The Thomas Jefferson National Accelerator Facility
Theory Group Preprint Series

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The Southeastern Universities Research Association (SURA) operates the Thomas Jefferson National Accelerator Facility for the United States Department of Energy under contract DE-AC05-84ER40150.

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WM-99-107

JLAB-THY-99-11

A Hexagonal Theory of Flavor

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(May, 1999)

Abstract

We construct a supersymmetric theory of flavor based on the discrete gauge group $(D_6)^2$, where D_6 describes the symmetry of a regular hexagon under proper rotations in three dimensions. The representation structure of the group allows one to distinguish the third from the lighter two generations of matter fields, so that in the symmetry limit only the top quark Yukawa coupling is allowed and scalar superpartners of the first two generations are degenerate. Light fermion Yukawa couplings arise from a sequential breaking of the flavor symmetry, and supersymmetric flavor-changing processes remain adequately suppressed. We contrast our model with others based on non-Abelian discrete gauge symmetries described in the literature, and discuss the challenges in constructing more minimal flavor models based on this approach.

11.30.Hv, 12.15.Ff, 12.60.Jv

Typeset using REVTEX

I. INTRODUCTION

Perhaps the most puzzling issue in the physics of elementary particles is the origin of the fermion mass hierarchy. The top quark mass is observed to be 350,000 times larger than that of the electron, although both originate at the scale at which electroweak symmetry is spontaneously broken. Other quark and lepton mass eigenvalues, as well as the off-diagonal elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix exhibit hierarchies that are no less mysterious. While many models have been proposed to explain the flavor structure of the standard model, a theory that is economical and compelling has yet to be found.

The naturalness of small couplings, in this case the Yukawa couplings of the light fermions, can be understood in a field-theoretic context if a symmetry of the Lagrangian is restored when these couplings are set to zero. Thus, one approach to the flavor problem is to propose new symmetries acting horizontally across standard model generations, that forbid all but the top quark Yukawa coupling in the flavor symmetric limit. If the flavor symmetry group G_f is spontaneously broken by a set of "flavon" fields Φ, then light fermion masses arise generically via higher-dimension operators involving the Φ fields, together with the matter and Higgs fields of the standard model. These operators may have a renormalizable origin, as in the Froggatt-Nielsen mechanism [1], or may simply be present at the Planck scale. A hierarchy in the light fermion Yukawa couplings may be explained if G_f is broken sequentially through a series of smaller and smaller subgroups, at successively lower energy scales. In this paper we consider a model that explains the quark and lepton mass hierarchies in precisely this way. In addition, we work within the context of supersymmetry so that the hierarchy between the weak scale and the flavor physics scale M_F is stable under radiative corrections.

One must now choose a type of symmetry, either global or gauged, for building a model of flavor. Global symmetries are thought to be violated by quantum gravitational effects, and may not be consistent as fundamental symmetries of nature [2]. We therefore focus on gauge symmetries in this paper. A complication in supersymmetry is that continuous gauge symmetries lead to D-term interactions between the flavon fields and the scalar superpartners of the minimal supersymmetric standard model (MSSM). These couplings lead to large off-diagonal squark masses, and unacceptable flavor-changing neutral current (FCNC) effects [3]. Fortunately, discrete gauge symmetries provide a viable alternative. A discrete gauge symmetry can arise, for instance, when a continuous gauge symmetry group is spontaneously broken to a discrete subgroup. In this case, the discrete charges of fields in the low-energy effective theory must satisfy constraints that follow from the anomaly freedom of the original gauge group [4]. More importantly, however, discrete gauge symmetries can be defined independently of any embedding [5]. For example, such symmetries might arise directly in string compactifications. We construct a model based on a discrete gauge symmetry satisfying the appropriate low-energy constraints, independent of the origin of the symmetry.

In addition, we restrict our attention to non-Abelian discrete gauge groups, since these groups can have two-dimensional irreducible representations. By placing the first two generations of quark superfields into a doublet, for example, we achieve exact degeneracy of the corresponding squarks in the flavor symmetry limit [6]. Since G_f is broken only by light fermion Yukawa couplings, a high degree of degeneracy is preserved in the low-energy theory.

In this way, one greatly suppresses the FCNC effects that one would expect otherwise if the superparticle spectrum were generic.

Despite the expansive literature on models of flavor [7–12], relatively few successful supersymmetric models based on discrete non-Abelian gauged flavor symmetries exist [7–9]. It seems that most models fall into two categories: (a) models with relatively large discrete groups containing so many subgroups that they can easily accommodate the observed fermion mass hierarchy via a sequential breaking of the symmetry, and (b) models based on smaller discrete symmetries that require fine tuning to compensate for the fact that the flavor symmetry is a bit too restrictive. The $(S_3)^3$ model [7], for example, is interesting in that it is based on the smallest non-Abelian group S_3, and has a simple representation structure. However, the replication of S_3 factors yields a discrete group with $(3!)^3 = 216$ elements and numerous subgroups, and so it is not surprising that one can find a breaking pattern that accommodates viable Yukawa textures. On the other hand the $Q_{12} \times U(1)_H$ model [8] has a smaller discrete group (one with 120 elements, when one takes into account that only a Z_5 subgroup of the U(1) is relevant), but has quark mass textures that would not be viable without tuning of order one operator coefficients. Our goal is to find a model that lives somewhere between the two extremes defined above, and believe that the model we present in this paper is a step in the right direction. We hope that it will lead to interest in finding an optimal theory of flavor based on a non-Abelian discrete gauge group.

Our model is based on the group $D_6 \times D_6$, where D_6 describes the symmetry of a regular hexagon under proper rotations in three dimensions. In Sec. II, we discuss this symmetry group in more detail, and explain why it is promising for model building. The model itself is presented in Sec. III. Left- and right-handed matter fields transform under separate D_6 factors, and the three generations of a given field have a simple doublet-plus-singlet representation structure, with quarks and leptons distinguished by their transformation properties under a parity subgroup. In Sec. IV we consider bounds on the model from flavor-changing processes, and in the final section we summarize our conclusions. Fundamental properties of the theory of discrete groups are summarized in Appendix A, and details of combining D_6 representations appear in Appendix B.

II. THE GROUP D_6

In order to obtain the degree of degeneracy among the scalar superpartners necessary to suppress FCNC's at a phenomenologically acceptable level, we require that the light two generations of a given MSSM matter field appear in at least a doublet. All Abelian groups can be shown to possess only one-dimensional irreducible representations, and therefore the desired horizontal symmetry group G_f must be non-Abelian. Although it is tempting to explain the three-family replication by placing all quarks in triplet representations of G_f, it is also true that the top quark appears to be distinguished from the others by its large mass. Indeed, the top quark is the only one in the standard electroweak theory with an $O(1)$ Yukawa coupling, suggesting that it may have different transformation properties than fields of the first two generations. Thus, it is natural to assign the top quark to a (possibly nontrivial) one-dimensional representation of G_f. If one works only with triplet quark representations as in Ref. [9], one must explain with dynamics rather than symmetry why m_t is so large.

