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Beyond the Adiabatic Approximation:

the impact of thresholds on the hadronic spectrum

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Abstract

In the adiabatic approximation, most of the effects of $q\bar{q}$ loops on spectroscopy can be absorbed into a static interquark potential. I first develop a formalism which can be used to treat the residual nonadiabatic effects associated with the presence of nearby hadronic thresholds for heavy quarks. I then define a potential which includes additional high energy corrections to the adiabatic limit which would be present for finite quark masses. This "improved" adiabatic potential allows a systematic low energy expansion of the impact of thresholds on hadronic spectra.

I. INTRODUCTION

The valence quark model is surprisingly successful at describing mesons and baryons as $q\bar{q}$ and qqq systems moving in effective potentials. The surprise comes in part because hadrons are so strongly coupled to their (real and virtual) decay channels that each nearby channel ought to shift a hadron's mass by $\Delta m \sim \Gamma_{typical}$, thereby totally disrupting the valence quark model's spectroscopy.

A simple resolution of this conundrum has been proposed in a series of papers [?,?] examining the effects of "unquenching the quark model", *i.e.*, allowing extra $q\bar{q}$ pairs to bubble up in valence quark states. This bubbling dresses the valence hadrons with a certain class of meson loop diagrams [?]. (These papers also address how the OZI rule [?] survives unquenching; in this paper I will exclusively consider flavor nonsinglet states for which such OZI-violation is not an issue.) The proposed resolution is an extension of the idea [?] that in the absence of light quarks the heavy quarkonium potential $V_0^{adiabatic}(r) \sim b_0 r$ is the adiabatically evolving ground state energy $E_0(r)$ of the purely gluonic QCD Hamiltonian in the presence of a static color triplet source Q and color anti-triplet sink $\bar{Q}$ separated by a distance r . Once n_f light quarks are introduced into this Hamiltonian, two major changes occur:

1. $E_0(r)$ will be shifted to $E_{n_f}(r)$ by ordinary second order perturbation theory, and

2. $E_{n_f}(r)$ will no longer be isolated from all other adiabatic surfaces: once pair creation can occur, the $Q\bar{Q}$ flux tube can break to create states $(Q\bar{q})_\alpha(q\bar{Q})_\beta$ with adiabatic energy surfaces that are constant in r at the values $\epsilon_\alpha + \epsilon_\beta$ (ϵ_i is the i^{th} eigenvalue of the $Q\bar{q}$ system, with the heavy quark mass m_Q subtracted).

Despite the latter complication, in the weak pair creation limit the flux-tube-like adiabatic surface $E_{n_f}(r)$ can be tracked through the level crossings that occur when $E_{n_f}(r) = \epsilon_\alpha + \epsilon_\beta$ and *identified as the renormalized $Q\bar{Q}$ adiabatic potential* $V_{n_f}^{adiabatic}(r) = E_{n_f}(r)$. In Ref. [?]

it is shown that for large r, $V_{n_f}^{adiabatic}(r)$ remains linear, so that the net effect of the pai is simply to renormalize the string tension. Since quark modellers determined their stri tension from experiment, *the quark model heavy quarkonium potential already included t effect of meson loops to leading order in the adiabatic approximation, i.e., $b_{n_f} = b$, t physical string tension.*

Note that a similar renormalization occurs at short distances: in lowest order $\alpha_s^{(0)}$ · $\alpha_s^{(n_f)} = 12\pi/[(33-2n_f)ln(Q^2/\Lambda_{QCD}^2)]$. The renormalization of the string tension by $q\bar{q}$ loo is quite similar, though complicated by the existence of the open channels corresponding adiabatic level crossings. It should also be stressed that the possibility of subsuming loops into b_{n_f} only occurs if one sums over a huge set of hadronic loop diagrams (real ar virtual) [?]. No simple truncation of the sum over loops, as is often attempted in hadron effective theories, is generally possible. Consider, for example, the simplest orbital splitti $a_2(1320)-\rho(770)$. Summing the Δm_i associated with the known decay modes of these stat would totally change their absolute masses and violently alter their splittings. Preservi them requires a large renormalization of the string tension and summing over loop grap involving many high mass (*i.e.*, virtual) channels. The reason is that $q\bar{q}$ creation inside t original $Q\bar{Q}$ state is dual to a very large tower of $(Q\bar{q})_\alpha(q\bar{Q})_\beta$ intermediate states.

Although the renormalization $V_0^{adiabatic} \to V_{n_f}^{adiabatic}$ will capture the bulk of the effect "unquenching" in heavy quarkonia, $E_{n_f}(r)$ deviates quite substantially from linearity ne level crossings [?] . Both this fact and explicit modelling suggest that for phenomenological relevant quark masses substantial nonadiabatic effects will remain after renormalizatio and in particular that states near thresholds to which they are strongly coupled should expected to deviate from their potential model positions. This paper is devoted to developi a method for addressing these residual effects. This is straightforward as $m_Q \to \infty$, but will show that for finite m_Q it is essential to go beyond the naive adiabatic approximati to define an "improved" interquark potential which includes the high energy part of t corrections to the adiabatic limit.

