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Symmetries and Structure of Skewed and Double Distributions

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Extending the concept of parton densities onto nonforward matrix elements $\langle p' \,|\, \mathcal{O}(0,z) \,|\, p\rangle$ of quark and gluon light-cone operators, one can use two types of nonperturbative functions: double distributions (DDs) $f(x,\alpha;t), F(x,y;t)$ and skewed (off&nonforward) parton distributions (SPDs) $H(\tilde{x},\xi;t), \mathcal{F}_\zeta(X,t)$. We treat DDs as primary objects producing SPDs after integration. We emphasize the role of DDs in understanding interplay between X ($\tilde{x}$) and ζ (ξ) dependences of SPDs. In particular, the use of DDs is crucial to secure the polynomiality condition: Nth moments of SPDs are Nth degree polynomials in the relevant skewedness parameter ζ or ξ. We propose simple ansätze for DDs having correct spectral and symmetry properties and derive model expressions for SPDs satisfying all known constraints. Finally, we argue that for small skewedness, one can obtain SPDs from the usual parton densities by averaging the latter with an appropriate weight over the region $[X-\zeta, X]$ (or $[\tilde{x}-\xi, \tilde{x}+\xi]$).

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I. INTRODUCTION

Nonforward matrix elements $\langle p-r \,|\, \mathcal{O}(0,z) \,|\, p\rangle |_{z^2=0}$ of quark and gluon light-cone operators which appear in applications of perturbative QCD to deeply virtual Compton scattering (DVCS) and hard exclusive electroproduction processes [1–5] can be parametrized by two basic types of nonperturbative functions. The double distributions (DDs) $F(x,y;t)$ [2,3] specify the Sudakov light-cone "plus" fractions xp^+ and yr^+ of the initial hadron momentum p and the momentum transfer r carried by the initial parton. Treating the proportionality coefficient ζ as an independent parameter one can introduce an alternative description in terms of the nonforward parton distributions (NFPDs) $\mathcal{F}_\zeta(X;t)$ with $X = x + y\zeta$ being the total fraction of the initial hadron momentum taken by the initial parton. The shape of NFPDs explicitly depends on the parameter ζ characterizing the *skewedness* of the relevant nonforward matrix element. This parametrization of nonforward matrix elements by $\mathcal{F}_\zeta(X;t)$ is similar to that proposed by X. Ji [1] who introduced off-forward parton distributions (OFPDs) $H(\tilde{x},\xi;t)$ in which the parton momenta and the skewedness parameter $\xi \equiv r^+/P^+$ are measured in units of the average hadron momentum $P = (p+p')/2$. There are one-to-one relations between OFPDs and NFPDs [3], so it is convenient to treat them as particular forms of *skewed* parton distributions (SPDs).

In our approach, DDs are primary objects producing SPDs after an appropriate integration. Our main goal in this letter is to show that using the formalism of DDs (in particular, their support and symmetry properties) one can easily establish important features of SPDs such as nonanalyticity at border points $X = \zeta, 0$ and $\tilde{x} = \pm\xi$, polynomiality of their X^N and $\tilde{x}^N$ moments in skewedness parameters ζ and ξ, etc. We also discuss simple models for DDs which result in realistic models for SPDs.

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II. DOUBLE DISTRIBUTIONS AND THEIR SYMMETRIES

In the pQCD factorization treatment of hard electroproduction processes, the nonperturbative information is accumulated in the nonforward matrix elements $\langle p - r \,|\, \mathcal{O}(0,z) \,|\, p \rangle$ of light cone operators $\mathcal{O}(0,z)$. For $z^2 = 0$ the matrix elements depend on the relative coordinate z through two Lorentz invariant variables (pz) and (rz). In the forward case, when $r = 0$, one obtains the usual quark helicity-averaged densities by Fourier transforming the relevant matrix element with respect to (pz)

