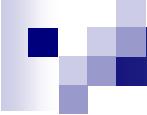


# Theory : phenomenology support JLab @ 12 GeV

Marc Vanderhaeghen  
College of William & Mary / JLab

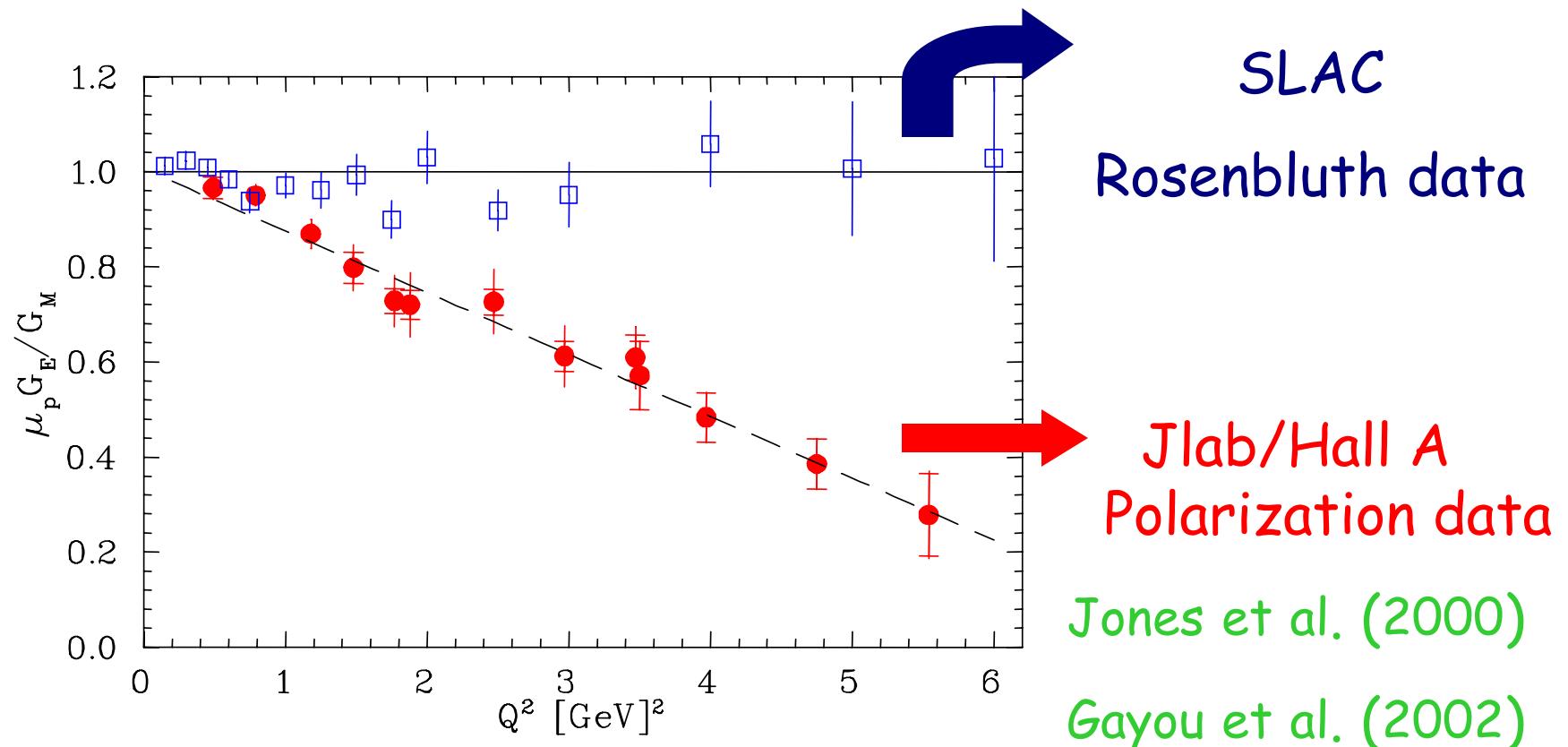
DOE Review of Science Program for  
12 GeV upgrade, April 6-8, 2005, JLab



# Outline

- Form factors at large momentum transfer,
  - two-photon exchange observables
- Hard exclusive processes :
  - deeply virtual Compton scattering
  - mapping out nucleon Generalized Parton Distributions
  - GPDs at large  $-t$  / transition to PQCD

# Rosenbluth vs polarization transfer measurements of $G_E/G_M$ of proton



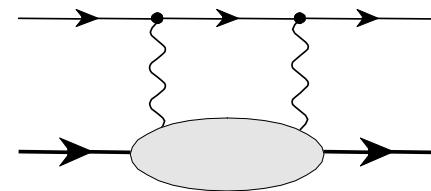
Two methods, two different results !