II. THE FORMALISM IN THE ADIABATIC LIMIT

To deal with violations of the adiabatic approximation, we can closely imitate the normal methods of mass renormalization. For very massive quarks Q and $\bar{Q}$, the effects of all hadronic loop graphs can be subsumed into

$$V_{n_f}^{adiabatic}(r) = V_0^{adiabatic}(r) + \sum_{\alpha\beta} \Delta V_{\alpha\beta}^{adiabatic}(r) \tag{1}$$

where $V_0^{adiabatic}(r)$ is the "purely gluonic" static $Q\bar{Q}$ potential, and $\Delta V_{\alpha\beta}^{adiabatic}(r)$ is the shift in this static potential generated by the channel $\alpha\beta$. Here the subscript on $V_0^{adiabatic}$ is used to denote that it *is* purely gluonic; we have suppressed additional labels to identify which gluonic adiabatic surface $V_0^{adiabatic}$ represents (the normal meson surface, the first $\Lambda = \pm 1$ hybrid surface, *etc.*) since our discussion applies identically to them all. For the low-lying thresholds of interest to us here, $\Delta V_{\alpha\beta}^{adiabatic}(r)$ will typically have a strength of order Λ_{QCD} and a range of order Λ_{QCD}^{-1}. This range arises because $\psi_\alpha(\vec{r}_{\bar{q}Q})$ and $\psi_\beta(\vec{r}_{q\bar{Q}})$ are localized at relatively small $|\vec{r}_{\bar{q}Q}|$ and $|\vec{r}_{q\bar{Q}}|$ for low-lying states so that for large r the production of such states by the point-like creation of a $q\bar{q}$ pair is strongly damped by the rapidly falling tails of their confined wavefunctions; conversely, for small r the created q and $\bar{q}$ are easily accommodated into the "heart" of their respective wavefunctions.

Let us now compare the adiabatic Hamiltonian for the $Q\bar{Q}$ system

$$H_{adiabatic} = \frac{p^2}{2\mu_{Q\bar{Q}}} + V_{n_f}^{adiabatic} \tag{2}$$

(with μ_{ij} the reduced mass of m_i and m_j) with the two channel Hamiltonian $H^{(\alpha\beta)}$ that is the penultimate step in generating $H_{adiabatic}$ in the sense that all channels *except* $\alpha\beta$ have been integrated out:

$$H^{(\alpha\beta)} = \begin{bmatrix} \frac{p^2}{2\mu_{Q\bar{Q}}} + V_{n_f}^{adiabatic} - \Delta V_{\alpha\beta}^{adiabatic} & H_{(\alpha\beta)}^{q\bar{q}} \\ H_{(\alpha\beta)}^{q\bar{q}} & \frac{p_\rho^2}{2\mu_{\alpha\beta}} + \epsilon_\alpha + \epsilon_\beta \end{bmatrix} \tag{3}$$

where $H_{(\alpha\beta)}^{q\bar{q}}$ is an interaction which couples the $Q\bar{Q}$ system to *the single channel* $(Q\bar{q})_\alpha(q\bar{Q})_\beta$ with the matrix elements dictated by the underlying pair creation Hamiltonian $H_{pc}^{q\bar{q}}$. In the adiabatic limit we must recover $H_{adiabatic}$ from $H^{(\alpha\beta)}$, but $H^{(\alpha\beta)}$ contains the full dynamics o the coupling of the $Q\bar{Q}$ system to the channel $\alpha\beta$. With the superscript $H^{(\alpha\beta)}$ we are making explicit that $H^{(\alpha\beta)}$ has the channel $\alpha\beta$ removed from the $Q\bar{Q}$ adiabatic potential and addec back in full via $H_{(\alpha\beta)}^{q\bar{q}}$. We could in general remove any subset of n channels from $V_{n_f}^{adiabatic}(r)$ and add them back in dynamically as part of an $(n+1)-$channel problem. In the limit o taking all channels we would recover the original full unquenched Hamiltonian. However since our treatment is in lowest-order perturbation theory, the effects of the individua channels are additive, and Eq. (??) with just an individual channel $(\alpha\beta)$ selected for study is sufficient for our purposes.