$$\langle p, s' \mid \bar\psi_a(0) \hat z E(0,z;A) \psi_a(z) \mid p, s \rangle \,|_{z^2=0} = \bar u(p,s') \hat z u(p,s) \int_0^1 \left(e^{-ix(pz)} f_a(x) - e^{ix(pz)} f_{\bar a}(x) \right) dx \,, \qquad (1)$$

where $E(0,z;A)$ is the gauge link, $\bar u(p',s'), u(p,s)$ are the Dirac spinors and we use the notation $\gamma_\alpha z^\alpha \equiv \hat z$. In the nonforward case, we can use the double Fourier representation with respect to both (pz) and (rz):

$$\begin{aligned} &\langle p', s' \mid \bar\psi_a(0) \hat z E(0,z;A) \psi_a(z) \mid p, s \rangle \,|_{z^2=0} \qquad (2) \\ &= \bar u(p',s') \hat z u(p,s) \int_0^1 dy \int_{-1}^1 e^{-ix(pz) - iy(rz)} \tilde F_a(x,y;t)\, \theta(0 \le x + y \le 1)\, dx \,+\, \text{“}\tilde K\text{”} - \text{term}\,, \end{aligned}$$

where the “$\tilde K$”-term stands for the hadron helicity-flip part [1,3]. For any Feynman diagram, the spectral constraints $-1 \le x \le 1$, $0 \le y \le 1$, $0 \le x + y \le 1$ were proved in the α-representation [3] using the approach of Ref. [6]. The support area for the *double distribution* $\tilde F_a(x,y;t)$ is shown on Fig.1a.

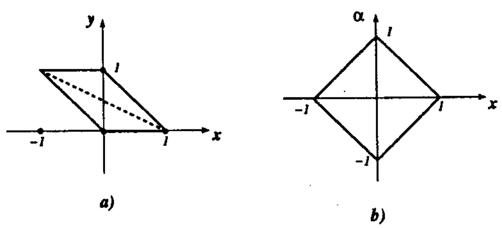

FIG. 1. a) Support region and symmetry line $y = \bar x/2$ for y-DDs $\tilde F(x,y;t)$; b) Support region for α-DDs $\tilde f(x,\alpha)$.

Comparing Eq. (1) with the $r = 0$ limit of the DD definition (2) gives the “reduction formulas” relating the double distribution $\tilde F_a(x,y;t=0)$ to the quark and antiquark parton densities

$$\int_0^{1-x} \tilde F_a(x,y;t=0)|_{x>0}\, dy = f_a(x) \qquad ; \qquad \int_{-x}^{1} \tilde F_a(x,y;t=0)|_{x<0}\, dy = -f_{\bar a}(-x) \,. \qquad (3)$$

Hence, the positive-x and negative-x components of the double distribution $\tilde F_a(x,y;t)$ can be treated as nonforward generalizations of quark and antiquark densities, respectively. If we define the “untilded” DDs by

$$F_a(x,y;t) = \tilde F_a(x,y;t)|_{x>0} \qquad ; \qquad F_{\bar a}(x,y;t) = -\tilde F_a(-x, 1-y;t)|_{x<0} \,, \qquad (4)$$

then x is always positive and the reduction formulas have the same form

$$\int_0^{1-x} F_{a,\bar a}(x,y;t=0)|_{x\neq 0}\, dy = f_{a,\bar a}(x) \qquad (5)$$

in both cases. The new antiquark distributions also “live” on the triangle $0 \le x, y \le 1$, $0 \le x + y \le 1$. Taking z in the lightcone “minus” direction, we arrive at the parton interpretation of functions $F_{a,\bar a}(x,y;t)$ as probability amplitudes for an outgoing parton to carry the fractions xp^+ and yr^+ of the external momenta r and p. The double distributions $F(x,y;t)$ are universal functions describing the flux of p^+ and r^+ independently of the ratio r^+/p^+. Note, that extraction of two separate components $F_a(x,y;t)$ and $F_{\bar a}(x,y;t)$ from the quark DD $\tilde F_a(x,y;t)$ as its positive-x and negative-x parts is unambiguous.