The smallest non-Abelian group is S_3, the group of permutations among three objects. It has $3! = 6$ elements, and possesses one doublet and two singlet representations, thus satisfying Eq. (A1). S_3 is isomorphic to the dihedral group D_3, which is the group of proper rotations in three dimensions that leave an equilateral triangle invariant. To picture this geometrically (Fig. 1), let the triangle lie in the xy-plane with its centroid at the origin, so that $\hat{\mathbf{z}}$ is the normal vector. Then D_3 is generated by a rotation (C_3) of $2\pi/3$ radians about $\hat{\mathbf{z}}$ and a rotation (C_x) of π radians about $\hat{\mathbf{x}}$ that turns the triangle upside-down. Note that $C_3C_x \neq C_xC_3$, so that the group is non-Abelian. The three irreducible representations in this picture are $\mathbf{1_S}$, the trivial singlet left invariant by all actions of the group; $\mathbf{1_A}$, a singlet invariant under C_3 but odd under C_x (such as the vector $\hat{\mathbf{z}}$); and a doublet $\mathbf{2}$, one realization of which is the pair $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$.

The simplicity of this structure was used to great effect in the $(S_3)^3$ model [7]; there, the three generations of left-handed quarks Q, the right-handed up-type quarks U, and the right-handed down-type quarks D were each assigned to $\mathbf{2} + \mathbf{1_A}$ representations of different S_3 factor groups, while remaining trivial (transforming as $\mathbf{1_S}$) under the others. The left-handed leptons L and their right-handed partners E were assigned to $\mathbf{2}+\mathbf{1_A}$ representations of S_3^D and S_3^Q, respectively. As one sees from the previous paragraph, the combination $\mathbf{2}+\mathbf{1_A}$ has the natural geometric interpretation of forming a complete vector in three-dimensional space, if one pictures D_3 embedded in the group O(3) of all rotations in three dimensions. This fact is useful in understanding anomaly cancellation conditions, as is discussed in Section III.

The $(S_3)^3$ model successfully explains the light squark degeneracy and the unique largeness of m_t, and also accommodates a phenomenologically viable texture of Yukawa matrices through sequential spontaneous symmetry breaking in which flavons, transforming nontrivially under G_f, obtain vacuum expectation values (VEV's). For example, the flavon VEV that gives rise to the bottom Yukawa coupling appears at a higher scale than the one that gives rise to that of the strange quark, explaining why $m_s < m_b$. The threefold replication of S_3 factors is necessary to allow for a sufficient number of stages of symmetry breaking to accommodate all the distinct Yukawa couplings. At each stage of symmetry breaking there remains some smaller set of symmetries still obeyed by the Lagrangian of the theory, which is specified by some subgroup of the original group G_f. Because of its small size, a single S_3 factor is insufficient since S_3 contains few nontrivial subgroups and therefore allows few distinct stages of symmetry breaking. While it is clearly possible to choose arbitrarily large groups that are suitable for our purposes, such choices violate the desire for minimality in particle physics. Therefore, our goal is to find a group large enough to accommodate all the properties in which we are interested but small enough to be compelling for study as a potentially real symmetry of nature. We seek a model satisfying the following criteria:

- Degeneracy of first two generations of squarks in the symmetry limit: a $\mathbf{2}+\mathbf{1}$ representation structure for the up-type quark superfields, and either a $\mathbf{2}+\mathbf{1}$ or $\mathbf{3}$ for the down-type quark and lepton superfields;
- Invariance of only the top quark Yukawa coupling under G_f;
- A simple pattern of anomaly cancellation;

- Enough subgroups to accommodate a viable Yukawa texture via sequential symmetry breaking;
- Absence of unnatural fine tuning of parameters;
- As small a group as possible, with minimal replication of factor groups.

In order to maintain the attractive features of the group D_3, we consider the 12-element dihedral group D_6, which is the smallest group that nontrivially contains D_3 as a subgroup. D_6 is the symmetry group of a regular hexagon under proper rotations in three dimensions, and indeed, is isomorphic to $D_3 \times Z_2$, as may be seen in Fig. 1. By adjoining C_2, the odd element of the Z_2 group, which is a rotation by π in the xy-plane, one generates the complete D_6: The sixfold rotation ($\pi/3$ radians) necessary for the hexagonal symmetry is $C_6 = C_2C_3^{-1}$.

D_6 possesses four singlet and two doublet inequivalent irreducible representations. Using the isomorphism $D_6 \cong D_3 \times Z_2$, these may be labeled $\mathbf{R}^P$, where $\mathbf{R}$ is a representation of D_3 and P is a parity indicating the action of C_2 on the elements of $\mathbf{R}$. For example, the original representations of D_3 transforming like $\hat{\mathbf{z}}$ and $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ are identified with the D_6 representations $\mathbf{1_A^+}$ and $\mathbf{2^-}$, respectively. The other representations of D_6 are $\mathbf{1_S^+}$ (the trivial singlet), $\mathbf{1_S^-}$, $\mathbf{1_A^-}$, and $\mathbf{2^+}$. Explicit matrix representations and the rules for combining them are relegated to Appendix B.

A real advantage of the dihedral groups D_n is that, in each doublet representation, the rotation matrix C_x in a particular basis (corresponding geometrically to a rotation of π radians about the x-axis) has the form

$$C_x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{2.1}$$

which means that the first component of the doublet in this basis is left invariant under the Z_2 subgroup of D_n generated by the twofold rotation C_x. In particular, breaking $D_n \to Z_2$ permits a VEV in the first component of the doublet while forbidding one in the second. This feature goes a long way toward building a hierarchical Yukawa texture.

There exists another 12-element group that generalizes D_3, the "dicyclic" group Q_6. Just as the full rotation group SU(2) in quantum mechanics forms a double-covering of the classical rotation group O(3), Q_6 may be thought of as the quantum-mechanical generalization of D_3: In either SU(2) or Q_6, a 2π rotation is not the identity operator for all representations, but rather there exist "spinorial" representations for which such a rotation produces a phase of -1. In particular, one of the doublet representations of Q_6 is spinorial. Unfortunately, one can show that there is no basis in which only one of the components of this doublet receives a VEV, leaving an unbroken subgroup. Therefore, the group D_6 is more convenient than Q_6 for building a viable Yukawa texture.

Finally, we have found that including only two factors of D_6 is enough to satisfy all the listed criteria; in particular, we have assigned Q and L to transform nontrivially under a single D_6, called D_6^L to denote chirality, and U, D, and E under a corresponding D_6^R. With $12^2 = 144$ elements, $D_6^L \times D_6^R$ is the smallest group yet discussed in the literature that satisfies our itemized criteria. While we cannot rule out that some single, smaller discrete

group might be found that works equally well in meeting our requirements, the simple, left-right symmetric form of the $D_6^L \times D_6^R$ model presented in the next section is intriguing, and suggests that this and similar group structures may be worthy of further consideration.