Note that the hadronic multichannel version of our unquenched Hamiltonian is an appropriate representation of $q\bar{q}$ pair creation in a confined system. When the pair is created the $(Q\bar{q}q\bar{Q})$ system has three relative coordinates which we may take to be $\vec{\rho}$, the separation between the center of mass of meson β and that of meson α, and the two intrameson coordinates $\vec{r}_{\bar{q}Q} \equiv \vec{r}_{\bar{q}} - \vec{r}_Q$ and $\vec{r}_{q\bar{Q}} \equiv \vec{r}_q - \vec{r}_{\bar{Q}}$. Since we ignore the residual $(Q\bar{q})_\alpha - (q\bar{Q})_\beta$ intermeson interaction, the eigenstates of this sector are confined $(Q\bar{q})_\alpha$ and $(q\bar{Q})_\beta$ mesons in relative plane waves, corresponding to the entry $H_{22}^{(\alpha\beta)}$ in Eq. (??), *i.e.*, with $\vec{p}_\rho$ canonically conjugate to $\vec{\rho}$ the three quantum labels $(\vec{p}_\rho, \alpha, \beta)$ replace the three labels $(\vec{\rho}, \vec{r}_{\bar{q}Q}, \vec{r}_{q\bar{Q}})$.

The main goal of this paper is to describe the relation between the eigenvalues of the adiabatic Hamiltonian (??) and the dynamic Hamiltonian (??). If we define

$$H_0 = \begin{bmatrix} \frac{p^2}{2\mu_{Q\bar{Q}}} + V_{n_f}^{adiabatic} & 0 \\ 0 & \frac{p_\rho^2}{2\mu_{\alpha\beta}} + \epsilon_\alpha + \epsilon_\beta \end{bmatrix} \tag{4}$$

and

$$H_{pert} = \begin{bmatrix} -\Delta V_{\alpha\beta}^{adiabatic} & H_{(\alpha\beta)}^{q\bar{q}} \\ H_{(\alpha\beta)}^{q\bar{q}} & 0 \end{bmatrix} \tag{5}$$

and denote the $Q\bar{Q}$ eigenvalues of H_0 and $H^{(\alpha\beta)}$ by E_i^0 and $E_i^{(\alpha\beta)}$, respectively, then since $H^{(\alpha\beta)} = H_0 + H_{pert}$, by second order perturbation theory $\Delta E_i^{(\alpha\beta)} = E_i^{(\alpha\beta)} - E_i^0$ is given by

$$\Delta E_i^{(\alpha\beta)} = -\langle\psi_0^i|\Delta V_{\alpha\beta}^{adiabatic}|\psi_0^i\rangle + \int d^3q \frac{|\langle\alpha\beta(\vec{q})|H^{q\bar{q}}_{(\alpha\beta)}|\psi_0^i\rangle|^2}{E_i^0 - (\epsilon_\alpha + \epsilon_\beta + \frac{q^2}{2\mu_{\alpha\beta}})} \tag{6}$$

$$\equiv -\Delta E_i^{adiabatic(\alpha\beta)} + \Delta E_i^{dynamic(\alpha\beta)} , \tag{7}$$

where $|\psi_0^i\rangle$ is the i^{th} eigenstate of H_0. *This simple equation is the main focus of this paper.* It represents the correction to the adiabatic approximation for the $Q\bar{Q}$ energy eigenvalues from a full dynamical versus an adiabatic treatment of the channel $(\alpha\beta)$. In what follows I will first show explicitly that $\Delta E_i^{(\alpha\beta)} \to 0$ as expected in the limit $m_Q \to \infty$. I will then define an improved effective potential $V_{n_f}^{improved}$ which incorporates "trivial" high energy corrections to the adiabatic approximation, but which is essential for incorporating threshold effects in a systematic low energy expansion for finite m_Q.

I begin by defining precisely $\Delta V_{\alpha\beta}^{adiabatic}$ in Eq. (??). If $|\alpha\beta(\vec{\rho})\rangle$ denotes an $(\alpha\beta)$ state with relative coordinate $\vec{\rho}$, then as $m_Q \to \infty$

$$\langle\alpha\beta(\vec{\rho})|H_{pc}^{q\bar{q}}|Q\bar{Q}(\vec{r})\rangle = c_{\alpha\beta}(\vec{r})\delta^3(\vec{\rho}-\vec{r}) \tag{8}$$

since the $Q\bar{Q}$ relative coordinate is frozen in the adiabatic approximation by definition and since $\vec{r}_{\bar{Q}} - \vec{r}_Q \to \vec{\rho}$ as $m_Q \to \infty$. Thus

$$\langle Q\bar{Q}(\vec{r}\,')|\Delta V_{\alpha\beta}^{adiabatic}|Q\bar{Q}(\vec{r})\rangle = \int d^3\rho \frac{\langle Q\bar{Q}(\vec{r}\,')|H_{pc}^{q\bar{q}}|\alpha\beta(\vec{\rho})\rangle\langle\alpha\beta(\vec{\rho})|H_{pc}^{q\bar{q}}|Q\bar{Q}(\vec{r})\rangle}{br - (\epsilon_\alpha + \epsilon_\beta)} \tag{9}$$

$$= \delta^3(\vec{r}\,' - \vec{r})\frac{|c_{\alpha\beta}(\vec{r})|^2}{br - (\epsilon_\alpha + \epsilon_\beta)} \tag{10}$$

$$\equiv \delta^3(\vec{r}\,' - \vec{r})\Delta V_{\alpha\beta}^{adiabatic}(\vec{r}) \tag{11}$$

and so by definition

$$\Delta E_i^{adiabatic(\alpha\beta)} = \langle\psi_0^i|\Delta V_{\alpha\beta}^{adiabatic}|\psi_0^i\rangle \tag{12}$$

$$= \int d^3r \frac{|\psi_0^i(\vec{r})|^2|c_{\alpha\beta}(\vec{r})|^2}{br - (\epsilon_\alpha + \epsilon_\beta)} . \tag{13}$$