# Observables including two-photon exchange

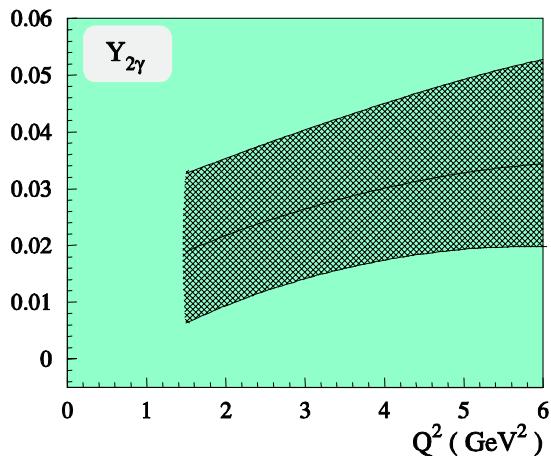
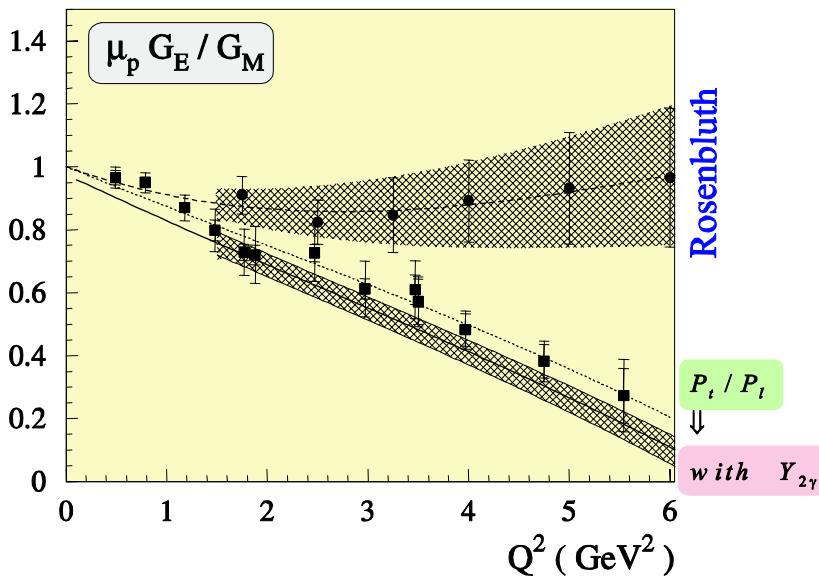
## Real parts of two-photon amplitudes

$$\begin{aligned}
 \rightarrow \quad \sigma_R &= G_M^2 \left( 1 + 2 \frac{\mathcal{R}(\delta \tilde{G}_M)}{G_M} \right) \\
 &+ \varepsilon \left\{ \frac{1}{\tau} G_E^2 \left( 1 + 2 \frac{\mathcal{R}(\delta \tilde{G}_E)}{G_E} \right) + 2G_M^2 \left( 1 + \frac{1}{\tau} \frac{G_E}{G_M} \right) \underbrace{\frac{\nu}{M^2} \frac{\mathcal{R}(\tilde{F}_3)}{G_M}}_{Y_{2\gamma}} \right\} \\
 &+ \mathcal{O}(e^4)
 \end{aligned}$$
  

$$\rightarrow \quad \frac{P_t}{P_l} = -\sqrt{\frac{2\varepsilon}{\tau(1+\varepsilon)}} \quad \left\{ \frac{G_E}{G_M} \left( 1 - \frac{\mathcal{R}(\delta \tilde{G}_M)}{G_M} \right) + \frac{\mathcal{R}(\delta \tilde{G}_E)}{G_M} \right. \\
 \left. + \left( 1 - \frac{2\varepsilon}{1+\varepsilon} \frac{G_E}{G_M} \right) \underbrace{\frac{\nu}{M^2} \frac{\mathcal{R}(\tilde{F}_3)}{G_M}}_{Y_{2\gamma}} \right\} \\
 + \mathcal{O}(e^4)$$



