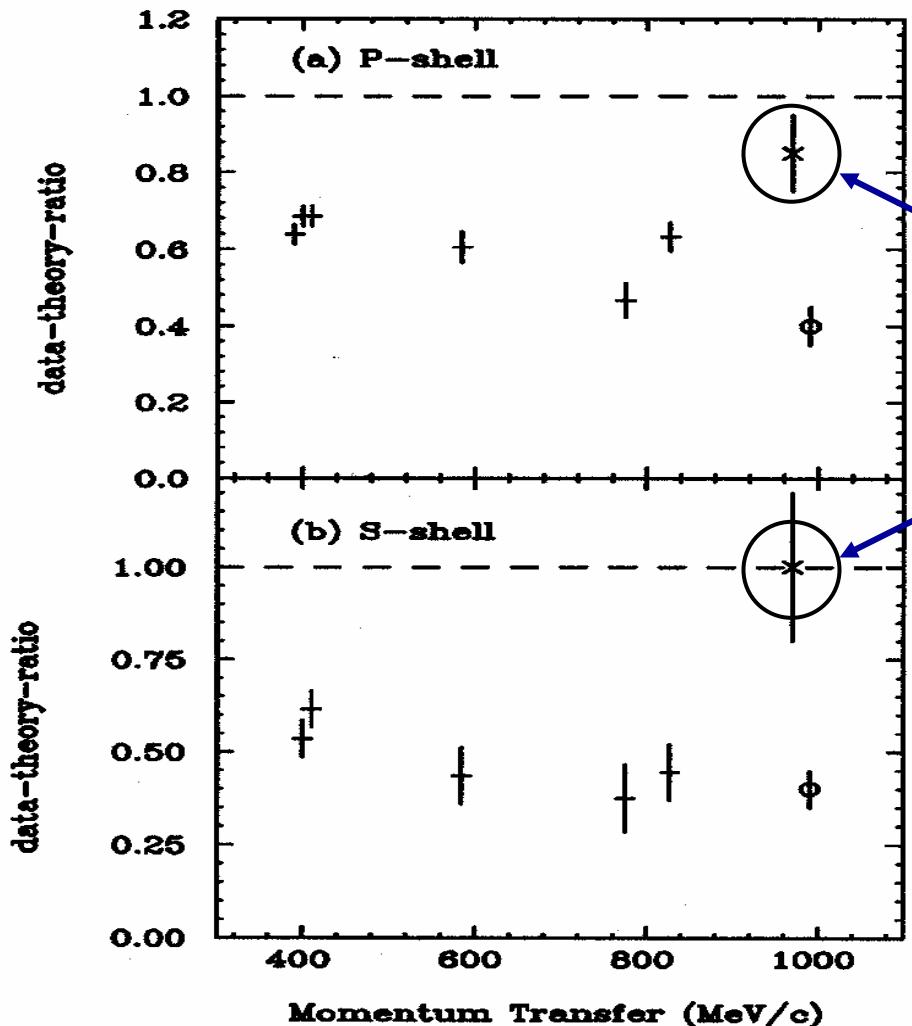
$$\frac{d^6\sigma}{d\Omega_e d\Omega_p d\omega d\varepsilon_m} = \sum_{l=0}^{l_{\max}} \alpha_l(\varepsilon_m) P_l\left(\frac{\omega - \omega_0}{\Delta\omega/2}\right)$$

Continuum strength increases strongly with  $\omega$ .  
Continuum cross section is smaller at high  $\varepsilon_m$ .

Figure adapted from J.H. Morrison *et al.*,  
Phys. Rev. C **59**, 221 (1999).

Bates Linear Accelerator

# $^{12}\text{C}(\text{e},\text{e}'\text{p})$

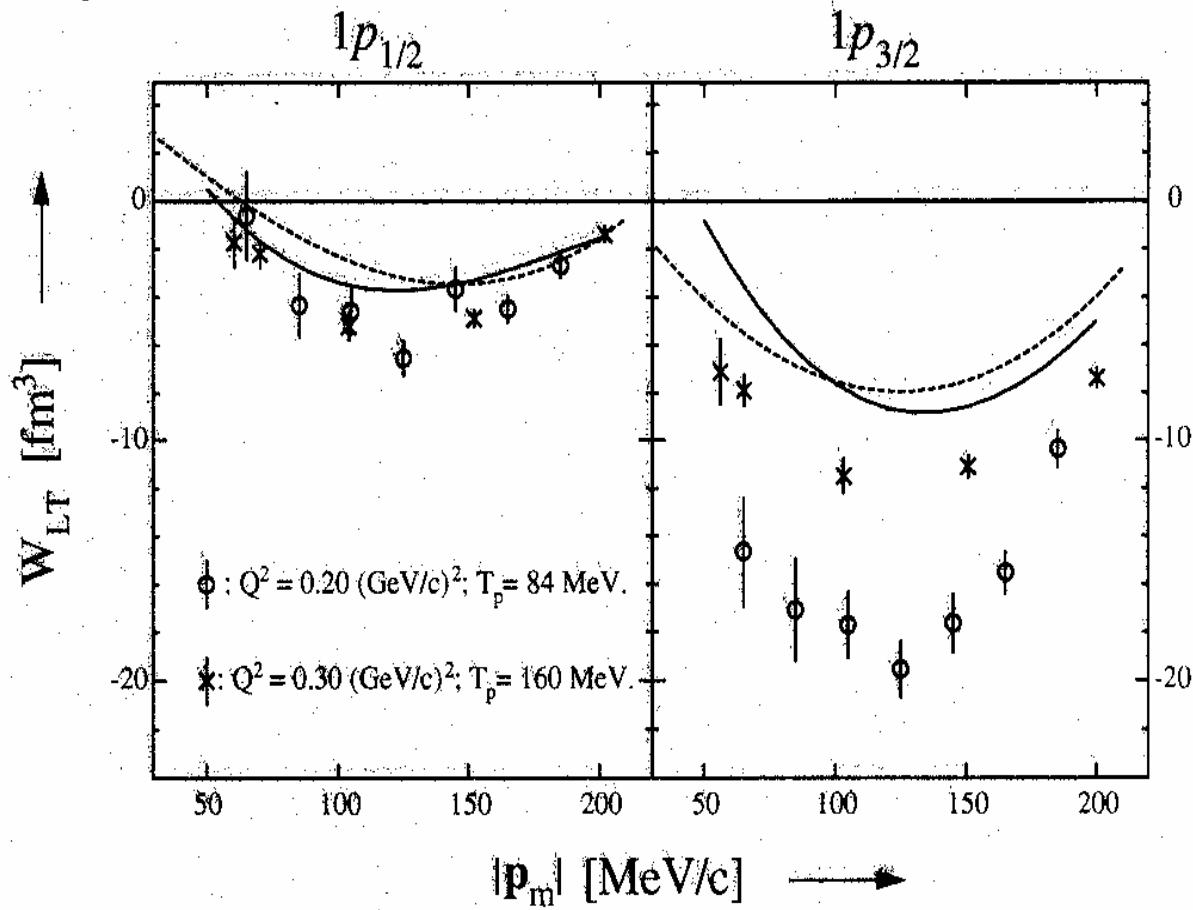


For  $\omega < \omega_{QE}$ ,  
spectroscopic  
factors  
consistent with  
naïve  
expectations.

Bates Linear  
Accelerator

J.H. Morrison *et al.*, Phys. Rev. C 59, 221 (1999).

# $^{16}\text{O}(\text{e},\text{e}'\text{p})$



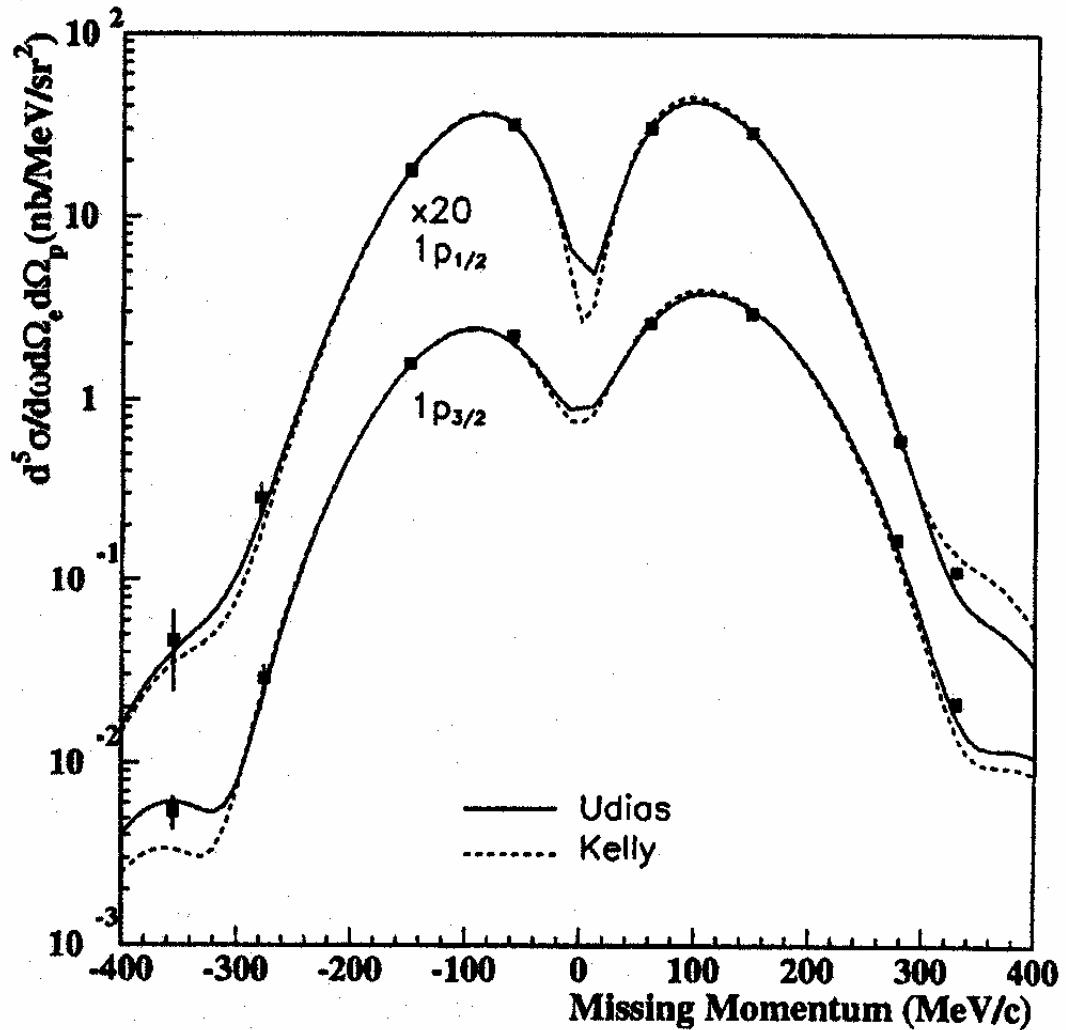
Large discrepancy for  $1p_{3/2}$ .  
 Relativistic effects predicted to be small here.  
 Two-body currents responsible??

C.M. Spaltro *et al.*, Phys. Rev. C **48**, 2385 (1993).

Circles (solid) – NIKHEF-K

Crosses (dashed) - Saclay

# $^{16}\text{O}(\text{e},\text{e}'\text{p})$ $Q^2=0.8 \text{ GeV}^2$ Quasielastic

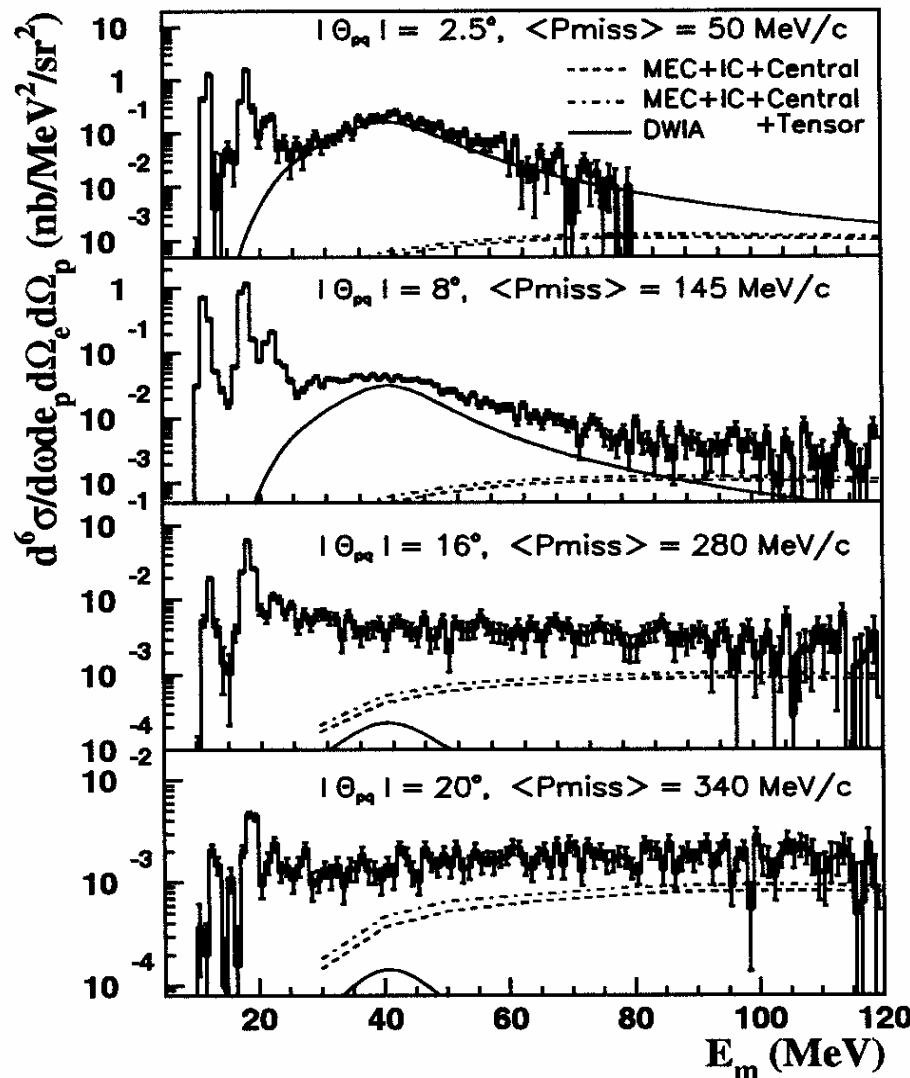


Relativistic  
DWIA gives  
good  
agreement  
with data.

JLab Hall A

J. Gao *et al.*, Phys. Rev. Lett. **84**, 3265 (2000).

# $^{16}\text{O}(\text{e},\text{e}'\text{p})$ $Q^2=0.8 \text{ GeV}^2$ Quasielastic



Two-body calculations of Ryckebusch *et al.*, give flat distribution, as seen in the data, but underpredict by a factor of two.