III. THE $D_6^L \times D_6^R$ MODEL

A. Field Content

We propose the flavor symmetry group $D_6^L \times D_6^R$, where the three generations of matter fields transform as $\mathbf{2}+\mathbf{1}$-dimensional combinations of D_6. Specifically,

$$\begin{aligned} Q &\sim ([\mathbf{2}^-, \mathbf{1}_\mathbf{S}^+], [\mathbf{1}_\mathbf{A}^+, \mathbf{1}_\mathbf{S}^+]), \\ U &\sim ([\mathbf{1}_\mathbf{S}^+, \mathbf{2}^-], [\mathbf{1}_\mathbf{S}^+, \mathbf{1}_\mathbf{A}^+]), \\ D &\sim ([\mathbf{1}_\mathbf{S}^+, \mathbf{2}^-], [\mathbf{1}_\mathbf{S}^+, \mathbf{1}_\mathbf{A}^+]), \\ L &\sim ([\mathbf{2}^+, \mathbf{1}_\mathbf{S}^+], [\mathbf{1}_\mathbf{A}^-, \mathbf{1}_\mathbf{S}^+]), \\ E &\sim ([\mathbf{1}_\mathbf{S}^+, \mathbf{2}^+], [\mathbf{1}_\mathbf{S}^+, \mathbf{1}_\mathbf{A}^-]), \end{aligned} \tag{3.1}$$

where the two entries inside square brackets distinguish the D_6^L and D_6^R transformation properties of each field, and the two sets of square brackets themselves refer to the transformation properties of the first two and third generation matter fields, in that order. Note that the left-handed fields Q and L transform nontrivially only under D_6^L, while the right-handed fields U, D, and E transform nontrivially only under D_6^R. Note also that quark and lepton fields are distinguished by their D_6 parity properties: Quarks transform as $\mathbf{2}^- + \mathbf{1}_\mathbf{A}^+$, while leptons transform as $\mathbf{2}^+ + \mathbf{1}_\mathbf{A}^-$.

In contrast, the MSSM Higgs fields must transform under both D_6 symmetries:

$$\begin{aligned} H_U &\sim (\mathbf{1}_\mathbf{A}^+, \mathbf{1}_\mathbf{A}^+), \\ H_D &\sim (\mathbf{1}_\mathbf{A}^-, \mathbf{1}_\mathbf{A}^+). \end{aligned} \tag{3.2}$$

The differing parity assignments assure that the top quark Yukawa coupling is invariant under the flavor symmetry, while that of the bottom quark is forbidden in the symmetry limit.

It is worth mentioning that the assignment of, for example, L and D to different D_6 factors implies that this model cannot be embedded easily into a unified field theory containing SU(5) as a subgroup. However, our model is perfectly consistent with gauge unification in a string theoretic context, which some may argue is preferable, given the absence of the usual doublet-triplet and proton decay problems associated with the existence of color-triplet Higgs fields.

B. Anomaly Cancellation

To understand anomaly cancellation in this model, it is useful to study first a simpler example. Consider a toy model based on the flavor symmetry $G_f = S_3$. Let us assign the three generations of matter fields to $\mathbf{2}+\mathbf{1}_\mathbf{A}$ representations, and assign the two Higgs fields

of the MSSM, H_U and H_D, both to $\mathbf{1}_\mathbf{A}$'s. We wish to show that S_3 is a consistent discrete gauge symmetry in this low-energy theory. To do so, imagine adding two additional Higgs fields, $H_U^{(2)}$ and $H_D^{(2)}$, which are doublets under S_3. Now one can embed all the fields into 3-dimensional representations of O(3), since a $\mathbf{3}$ decomposes into $\mathbf{2}+\mathbf{1}_\mathbf{A}$. It is clear in this case that the O(3)×SU(2)^2 and O(3)×SU(3)^2 anomalies vanish trivially, since the O(3) generators are antisymmetric, and hence traceless. It was demonstrated by Banks and Dine [5] that the only anomaly cancellation conditions needed for the consistency of the low-energy effective theory are those derived by Ibáñez and Ross [4] that are linear in the embedding group (in this case O(3)), and that involve only the non-Abelian low-energy continuous gauge groups. Furthermore, they point out that this is equivalent to the requirement that any effective interaction $\mathcal{O}_{\rm eff}$ generated by instantons of these continuous gauge groups remains invariant under the action of the discrete group. Since this latter criterion does not depend on any particular embedding of the discrete group into a continuous one at higher energies, it is the actual constraint we demand on the low-energy effective theory. Since we have seen that the linear O(3) anomalies vanish, we conclude that S_3 is not anomalous in the extended version of the toy model. Now notice that the Higgs fields in the extended theory form a vector-like pair, so that one can give a large mass to the unwanted doublet components, thus decoupling them from the low-energy theory. (In the O(3) embedding, this can be accomplished by breaking O(3) through giving a large VEV to only the linear combination of 1+5 coupling to the doublets.) Since S_3 remains unbroken when integrating out the unwanted states, the set of operators generated from $\mathcal{O}_{\rm eff}$ involving just the light fields, $\mathcal{O}'_{\rm eff}$, also remains invariant under S_3. Thus, one concludes that desired low-energy theory is also free of discrete gauge anomalies. This is the basis of the S_3 charge assignments in the $(S_3)^3$ model described in Refs. [7].

In our model, we maintain the multiplicity of $\mathbf{2}$ and $\mathbf{1}_\mathbf{A}$ representations described in the toy model above, for any given S_3 factor. However, the group D_6 is isomorphic to $S_3 \times Z_2$, and so one must also check that the Z_2 factor is not anomalous. To do so, one can embed Z_2 into a U(1) and check that the U(1) charges satisfy the linear Ibáñez-Ross condition for the $Z_2 \times \mathrm{SU}(N)^2$ anomaly [4]:

$$\sum_i T_i(q_i) = 2r\ , \quad \text{integer r} \tag{3.3}$$

where T_i is the SU(N) invariant defined by $\mathrm{Tr}\, t^a t^b = T\delta^{ab}$ and q_i is the U(1) charge of the ith field. For example, the right-handed quark fields might be embedded into $\mathbf{2}+\mathbf{1}_\mathbf{A}$, where we assign U(1) charges +1 and +2 to the $\mathbf{2}$ and $\mathbf{1}_\mathbf{A}$, respectively. Then Eq. (3.3) tells us

$$\frac{1}{2}(2\cdot 1 + 2) = 2r$$

which is satisfied for $r = 1$. Thus, the D_6 representation $\mathbf{2}^- + \mathbf{1}_\mathbf{A}^+$ is anomaly free. It is straightforward to verify that Eq. (3.3) is satisfied for SU(2) and SU(3) given the Z_2 charge assignments presented in (3.1) and (3.2). Note that the anomaly from the Z_2 subgroup of D_6^L cancels between the third-generation L field and H_D.