# Phenomenological analysis

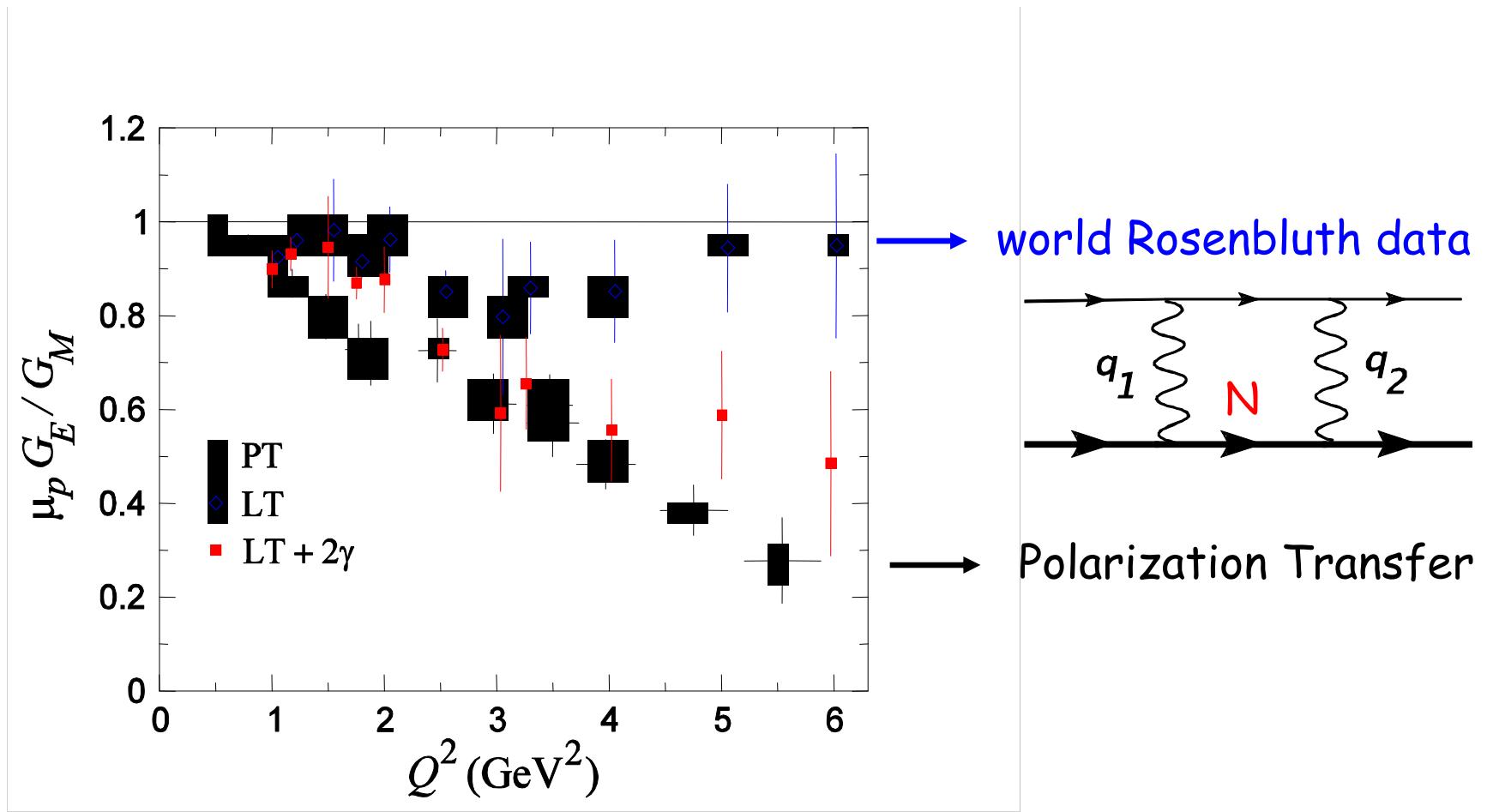


2-photon exchange is a good candidate to explain the discrepancy between both experimental methods

→ relevance when extracting form factors at large  $Q^2$

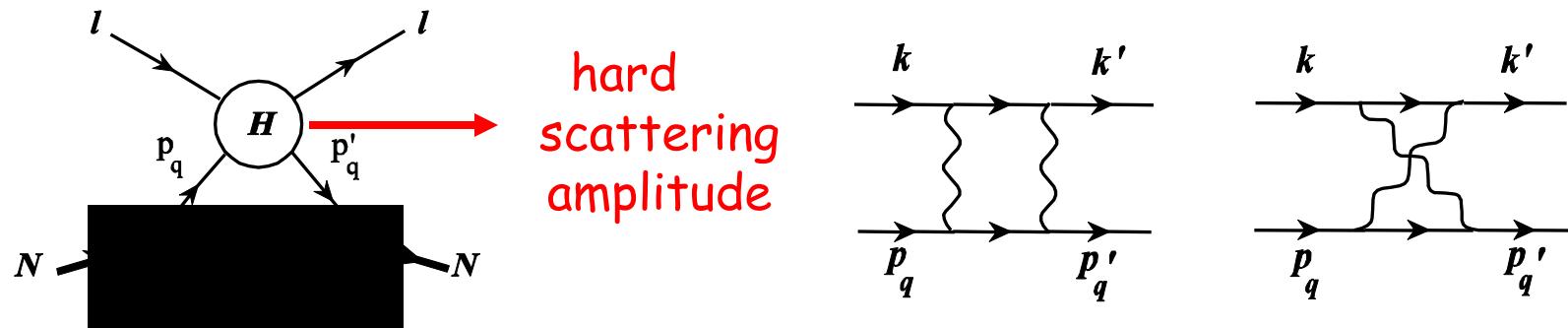
Guichon, Vdh  
(2003)

# Two-photon exchange calculation : elastic contribution



Blunden, Tjon, Melnitchouk (2003, 2005)