JLab Hall A

N. Liyanage *et al.*, Phys. Rev. Lett. **86**, 5670 (2001).

**At high energies,  $R_{LT}$   
interference response  
function sensitive to  
relativistic effects.**

**For example, spinor  
distortion ...**

# Spinor Distortions

$$\Psi = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix}$$

$$\Psi_- = \frac{\sigma \cdot p}{E + m + S - V} \Psi_+$$

**N.R. reduction**

$S + V \rightarrow$  Mean field

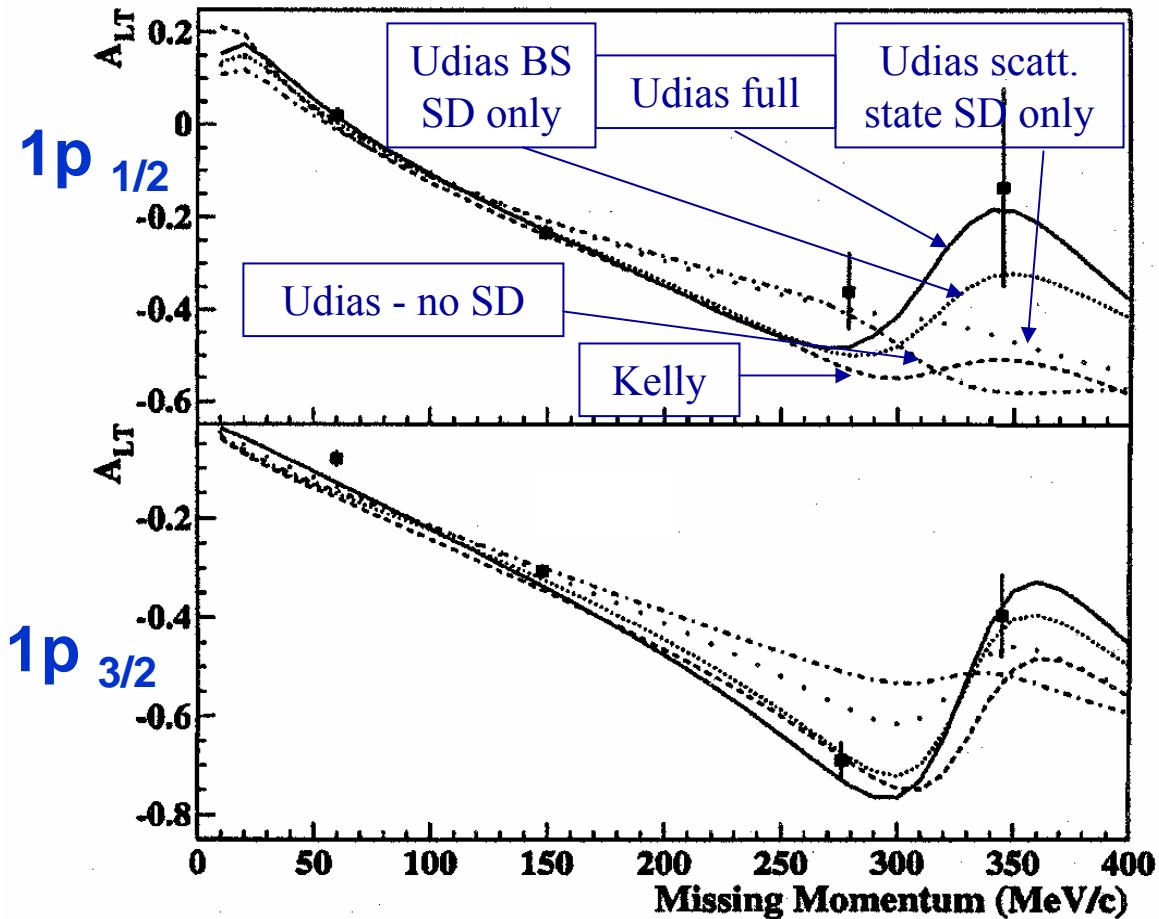
$S + V$  relatively small

**Dirac spinor**

$S - V$  affects lower components

$S - V$  large

# $^{16}\text{O}(\text{e},\text{e}'\text{p})$ $Q^2=0.8 \text{ GeV}^2$ Quasielastic



Sensitive  
to “spinor  
distortions”

JLab Hall A

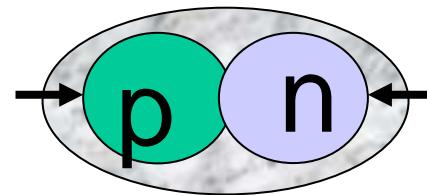
J. Gao *et al.*, Phys. Rev. Lett. **84**, 3265 (2000).

Few-body  
Nuclei ...

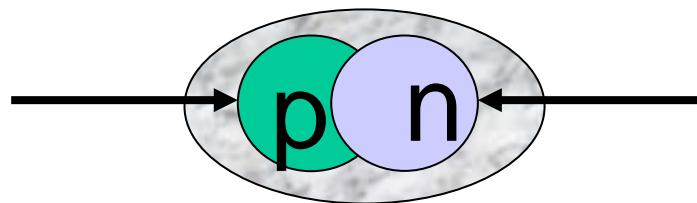
# The Deuteron

# Short-distance Structure

Low  $p_m$

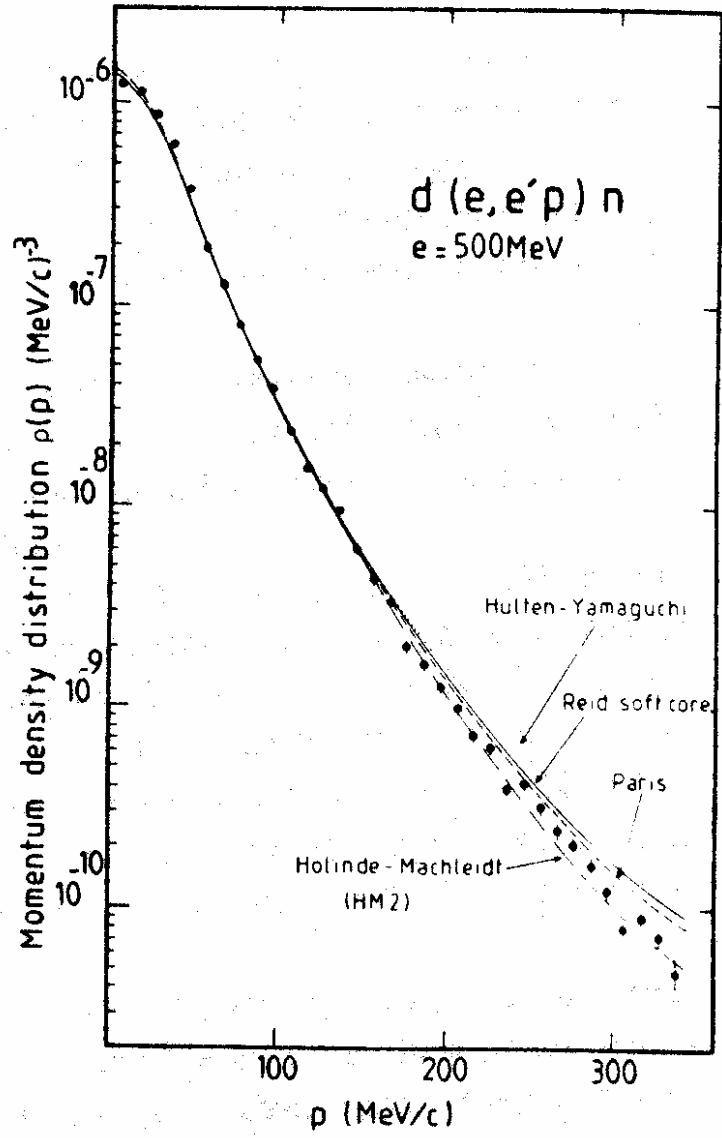


High  $p_m$



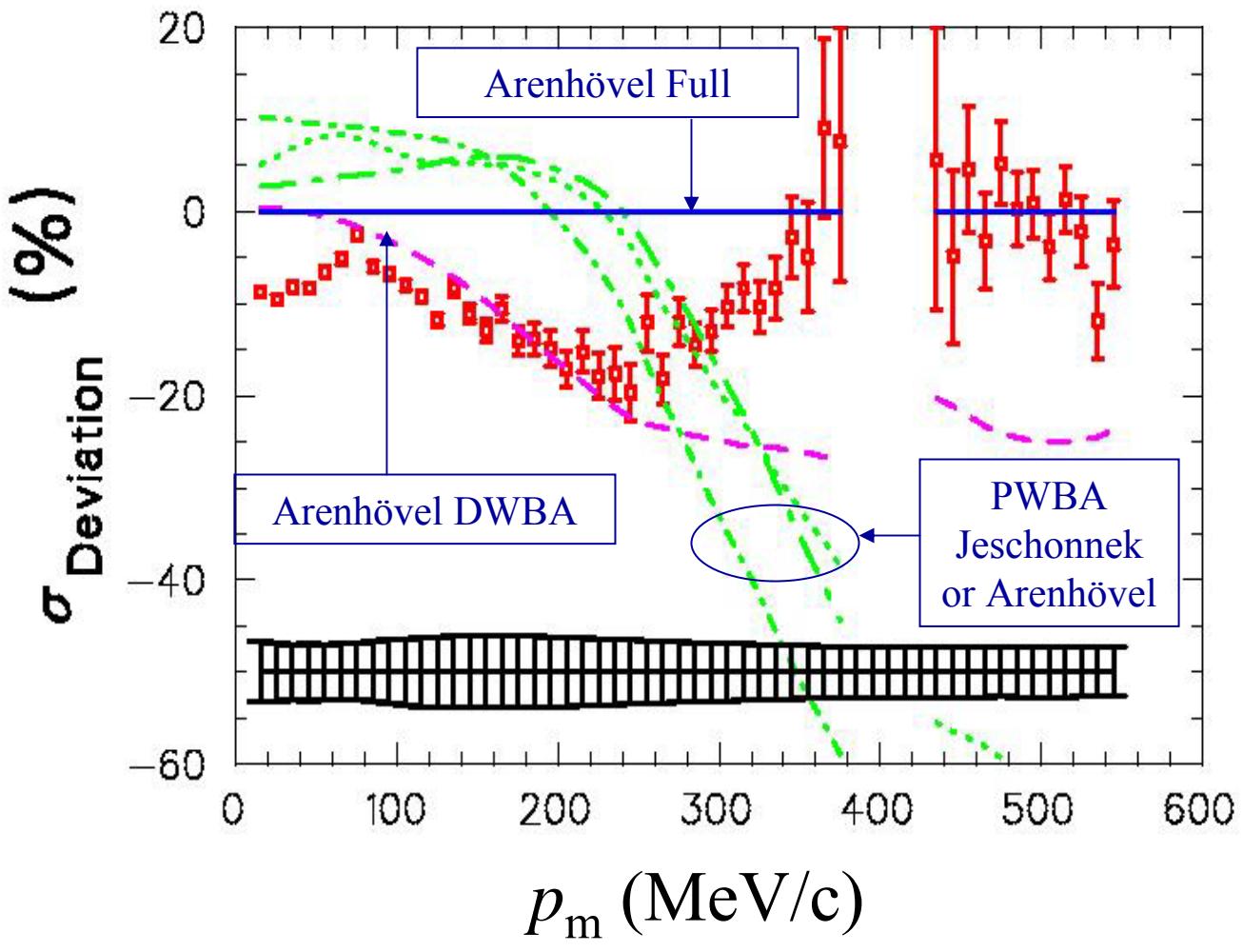
For large overlap, nucleons may lose individual identities:

**Quark/gluon d.o.f.?**



Saclay  
Linac,  
France

M. Bernheim *et al.*, Nucl. Phys. **A365**, 349 (1981).



Large  
 FSI/non-  
 nucleonic  
 effects.  
 Problem  
 at  $p_m=0$ .

JLab  
Hall A

P.E. Ulmer *et al.*, Phys. Rev. Lett. **89**, 062301 (2002).

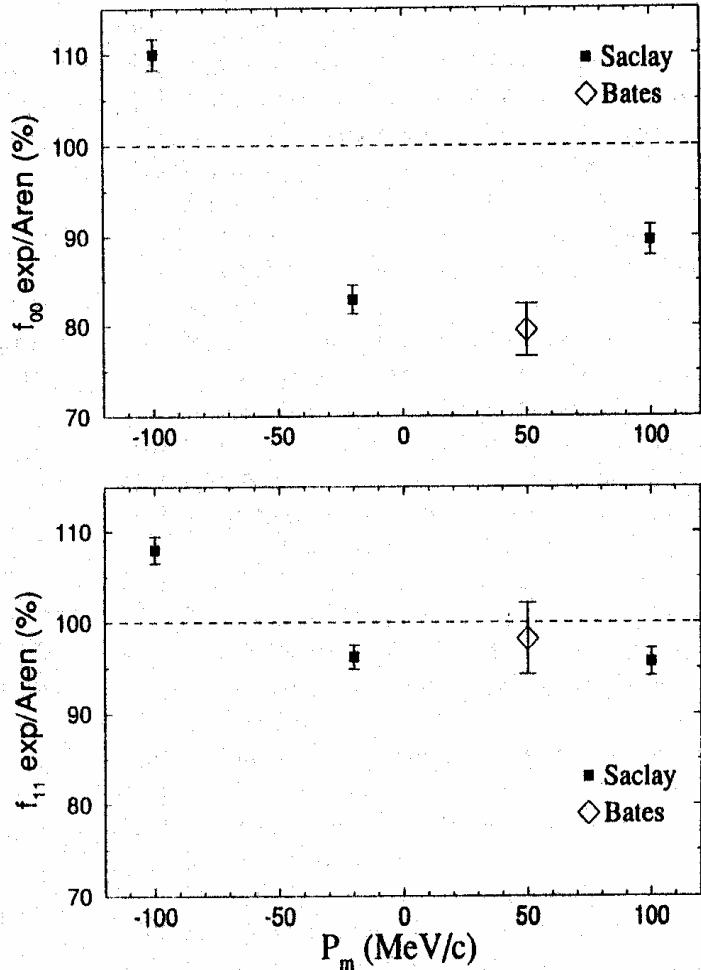


FIG. 1. Ratio of measured  $f_{00}$  and  $f_{11}$  structure functions to Arenhövel's calculation for this experiment and the Saclay experiment of Ducret *et al.* [6]. Only statistical errors are shown.