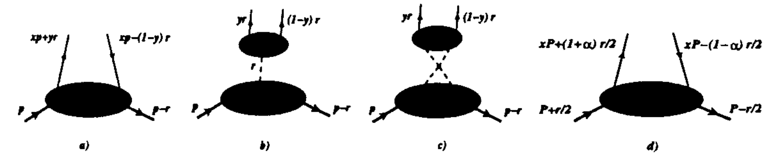

FIG. 2. a) Parton picture in terms of y-DDs; b, c) F_M-type contributions; d) parton picture in terms of α-DDs.

In principle, we cannot exclude the third possibility that the functions $\tilde F(x,y;t)$ have singular terms at $x = 0$ proportional to $\delta(x)$ or its derivative(s). Such terms would have no projection onto the usual parton densities. We will denote them by $F_M(x,y;t)$ – they may be interpreted as coming from the t-channel meson-exchange type contributions (see Fig.2b). In this case, the partons just share the plus component of the momentum transfer r: information about the magnitude of the initial hadron momentum is lost if the exchanged particle can be described by a pole propagator $\sim 1/(t - m_M^2)$. Hence, the meson-exchange contributions to a double distribution may look like

$$\tilde F_M^+(x,y;t) \sim \delta(x)\, \frac{\varphi_M^+(y)}{m_M^2 - t} \quad \text{or} \quad \tilde F_M^-(x,y;t) \sim \delta'(x)\, \frac{\varphi_M^-(y)}{m_M^2 - t} \;\;, \text{ etc.}\,, \qquad (6)$$

where $\varphi_M^\pm(y)$ are the functions related to the distribution amplitudes of the relevant mesons $M^\pm$. The two examples above correspond to x-even and x-odd parts of the double distribution $\tilde F(x,y;t)$. The singular terms can also be produced by diagrams containing a quartic pion vertex (Fig.2c) [7].

To make the description more symmetric with respect to the initial and final hadron momenta, we can treat nonforward matrix elements as functions of (Pz) and (rz), where $P = (p+p')/2$ is the average hadron momentum. The relevant double distributions $\tilde f_a(x,\alpha;t)$ [which we will call α-DDs to distinguish them from y-DDs $F(x,y;t)$] are defined by

$$\langle p' | \bar\psi_a(-z/2) \hat z \psi_a(z/2) | p \rangle = \bar u(p') \hat z u(p) \int_{-1}^1 dx \int_{-1+|x|}^{1-|x|} e^{-ix(Pz) - i\alpha(rz)/2}\, \tilde f_a(x,\alpha;t)\, d\alpha \,+\, \text{“}\tilde k\text{”} - \text{terms}\,. \qquad (7)$$

The support area for $\tilde f_a(x,\alpha;t)$ is shown in Fig.1b. Again, the usual forward densities $f_a(x)$ and $f_{\bar a}(x)$ are given by integrating $\tilde f_a(x,\alpha\,;\,t=0)$ over vertical lines $x = \text{const}$ for $x > 0$ and $x < 0$, respectively. Hence, we can split $\tilde f_a(x,\alpha\,;\,t)$ into three components

$$\tilde f_a(x,\alpha\,;\,t) = f_a(x,\alpha\,;\,t)\, \theta(x > 0) - f_{\bar a}(-x,-\alpha\,;\,t)\, \theta(x < 0) + f_M(x,\alpha\,;\,t)\,, \qquad (8)$$

where $f_M(x,\alpha\,;\,t)$ is a singular term with support at $x = 0$ only†. Due to hermiticity and time-reversal invariance properties of nonforward matrix elements, the α-DDs are even functions of α:

$$\tilde f_a(x,\alpha;t) = \tilde f_a(x,-\alpha;t)\,.$$

For our original y-DDs $F_{a,\bar a}(x,y;t)$, this corresponds to symmetry with respect to the interchange $y \leftrightarrow 1 - x - y$ established in Ref. [8]. In particular, the functions $\varphi_M^\pm(y)$ for singular contributions $F_M^\pm(x,y;t)$ are symmetric $\varphi_M^\pm(y) = \varphi_M^\pm(1-y)$ both for x-even and x-odd parts. The a-quark contribution