# Two-photon exchange : partonic calculation



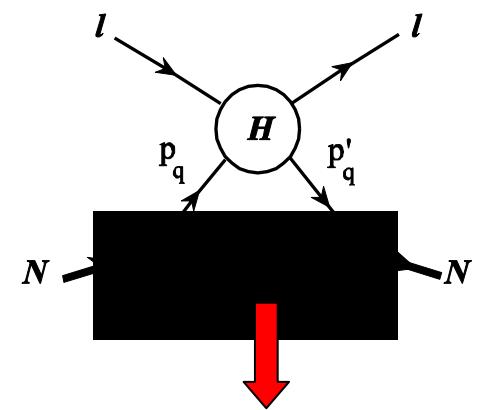
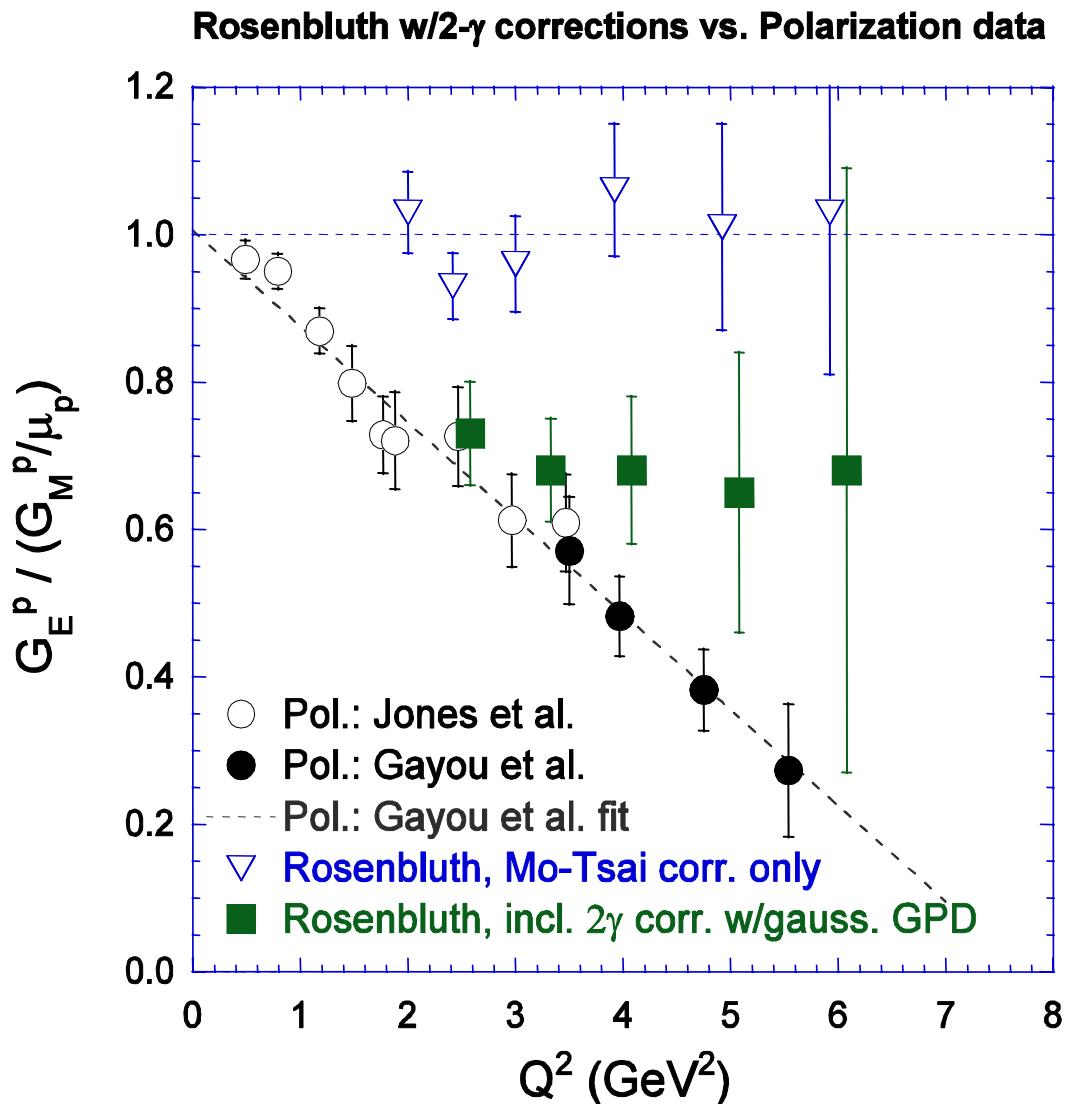
→ GPD integrals

$$A \equiv \int_{-1}^1 \frac{dx}{x} \frac{[(\hat{s} - \hat{u})\tilde{f}_1^{hard} - \hat{s}\hat{u}\tilde{f}_3]}{(s-u)} \sum_q e_q^2 (H^q + E^q) \quad \text{"magnetic" GPD}$$

$$B \equiv \int_{-1}^1 \frac{dx}{x} \frac{[(\hat{s} - \hat{u})\tilde{f}_1^{hard} - \hat{s}\hat{u}\tilde{f}_3]}{(s-u)} \sum_q e_q^2 (H^q - \tau E^q) \quad \text{"electric" GPD}$$

$$C \equiv \int_{-1}^1 \frac{dx}{x} \tilde{f}_1^{hard} \operatorname{sgn}(x) \sum_q e_q^2 \tilde{H}^q \quad \text{"axial" GPD}$$

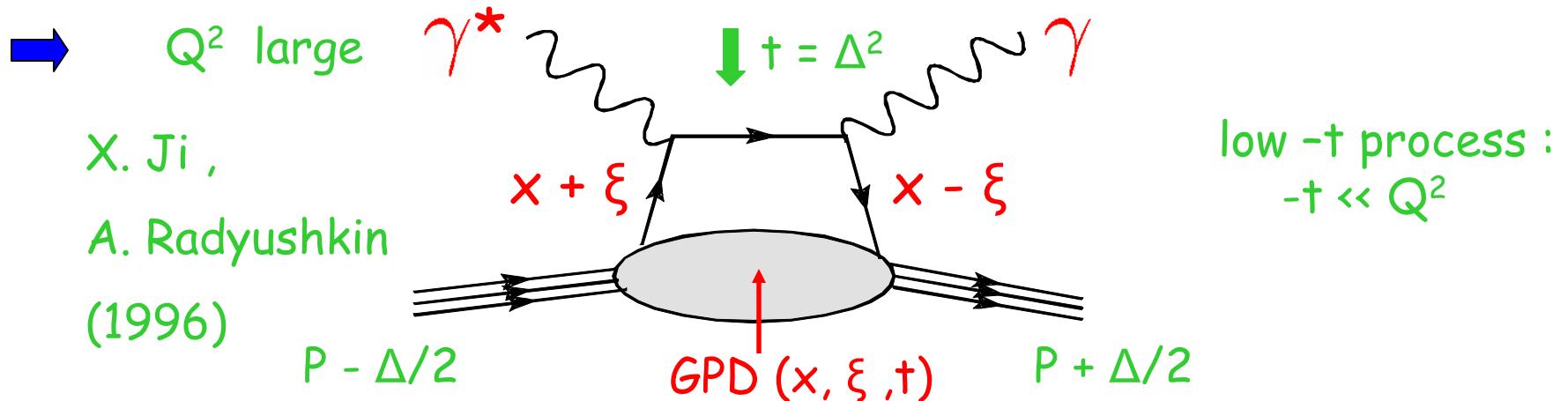
# Two-photon exchange : partonic calculation



GPDs

Chen, Afanasev,  
Brodsky, Carlson, Vdh  
(2004)