I now show how $\Delta E_i^{adiabatic(\alpha\beta)}$ approximates the true shift

$$\Delta E_i^{dynamic(\alpha\beta)} \equiv \int d^3q \frac{|\langle\alpha\beta(\vec{q})|H^{q\bar{q}}_{(\alpha\beta)}|\psi_0^i\rangle|^2}{E_i^0 - (\epsilon_\alpha + \epsilon_\beta + \frac{q^2}{2\mu_{\alpha\beta}})} \tag{1}$$

even for "nearby" thresholds as $m_Q \to \infty$. Denote by $\langle v\rangle_i$ the expectation value of t variable v in the state $|\psi_0^i\rangle$. In the limit $m_Q \to \infty$, each of $\frac{\langle p^2\rangle_i}{2\mu_{Q\bar{Q}}}$, $b\langle r\rangle_i$, and $\frac{q^2}{2\mu_{\alpha\beta}}$ vanish like $(\frac{\Lambda_{QCD}}{m_Q})^{1/3}\Lambda_{QCD}$ and so is small compared to $\epsilon_\alpha + \epsilon_\beta$ which is of order Λ_{QCD}. (In t general power law potential $c_n r^n$, they each behave like $(\frac{\Lambda_{QCD}}{m_Q})^{n/n+2}\Lambda_{QCD}$, *i.e.*, they vani for any confining $(n > 0)$ potential). For $\frac{q^2}{2\mu_{\alpha\beta}}$, this statement is nontrivial: it relies on t behavior of the numerator of Eq. (??). Using Eq. (??), which is valid so long as $m_Q \to \infty$

$$\langle\alpha\beta(\vec{q})|H^{q\bar{q}}_{(\alpha\beta)}|Q\bar{Q}(\vec{p})\rangle = \frac{1}{(2\pi)^3}\int d^3r e^{i(\vec{p}-\vec{q})\cdot\vec{r}} c_{\alpha\beta}(\vec{r}) \tag{1}$$

$$\equiv \tilde{c}_{\alpha\beta}(\vec{p}-\vec{q}) , \tag{1}$$

so $|\vec{p}-\vec{q}|$ must be of order Λ_{QCD} even though $|\vec{p}| \sim (\Lambda_{QCD}^2 m_Q)^{1/3} \to \infty$ since $\tilde{c}_{\alpha\beta}$ is a lig quark object. After writing $E_i^0 = \frac{\langle p^2\rangle_i}{2\mu_{Q\bar{Q}}} + b\langle r\rangle_i$, we can therefore Taylor series expand:

$$\Delta E_i^{dynamic(\alpha\beta)} \simeq -(\frac{1}{\epsilon_\alpha+\epsilon_\beta})\int d^3q|\langle\alpha\beta(\vec{q})|H^{q\bar{q}}_{(\alpha\beta)}|\psi_0^i\rangle|^2$$

$$(1 + (\frac{1}{\epsilon_\alpha+\epsilon_\beta})[\frac{\langle p^2\rangle_i - q^2}{2\mu_{Q\bar{Q}}} + b\langle r\rangle_i] + \cdots) \tag{1}$$

$$\simeq -(\frac{1}{\epsilon_\alpha+\epsilon_\beta})\int d^3p'\int d^3q\int d^3p\; \phi_0^{i*}(\vec{p}\,')\tilde{c}^*_{\alpha\beta}(\vec{p}\,'-\vec{q})\tilde{c}_{\alpha\beta}(\vec{p}-\vec{q})\phi_0^i(\vec{p})$$

$$(1 + (\frac{1}{\epsilon_\alpha+\epsilon_\beta})[\frac{\langle p^2\rangle_i - q^2}{2\mu_{Q\bar{Q}}} + b\langle r\rangle_i] + \cdots) . \tag{1}$$