$$\mathcal{O}_a^S(-z/2, z/2) = \frac{i}{2}[\bar\psi_a(-z/2) \hat z E(-z/2, z/2; A) \psi_a(z/2) - \{z \to -z\}]$$

into the flavor-singlet operator can be parametrized either by y-DDs $\tilde F_a^S(x,y;t)$ or by α-DDs $\tilde f_a^S(x,\alpha\,;\,t)$

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†As argued by M. Polyakov and C. Weiss [7], in the case of pion distributions it makes sense to write the (Pz)-independent terms as a separate integral over a single variable y rather than to include them into a singular part of DDs.

$$
\begin{aligned}
&\langle p', s' \mid O_a^S(-z/2, z/2) \mid p, s\rangle \,|_{z^2=0} \\
&= \bar{u}(p', s')\hat{z}u(p, s) \int_0^1 dx \int_0^{1-x} \frac{1}{2}\left(e^{-ix(pz)-i(y-1/2)(rz)} - e^{ix(pz)+i(y-1/2)(rz)}\right) F_a^S(x, y; t)\, dy + \text{“}K_a^S\text{”}-\text{term} \\
&= \bar{u}(p', s')\hat{z}u(p, s) \int_{-1}^1 dx \int_{-1+|x|}^{1-|x|} e^{-ix(Pz)-i\alpha(rz)/2}\, \tilde{f}_a^S(x, \alpha\,;\, t)\, d\alpha + \text{“}\tilde{k}_a^S\text{”}-\text{term.}
\end{aligned}
\tag{9}
$$

In the second line here we have used the fact that positive-x and negative-x parts in this case are described by the same untilded function

$$F_a^S(x, y; t)|_{x \neq 0} = F_a(x, y; t) + F_{\bar{a}}(x, y; t).$$

The α-DDs $\tilde{f}_a^S(x, \alpha\,;\, t)$ are even functions of α and, according to Eq. (9), odd functions of x:

$$\tilde{f}_a^S(x, \alpha; t) = \{f_a(|x|, |\alpha|; t) + f_{\bar{a}}(|x|, |\alpha|; t)\}\, \mathrm{sign}(x) + f_M^S(x, \alpha; t)\ . \tag{10}$$

Finally, the valence quark functions $\tilde{f}_a^V(x, \alpha\,;\, t)$ related to the operators

$$O_a^V(-z/2, z/2) = \frac{1}{2}[\bar{\psi}_a(-z/2)\hat{z}E(-z/2, z/2; A)\psi_a(z/2) + \{z \to -z\}]$$

are even functions of both α and x:

$$\tilde{f}_a^V(x, \alpha; t) = f_a(|x|, |\alpha|; t) - f_{\bar{a}}(|x|, |\alpha|; t) + f_M^V(x, \alpha; t)\ . \tag{11}$$

III. PARTON INTERPRETATION AND MODELS FOR DOUBLE DISTRIBUTIONS

The structure of the integral (5) relating double distributions with the usual ones has a simple graphic illustration (see Fig.3): integrating DDs over a line orthogonal to the x axis, we get $f(X)$.

The reduction formulas and interpretation of the x-variable as the fraction of the p (or P) momentum suggest that the profile of $F(x, y)$ (or $f(x, \alpha)$) in x-direction is basically determined by the shape of $f(x)$. On the other hand, the profile in y (or α) direction characterizes the spread of the parton momentum induced by the momentum transfer r. In particular, since the α-DDs $f(x, \alpha)$ are even functions of α, it make sense to write