# Generalized Parton Distributions



$(x + \xi)$  and  $(x - \xi)$  : longitudinal momentum fractions of quarks

→ at large  $Q^2$  : QCD factorization theorem → hard exclusive process can be described by 4 transitions (GPDs) :

Vector :  $H(x, \xi, t)$

Tensor :  $E(x, \xi, t)$

Axial-Vector :  $\tilde{H}(x, \xi, t)$

Pseudoscalar :  $\tilde{E}(x, \xi, t)$

# known information on GPDs

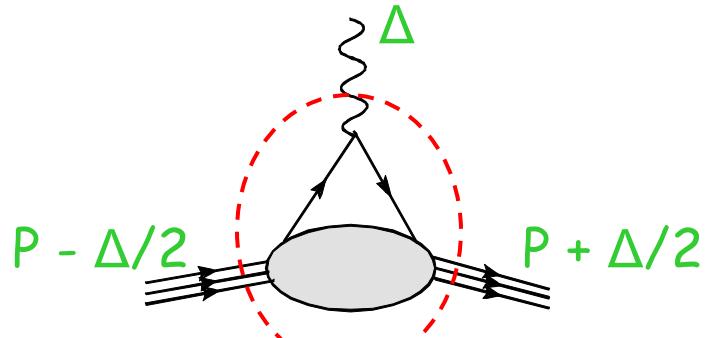
→ forward limit : ordinary parton distributions

$$H^q(x, \xi = 0, t = 0) = q(x) \quad \text{unpolarized quark distr}$$

$$\tilde{H}^q(x, \xi = 0, t = 0) = \Delta q(x) \quad \text{polarized quark distr}$$