Noting that

$$\int d^3s\; \tilde{c}_{\alpha\beta}(\vec{s}) = c_{\alpha\beta}(\vec{0}) \tag{1}$$

and that except for the $\tilde{c}$ the factors of the integrand are slowly varying functions, we ca approximate

$$\tilde{c}_{\alpha\beta}(\vec{s}) \simeq c_{\alpha\beta}(\vec{0})\delta^3(\vec{s}) + \cdots \tag{2}$$

to obtain

$$\Delta E_i^{dynamic(\alpha\beta)} \simeq -(\frac{|c_{\alpha\beta}(\vec{0})|^2}{\epsilon_\alpha+\epsilon_\beta}) \int d^3p |\phi_0^i(\vec{p})|^2 (1+(\frac{1}{\epsilon_\alpha+\epsilon_\beta})[\frac{\langle p^2\rangle_i - p^2}{m_Q} + b\langle r\rangle_i] + \cdots) \tag{21}$$

$$\simeq -(\frac{|c_{\alpha\beta}(\vec{0})|^2}{\epsilon_\alpha+\epsilon_\beta}) \int d^3p |\phi_0^i(\vec{p})|^2 [1+(\frac{br}{\epsilon_\alpha+\epsilon_\beta})] \tag{22}$$

$$\simeq \int d^3p \frac{|\phi_0^i(\vec{p})|^2 |c_{\alpha\beta}(\vec{0})|^2}{br-(\epsilon_\alpha+\epsilon_\beta)} \tag{23}$$

$$\simeq \int d^3r \frac{|\psi_0^i(\vec{r})|^2 |c_{\alpha\beta}(\vec{0})|^2}{br-(\epsilon_\alpha+\epsilon_\beta)} \ . \tag{24}$$

This expression differs slightly from $\Delta E_i^{adiabatic(\alpha\beta)}$ in Eq. (??): it has $|c_{\alpha\beta}(\vec{r})|^2 \to |c_{\alpha\beta}(\vec{0})|^2$. However, $\frac{|c_{\alpha\beta}(\vec{r})|^2}{|c_{\alpha\beta}(\vec{0})|^2} - 1 \sim (\frac{\Lambda_{QCD}}{m_Q})^{2/3}$ which is negligible as $m_Q \to \infty$ compared to $\frac{br}{\epsilon_\alpha+\epsilon_\beta} \sim (\frac{\Lambda_{QCD}}{m_Q})^{1/3}$ which we retained. (The physics behind this approximation is simply that $|c_{\alpha\beta}(\vec{r})|^2$ reflects light quark scales while $|\psi_0^i(\vec{r})|^2$ reflects short distance scales as $m_Q \to \infty$.) Thus to leading order as $m_Q \to \infty$, $\Delta E_i^{adiabatic(\alpha\beta)} = \Delta E_i^{dynamic(\alpha\beta)}$ as we set out to prove.

III. AN IMPROVED QUARKONIUM POTENTIAL

Eq. (??) provides the deviation $\Delta E_i^{(\alpha\beta)}$ of the energy of the state i from its value in the true adiabatic potential $V_{n_f}^{adiabatic}$ due to the residual dynamical effects of the channel $(Q\bar{q})_\alpha(q\bar{Q})_\beta$. While $\Delta E_i^{(\alpha\beta)} \to 0$ as $m_Q \to \infty$, and while large "random" mass shifts will come from the impact of strategically placed low mass channels, the full shift $\Delta E_i \equiv \sum_{\alpha\beta} \Delta E_i^{(\alpha\beta)}$ for finite m_Q will in general receive a significant accumulation of contributions from distant high mass channels which therefore need to be treated more economically.

I will now show that it is possible to define an improved effective quarkonium potential $V_{n_f}^{improved}$ which leads to energy shifts $\delta E_i^{(\alpha\beta)}$ which vanish as $\epsilon_\alpha+\epsilon_\beta \to \infty$ *for any* m_Q. The price to be paid for this important feature is that the universal (flavor-independent) adiabatic quarkonium potential $V_{n_f}^{adiabatic}$ must be replaced by a flavor-dependent effective potential $V_{n_f}^{improved}$ built out of $V_0^{adiabatic}$ plus flavor-dependent contributions $\Delta V_{\alpha\beta}^{improved}$.

The basic idea is very simple. For any m_Q (I will continue to refer to a $Q\bar{Q}$ system since the extension to $Q_1\bar{Q}_2$ with masses m_1 and m_2 is completely trivial), the shift in the energy of the state $|\psi_0^{i\ (m_Q)}\rangle$ due to channel $\alpha\beta$ is given by the generalization of Eq. (??), namely

$$\Delta E_i^{dynamic(\alpha\beta)(m_Q)} \equiv \int d^3q \frac{|\langle \alpha^{(m_Q)}\beta^{(m_Q)}(\vec{q})|H_{(\alpha\beta)}^{q\bar{q}}|\psi_0^{i\ (m_Q)}\rangle|^2}{E_i^{0\ (m_Q)} - (\epsilon_\alpha^{(m_Q)}+\epsilon_\beta^{(m_Q)}+\frac{q^2}{2\mu_{\alpha\beta}})} \tag{25}$$