C. Yukawa Textures

To derive the Yukawa textures, one requires the rules for combining D_6 representations; these are presented in Appendix B. Treating the Yukawa matrices Y as flavor spurions, the superpotential terms QY_UH_UU, QY_DH_DD, and LY_LH_DE must be trivial singlets under G_f. Thus, the flavon fields contributing to the Yukawa matrices must transform as

$$
\begin{aligned}
Y_U &\sim \begin{pmatrix} [\tilde{\mathbf{2}}^-,\tilde{\mathbf{2}}^-] & [\tilde{\mathbf{2}}^-,\mathbf{1}_S^+] \\ [\mathbf{1}_S^+,\tilde{\mathbf{2}}^-] & [\mathbf{1}_S^+,\mathbf{1}_S^+] \end{pmatrix}, \\
Y_D &\sim \begin{pmatrix} [\tilde{\mathbf{2}}^+,\tilde{\mathbf{2}}^-] & [\tilde{\mathbf{2}}^+,\mathbf{1}_S^+] \\ [\mathbf{1}_S^-,\tilde{\mathbf{2}}^-] & [\mathbf{1}_S^-,\mathbf{1}_S^+] \end{pmatrix}, \\
Y_L &\sim \begin{pmatrix} [\tilde{\mathbf{2}}^-,\tilde{\mathbf{2}}^+] & [\tilde{\mathbf{2}}^-,\mathbf{1}_S^-] \\ [\mathbf{1}_S^+,\tilde{\mathbf{2}}^+] & [\mathbf{1}_S^+,\mathbf{1}_S^-] \end{pmatrix}.
\end{aligned}
\tag{3.4}
$$

Given a doublet $\mathbf{2} = (a, b)$, the conjugate doublet $\tilde{\mathbf{2}} = (-b, a)$ arises through the product of a $\mathbf{2}$ with a $\mathbf{1}_A$, as described in Appendix B.

For comparison, the squark and slepton mass matrices, which appear in the Lagrangian as terms of the form $\tilde{\phi}^\dagger m^2_{\tilde{\phi}}\tilde{\phi}$, where $\tilde{\phi}$ is the scalar component of the superfield $\phi = Q, U, D, L$, or E, transform as

$$
\begin{aligned}
m^2_{\tilde{Q}}, m^2_{\tilde{L}} &\sim \left(\begin{array}{c|c} [\mathbf{2}^+,\mathbf{1}_S^+] \oplus [\mathbf{1}_A^+,\mathbf{1}_S^+] \oplus [\mathbf{1}_S^+,\mathbf{1}_S^+] & [\tilde{\mathbf{2}}^-,\mathbf{1}_S^+] \\ \hline [\tilde{\mathbf{2}}^-,\mathbf{1}_S^+] & [\mathbf{1}_S^+,\mathbf{1}_S^+] \end{array}\right), \\
m^2_{\tilde{U}}, m^2_{\tilde{D}}, m^2_{\tilde{E}} &\sim \left(\begin{array}{c|c} [\mathbf{1}_S^+,\mathbf{2}^+] \oplus [\mathbf{1}_S^+,\mathbf{1}_A^+] \oplus [\mathbf{1}_S^+,\mathbf{1}_S^+] & [\mathbf{1}_S^+,\tilde{\mathbf{2}}^-] \\ \hline [\mathbf{1}_S^+,\mathbf{2}^-] & [\mathbf{1}_S^+,\mathbf{1}_S^+] \end{array}\right),
\end{aligned}
\tag{3.5}
$$

where the internal lines divide the first two generations from the third.

One sees from Eq. (3.4) that of 35 possible nontrivial flavon structures, only 11 appear in the Yukawa matrices. Two more (see Eq.(3.5)) appear in the squark mass matrices, and the remaining 22 cannot appear at tree level in any of the matrices. Indeed, we claim that a phenomenologically viable structure may be obtained through the introduction of only 5 types of flavon fields,

$$
\begin{aligned}
\sigma_b &\sim [\mathbf{1}_S^-,\mathbf{1}_S^+], \\
\sigma_\tau &\sim [\mathbf{1}_S^+,\mathbf{1}_S^-], \\
\phi &\sim [\tilde{\mathbf{2}}^-,\mathbf{1}_S^+], \\
\psi &\sim [\mathbf{1}_S^+,\tilde{\mathbf{2}}^-], \\
\Sigma &\sim [\tilde{\mathbf{2}}^+,\tilde{\mathbf{2}}^+].
\end{aligned}
\tag{3.6}
$$

All of the other flavon fields necessary to achieve a satisfactory accounting of phenomenology arise as products of these, at higher order in the effective Lagrangian.

The pattern of symmetry breaking is likewise chosen to be rather minimal; it consists of only three steps, and is summarized in Table I. The level of symmetry breaking is described in powers of a dimensionless parameter λ, and describes the size of the corresponding VEV arising at that point relative to some high energy scale M_F at which perfect flavor symmetry is restored. In phenomenological terms, we choose λ to be approximately the size of the

Cabibbo angle, $\lambda \approx 0.22$, and assign powers of λ consistent with fermion masses renormalized at a high ($\gtrsim 10^{16}$ GeV) scale [7].

The full symmetry $D_6^L \times D_6^R$ is broken at the scale $\lambda^2 M_F$ down to $Z_2^L \times Z_2^R$. Here, the Z_2 factors refer to the subgroup of $D_3 \subset D_6$ described in and after Eq. (2.1), which allow one to give VEV's to the first components of $\mathbf{2}$'s, or equivalently the second components of $\tilde{\mathbf{2}}$'s, while disallowing VEV's in the other components; it is not to be confused with the Z_2 parity subgroup of D_6, which is already broken after the first step. Since this step breaks both D_6's to Z_2's, only $\mathbf{1}_S$ singlets, the second components of $\tilde{\mathbf{2}}$'s, and the (22) component of $[\tilde{\mathbf{2}},\tilde{\mathbf{2}}]$ obtain $O(\lambda^2)$ VEV's at this stage. In the second stage, Z_2^L is broken to Z_1^L (*i.e.*, no symmetry), which gives VEV's of $O(\lambda^3)$ to the first component of D_6^L doublet ϕ and the first component in the second column of Σ (and also to the second components, but these are subleading). In the third stage, all remaining components previously protected receive $O(\lambda^4)$ VEV's. In summary, the textures of leading VEV's in the flavon fields read

$$
\begin{aligned}
&\sigma_b, \sigma_\tau \sim \lambda^2, \\
&\phi_a^T \sim (\lambda^3, \lambda^2), \quad a = 1, 2, \\
&\psi \sim (\lambda^4, \lambda^2), \\
&\Sigma \sim \begin{pmatrix} \lambda^4 & \lambda^3 \\ \lambda^4 & \lambda^2 \end{pmatrix}.
\end{aligned}
\tag{3.7}
$$