$E^q, \tilde{E}^q$  : do NOT appear in DIS → additional information

→ first moments : nucleon electroweak form factors



$\xi$  independence :  
Lorentz invariance

$$\int_{-1}^1 dx H^q(x, \xi, t) = F_1^q(t)$$

$$\int_{-1}^1 dx E^q(x, \xi, t) = F_2^q(t)$$

$$\int_{-1}^1 dx \tilde{H}^q(x, \xi, t) = G_A^q(t)$$

$$\int_{-1}^1 dx \tilde{E}^q(x, \xi, t) = G_P^q(t)$$

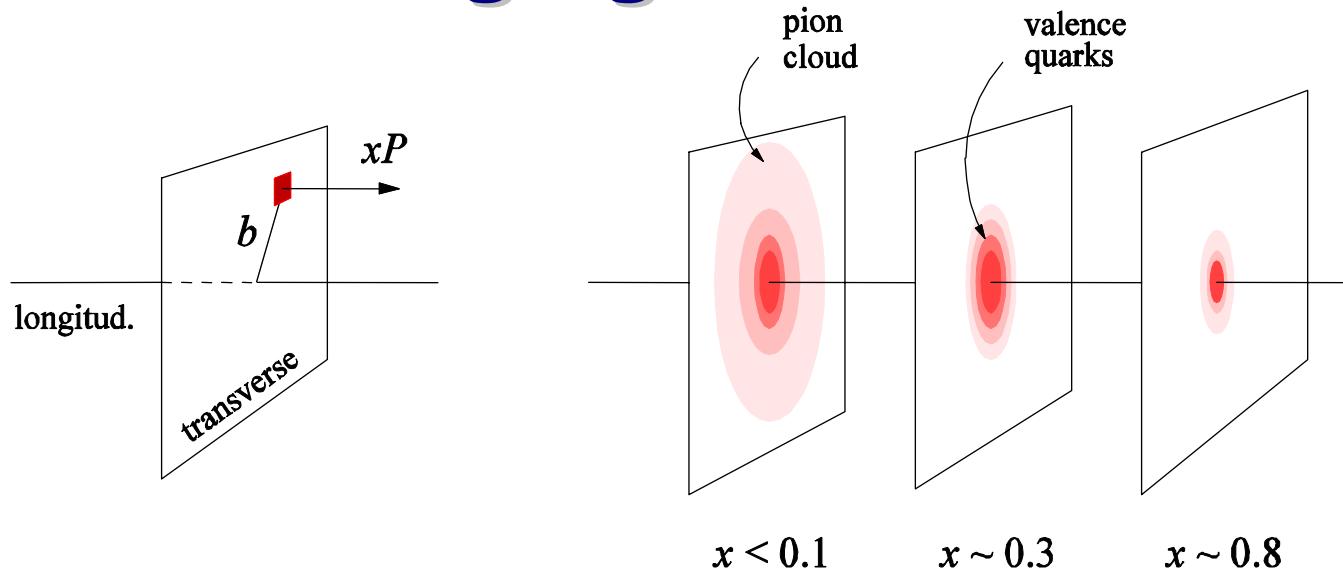
Dirac

Pauli

axial

pseudo-scalar

# GPDs : 3D quark/gluon imaging of nucleon

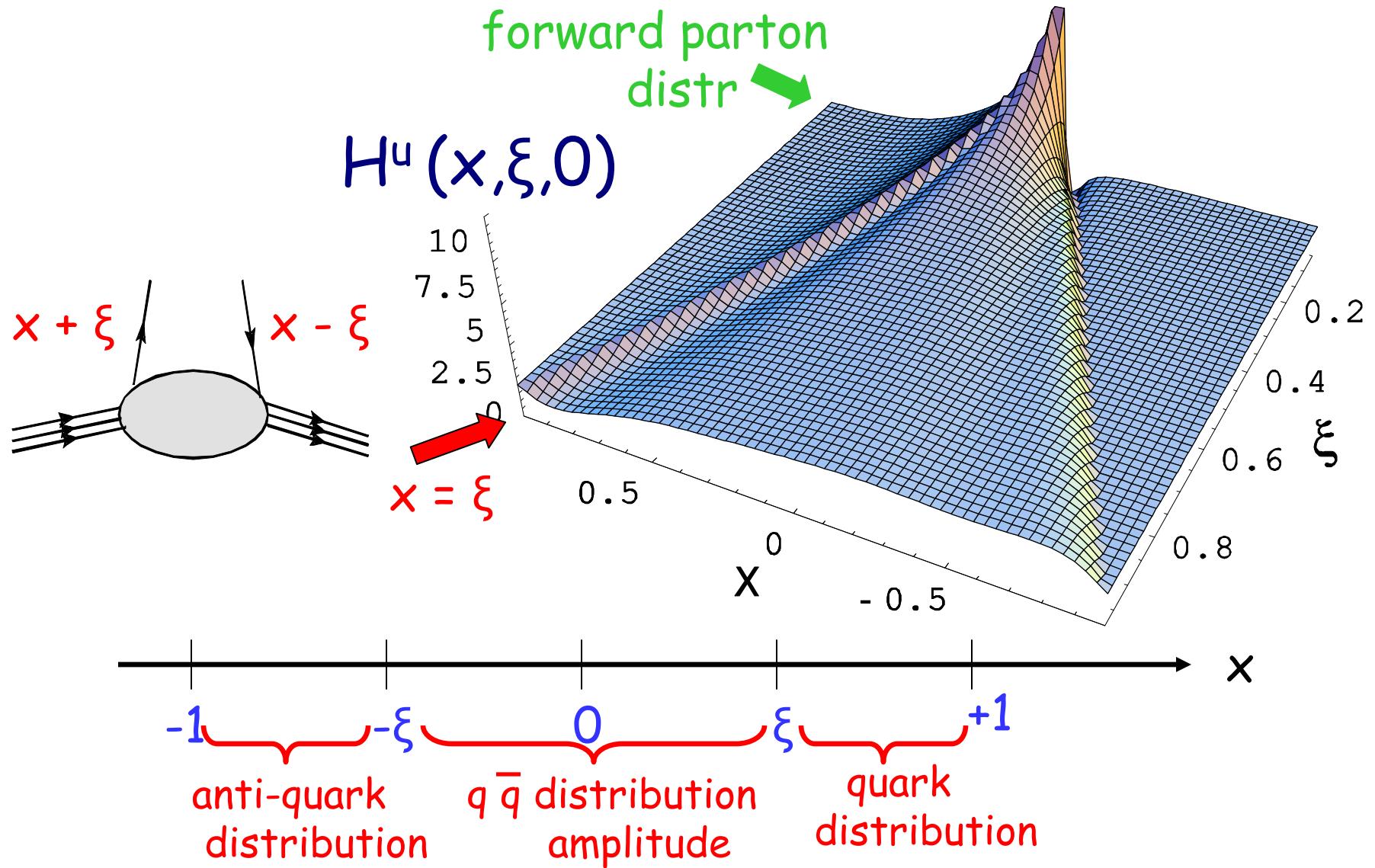


Fourier transform of GPDs :

simultaneous distributions of quarks w.r.t. longitudinal momentum  $xP$  and transverse position  $b$

→ theoretical parametrization needed

# GPDs : $x$ and $\xi$ dependence



# quark contribution to proton spin

→  $2 J^q = \int_{-1}^1 dx x \left\{ H^q(x, 0, 0) + E^q(x, 0, 0) \right\}$

X. Ji  
(1997)

$$2 J^q = M_2^q + \int_{-1}^1 dx x E^q(x, 0, 0)$$

with  $M_2^q = \int_0^1 dx x [q(x) + \bar{q}(x)]$

→ parametrizations for  $E^q$

- M1 :  $E^q(x, 0, 0) = \kappa_q q_v(x)$
- M2 :  $E^q(x, 0, 0) = \kappa_q / N_q (1 - x)^{\eta_q} q_v(x)$

PROTON	$M_2^q$	$2 J^q$ valence model M1 (GPV 01)	$2 J^q$ valence model M2 (GPRV 04)	$2 J^q$ Lattice QCDSF 03
u	0.40	0.69	0.63	$0.734 \pm 0.135$
d	0.22	-0.07	-0.06	$-0.085 \pm 0.088$
s	0.03	0.03	0.03	
u + d + s	<b>0.65</b>	<b>0.65</b>	<b>0.60</b>	<b><math>0.65 \pm 0.16</math></b>

# orbital angular momentum carried by quarks : resolving the spin crisis

$$J^q = \frac{1}{2} \Delta q + L^q$$

evaluated at  $\mu^2 = 2.5 \text{ GeV}^2$

PROTON	<b>2 J<sup>q</sup></b> valence model M1 (GPV 01)	<b>Δq</b> HERMES (1999)	<b>2 L<sup>q</sup></b>
u	0.61	$0.57 \pm 0.04$	$0.04 \mp 0.04$
d	-0.05	$-0.25 \pm 0.08$	$0.20 \mp 0.08$
s	0.04	$-0.01 \pm 0.05$	$0.05 \mp 0.05$
u + d + s	<b>0.60</b>	<b><math>0.30 \pm 0.10</math></b>	<b><math>0.30 \mp 0.10</math></b>

# GPDs : $t$ dependence ( small $-t$ )

→ small  $-t$  ( $-t < 1 \text{ GeV}^2$ ) : Regge parametrization Goeke, Polyakov, Vdh (2001)

- $t = 0$  :  $H^q(x, 0, 0) + H^q(-x, 0, 0) = q_v(x) \sim \frac{1}{x^{\alpha(0)}}$   $\alpha(0) \simeq 0.5$   
 valence quarks 