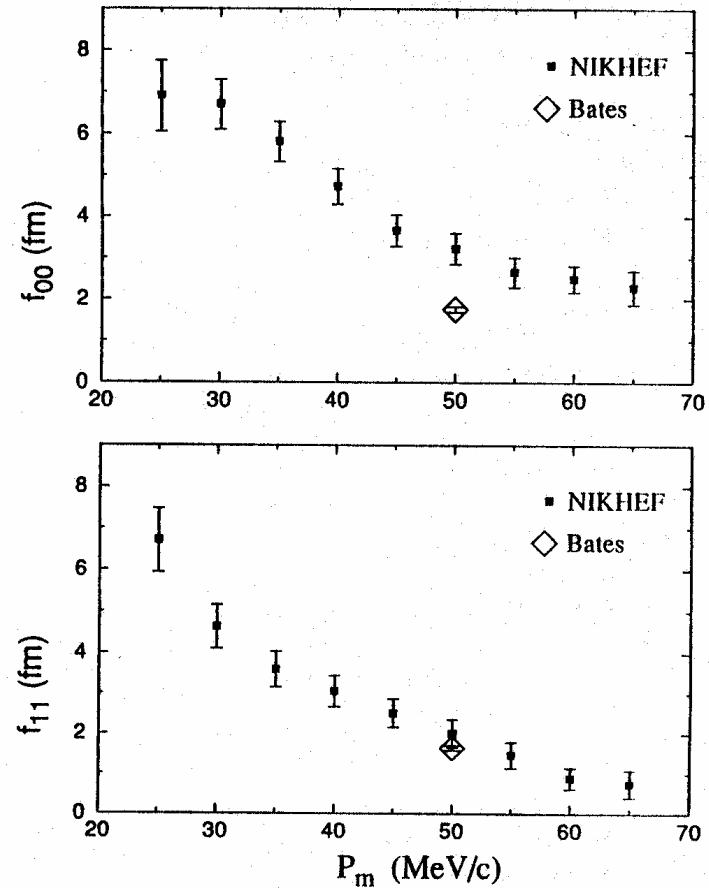
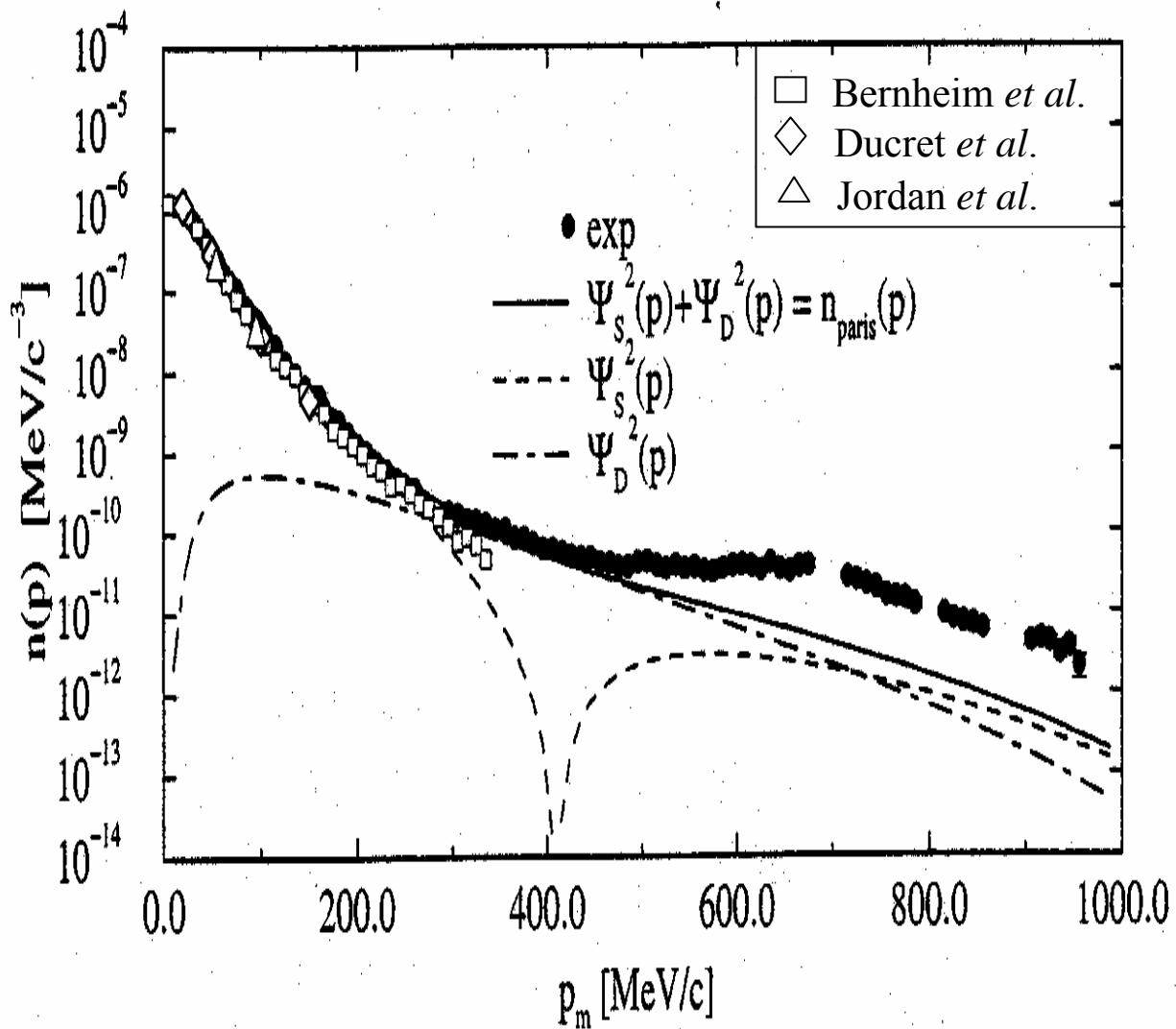


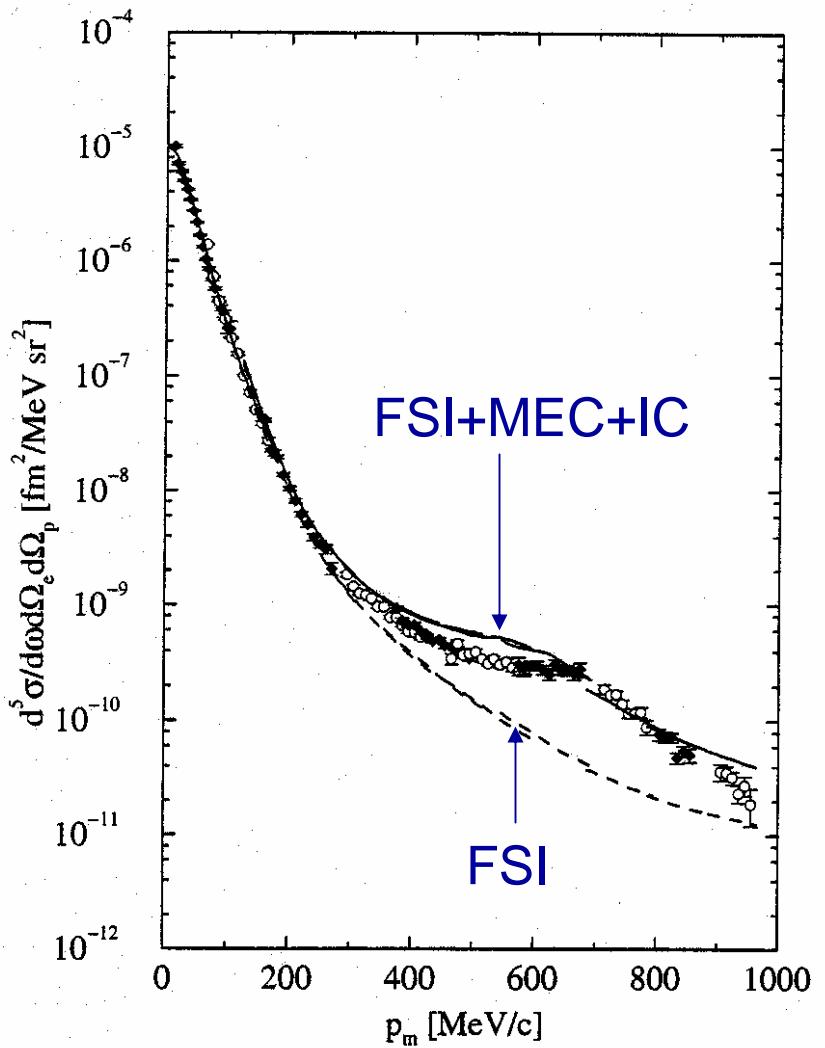
FIG. 2. Separated  $f_{00}$  and  $f_{11}$  structure functions for this experiment and the NIKHEF experiment of van der Schaaf *et al.* [5]. The NIKHEF data ( $q = 380 \text{ MeV}/c$ ) are averaged over 5  $\text{MeV}/c$  bins in  $p_m$ . The Bates data ( $q = 400 \text{ MeV}/c$ ) are averaged over the range of 30 to 70  $\text{MeV}/c$  in  $p_m$ . Only statistical errors are shown.



Blomqvist *et al.*  
data cover  
kinematics  
beyond  $\Delta$ .  
Also neutron  
exchange  
diagram  
important.

MAMI  
Mainz,  
Germany

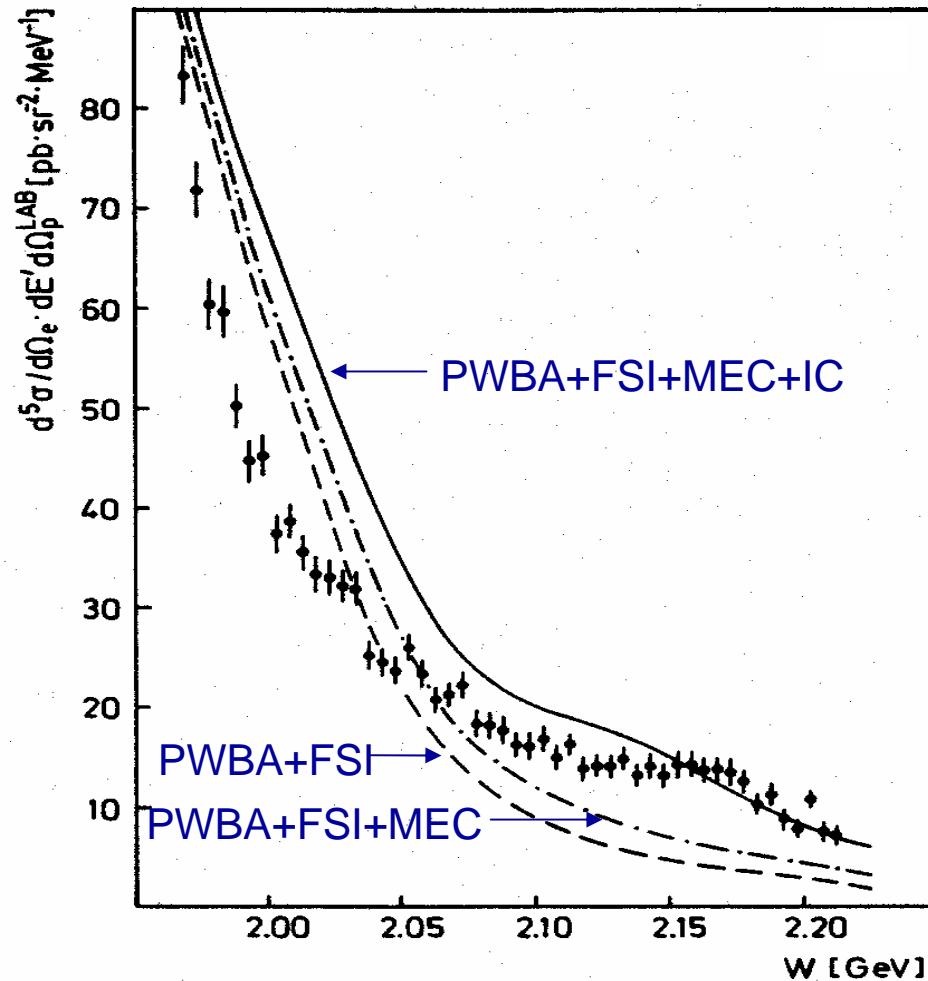
K.I. Blomqvist *et al.*, Phys. Lett. B 424, 33 (1998).



K.I. Blomqvist *et al.*, Phys. Lett. B **424**, 33 (1998).

Calculations: H. Arenhövel

## $^2\text{H}(\text{e},\text{e}'\text{p})$ $Q^2=0.23 \text{ GeV}^2$ near $\Delta$

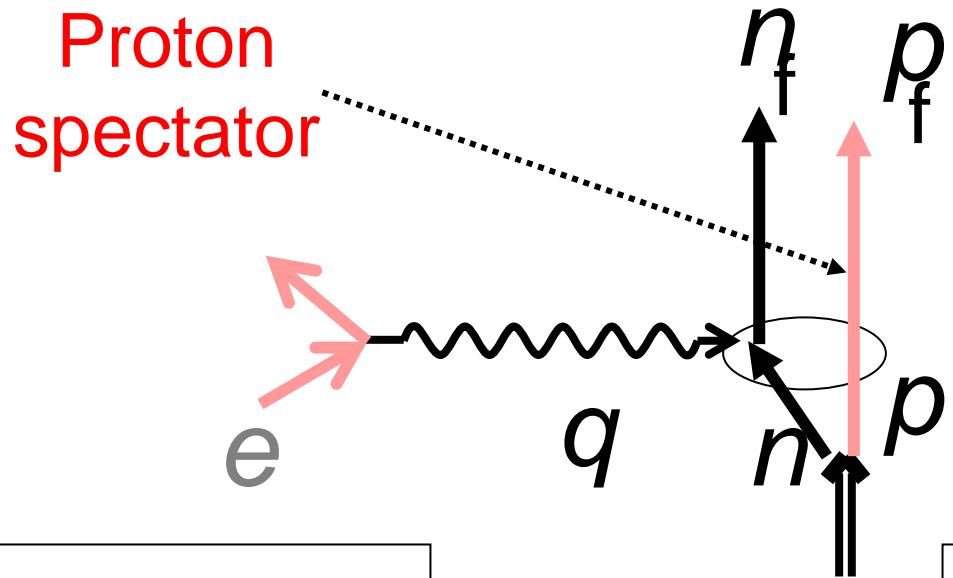


$\Delta$  clearly  
important

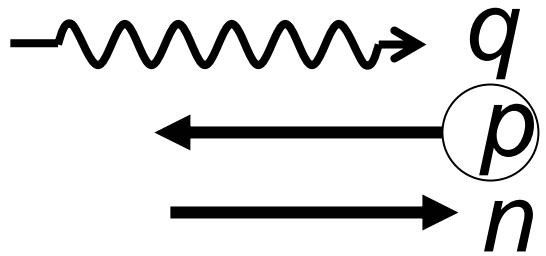
Bonn Electron  
Synchrotron,  
Germany

H. Breuker *et al.*, Nucl. Phys. **A455**, 641 (1986).

Calculations: Leidemann and Arenhövel



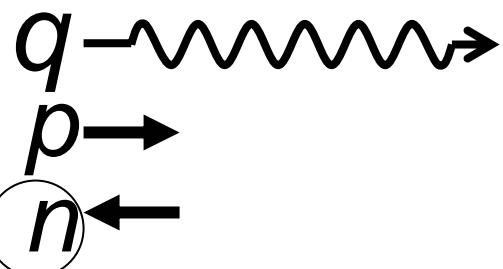
Proton hit (high  $p_m$ )