The presence of two ϕ doublets but only one ψ doublet leads to a viable phenomenology, as described below; The generic label ϕ used henceforth refers to a potentially different linear combination of ϕ_1 and ϕ_2 each time ϕ appears. With these assignments, it is a straightforward matter to find the leading contributions to the Yukawa and scalar mass-squared matrices with transformation properties given by Eqs. (3.4) and (3.5), and then to perform the tensor products as explained in Appendix B to obtain the textures for these matrices:

$$
\begin{aligned}
Y_U &\sim \left(\begin{array}{c|c} \phi\psi & \phi \\ \hline \psi & 1 \end{array}\right) \to \begin{pmatrix} \lambda^7 & \lambda^5 & \lambda^3 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}, \\
Y_D &\sim \left(\begin{array}{c|c} \sigma_\tau\Sigma + \psi\Sigma & \sigma_b\phi + \phi\phi + \Sigma\Sigma \\ \hline \sigma_b\psi & \sigma_b \end{array}\right) \to \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^5 \\ \lambda^6 & \lambda^4 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \end{pmatrix}, \\
Y_L &\sim \left(\begin{array}{c|c} \sigma_b\Sigma + \phi\Sigma & \sigma_\tau\phi \\ \hline \sigma_\tau\psi + \psi\psi + \Sigma\Sigma & \sigma_\tau \end{array}\right) \to \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^5 \\ \lambda^6 & \lambda^4 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \end{pmatrix}, \\
m^2_{\tilde{Q}}, m^2_{\tilde{L}} &\sim \left(\begin{array}{c|c} \begin{array}{c} \{\sigma_b\phi + \phi\phi + \Sigma\Sigma\} \\ \oplus \{\Sigma\Sigma + \phi\phi\} \oplus 1 \end{array} & \phi \\ \hline \phi^T & 1 \end{array}\right) \to M^2 \begin{pmatrix} \lambda^0 + \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^0 + \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^0 \end{pmatrix}, \\
m^2_{\tilde{U}}, m^2_{\tilde{D}}, m^2_{\tilde{E}} &\sim \left(\begin{array}{c|c} \begin{array}{c} \{\sigma_\tau\psi + \psi\psi + \Sigma\Sigma\} \\ \oplus \{\psi\psi + \Sigma\Sigma\} \oplus 1 \end{array} & \psi^T \\ \hline \psi & 1 \end{array}\right) \to M^2 \begin{pmatrix} \lambda^0 + \lambda^4 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^0 + \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & \lambda^0 \end{pmatrix},
\end{aligned}
\tag{3.8}
$$

where M is the squark/slepton mass scale, assumed to be a few hundred GeV. The $O(\lambda^0)$ entries in the first two generations for each superfield are identical as a consequence of the flavor symmetry, while the $O(\lambda^0)$ entry in the third generation may be different.

In writing these matrices we have neglected all signs in the exact Clebsch-Gordan couplings, which are irrelevant in an effective theory unless there is a special correlation of the VEV's. For our purposes, this occurs only in the upper 2×2 block of Y_U, where one sees that only one combination of flavons, $\phi \times \psi$, occurs at leading order. Observe that the $n \times n$ matrix obtained from the product of an n-dimensional column vector with an n-dimensional row vector has $n-1$ zero eigenvalues and only one nonzero eigenvalue, the dot product of the two vectors. In the present case, a zero eigenvalue for the 2×2 matrix $\phi \times \psi$ means that the up quark Yukawa coupling remains exactly zero at this order. Note that this would not be the case if more than one ψ doublet were present. On the other hand, at least two ϕ doublets are needed to guarantee that Y_U does not have two zero eigenvalues. Indeed, this eigenvalue zero is lifted only at third order in the flavons, where contributions appear from

$$\sigma_b \sigma_\tau \Sigma, \quad \sigma_b \psi \Sigma, \quad \sigma_\tau \phi \Sigma, \quad \text{and} \quad \phi \psi \Sigma. \tag{3.9}$$

The computation of these flavon products in all orderings (Clebsch-Gordan coefficients are not in general associative) produces the same texture in each case,

$$\Delta Y_U|_{\text{upper } 2\times 2} = \begin{pmatrix} \lambda^8 & \lambda^7 \\ \lambda^8 & \lambda^6 \end{pmatrix}, \tag{3.10}$$

thus lifting the zero eigenvalue and providing an up quark Yukawa coupling of $O(\lambda^8)$.

D. Non-Supersymmetric Phenomenology

Now let us consider how this assignment of fields and pattern of VEV's leads to a set of quark masses and mixing angles that are phenomenologically acceptable.

First, observe that the Yukawa coupling h_t giving rise to the top quark mass is unique in surviving in the symmetry-conserving $\lambda \to 0$ limit. The largest parameters breaking the symmetry arise at $O(\lambda^2)$, and give rise to h_b, h_τ, and V_{cb}. In typical flavor models with $\tan\beta \approx 1$, h_b and h_τ appear at $O(\lambda^3)$; their slightly larger size in this model may easily be accommodated choosing $\tan\beta \approx 6$ or a smaller $\tan\beta$ combined with some adjustment of the undetermined order unity coefficients.

The next parameters of interest are V_{ub}, arising at $O(\lambda^3)$ and h_c, h_s, and h_μ at $O(\lambda^4)$. These give rise to phenomenologically acceptable values for our choice of $\tan\beta$. Note at this point that the Yukawa matrices Y_D and Y_L, although arising from different flavon structures, have the same basic texture; in order to accommodate the detailed differences between the down-type quark and lepton textures, one must choose coefficients such that $(Y_L)_{22}/(Y_D)_{22} \approx 3$ and $(Y_L)_{11}/(Y_D)_{11} \approx 1/3$.

The entries giving rise to V_{us}, $(Y_U)_{12}$ or $(Y_D)_{12}$, appear at $O(\lambda^5)$, and indeed give rise upon diagonalization to a Cabibbo angle of $O(\lambda)$, since the (12) entries are suppressed by one power of λ compared to the (22) entries. The couplings h_d and h_e appear at $O(\lambda^6)$, and have appropriate magnitudes, taking into account the factor 1/3 previously mentioned. Finally, h_u arises only at third order in the flavon fields as described in the previous subsection, and appears (as desired) at $O(\lambda^8)$.

IV. PHENOMENOLOGY

In the previous section, we showed that there is a three-step symmetry breaking pattern for $D_6^L \times D_6^R$ that yields viable Yukawa textures, given our choice of flavon fields. It remains to be addressed whether the scalar superpartner mass-squared matrices retain a sufficient degeneracy after the flavor symmetry group is broken to suppress supersymmetric FCNC effects adequately. We consider this question in the current section. First, note that scalar degeneracy is still approximate in the low energy theory, so one may treat off-diagonal scalar masses in an insertion approximation. This is clearly justified for scalars of the first two generations, where approximate degeneracy is predicted by the theory, and we make the mild assumption that the third generation scalar masses do not differ wildly from the first two. Then, for any given scalar mass-squared matrix, let us define an average diagonal entry by $\tilde{m}^2$, and parameterize the size of the off-diagonal elements by

$$(\delta_{ij})_{AB} \equiv \frac{(\tilde{m}^2_{ij})_{AB}}{\tilde{m}^2}, \tag{4.1}$$

where A and B indicate chirality (L or R), and the other indices run over generation space. Note that we evaluate the δ_{ij} in the basis where the quark and lepton mass matrices are diagonal. This parameterization is especially useful since the relevant experimental bounds on the δ_{ij} have been compiled systematically by Gabbiani *et al.* [13], allowing one to put our model to the test. In Tables II through VII, we give a comparison between the predictions of our model and the experimental bounds. The predictions follow directly from λ power counting with $\lambda \approx 0.22$, and do not take into account the undetermined order one coefficients. Thus, the predictions of our model may in fact be significantly smaller. Table II follows from K-$\overline{K}$, D-$\overline{D}$, and B-$\overline{B}$ mixing constraints, Table III from the decay $b \to s\gamma$, and Table IV from the lepton flavor-violating decays $\ell_i \to \ell_j \gamma$. Note that the left-right scalar mass terms are determined by a Higgs VEV, as well as the value of a dimensionful coefficient A. The ratio of Higgs VEV's is determined by the value of the bottom quark Yukawa coupling that is predicted in our model; again, we take $\tan\beta \approx 6$ for our estimates. The A parameter is of the same order as the supersymmetry-breaking scale, but need not be equal to the average scalar superpartner mass. We give our assumed values for A and the relevant superparticle masses in the table captions. Of course, taking larger values for the superparticle masses makes the constraints less severe.