- $t \neq 0$  :  $H^q(x, 0, t) + H^q(-x, 0, t) \sim \frac{1}{x^{\alpha(t)}}$   $\alpha(t) = \alpha(0) + \alpha' t$   


Regge trajectory  $\alpha' \simeq 0.9 \text{ GeV}^{-2}$

$$\rightarrow F_1^q(t) = \int_0^1 dx q_v(x) \frac{1}{x^{\alpha'_1 t}} \quad F_2^q(t) = \int_0^1 dx \kappa_q q_v(x) \frac{1}{x^{\alpha'_2 t}}$$

Regge slopes :  $\alpha'_1, \alpha'_2$  determined from rms radii valence model  
 for E 

# GPDs : $t$ dependence ( large $-t$ )

modified Regge parametrization : Guidal, Polyakov, Radyushkin, Vdh (2004)

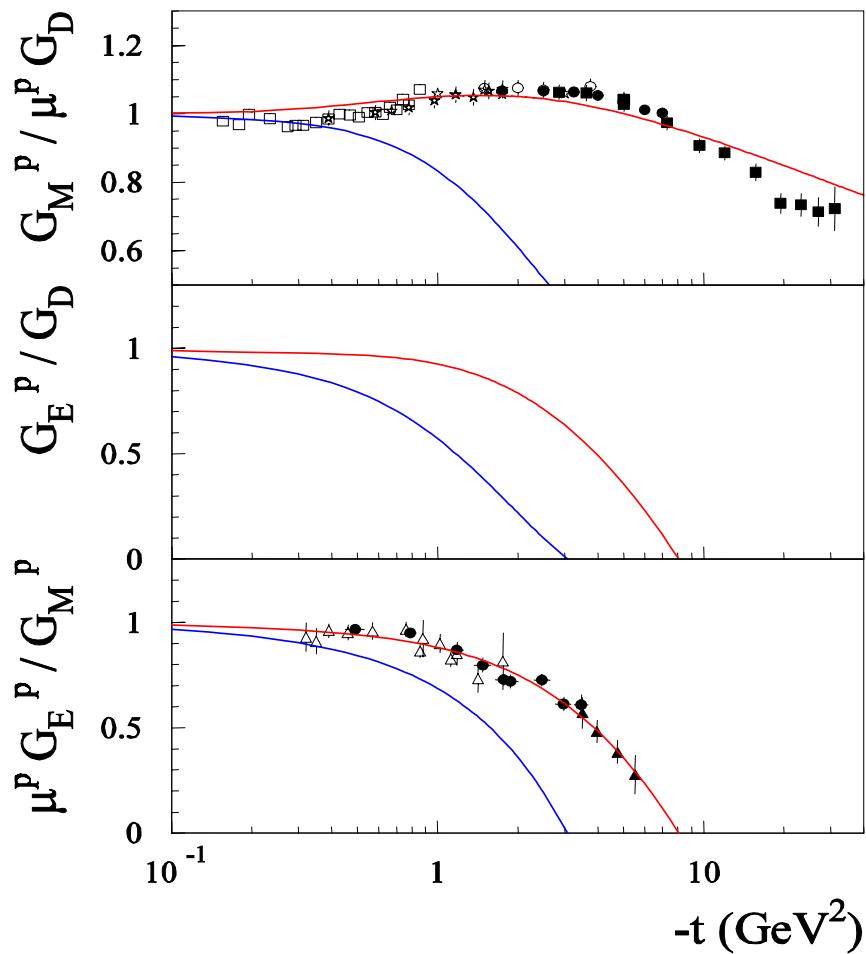
$$H^q(x, 0, t) = q_v(x) x^{-\alpha'_1 (1-x) t}$$

$$E^q(x, 0, t) = \frac{\kappa_q}{N_q} (1 - x)^{\eta_q} q_v(x) x^{-\alpha'_2 (1-x) t}$$

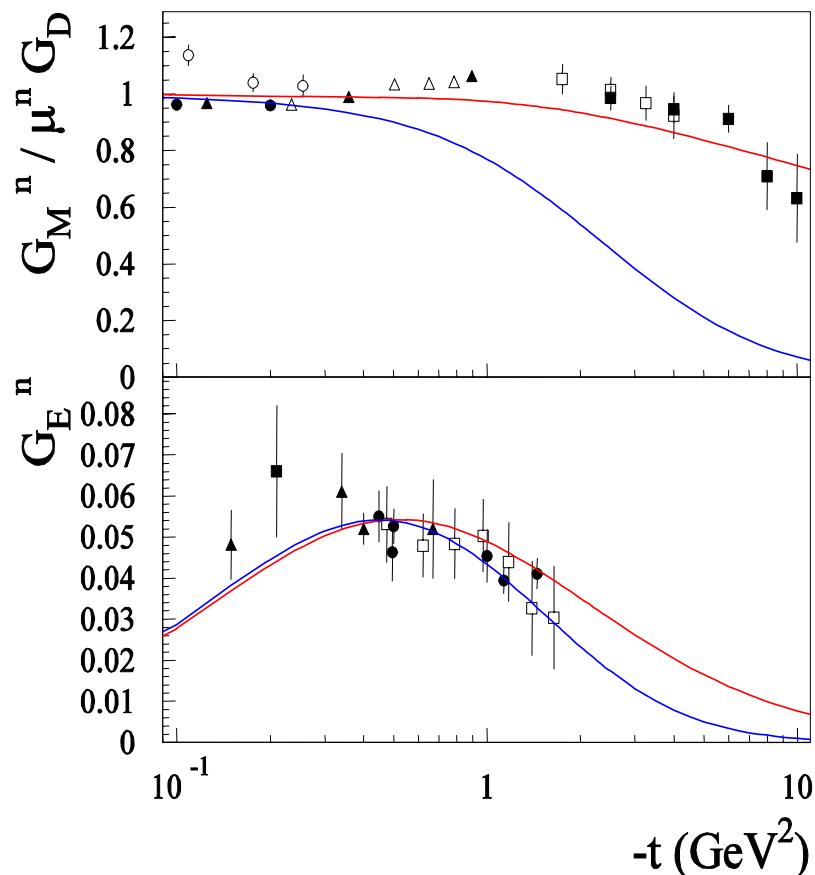
- Input : forward parton distributions at  $\mu^2 = 1 \text{ GeV}^2$  (MRST2002 NNLO)
- Drell-Yan-West relation :  $\exp(-\alpha' t) \rightarrow \exp(-\alpha' (1 - x) t)$  M. Burkardt (2001)
- parameters :
  - regge slopes :  $\alpha'_1, \alpha'_2$  determined from rms radii
  - $\eta_u, \eta_d$  determined from  $F_2 / F_1$  at large  $-t$
- future constraints : moments from lattice QCD see D. Richards