Final State

$$p_f \quad n_f$$

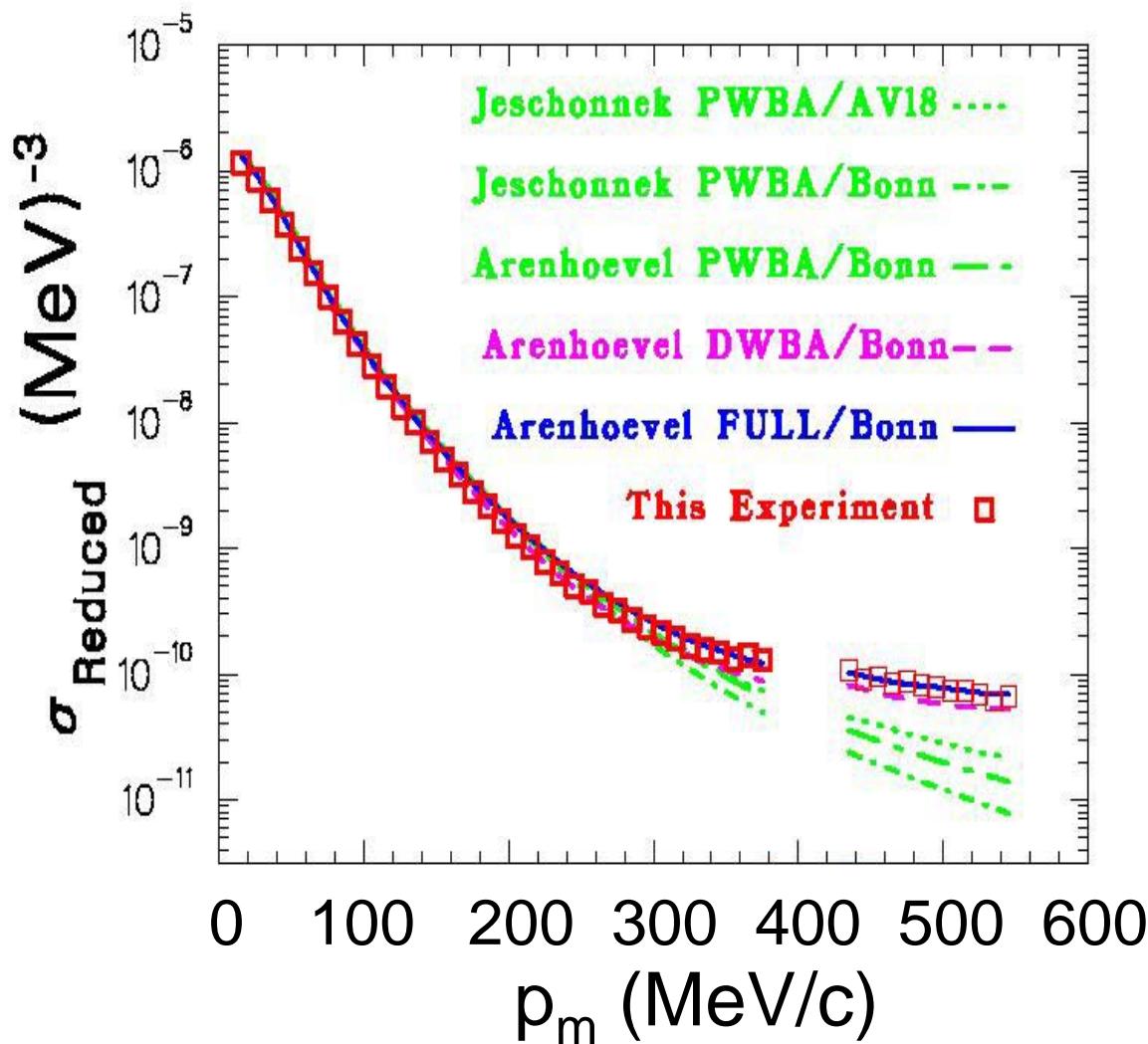
Neutron hit (low  $p_m$ )



Final State

$$p_f \quad n_f$$

# $Q^2=0.67 \text{ GeV}^2$ Quasielastic



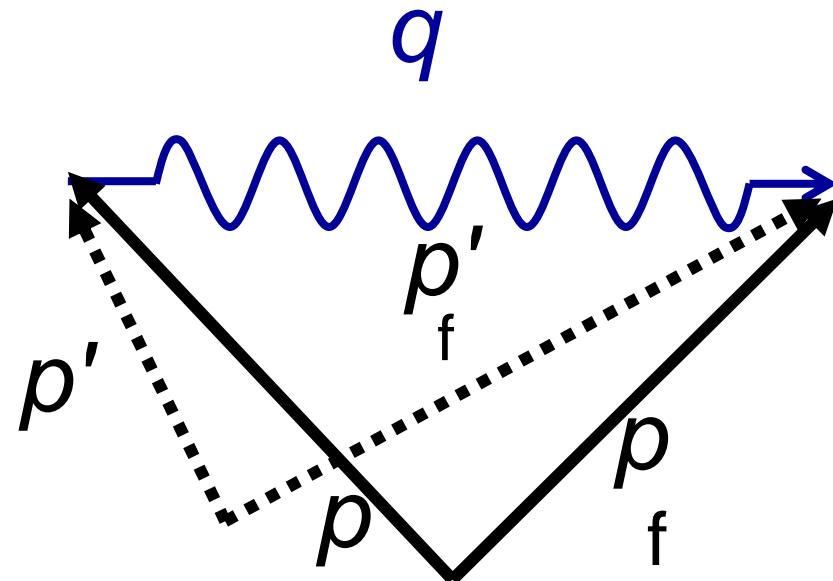
Large FSI effects.  
Also, substantial non-nucleonic effects.

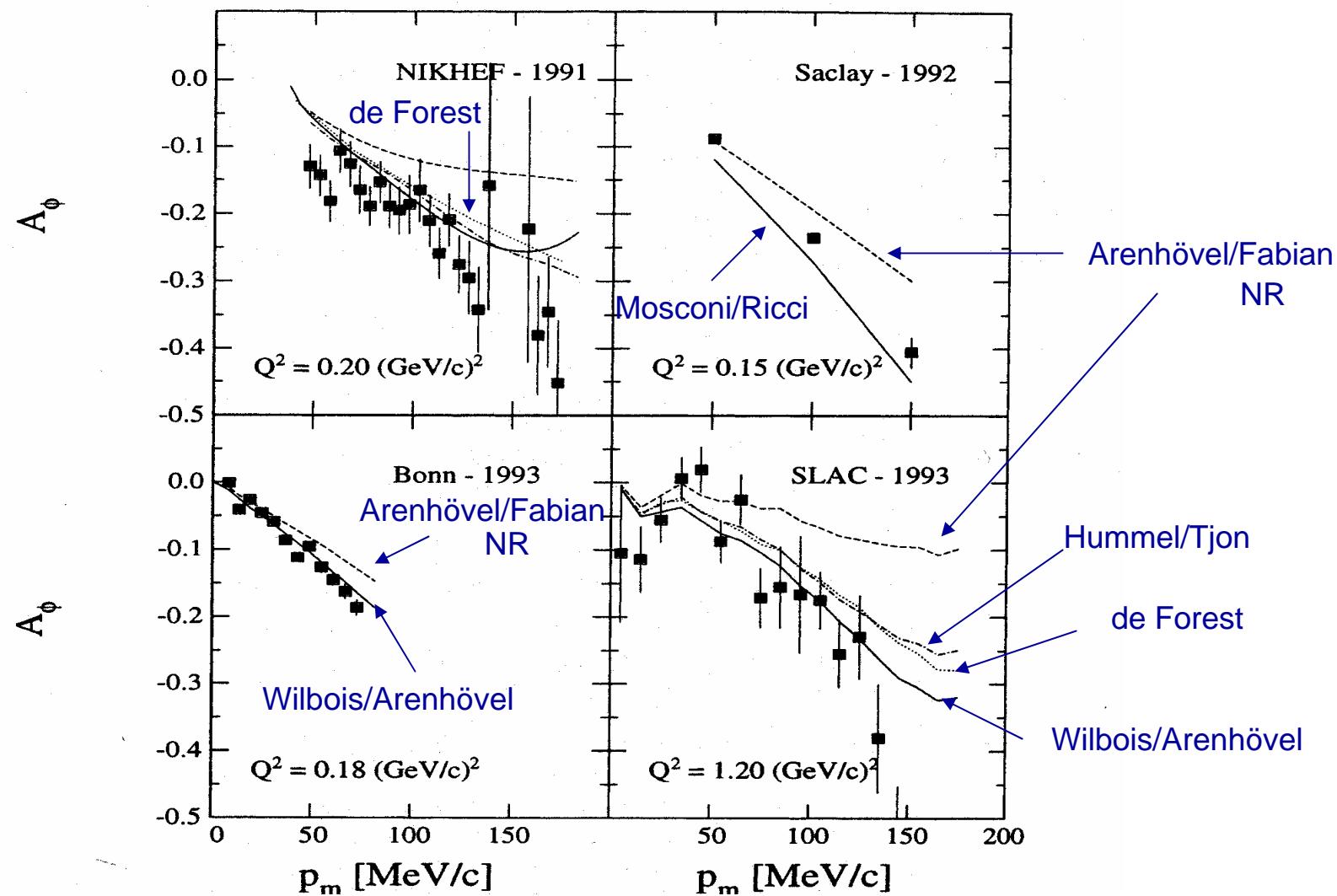
JLab  
Hall A

P.E. Ulmer *et al.*, Phys. Rev. Lett. **89**, 062301 (2002).

# Final State Interactions Can be LARGE

The diagram consists of two words at the top: "actual" on the left and "inferred" on the right. Each word has a vertical arrow pointing downwards to a yellow rectangular box. Inside the box is the mathematical expression  $p' < p$ .



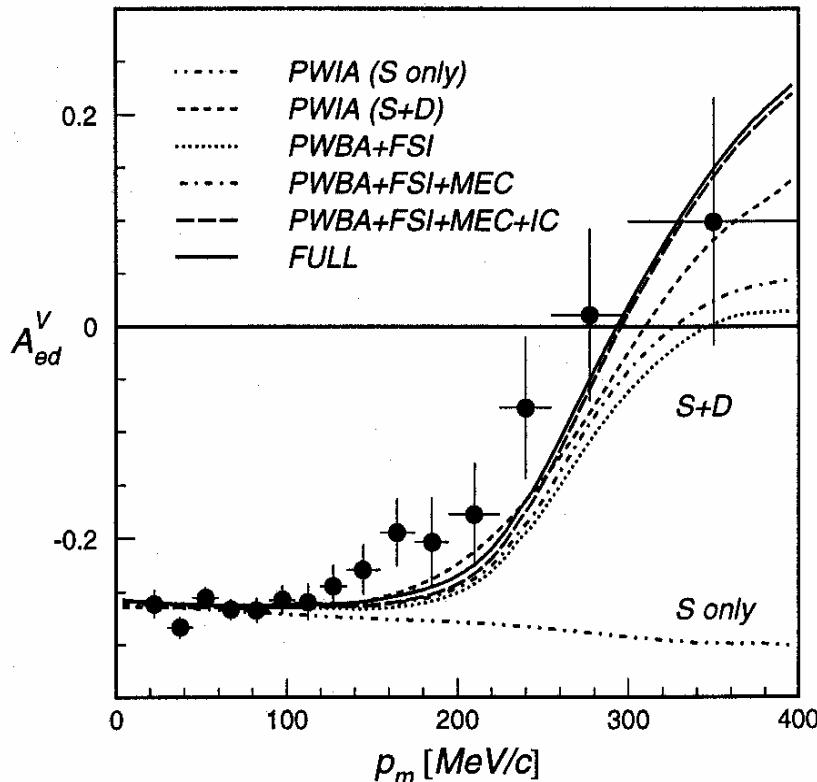


G. van der Steenhoven, Few-Body Syst. 17, 79 (1994).

# What do all these data and curves suggest?

- Relativistic effects substantial in  $A_\phi$  (and  $R_{LT}$ ).
- de Forest “CC1” nucleon cross section gives same qualitative features as more complete calculations → here, relativity more related to nucleonic current, as opposed to deuteron structure.

# $^2\vec{\text{H}}(\vec{e}, e' p)$



**D-state  
important**

AmPS  
NIKHEF-K  
Amsterdam

I. Passchier *et al.*, Phys. Rev. Lett. **88**, 102302 (2002).

$$\sigma = \sigma_0 \left[ 1 + P_1^d A_d^V + P_2^d A_d^T + h \left( A_e + P_1^d \boxed{A_{ed}^V} + P_2^d A_{ed}^T \right) \right]$$

**Lots more  $d(e,e'p)$   
data on the way!**

# $^2\text{H}(\text{e}, \text{e}'\text{p})\text{n}$ E01-020 Hall A

Perpendicular:  $R_{LT}$

$Q^2 : 0.80, 2.10, 3.50 \text{ (GeV/c)}^2$

$x=1: p_m$  from 0 to  $\pm 0.5 \text{ GeV/c}$

Parallel/Anti-parallel

$Q^2 : 2.10 \text{ (GeV/c)}^2$

vary  $x: p_m$  from 0 to 0.5 GeV/c

Neutron angular distribution

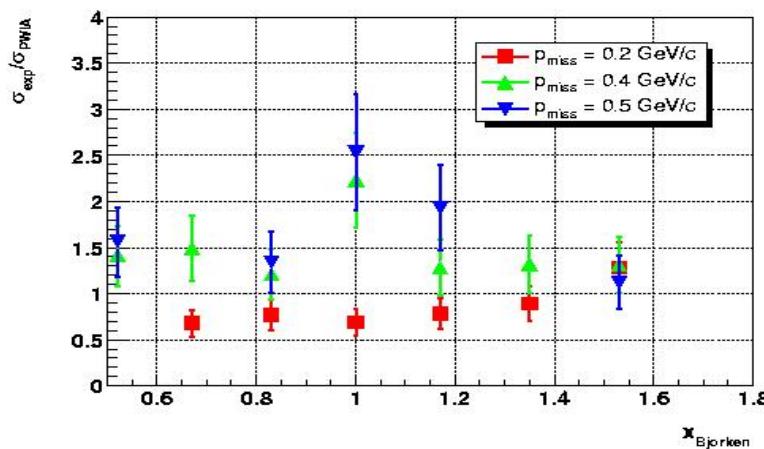
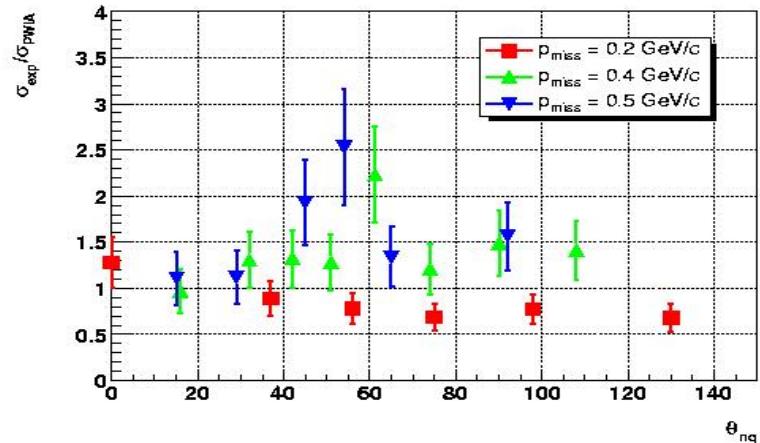
$Q^2 : 0.80, 2.10, 3.50 \text{ (GeV/c)}^2$