$$f(x, \alpha) = h(x, \alpha)\, f(x)\,, \tag{12}$$

where $h(x, \alpha)$ is an even function of α normalized by

$$\int_{-1+x}^{1-x} h(x, \alpha)\, d\alpha = 1. \tag{13}$$

We may expect that the α-profile of $h(x, \alpha)$ is similar to that of a symmetric distribution amplitude (DA) $\varphi(\alpha)$. Since $|\alpha| \leq \bar{x}$, to get a more complete analogy with DA's, it makes sense to rescale α as $\alpha = \bar{x}\beta$ introducing the variable β with x-independent limits: $-1 \leq \beta \leq 1$. The simplest model is to assume that the profile in the β-direction is a universal function $g(\beta)$ for all x. Possible simple choices for $g(\beta)$ may be $\delta(\beta)$ (no spread in β-direction), $\frac{3}{4}(1 - \beta^2)$ (characteristic shape for asymptotic limit of nonsinglet quark distribution amplitudes), $\frac{15}{16}(1 - \beta^2)^2$ (asymptotic shape of gluon distribution amplitudes), etc. In the variables x, α, this gives

$$h^{(0)}(x, \alpha) = \delta(\alpha)\ ,\ h^{(1)}(x, \alpha) = \frac{3}{4}\frac{(\bar{x}^2 - \alpha^2)}{(1 - x)^3}\ ,\ h^{(2)}(x, \alpha) = \frac{15}{16}\frac{(\bar{x}^2 - \alpha^2)^2}{(1 - x)^5}\ . \tag{14}$$

It is straightforward to generalize these models onto the "tilded" DDs $\tilde{f}(x, \alpha)$ with x ranging between -1 and 1: $\tilde{f}(x, \alpha)$ should be even in x for the gluon and nonsinglet quark distributions and odd in x for the singlet quark case. Furthermore, one can construct ansätze for functions $f(x, \alpha; t)$ involving nonzero t values, e.g., the model

$$f_i(x, \alpha; t) = h(x, \alpha)\, f_i(x) \exp\left\{\frac{(\bar{x}^2 - \alpha^2)t}{4x\bar{x}\lambda^2}\right\} \tag{15}$$

with $h(x, \alpha) = \delta(\alpha)$ and experimental valence densities $f_{u,d}^V(x)$ was used in ref. [9] to describe the $F_1(t)$ form factor and wide-angle Compton scattering.

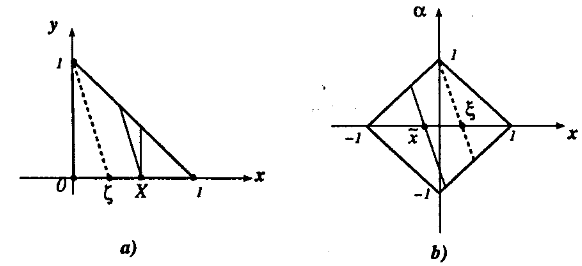

FIG. 3. Integration lines for integrals relating SPDs and DDs.

IV. RELATIONS BETWEEN DOUBLE AND SKEWED DISTRIBUTIONS

An important parameter for nonforward matrix elements is the coefficient of proportionality $\zeta = r^+/p^+$ (or $\xi = r^+/P^+$) between the plus components of the momentum transfer and initial (or average) hadron momentum. It specifies the *skewedness* of the matrix elements. The characteristic feature implied by representations for double distributions [see, e.g., Eq.(2)] is the absence of the ζ-dependence in the DDs $F(x, y)$ and ξ-dependence in $f(x, \alpha)$. An alternative way to parametrize nonforward matrix elements of light-cone operators is to use ζ (or ξ) and the *total* momentum fractions $X \equiv x + y\zeta$ (or $\tilde{x} \equiv x + \xi\alpha$) as independent variables. If we require that the light-cone plus components of both the momentum transfer r and the final hadron momentum $p - r$ are positive, then $0 \leq \zeta \leq 1$ and $0 \leq \xi \leq 1$. Using the spectral property $0 \leq x + y \leq 1$ of double distributions, we obtain that the NFPD variable X satisfies the "parton" constraint $0 \leq X \leq 1$. Integrating each particular double distribution over y gives the nonforward parton distributions