where the superscripts (m_Q) denote quantities at finite m_Q in contrast to those previously defined for $m_Q \to \infty$. For $\epsilon_\alpha^{(m_Q)}+\epsilon_\beta^{(m_Q)}+\frac{q^2}{2\mu_{\alpha\beta}} >> E_i^{0\ (m_Q)}$, this can be written in the form

$$\Delta E_i^{dynamic(\alpha\beta)(m_Q)} \ \xrightarrow{\epsilon_\alpha^{(m_Q)}+\epsilon_\beta^{(m_Q)}+\frac{q^2}{2\mu_{\alpha\beta}}>>E_i^{0\ (m_Q)}} \ \langle \psi_0^{i\ (m_Q)}|\Delta\tilde{V}_{\alpha\beta}^{(m_Q)}|\psi_0^{i\ (m_Q)}\rangle \tag{26}$$

where

$$\Delta\tilde{V}_{\alpha\beta}^{(m_Q)} = -\int d^3q \frac{H_{(\alpha\beta)}^{q\bar{q}}|\alpha^{(m_Q)}\beta^{(m_Q)}(\vec{q})\rangle\langle\alpha^{(m_Q)}\beta^{(m_Q)}(\vec{q})|H_{(\alpha\beta)}^{q\bar{q}}}{(\epsilon_\alpha^{(m_Q)}+\epsilon_\beta^{(m_Q)}+\frac{q^2}{2\mu_{\alpha\beta}})} \tag{27}$$

is an m_Q-dependent but $|\psi_0^{i\ (m_Q)}\rangle$-independent effective potential operator. Eq. (??) is thus an optimized expression for the $\alpha\beta$ contribution to an effective quarkonium potential. However, since it is in general nonlocal it is not a very useful representation for quark models. I now show that the limit $\epsilon_\alpha^{(m_Q)}+\epsilon_\beta^{(m_Q)}+\frac{q^2}{2\mu_{\alpha\beta}} >> E_i^{0\ (m_Q)}$ leads naturally to a local approximation to $\Delta\tilde{V}_{\alpha\beta}^{(m_Q)}$ which should be identified with the effective potentials of quark models. In a coordinate representation

$$\Delta\tilde{V}_{\alpha\beta}^{(m_Q)}(\vec{r}\,',\vec{r}) = -\int d^3q \frac{\langle Q\bar{Q}(\vec{r}\,')|H_{(\alpha\beta)}^{q\bar{q}}|\alpha^{(m_Q)}\beta^{(m_Q)}(\vec{q})\rangle\langle\alpha^{(m_Q)}\beta^{(m_Q)}(\vec{q})|H_{(\alpha\beta)}^{q\bar{q}}|Q\bar{Q}(\vec{r})\rangle}{(\epsilon_\alpha^{(m_Q)}+\epsilon_\beta^{(m_Q)}+\frac{q^2}{2\mu_{\alpha\beta}})} \tag{28}$$

$$= -\int d^3q\, d^3\rho\,'\, d^3\rho\, \langle Q\bar{Q}(\vec{r}\,')|H_{(\alpha\beta)}^{q\bar{q}}|\alpha^{(m_Q)}\beta^{(m_Q)}(\vec{\rho}\,')\rangle \frac{e^{i\vec{q}\cdot(\vec{\rho}\,'-\vec{\rho})}}{(2\pi)^3(\epsilon_\alpha^{(m_Q)}+\epsilon_\beta^{(m_Q)}+\frac{q^2}{2\mu_{\alpha\beta}})}\langle\alpha^{(m_Q)}\beta^{(m_Q)}(\vec{\rho})|H_{(\alpha\beta)}^{q\bar{q}}|Q\bar{Q}(\vec{r})\rangle \tag{29}$$

$$= -\int d^3\rho\,'\, d^3\rho\, \langle Q\bar{Q}(\vec{r}\,')|H_{(\alpha\beta)}^{q\bar{q}}|\alpha^{(m_Q)}\beta^{(m_Q)}(\vec{\rho}\,')\rangle \frac{e^{-\kappa|\vec{\rho}\,'-\vec{\rho}|}}{4\pi\kappa^2|\vec{\rho}\,'-\vec{\rho}|}\langle\alpha^{(m_Q)}\beta^{(m_Q)}(\vec{\rho})|H_{(\alpha\beta)}^{q\bar{q}}|Q\bar{Q}(\vec{r})\rangle \tag{30}$$

where $\kappa^2 = 2\mu_{\alpha\beta}(\epsilon_\alpha^{(m_Q)}+\epsilon_\beta^{(m_Q)})$. Since we need $\Delta\tilde{V}_{\alpha\beta}^{(m_Q)}$ only over the distance scales corresponding to low-lying $Q\bar{Q}$ states, in the limit being considered $\kappa|\vec{\rho}\,'-\vec{\rho}| >> 1$ for confined quarks of any mass so that