$$Q^2 = 0.8 \text{ (GeV/c)}^2$$

PRELIMINARY

20% error added to statistical error

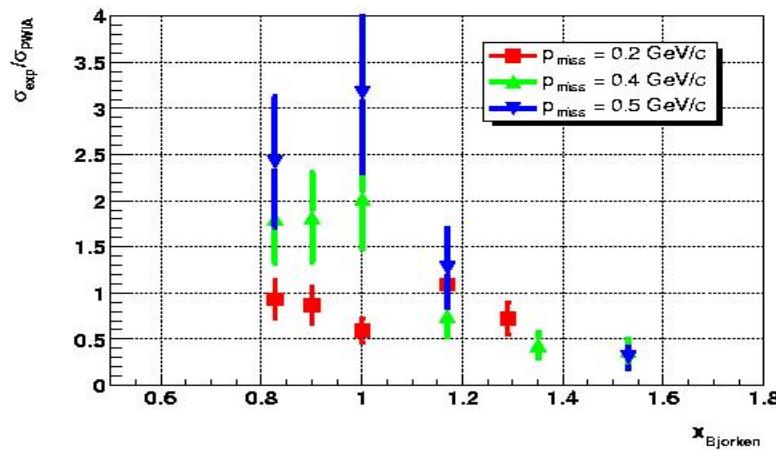
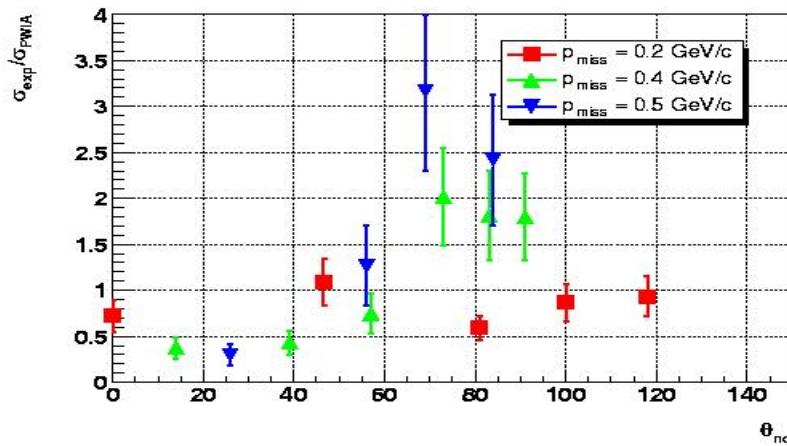
${}^2\text{H}(\text{e},\text{e}'\text{p})\text{n}$   
E01-020 Hall A



$$Q^2 = 3.5 \text{ (GeV/c)}^2$$

PRELIMINARY

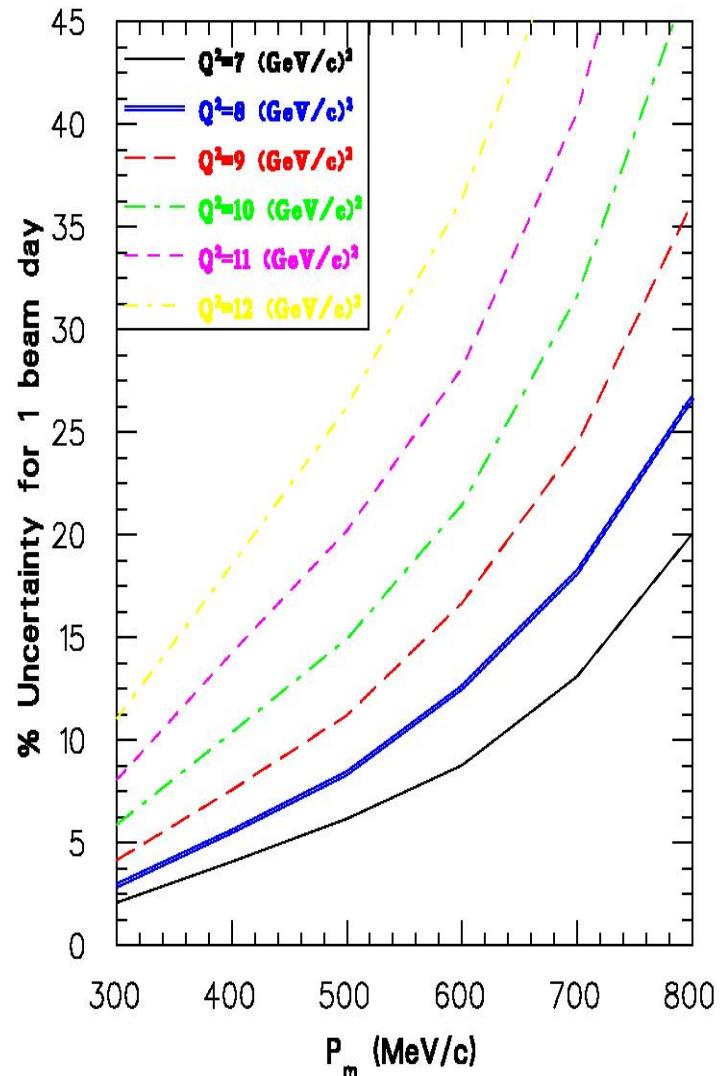
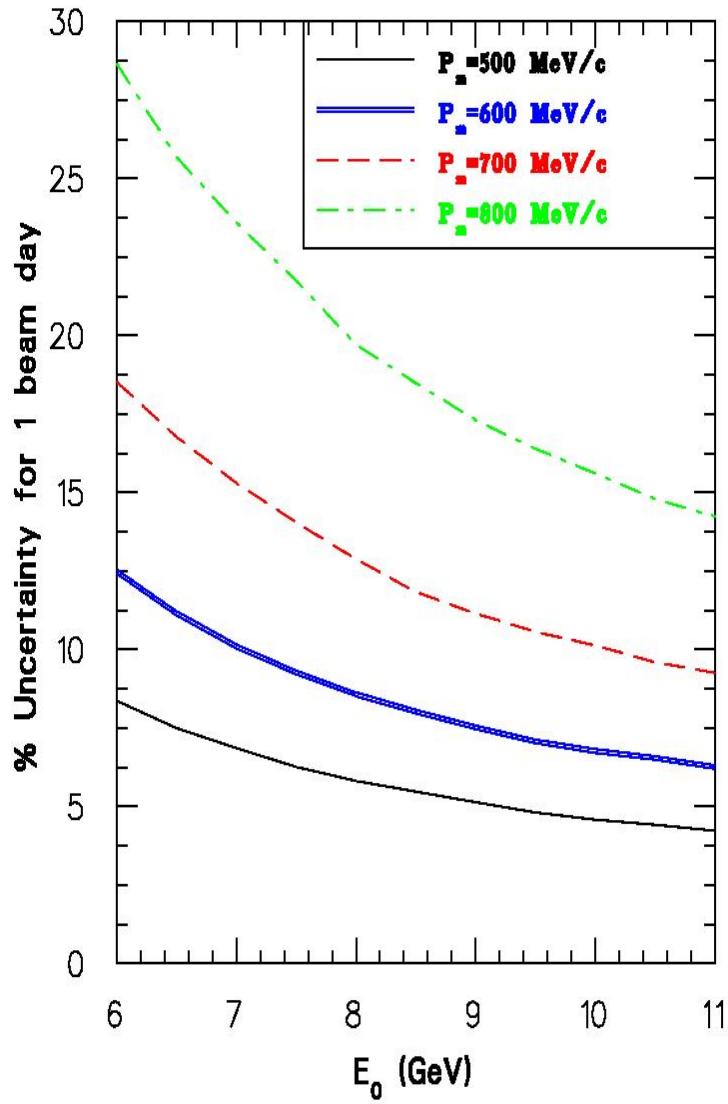
20% error added to statistical error



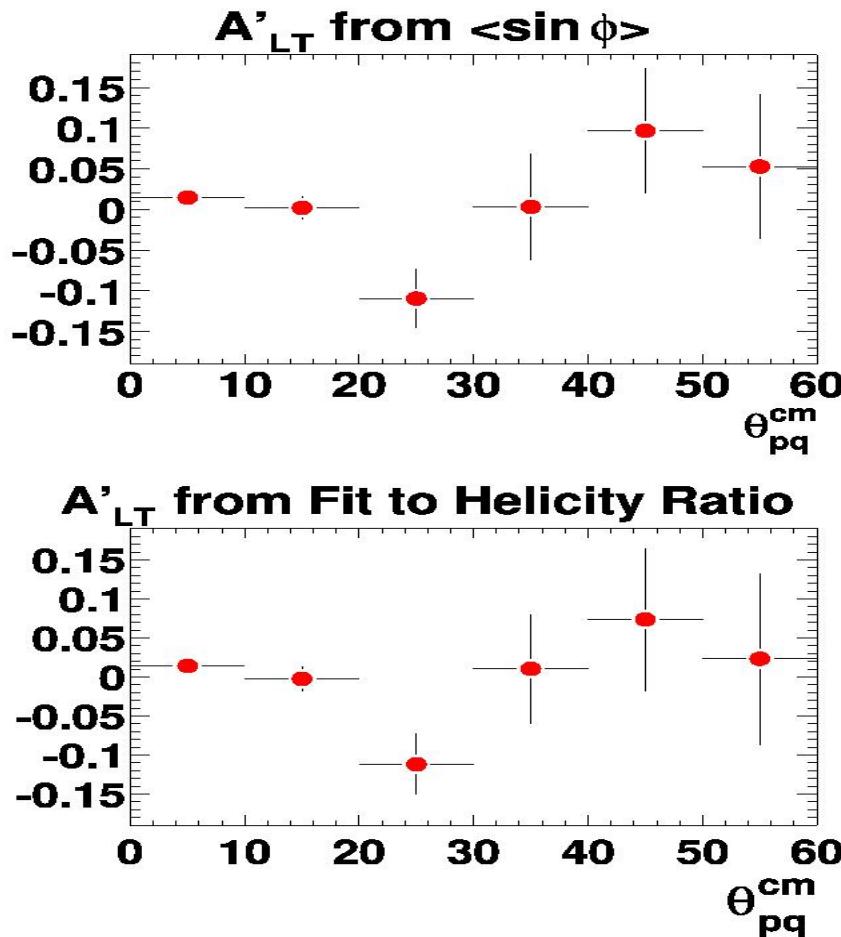
${}^2\text{H}(\text{e},\text{e}'\text{p})\text{n}$

E01-020 Hall A

# $^2\text{H}(\text{e},\text{e}'\text{p})\text{n}$ with JLab 12 GeV upgrade



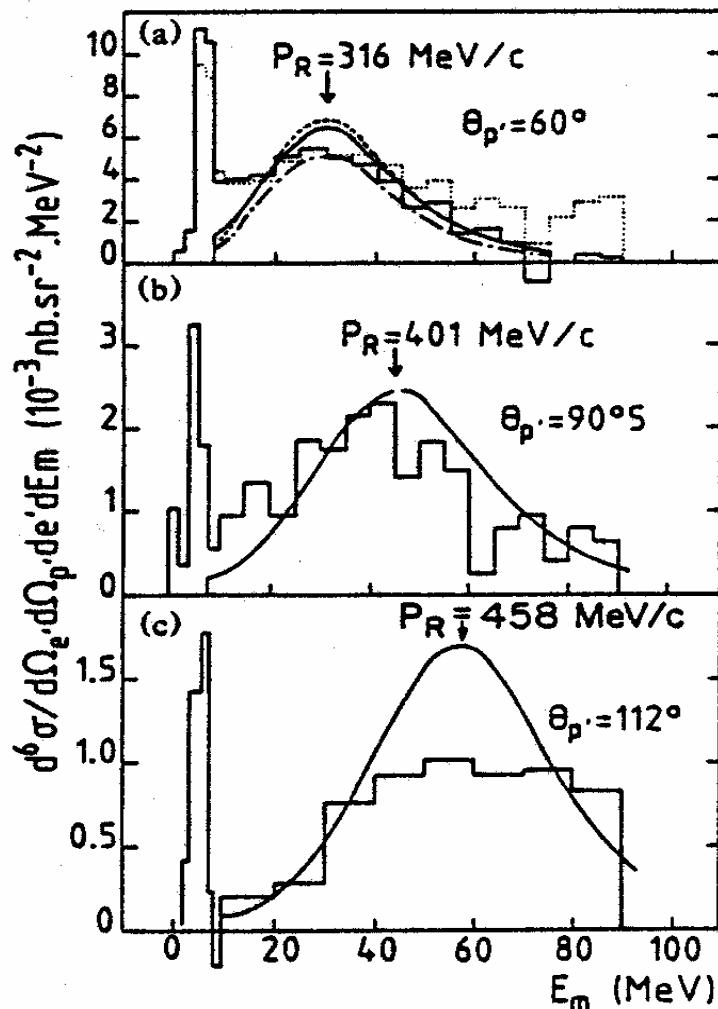
# Preliminary Hall B E5 Data – ${}^2\text{H}(\text{e},\text{e}'\text{p})$



Hall B data covers large range of  $Q^2$  and excitation as well as  $\phi$  coverage to separate  $R_{LT}$ ,  $R_{LT}'$  and  $R_{TT}$ .

$^{3,4}\text{He}$

# $^3\text{He}(e, e'p)$



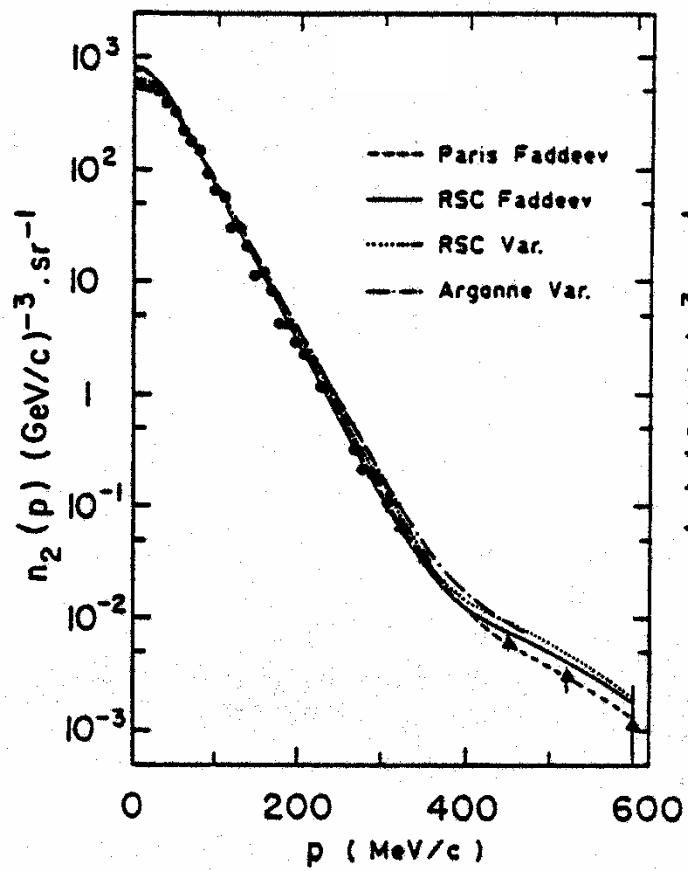
C. Marchand *et al.*,  
Phys. Rev. Lett. **60**, 1703 (1988).