$$
\begin{aligned}
\mathcal{F}_\zeta^i(X) &= \int_0^1 dx \int_0^{1-x} \delta(x + \zeta y - X)\, F_i(x, y)\, dy \\
&= \theta(X \geq \zeta) \int_0^{\bar{X}/\bar{\zeta}} F_i(X - y\zeta, y)\, dy + \theta(X \leq \zeta) \int_0^{X/\zeta} F_i(X - y\zeta, y)\, dy\,,
\end{aligned}
\tag{16}
$$

where $\bar{\zeta} \equiv 1 - \zeta$. The two components of NFPDs correspond to positive ($X > \zeta$) and negative ($X < \zeta$) values of the fraction $X' \equiv X - \zeta$ associated with the "returning" parton. As explained in refs. [2,3], the second component can be interpreted as the probability amplitude for the initial hadron with momentum p to split into the final hadron with momentum $(1 - \zeta)p$ and two-parton state with total momentum $r = \zeta p$ shared by the partons in fractions Yr and $(1 - Y)r$, where $Y = X/\zeta$.

The relation between NFPDs and DDs can be illustrated on the "DD-life triangle" defined by $0 \leq x, y, x + y \leq 1$ (see Fig.3a). Specifically, to get $\mathcal{F}_\zeta(X)$, one should integrate $F(x, y)$ over y along a straight line $x = X - \zeta y$. Fixing some value of ζ, one deals with a set of parallel lines intersecting the x-axis at $x = X$. The upper limit of the y-integration is determined by intersection of this line either with the line $x + y = 1$ (this happens if $X > \zeta$) or with the y-axis (if $X < \zeta$). The line corresponding to $X = \zeta$ separates the triangle into two parts generating the two components of the nonforward parton distribution.

In a similar way, we can write the relation between OFPDs $\tilde{H}(\tilde{x}, \xi; t)$ [1] and the α-DDs $\tilde{f}(x, \alpha; t)$

$$\tilde{H}(\tilde{x}, \xi; t) = \int_{-1}^1 dx \int_{-1+|x|}^{1-|x|} \delta(x + \xi\alpha - \tilde{x})\, \tilde{f}(x, \alpha; t)\, d\alpha\,. \tag{17}$$

We use here the tilded notation $\tilde{H}(\tilde{x}, \xi; t)$ to emphasize that OFPDs as defined by X. Ji [1] correspond to parametrization of the nonforward matrix element by a Fourier integral with a single common exponential. Note that Eq. (17) allows to construct $\tilde{H}(\tilde{x}, \xi; t)$ both for positive and negative values of ξ. Since DDs $\tilde{f}(x, \alpha; t)$ are even functions of α, the OFPDs $\tilde{H}(\tilde{x}, \xi; t)$ are even functions of ξ:

$$\tilde{H}(\tilde{x}, \xi; t) = \tilde{H}(\tilde{x}, -\xi; t) .$$

This result was originally obtained by X. Ji [10] by directly using hermiticity and time reversal invariance properties in his definition of OFPDs.

The delta-function in Eq.(17) specifies the line of integration in the $\{x, \alpha\}$ plane. For definiteness, we will assume below that ξ is positive. The integration line $x = \tilde{x} - \xi\alpha$ consists of two parts corresponding to positive and negative values of x. In the case of quarks with flavor a, substituting $\tilde{f}_a(x, \alpha)$ by $f_a(x, \alpha)$ or $f_{\bar{a}}(x, \alpha)$, respectively (see Eq.(8)), we get OFPD $\tilde{H}_a(\tilde{x}, \xi; t)$ as the sum of three components

$$\tilde{H}_a(\tilde{x}, \xi; t) = H_a(\tilde{x}, \xi; t)\, \theta(-\xi \le \tilde{x} \le 1) - H_{\bar{a}}(-\tilde{x}, \xi; t)\, \theta(-1 \le \tilde{x} \le \xi) + H_M(\tilde{x}, \xi; t)\, \theta(-\xi \le \tilde{x} \le \xi) , \qquad (18)$$