$$\frac{e^{-\kappa|\vec{\rho}\,'-\vec{\rho}|}}{4\pi\kappa^2|\vec{\rho}\,'-\vec{\rho}|} \simeq \frac{\delta^3(\vec{\rho}\,'-\vec{\rho})}{\epsilon_\alpha^{(m_Q)}+\epsilon_\beta^{(m_Q)}} \tag{31}$$

and thus in this approximation $\Delta\tilde{V}_{\alpha\beta}^{(m_Q)} \to \Delta V_{\alpha\beta}^{(m_Q)}$ where

$$\Delta V_{\alpha\beta}^{(m_Q)}(\vec{r}\,',\vec{r}) = -(\frac{1}{\epsilon_\alpha^{(m_Q)}+\epsilon_\beta^{(m_Q)}}) \int d^3\rho \; \langle Q\bar{Q}(\vec{r}\,')|H_{(\alpha\beta)}^{q\bar{q}}|\alpha^{(m_Q)}\beta^{(m_Q)}(\vec{\rho})\rangle \tag{32}$$
$$\langle\alpha^{(m_Q)}\beta^{(m_Q)}(\vec{\rho})|H_{(\alpha\beta)}^{q\bar{q}}|Q\bar{Q}(\vec{r})\rangle \quad .$$

Next we note that in the approximation that the $q\bar{q}$ pair creation is point-like and instantaneous, $H_{pc}^{q\bar{q}}$ connects $Q\bar{Q}$ to a state $Q\bar{q}q\bar{Q}$ with

$$\vec{\rho} = (\frac{m_Q}{m_Q+m_{\bar{q}}})\vec{r}_{\bar{Q}Q} \tag{33}$$

where $\vec{r}_{\bar{Q}Q}$ is the $Q\bar{Q}$ separation $\vec{r}_{\bar{Q}} - \vec{r}_Q$ inside the $Q\bar{q}q\bar{Q}$ state. Thus the finite m_Q generalization of Eq. (??) is

$$\langle\alpha^{(m_Q)}\beta^{(m_Q)}(\vec{\rho})|H_{pc}^{q\bar{q}}|Q\bar{Q}(\vec{r})\rangle = c_{\alpha\beta}^{(m_Q)}(\vec{r})\delta^3(\vec{r}_{\bar{Q}Q}-\vec{r}) \tag{34}$$
$$= c_{\alpha\beta}^{(m_Q)}(\vec{r})\delta^3(\frac{m_Q+m_{\bar{q}}}{m_Q}\vec{\rho}-\vec{r}) \tag{35}$$

where

$$c_{\alpha\beta}^{(m_Q)}(\vec{r}) \quad \xrightarrow{m_Q\to\infty} \quad c_{\alpha\beta}(\vec{r}) \; , \tag{36}$$

the right hand side being the function defined in the adiabatic limit by Eq. (??). Note that $c_{\alpha\beta}^{(m_Q)}(\vec{r})$ involves at the microscopic level overlap integrals between $|Q\bar{Q}(\vec{r})\rangle$ and $|\alpha^{(m_Q)}\beta^{(m_Q)}(\vec{\rho})\rangle$ with wavefunctions $\psi_\alpha^{(m_Q)}(\vec{r}_{\bar{q}Q})$ and $\psi_\beta^{(m_Q)}(\vec{r}_{q\bar{Q}})$ for finite m_Q, while $c_{\alpha\beta}(\vec{r})$ involves the heavy quark limits $\psi_\alpha(\vec{r}_{\bar{q}Q})$ and $\psi_\beta(\vec{r}_{q\bar{Q}})$ of these wave functions. Though in most models the pair creation operator is taken to be point-like [?,?], it need not be [?]. Nevertheless, one can always make a point-like approximation to this operator so that we can use Eq. (??) to define a local approximation $\Delta V_{\alpha\beta}^{improved}$ to $\Delta V_{\alpha\beta}^{(m_Q)}$ where

$$\Delta V_{\alpha\beta}^{improved} = -(\frac{m_Q}{m_Q+m_{\bar{q}}})^3 \frac{|c_{\alpha\beta}^{(m_Q)}(\vec{r})|^2}{\epsilon_\alpha^{(m_Q)}+\epsilon_\beta^{(m_Q)}}\delta^3(\vec{r}\,'-\vec{r}) \tag{37}$$
$$\equiv \Delta V_{\alpha\beta}^{improved}(\vec{r})\delta^3(\vec{r}\,'-\vec{r}) \; . \tag{38}$$

Note that as expected

$$\Delta V_{\alpha\beta}^{improved}(\vec{r}) \quad \equiv \quad -(\frac{m_Q}{m_Q+m_{\bar{q}}})^3 \frac{|c_{\alpha\beta}^{(m_Q)}(\vec{r})|^2}{\epsilon_\alpha^{(m_Q)}+\epsilon_\beta^{(m_Q)}} \tag{39}$$
$$\xrightarrow{m_Q\to\infty} \quad \Delta V_{\alpha\beta}^{adiabatic}(\vec{r}) \; . \tag{40}$$