**Calculations by Laget:**  
dashed=PWIA  
dot-dashed=DWIA  
solid=DWIA+MEC

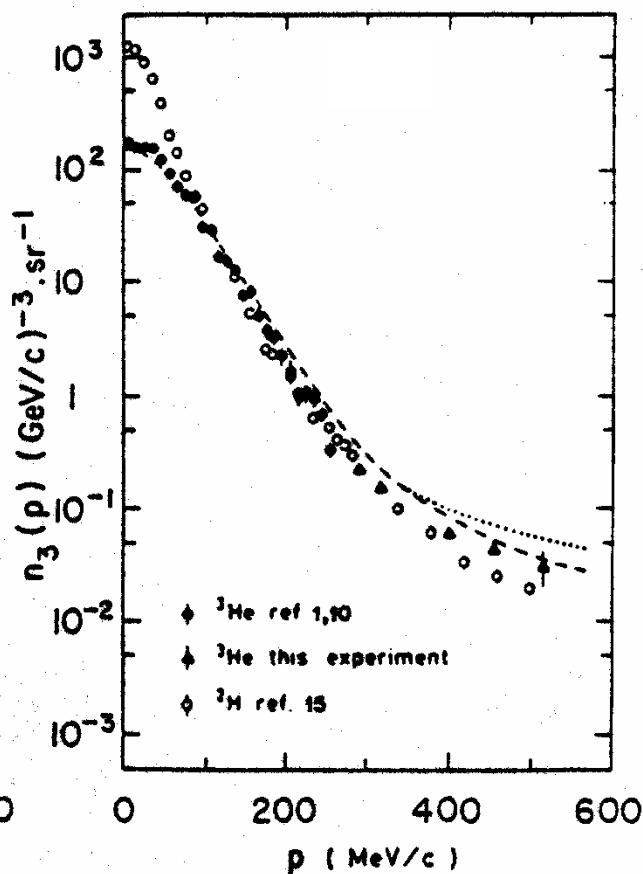
Arrows indicate expected position for correlated pair.

Saclay  
Linear  
Accelerator

### $^3\text{He}(\text{e},\text{e}'\text{p})\text{d}$

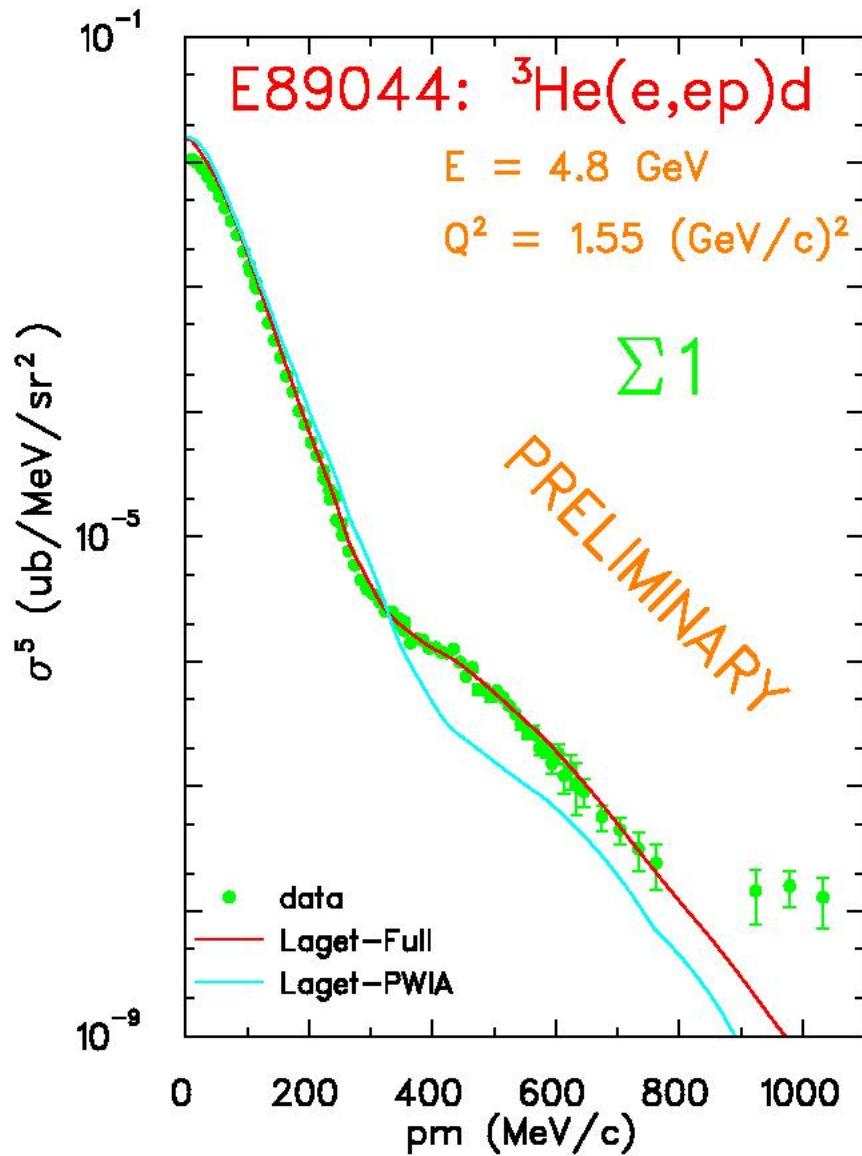


### $^3\text{He}(\text{e},\text{e}'\text{p})\text{np}$



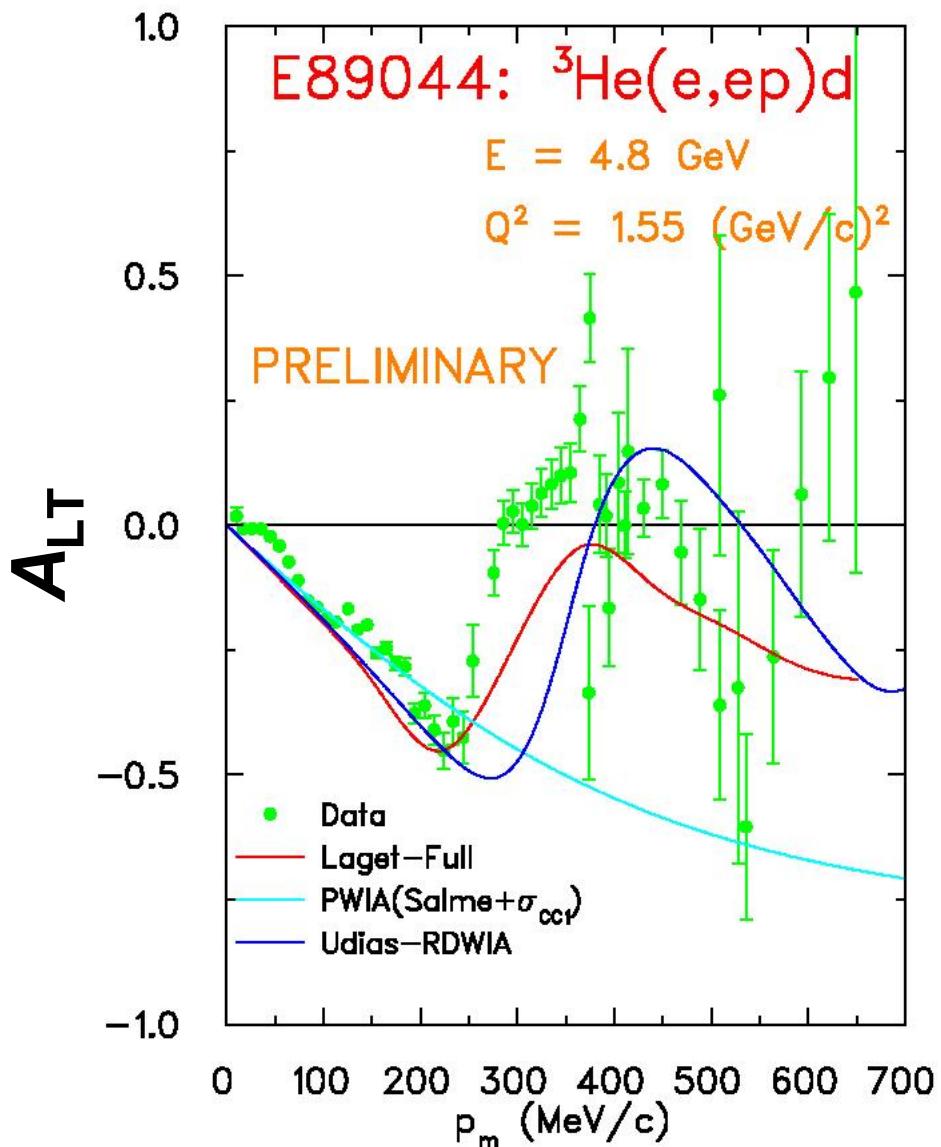
3BBU  
similar  
to  
 $\text{d} \rightarrow \text{np}$

C. Marchand *et al.*, Phys. Rev. Lett. **60**, 1703 (1988).



Large effects  
 from FSI and  
 non-nucleonic  
 currents.  
  
 Highest  $p_m$   
 shows excess  
 strength.

JLab  
Hall A



General features reproduced but not at correct values of  $p_m$ .

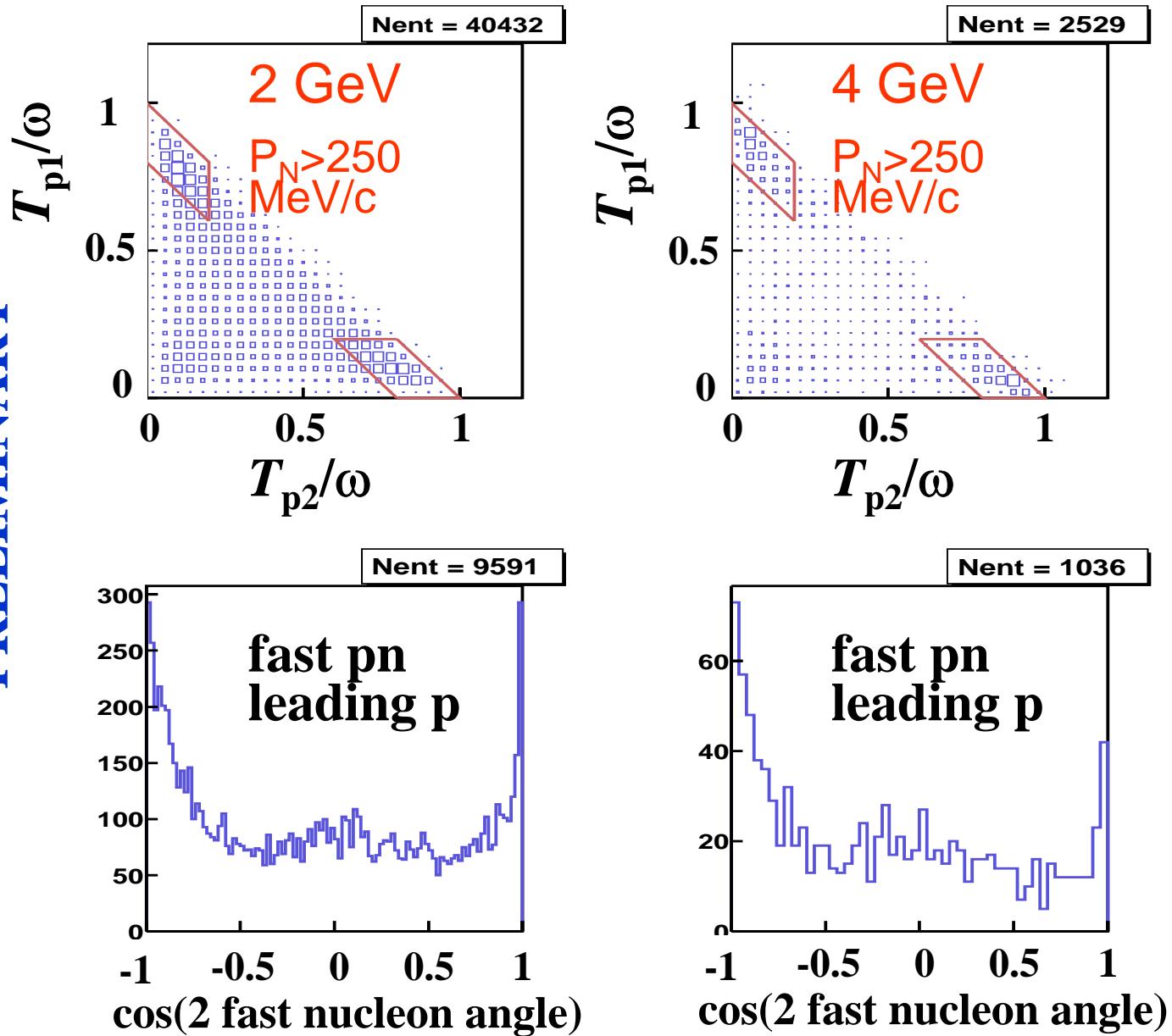
JLab  
 Hall A

The most direct way  
to look for correlated  
nucleons?