where $H_M(\tilde{x}, \xi; t)$ comes from integration of the singular term $f_M(\tilde{x} - \xi\alpha, \alpha)$ over $\tilde{x}/\xi - \epsilon < \alpha < \tilde{x}/\xi + \epsilon$ and

$$H_{a,\bar{a}}(\tilde{x}, \xi; t) = \theta(\xi \le \tilde{x} \le 1) \int_{-\frac{1-\tilde{x}}{1+\xi}}^{\frac{1-\tilde{x}}{1-\xi}} f_{a,\bar{a}}(\tilde{x} - \xi\alpha, \alpha)\, d\alpha + \theta(-\xi \le \tilde{x} \le \xi) \int_{-\frac{1-\tilde{x}}{1+\xi}}^{\tilde{x}/\xi - \epsilon} f_{a,\bar{a}}(\tilde{x} - \xi\alpha, \alpha)\, d\alpha . \qquad (19)$$

The OFPD $\tilde{H}_a(\tilde{x}, \xi; t)$ is in a one-to-one correspondence with the "tilded" NFPD $\tilde{\mathcal{F}}_\zeta^a(X)$ introduced in our paper [3]. It parametrizes the nonforward matrix element of the quark operator $\bar{\psi}_a \ldots \psi_a$ through a Fourier integral with a single common exponential. The support of $\tilde{\mathcal{F}}_\zeta^a(X)$ is $-1 + \zeta \le X \le 1$ and by

$$\tilde{\mathcal{F}}_\zeta^a(X) = \mathcal{F}_\zeta^a(X)\, \theta(0 \le X \le 1) - \mathcal{F}_\zeta^{\bar{a}}(\zeta - X)\, \theta(-1 + \zeta \le X \le \zeta) + \mathcal{F}_\zeta^M(X)\, \theta(0 \le X \le \zeta) \qquad (20)$$

it is related to the untilded components given by Eq. (16). In the middle region $0 \le X \le \zeta$, the components $\mathcal{F}_\zeta^{a,\bar{a}}(X)$ appear only through the difference $\mathcal{F}_\zeta^a(X) - \mathcal{F}_\zeta^{\bar{a}}(\zeta - X)$. In a recent paper [11], Golec-Biernat and Martin argued that the decomposition of $\tilde{\mathcal{F}}_\zeta^a(X)$ in the middle region into $\mathcal{F}_\zeta^a(X)$ and $\mathcal{F}_\zeta^{\bar{a}}(\zeta - X)$ parts made in Ref. [3] amounts to "doubling the quark degrees of freedom" ‡. Compared to Ref. [3], we have an extra function $\mathcal{F}_\zeta^M(X)$ in Eq.(20), so one may question now whether it make sense to represent $\tilde{\mathcal{F}}_\zeta^a(X)$ as a sum of three functions in the $0 \le X \le \zeta$ region. Of course, if there were only one value of ζ in the nature, one would never get an idea about how much of $\tilde{\mathcal{F}}_\zeta^a(X)$ should be attributed to $\mathcal{F}_\zeta^a(X), \mathcal{F}_\zeta^{\bar{a}}(X)$ or $\mathcal{F}_\zeta^M(X)$. The crucial missing element is the interplay between ζ and X dependences. We recall that our decomposition of $\tilde{\mathcal{F}}_\zeta^a(X)$ is based on the splitting of the underlying y-DDs $F^a(x, y)$ into positive-x, negative-x and zero-x parts. The DDs contain information about NFPDs for all possible ζ's and X's, and that is why the DDs produce an unambiguous decomposition: DDs "know" not only what is the shape of $\tilde{\mathcal{F}}_\zeta^a(X)$ for a particular ζ, but also how this shape would change if one would take another ζ. The simplest illustration of interplay between X and ζ dependences is provided by NFPDs $\mathcal{F}_\zeta^M(X) = \theta(0 \le X/\zeta \le 1)\varphi(X/\zeta)/|\zeta|$ corresponding to singular parts of DDs. Clearly, knowing $\mathcal{F}_\zeta^M(X)$ at some $\zeta = \zeta_0$, we can obtain its shape for any other ζ by rescaling. To demonstrate that NFPDs with such a behavior can be obtained only from singular DDs, we write a formal inversion of the basic relation (16)