To account for standard model CP violation, one must allow for the possibility of complex operator coefficients and flavon VEV's. Here one suffers from some ambiguity since the origins of CP violation are not specified in our theory. Nevertheless, one may compare the constraints from CP-violating processes to the predictions of our model assuming the worst-case scenario in which order one phases appear in *all* entries of the scalar mass matrices. The corresponding results are given in Tables V through VII.

One learns from these tables that the flavor symmetry provides a significant suppression of CP-conserving and CP-violating flavor-changing processes, so that it is not necessary to take any soft supersymmetry-breaking mass larger than 600 GeV. It is reasonable to view this as a positive result, especially when taking into account the possible alternatives: For example, if the (12) elements of all the $\tilde{m}^2$ matrices were of order the Cabibbo angle λ, one would be forced to take the average squark mass above 40 TeV to evade the K-$\overline{K}$ mixing

constraints, and above 500 TeV to evade the bounds on ϵ, assuming order one phases. The $D_6^L \times D_6^R$ symmetry is precisely what renders the model suitable for solving the hierarchy problem, by allowing a sufficiently low superparticle mass scale. In addition, we see that our results are compatible with the bounds from the flavor-conserving, CP-violating electric dipole moments. These bounds could be accommodated even in the worst-case scenario of ubiquitous order one phases, provided that A terms are taken somewhat smaller than the average scalar masses.

In addition, these tables give a clear indication of where the model is most likely to be tested experimentally. The bounds that are most marginally satisfied for light superparticle masses are those from $\mu \to e\gamma$ and the various CP-violating observables. Improved bounds on $\mu \to e\gamma$ combined with the discovery of light sleptons would provide the clearest means of excluding the model. In addition, improved bounds on electric dipole moments, as well as a measurement of CP violation in the B system, might exclude the generic order one phase picture described above. Depending on the precise bounds, the model may still be viable in such a circumstance if CP is spontaneously broken in only some of the flavon VEV's, since the pattern of phases in the soft masses would then be far from generic. However, we do not consider that possibility further in this article.

V. CONCLUSIONS

In this paper, we have explored the possibility of constructing a realistic theory of flavor based on a relatively small discrete gauge symmetry group. Our $D_6^L \times D_6^R$ model can explain the hierarchy of quark and lepton masses via a three-step sequential breaking pattern, and greatly alleviates the flavor-changing neutral current problem that is endemic to generic, softly-broken supersymmetric theories. In addition, the smallness of the flavor symmetry group prevents one from suppressing flavor-changing effects arbitrarily, so that the model remains falsifiable. In particular, improved bounds on $\mu \to e\gamma$ could be used to exclude the model if light sleptons are eventually discovered. Better measurements of electric dipole moments of the electron and neutron may demand modifying our assumption of a generic pattern of order one phases in operator coefficients and flavon VEV's, which we argued could account for standard model CP violation, at least for some range of the soft supersymmetry-breaking masses and A parameters. While in a small number of places we needed to assume some fluctuation in order one coefficients, for example to obtain the famous Georgi-Jarskog factors of 3 in the charged lepton Yukawa matrix, we do not view this as a serious shortcoming of the model. After all, we know of no low-energy effective theory in particle physics where symmetry considerations alone completely account for the particle phenomenology. From this point of view, our model works surprisingly well overall. Finally, there are a number of possible issues in our model that may be worth additional investigation, such as spontaneous CP violation and extension of the model to the neutrino sector. However, it seems to us that the more interesting problem is to find the most compelling model based on a minimal discrete gauged flavor-symmetry group. While we believe we have made progress in the current work, the possibility of constructing the most convincing theory of this type remains

an intriguing open question.

Acknowledgments

CC thanks the National Science Foundation for support under Grant No. PHY-9800741; RL thanks the Department of Energy for support under Contract No. DE-AC05-84ER40150.

APPENDIX A: GROUP THEORY FUNDAMENTALS

In this appendix we describe in general terms the group theory necessary to understand the manipulation of discrete horizontal symmetry groups, and re-acquaint the reader with the necessary terminology. The fundamentals of discrete group theory used here appear, for example, in the book by Hamermesh [14], particularly the first four chapters.

An abstract *group* G is defined as any set of objects closed under an associative binary operation, called multiplication; the set contains a multiplicative identity; and each element has a multiplicative inverse. Clearly, the structure of G is completely determined by a multiplication table of its elements. Two groups are *isomorphic* if they have the same number of elements and the same multiplication table.

A *representation* R of G is just a set of objects satisfying the same multiplication table as G, although it can happen that two or more distinct elements of G correspond to only one element of R (that is, R considered as a group need not be isomorphic to G); moreover, the actual operation corresponding to multiplication in R may be very different from that used by G. Operationally, the most useful group representations are in terms of $n \times n$ matrices under the usual matrix multiplication (*n-dimensional* or *n-plet representations*), where some of the representation matrices may fail to commute, providing a natural way to accommodate non-Abelian groups. A representation is *irreducible* if the corresponding matrices cannot be simultaneously block-diagonalized, and two matrix representations are *equivalent* if they can be related by a change of basis.

The most interesting feature of finite groups for our purposes is that the number of elements (*order*) g of G is related to the number and dimensions n_ν of its inequivalent irreducible representations ν,

$$g = \sum_\nu n_\nu^2, \tag{A1}$$

meaning that a given finite group has only a finite number of inequivalent irreducible representations, all of them finite-dimensional. It can be shown that all irreducible representations of an Abelian group are one-dimensional.

Complete information on the representation content of a given group is exhibited by its *character table*. The *character* of an element of R is just the trace of the corresponding matrix, which is invariant under basis changes. Reference [14] presents character tables for many common small groups, while Ref. [15] presents full information for all groups of order ≤ 31.

APPENDIX B: MANIPULATION OF D_6 REPRESENTATIONS

The group D_6 has four inequivalent singlet representations, $\mathbf{1}_{\mathbf{S}}^{+}$, $\mathbf{1}_{\mathbf{A}}^{+}$, $\mathbf{1}_{\mathbf{S}}^{-}$, and $\mathbf{1}_{\mathbf{A}}^{-}$, and two inequivalent doublet representations, $\mathbf{2}^{-}$ and $\mathbf{2}^{+}$. In the notation of Ref. [10], these are 1, $1'$, $1''$, $1'''$, $2_{(1)}$, and $2_{(2)}$, respectively.