Detect both of them  
→ JLab Hall B

PRELIMINARY

# ${}^3\text{He}(e,e'pp)n$ Hall B

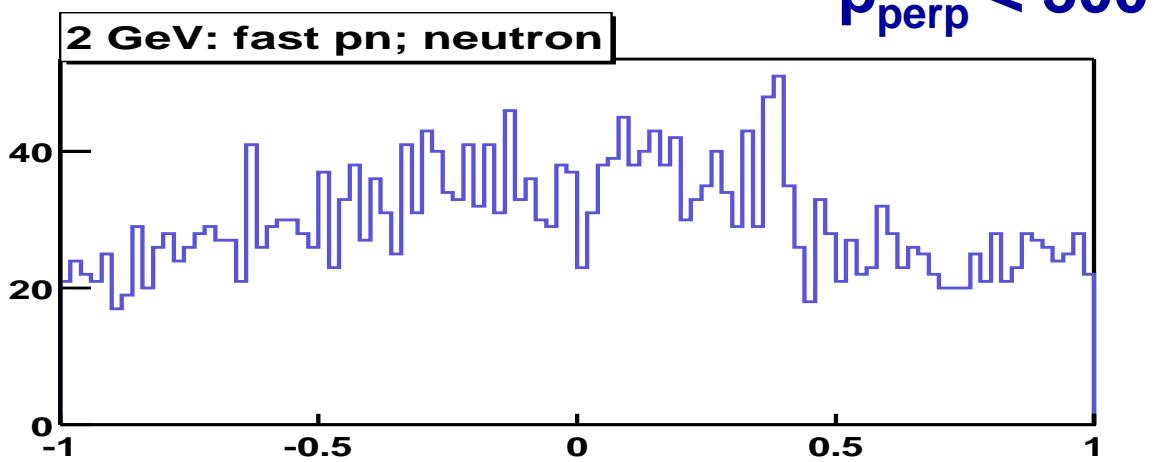


# Hall B

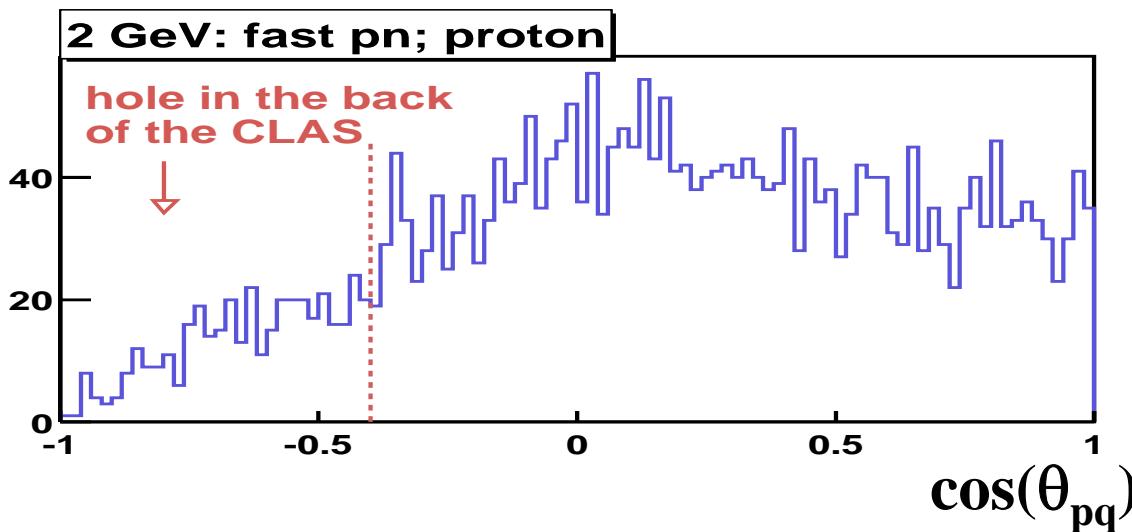
${}^3\text{He}(e,e'pp)n$  2 GeV

$p_{\text{perp}} < 300 \text{ MeV}/c$

PRELIMINARY



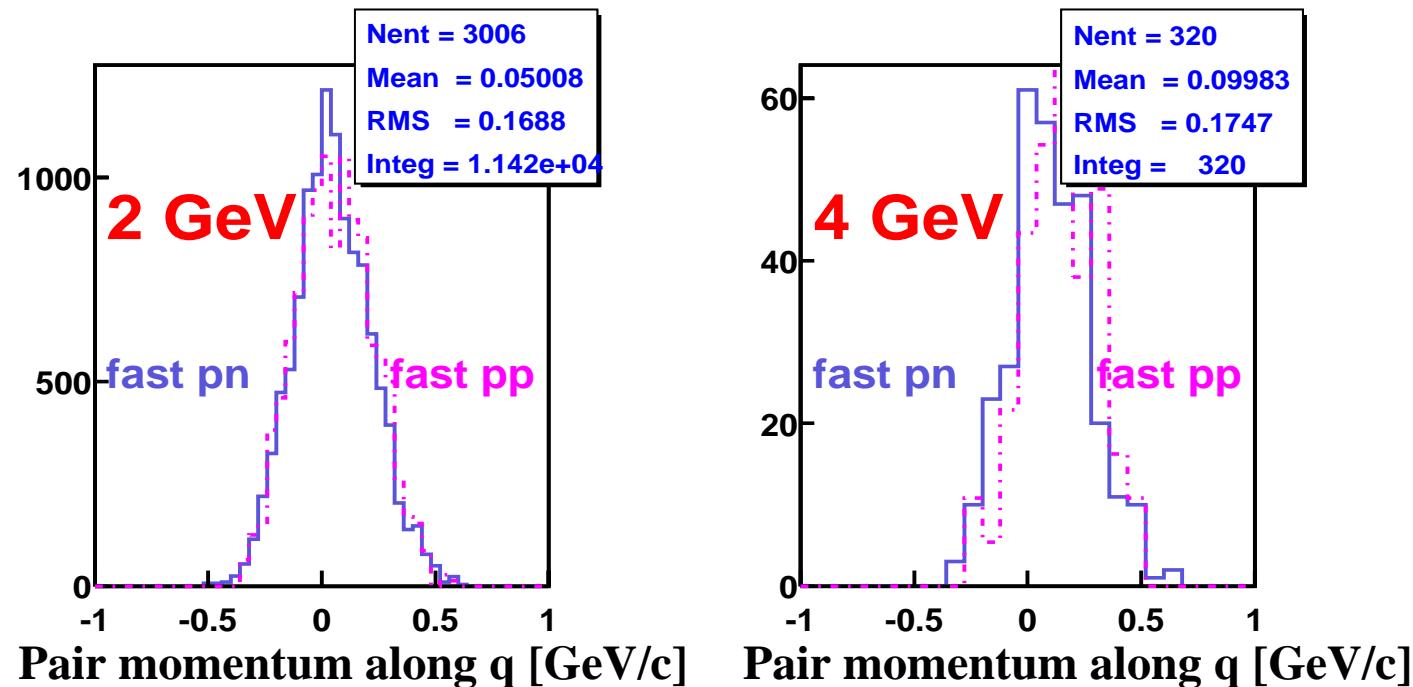
Isotropic fast pairs  
→ pair not involved in reaction.



# Hall B



PRELIMINARY



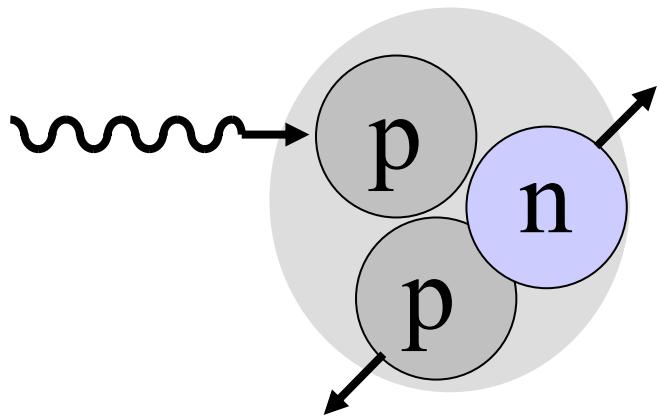
**Small momentum along  $q$**   
→ pair not involved in reaction.

**Little  $Q^2$  or isospin dependence.**

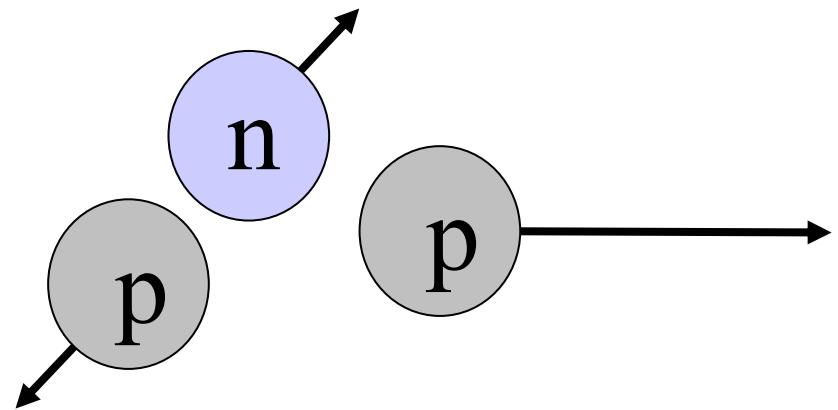
**2 GeV has acceptance corrections**

# Direct evidence of NN correlations

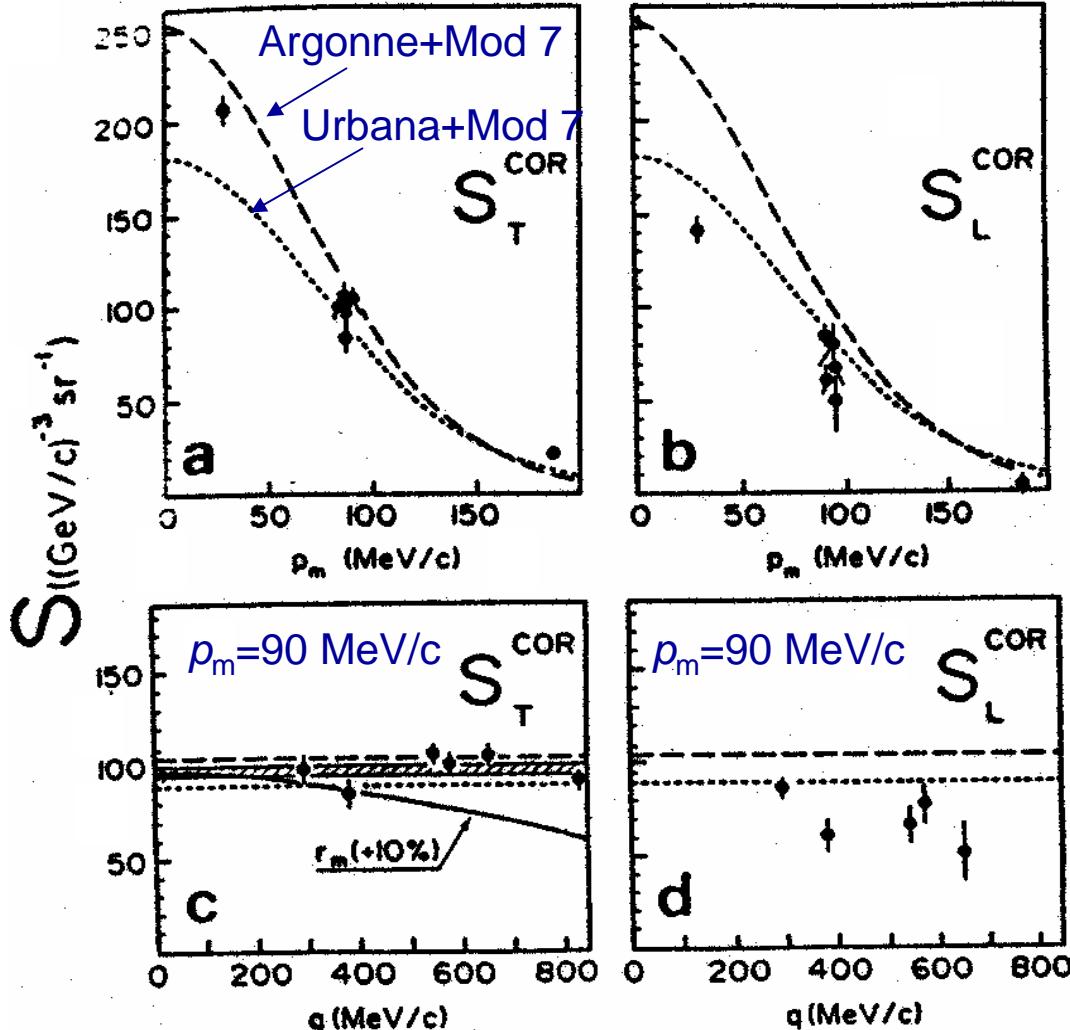
Before



After



# ${}^4\text{He}(\text{e}, \text{e}'\text{p}){}^3\text{H}$



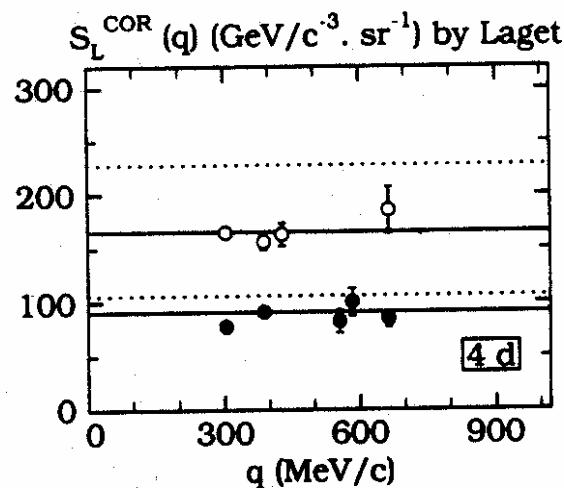
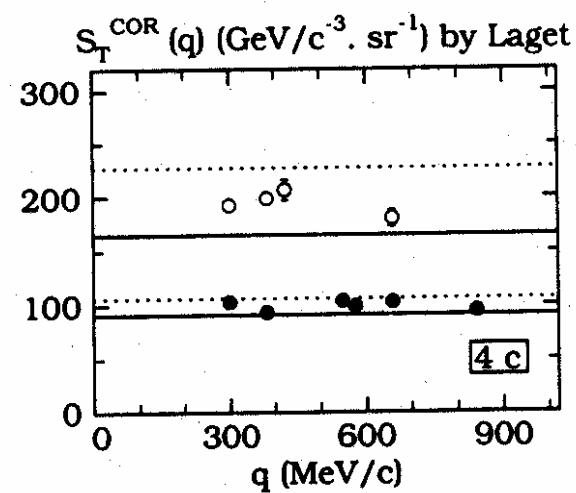
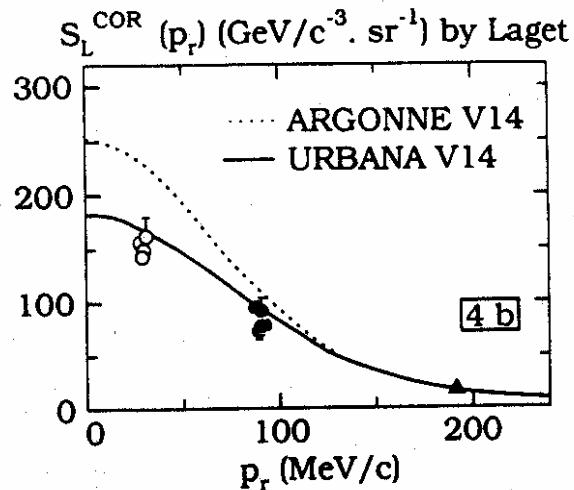
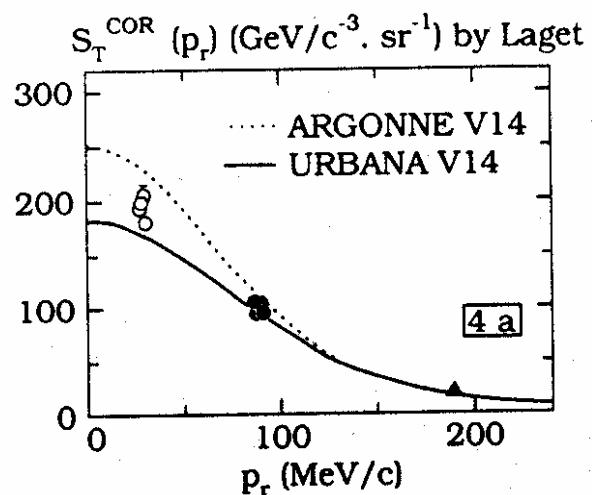
Data and calculations  
“corrected” for  
MEC+IC  
(Laget).

Longitudinal  
overpredicted.