# electromagnetic form factors

PROTON



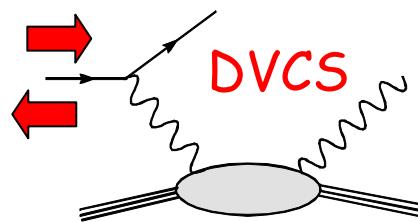
NEUTRON



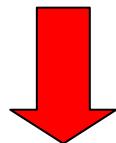
→ modified Regge parametrization  
 → Regge parametrization

# DVCS : beam spin asymmetry

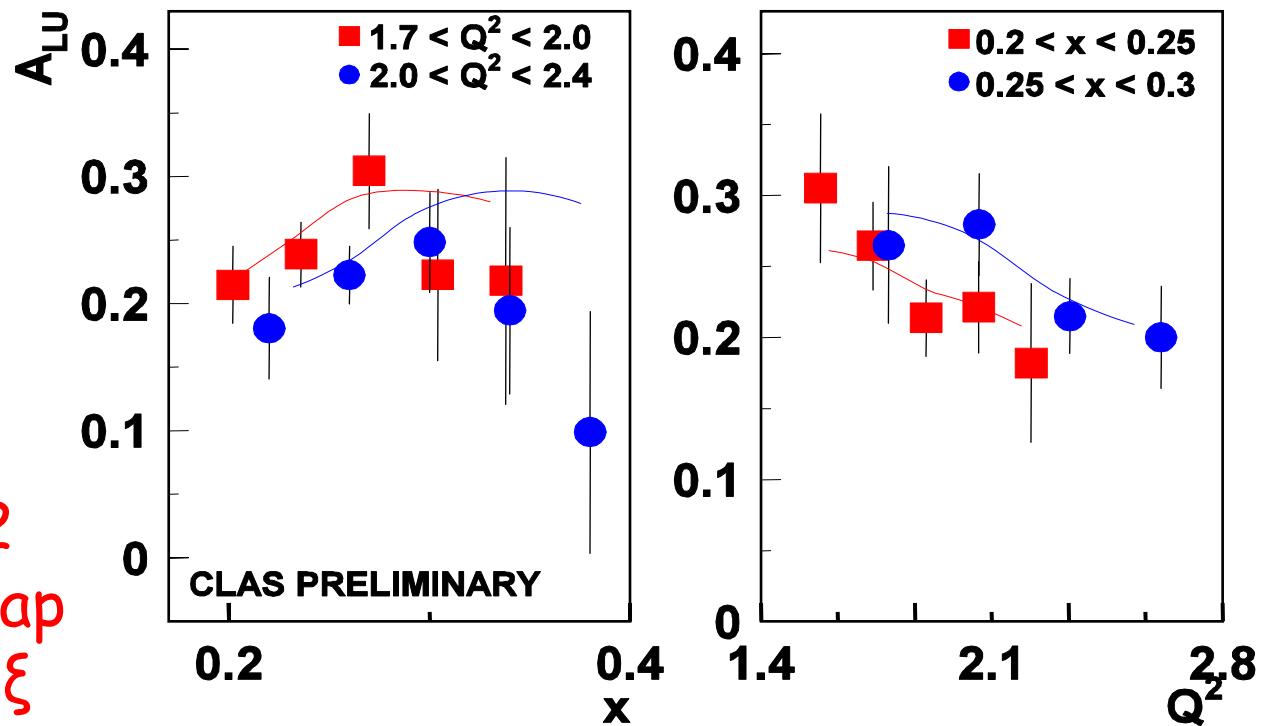
$$A_{LU} = (BH) * \text{Im}(DVCS) * \sin \Phi$$



Process under theoretical control including twist-3 corrections



Extract twist-2 component and map out GPDs at  $x = \xi$



twist-2 + twist-3 : Kivel, Polyakov, Vdh (2000)

# Summary and outlook

- Form factors at large momentum transfer :

- ➡ extend analysis of two-photon exchange observables to larger  $Q^2$  and to transition form factors , such as  $N \rightarrow \Delta$

- Hard exclusive processes :

- ➡ quantify power corrections (higher twist) to observables

- ➡ general parametrizations for Generalized Parton Distributions inclusion of all constraints possible : form factors, lattice QCD calculations for higher moments, positivity

- ➡ mapping out nucleon GPDs :

- get 3D image of transverse positions of quarks in nucleon for given longitudinal momentum (via Fourier transforms)