The singlet representations of D_6 obey the same multiplication rules as elements of $Z_2 \times Z_2$, where the even/odd elements of the first Z_2 are S/A, and those of the second are $+/-$. Thus, for example, $\mathbf{1}_{\mathbf{A}}^{-} \otimes \mathbf{1}_{\mathbf{S}}^{-} = \mathbf{1}_{\mathbf{A}}^{+}$. The full list of binary tensor products of singlet representations appears in Table VIII.

For doublet (and larger) representations, the question of combining two representations is less trivial, and constitutes the question of computing Clebsch-Gordan coefficients of a given group, in this case D_6. The general expression relating the product of an n_x-plet $\mathbf{x}$ and and an n_y-plet $\mathbf{y}$ to an n_z-plet $\mathbf{z}$ is given by

$$R_z^\dagger \mathcal{O}_i R_y = \sum_{j=1}^{n_z} (R_z)_{ij} \mathcal{O}_j, \qquad i = 1, \ldots, n_z, \tag{B1}$$

where R refers to each matrix in a given representation of D_6, and the n_z matrices $\mathcal{O}_i$ of dimensions $n_x \times n_y$ are the Clebsch-Gordan coefficients of the group. That is, $\mathbf{x}^\dagger \mathcal{O}_i \mathbf{y}$ transforms like the ith component of $\mathbf{z}$. Since the dihedral groups D_n are entirely generated by the rotations C_n and C_x, it is enough to obtain $\mathcal{O}$ satisfying this condition for just $R = C_n, C_x$. Explicit representations of the C_x, C_6 are given in Table IX. The tensor products of singlet with doublet representations are given by

$$\begin{aligned} \mathbf{1}_{\mathbf{S}}^{P_1} \otimes \mathbf{2}^{P_2} &= \mathbf{2}^{P_1 P_2}, \text{ with } \mathcal{O}_1 = (1\ 0),\ \mathcal{O}_2 = (\ 0\ 1), \\ \mathbf{1}_{\mathbf{A}}^{P_1} \otimes \mathbf{2}^{P_2} &= \mathbf{2}^{P_1 P_2}, \text{ with } \mathcal{O}_1 = (0\ 1),\ \mathcal{O}_2 = (-1\ 0), \end{aligned} \tag{B2}$$

while the products of two doublet representations are given by

$$\mathbf{2}^{P_1} \otimes \mathbf{2}^{P_2} = \mathbf{2}^{P_1 P_2} \oplus \mathbf{1}_{\mathbf{A}}^{P_1 P_2} \oplus \mathbf{1}_{\mathbf{S}}^{P_1 P_2}, \tag{B3}$$

where the Clebsch-Gordan coefficients are given by

$$\begin{aligned} \mathbf{2}^{P_1} \otimes \mathbf{2}^{P_2} &\supset \mathbf{2}^{P_1 P_2} : \mathcal{O}_1 = \sigma_3,\ \mathcal{O}_2 = -\sigma_1; \\ &\supset \mathbf{1}_{\mathbf{A}}^{P_1 P_2} : \mathcal{O} = i\sigma_2; \\ &\supset \mathbf{1}_{\mathbf{S}}^{P_1 P_2} : \mathcal{O} = \mathbb{1}. \end{aligned} \tag{B4}$$

Here σ_i are the standard Pauli matrices, and $\mathbb{1}$ is the identity matrix. That the parities P_i are trivially multiplicative when representations are combined is clear from the isomorphism $D_6 \cong D_3 \times Z_2$. Note also from Eq. (B2) that the effect of forming the tensor product between either $\mathbf{2}$ with $\mathbf{1}_{\mathbf{A}}^{\pm}$ has the effect of switching the order of the components in the doublet, while that with $\mathbf{1}_{\mathbf{S}}^{\pm}$ leaves the order unchanged. We refer to a doublet whose components have been interchanged in this way generically as a $\tilde{\mathbf{2}}$.

TABLES

Flavon receiving VEV	Magnitude of VEV	Remaining symmetry
σ_b, σ_τ, $\phi^{(2)}$, $\psi^{(2)}$, $\Sigma^{(2,2)}$	λ^2	$Z_2^L \times Z_2^R$
$\phi^{(1)}$, $\Sigma^{(1,2)}$	λ^3	$Z_1^L \times Z_2^R$
$\psi^{(1)}$, $\Sigma^{(1,1)}$, $\Sigma^{(2,1)}$	λ^4	$Z_1^L \times Z_1^R$

TABLE I. Pattern of sequential symmetry breaking in $D_6^L \times D_6^R$. Z_2 factors refer to the twofold symmetry of the D_3 subgroup and not the parity subgroup of D_6, while Z_1 is shorthand for no symmetry. Superscripts refer to particular tensor components, which receive separate VEV's as described in the text.

	$\sqrt{\lvert\mathrm{Re}(\delta_{12}^d)_{LL}^2\rvert}$	$\sqrt{\lvert\mathrm{Re}(\delta_{12}^d)_{LR}^2\rvert}$	$\sqrt{\lvert\mathrm{Re}(\delta_{12}^d)_{LL}(\delta_{12}^d)_{RR}\rvert}$
Model	5.2×10^{-4}	4.0×10^{-5}	2.4×10^{-4}
Expt.	4.0×10^{-2}	4.4×10^{-3}	2.8×10^{-3}
	$\sqrt{\lvert\mathrm{Re}(\delta_{13}^d)_{LL}^2\rvert}$	$\sqrt{\lvert\mathrm{Re}(\delta_{13}^d)_{LR}^2\rvert}$	$\sqrt{\lvert\mathrm{Re}(\delta_{13}^d)_{LL}(\delta_{13}^d)_{RR}\rvert}$
Model	1.1×10^{-2}	4.0×10^{-5}	5.0×10^{-3}
Expt.	9.8×10^{-2}	3.3×10^{-2}	1.8×10^{-2}
	$\sqrt{\lvert\mathrm{Re}(\delta_{12}^u)_{LL}^2\rvert}$	$\sqrt{\lvert\mathrm{Re}(\delta_{12}^u)_{LR}^2\rvert}$	$\sqrt{\lvert\mathrm{Re}(\delta_{12}^u)_{LL}(\delta_{12}^u)_{RR}\rvert}$
Model	5.2×10^{-4}	2.5×10^{-4}	2.4×10^{-4}
Expt.	1.0×10^{-1}	3.1×10^{-2}	1.7×10^{-2}

TABLE II. Bounds on the off-diagonal squark mixing parameters δ_{ij} from neutral meson mixing, assuming a gluino and average squark mass of 500 GeV. For left-right masses, an A parameter of 500 GeV was also assumed. Note that the experimental bounds scale linearly in the average squark mass, given a fixed ratio with the gluino mass.

	$\lvert(\delta_{23}^d)_{LL}\rvert$	$\lvert(\delta_{23}^d)_{LR}\rvert$
Model	4.8×10^{-2}	1.8×10^{-4}
Expt.	8.2	1.6×10^{-2}

TABLE III. Bounds on δ_{ij} from $b \to s\gamma$, assuming a gluino and average squark mass of 500 GeV. For left-right masses, an A parameter of 500 GeV was also assumed. Note that the experimental bounds scale quadratically in the average squark mass, given a fixed ratio with the gluino mass.