Since, for any finite m_Q, $\Delta V_{\alpha\beta}^{improved}(\vec{r})$ is a more accurate approximation to the effects o $H_{pc}^{q\bar{q}}$ than $\Delta V_{\alpha\beta}^{adiabatic}(\vec{r})$, it is appropriate to improve Eqs. (??) and (??) by defining

$$H_{improved} = \frac{p^2}{2\mu_{Q\bar{Q}}} + V_{n_f}^{improved} \tag{41}$$

and

$$H_{improved}^{(\alpha\beta)} = \begin{bmatrix} \frac{p^2}{2\mu_{Q\bar{Q}}} + V_{n_f}^{improved} - \Delta V_{\alpha\beta}^{improved} & H_{(\alpha\beta)}^{q\bar{q}} \\ H_{(\alpha\beta)}^{q\bar{q}} & \frac{p_\rho^2}{2\mu_{\alpha\beta}} + \epsilon_\alpha^{(m_Q)} + \epsilon_\beta^{(m_Q)} \end{bmatrix} \tag{42}$$

along with the analog of Eq. (??)

$$\delta E_i^{(\alpha\beta)} \equiv -\langle\psi_0^{i\;(m_Q)}|\Delta V_{\alpha\beta}^{improved}|\psi_0^{i\;(m_Q)}\rangle + \Delta E_i^{dynamic(\alpha\beta)} \; . \tag{43}$$

The $\delta E_i^{(\alpha\beta)}$ now approach zero both in the strict adiabatic limit $m_Q \to \infty$ and also in th limit $\epsilon_\alpha^{(m_Q)} + \epsilon_\beta^{(m_Q)} >> \Lambda_{QCD}$. They therefore allow a systematic *low energy* expansion o the impact of thresholds on the spectra of light quarkonia. When using a string tension fi to the spectroscopy of a particular system, there should be no practical difference betwee employing the "adiabatic" and "improved" potentials, but we have seen that the "improved" potential is required if one hopes to define, use, and systematically improve upon a quar model valid in all quark systems.

IV. CONCLUSIONS

I have presented here a formalism for calculating the nonadiabatic component $\Delta E_i^{\alpha\beta}$ o the mass shift of a valence state i from the hadronic loop process $i \to \alpha\beta \to i$, *i.e.*, th component of this process that cannot be absorbed into the renormalized heavy quarkoniu potential. The resulting formula was shown to have the expected property that $\Delta E_i^{(\alpha\beta)} \to 0$ as $m_Q \to \infty$. The formula is also very simple and, when combined with a pair creation model like the flux-tube-breaking model [?] or the $^3P_0$ model [?], should provide a quick method of estimating the influence of nearby thresholds on the spectra of heavy quarkonia.

I have also shown how to define an "improved" quarkonium potential which incorporates nonadiabatic effects associated with high mass thresholds for any m_Q. When this potential is identified with the quark model potential, the deviations $\delta E_i^{(\alpha\beta)}$ of the spectrum from the potential model predictions due to thresholds have the property that they vanish both as $m_Q \to \infty$ and also as the mass $\epsilon_\alpha^{(m_Q)} + \epsilon_\beta^{(m_Q)}$ of the threshold $\alpha\beta$ gets large. This improved potential therefore allows a systematic low energy expansion of the impact of thresholds on hadronic spectra.

This improved quark model potential has the characteristic that it violates the rule of flavor independence. While this rule is valid in the heavy quark limit and to leading order in perturbative QCD for light quarks, violations are to be expected. Indeed, though obscured by possible relativistic corrections, there are indications from quark models that the best effective potentials are system-dependent [?].

An important step not taken here is to calculate the $\Delta E_i^{(\alpha\beta)}$ and $\delta E_i^{(\alpha\beta)}$ for selected channels to assess numerically how rapidly each converges as $\epsilon_\alpha^{(m_Q)} + \epsilon_\beta^{(m_Q)} \to \infty$, and to quantify the m_Q-dependence of $V_{n_f}^{improved}$. Quark models seem to constrain this mass dependence to be surprisingly weak. Assuming that the approach defined here passes quantitative tests such as these, it will then be interesting to apply it to a number of outstanding phenomenological issues. Among these are the threshold shifts in the $c\bar{c}$ and $b\bar{b}$ systems and the $\Lambda(1520) - \Lambda(1405)$ problem. It will also be amusing to study heavy-light systems to see explicitly how groups of states conspire to maintain the spectroscopic relations required by heavy quark symmetry [?] as $m_Q \to \infty$, and to quantify the importance of symmetry-breaking pair creation effects residing in the $\delta E_i^{(\alpha\beta)}$ compared to their valence potential model counterparts [?].

Finally, I note that while this paper in couched in the language of the nonrelativistic quark model, there is nothing in the proposed general framework that would prevent its being transferred to either a relativistic quark model or to field theory.

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