Saclay

A. Magnon *et al.*, Phys. Lett. B **222**, 352 (1989).

# $^4\text{He}(\text{e}, \text{e}'\text{p})^3\text{H}$

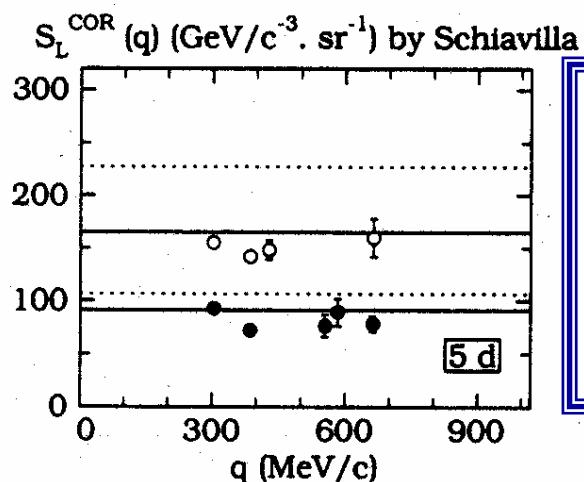
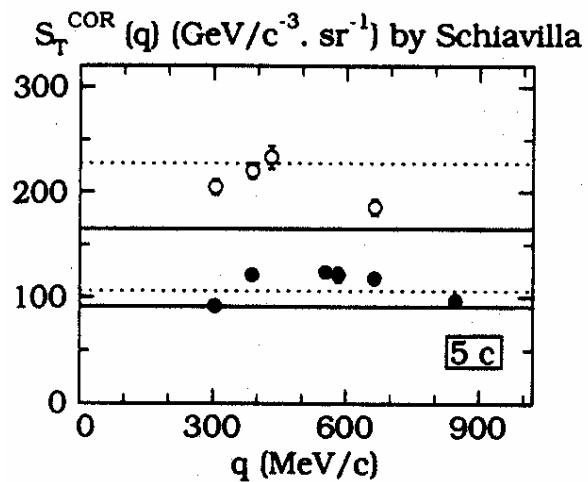
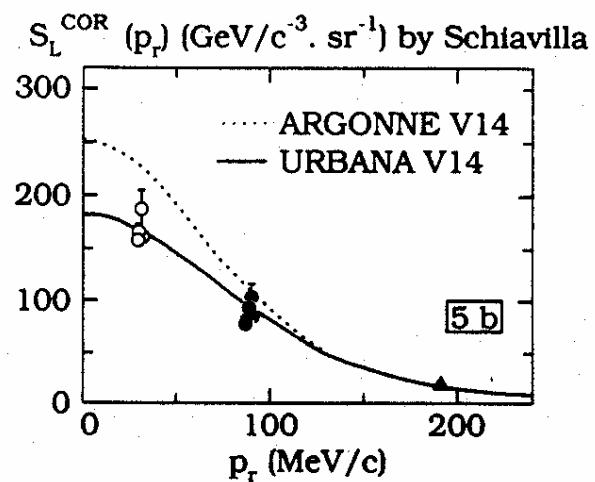
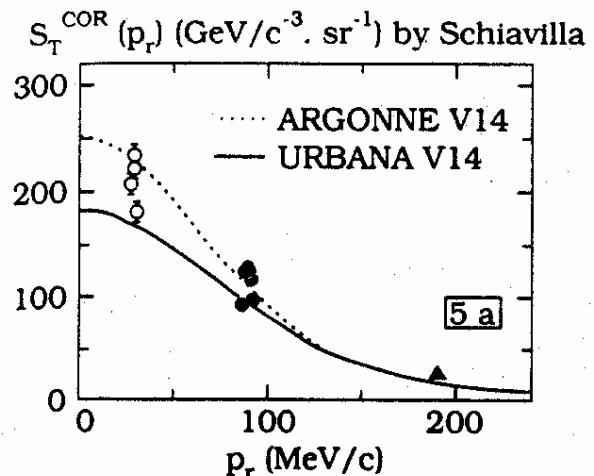


Calculations predict  $q$  dependence.

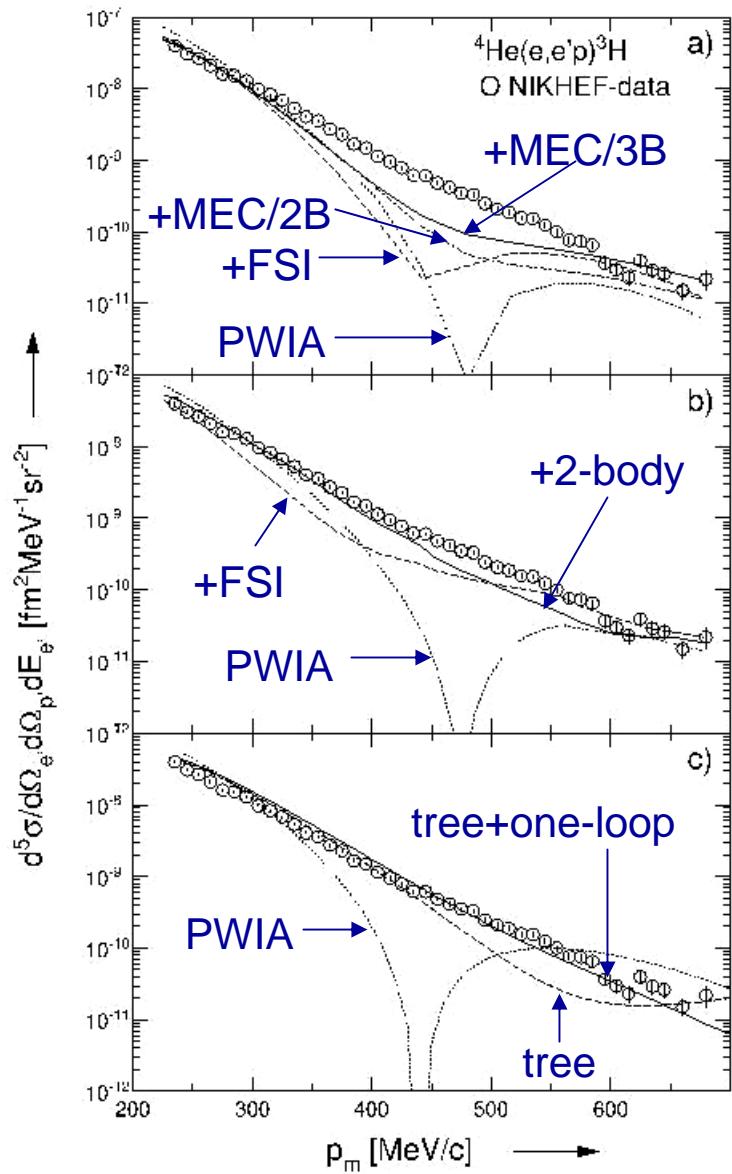
Saclay

J.E. Ducret *et al.*, Nucl. Phys. **A556**, 373 (1993).

# ${}^4\text{He}(\text{e}, \text{e}'\text{p}){}^3\text{H}$



Again,  
calculations  
predict  $q$   
dependence.



${}^4\text{He}(e, e'p){}^3\text{H}$

Laget

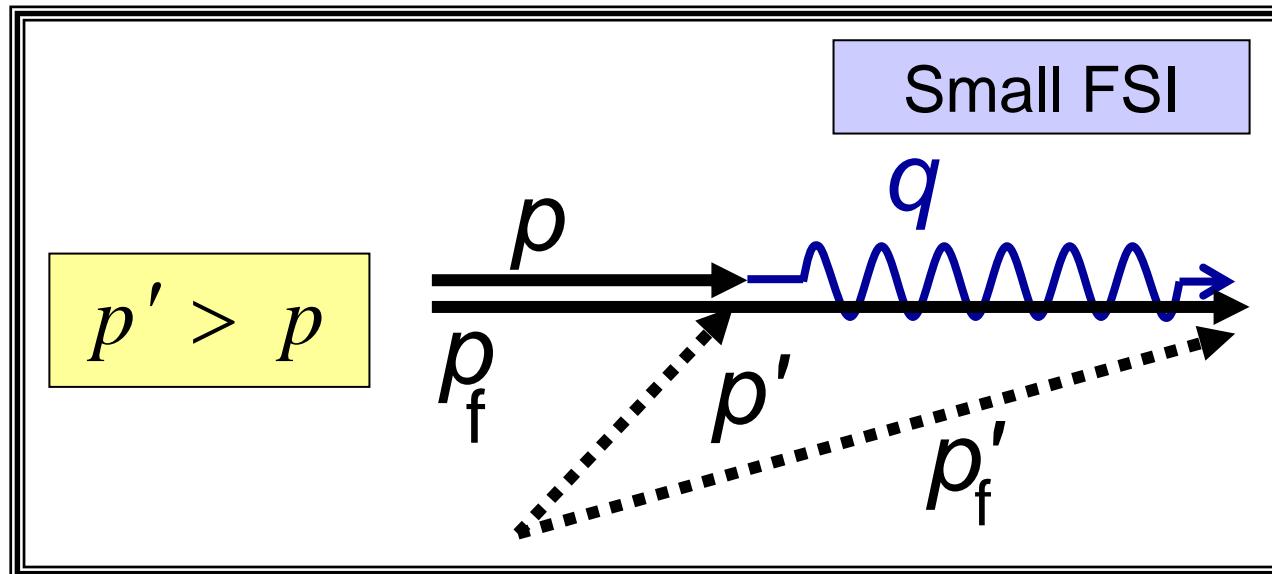
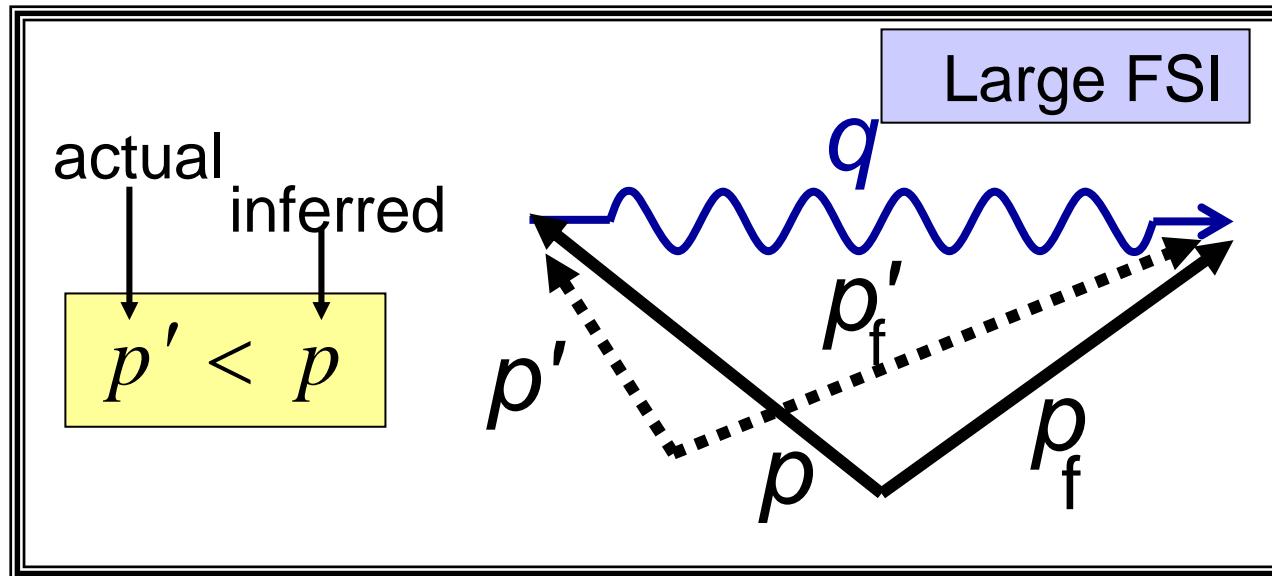
Minimum  
filled in by  
FSI and  
2&3-body  
currents.

Schiavilla

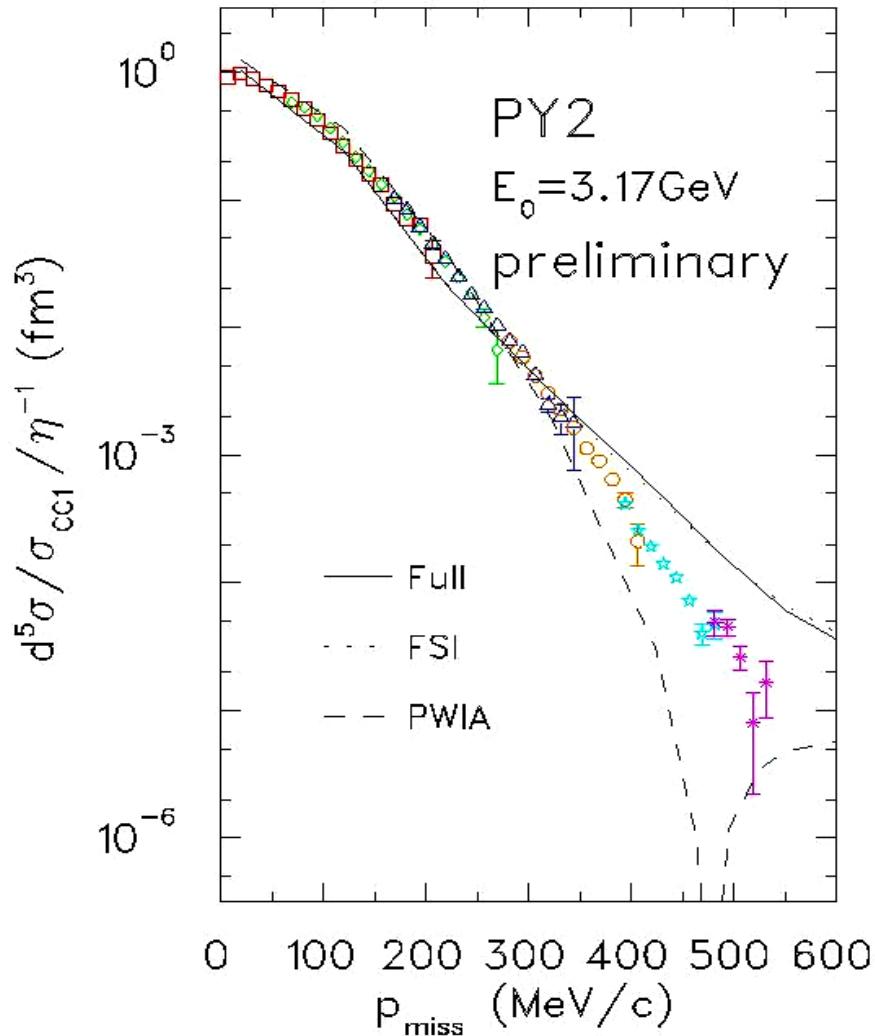
AmPS  
NIKHEF-K  
Amsterdam

Nagorny

# FSI: dependence on kinematics



# ${}^4\text{He}(\text{e},\text{e}'\text{p}){}^3\text{H}$



It looks like  
the minimum  
is filled in  
here as well.

JLab Hall A Experiment E97-111, J. Mitchell, B. Reitz,  
J. Templon, cospokesmen

# Summary

- $(e,e'p)$  sensitive to single-particle aspects of nucleus, but ...
- More complicated physics is clearly important.
- Spectroscopic factors reduced compared to naïve shell model (including FSI corrections).
- Missing strength at least partly due to interaction currents: direct interaction with exchanged mesons or interaction with correlated pairs (spreads strength over  $\varepsilon_m$ ).

# Summary cont'd.

- After several decades of experimental and theoretical effort, there are still unanswered questions.
- What is the nature of the interaction of the virtual photon with the “nucleon”: medium and offshell effects?
- Handling FSI and other reaction currents still problematic, though realistic calculations are now available for the lighter systems.
- High energy program is underway, pushing to shorter distance scales, emphasizing relativistic effects, ...