	$\lvert(\delta^\ell_{12})_{LL}\rvert$	$\lvert(\delta^\ell_{12})_{LR}\rvert$
Model	5.2×10^{-4}	1.6×10^{-5}
Expt.	9.4×10^{-2}	2.1×10^{-5}
	$\lvert(\delta^\ell_{13})_{LL}\rvert$	$\lvert(\delta^\ell_{13})_{LR}\rvert$
Model	1.1×10^{-2}	1.6×10^{-5}
Expt.	355	1.3
	$\lvert(\delta^\ell_{23})_{LL}\rvert$	$\lvert(\delta^\ell_{23})_{LR}\rvert$
Model	4.8×10^{-2}	7.3×10^{-5}
Expt.	65	2.5×10^{-1}

TABLE IV. Bounds on the δ_{ij} from $\ell_i \to \ell_j\gamma$ decays, assuming a photino and average slepton mass of 350 GeV. For left-right masses, an A parameter of 100 GeV was also assumed. Note that the experimental bounds scale quadratically in the average slepton mass, given a fixed ratio with the photino mass.

	$\sqrt{\lvert\mathrm{Im}(\delta^d_{12})^2_{LL}\rvert}$	$\sqrt{\lvert\mathrm{Im}(\delta^d_{12})^2_{LR}\rvert}$	$\sqrt{\lvert\mathrm{Im}(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}\rvert}$
Model	5.2×10^{-4}	3.3×10^{-5}	2.4×10^{-4}
Expt.	3.8×10^{-3}	4.2×10^{-4}	2.6×10^{-4}

TABLE V. Bounds on the δ_{ij} from ϵ assuming a gluino and average squark mass of 600 GeV. For left-right masses, an A parameter of 600 GeV was also assumed. Note that the experimental bounds scale linearly in the average squark mass, given a fixed ratio with the gluino mass. The model predictions assume a worst-case scenario in which order one phases appear in all entries of the scalar mass matrices.

	$\lvert\mathrm{Im}(\delta^d_{12})_{LL}\rvert$	$\lvert\mathrm{Im}(\delta^d_{12})_{LR}\rvert$
Model	5.2×10^{-4}	1.6×10^{-5}
Expt.	6.9×10^{-1}	2.9×10^{-5}

TABLE VI. Bounds on the δ_{ij} from ϵ'/ϵ, assuming a gluino and average squark mass of 600 GeV. For left-right masses, an A parameter of 300 GeV was also assumed. Note that the experimental bounds scale quadratically in the average squark mass, given a fixed ratio with the gluino mass. The model predictions assume a worst-case scenario in which order one phases appear in all entries of the scalar mass matrices.

	$\lvert\mathrm{Im}(\delta^d_{11})_{LR}\rvert$	$\lvert\mathrm{Im}(\delta^u_{11})_{LR}\rvert$	$\lvert\mathrm{Im}(\delta^\ell_{11})_{LR}\rvert$
Model	3.0×10^{-6}	4.2×10^{-6}	1.8×10^{-6}
Expt.	3.6×10^{-6}	7.1×10^{-6}	2.2×10^{-6}

TABLE VII. Bounds on the δ_{ii} from electric dipole moments, assuming a gluino, photino, average squark, and average slepton mass of 600 GeV. An A parameter of 250 GeV for the squarks and 150 GeV for the sleptons was assumed. The model predictions assume a worst-case scenario in which order one phases appear in all entries of the scalar mass matrices.

$\otimes$	1^+_S	1^+_A	1^-_S	1^-_A
1^+_S	1^+_S	1^+_A	1^-_S	1^-_A
1^+_A	1^+_A	1^+_S	1^-_A	1^-_S
1^-_S	1^-_S	1^-_A	1^+_S	1^+_A
1^-_A	1^-_A	1^-_S	1^+_A	1^+_S

TABLE VIII. Tensor products of two singlet representations of D_6.

	1^+_S	1^+_A	1^-_S	1^-_A	2^+	2^-
C_x	$+1$	-1	$+1$	-1	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
C_6	$+1$	$+1$	-1	-1	$\begin{pmatrix} -\frac{1}{2} & +\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$	$\begin{pmatrix} +\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ +\frac{\sqrt{3}}{2} & +\frac{1}{2} \end{pmatrix}$

TABLE IX. Explicit representations of the generating elements of D_6 for each inequivalent irreducible representation.

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FIGURES

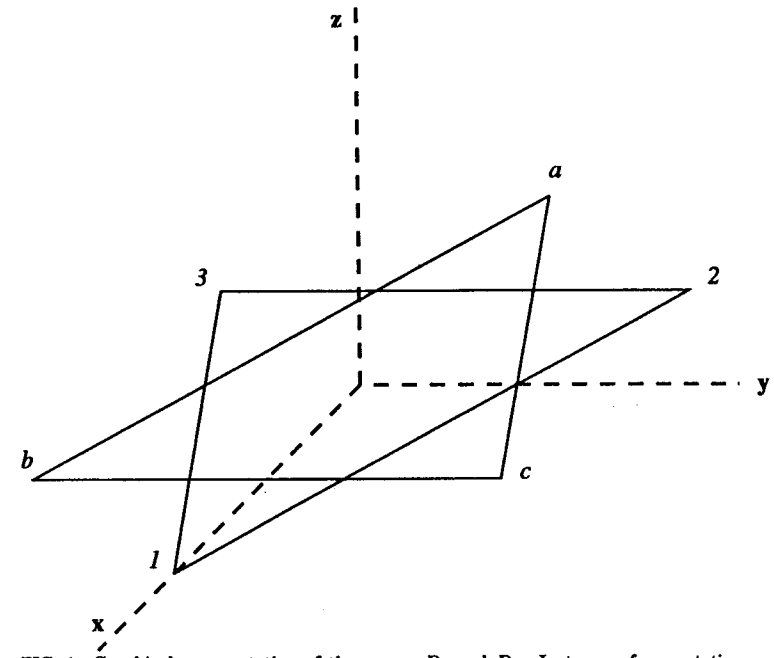

FIG. 1. Graphical representation of the groups D_3 and D_6. In terms of permutation cycle notation (*e.g.*, (132)(45) means $1 \to 3$, $3 \to 2$, $2 \to 1$, $4 \longleftrightarrow 5$), D_3 is generated by C_3, a $2\pi/3$ rotation about $\hat{\mathbf{z}}$, which is equivalently (123), and C_x, a π rotation about $\hat{\mathbf{x}}$, which is (1)(23). Similar statements apply to the triangle *abc*, which is disjoint from 123 as far as D_3 is concerned. D_6 adjoins the rotation C_2, a π rotation about $\hat{\mathbf{z}}$, which is $(1a)(2b)(3c)$. Alternately, D_6 is generated by the $\pi/6$ rotation $C_6 = C_2 C_3^{-1} = (1\,c\,2\,a\,3\,b)$ and $C_x = (1)(23)(a)(bc)$.