$$F(x, y) = \int_{-\infty}^{\infty} dX \int_{-\infty}^{\infty} \Delta[X - x - \zeta y]\, \mathcal{F}_\zeta(X)\, d\zeta , \qquad (21)$$

where the (mathematical) distribution $\Delta(z)$ is defined by

$$\Delta[z] = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} |\mu|\, e^{i\mu z} d\mu . \qquad (22)$$

Taking $\mathcal{F}_\zeta^M(X) = \theta(0 \le X/\zeta \le 1)\varphi(X/\zeta)/|\zeta|$ and using the following property of the Δ-function

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‡They also proposed to chop our function $\tilde{\mathcal{F}}_\zeta^a(X)$ into overlapping $0 \le X \le 1$ and $-1 + \zeta \le X \le \zeta$ parts to introduce "off-diagonal" "quark" $\hat{\mathcal{F}}_\zeta^a(X) = \theta(0 \le X \le 1)\tilde{\mathcal{F}}_\zeta^a(X)$ and "antiquark" $\hat{\mathcal{F}}_\zeta^{\bar{a}}(\zeta - X) = -\theta(0 \le X \le 1)\tilde{\mathcal{F}}_\zeta^a(\zeta - X)$ distributions both of which include the same middle part of $\tilde{\mathcal{F}}_\zeta^a(X)$.

$$\int_{-\infty}^{\infty} \Delta[a - \zeta b]\, d\zeta = \delta(a)\delta(b) , \qquad (23)$$

we obtain from Eq. (21) that $F_M(x, y) = \delta(x)\varphi(y)$.

Thus, information contained in SPDs originates from two physically different sources: meson-exchange type contributions $\mathcal{F}_\zeta^M(X)$ coming from the singular $x = 0$ parts of DDs and the functions $\mathcal{F}_\zeta^a(X)$, $\mathcal{F}_\zeta^{\bar{a}}(X)$ obtained by scanning the $x \ne 0$ parts of DDs $F^a(x, y)$, $F^{\bar{a}}(x, y)$. The support of exchange contributions is restricted to $0 \le X \le \zeta$. Up to rescaling, the function $\mathcal{F}_\zeta^M(X)$ has the same shape for all ζ. For any nonvanishing X, these exchange terms become invisible in the forward limit $\zeta \to 0$. On the other hand, interplay between X and ζ dependences of the functions $\mathcal{F}_\zeta^a(X)$, $\mathcal{F}_\zeta^{\bar{a}}(X)$ is quite nontrivial and their support in general covers the whole $0 \le X \le 1$ region for all ζ including the forward limit $\zeta = 0$ in which they convert into the usual (forward) densities $f^a(x)$, $f^{\bar{a}}(x)$. The latter are rather well known from inclusive measurements. Hence, information contained in $f^a(x)$, $f^{\bar{a}}(x)$ can be used to restrict the models for $\mathcal{F}_\zeta^a(X)$, $\mathcal{F}_\zeta^{\bar{a}}(X)$. Note that the functions $F^a(x, y)$ and $F^{\bar{a}}(x, y)$ are independent as are their ζ-sensitive scans $\mathcal{F}_\zeta^a(X)$ and $\mathcal{F}_\zeta^{\bar{a}}(X)$. Instead of $F^a(x, y)$ and $F^{\bar{a}}(x, y)$, one can use as independent functions their sum $F^a(x, y) + F^{\bar{a}}(x, y)$ which contributes to the quark singlet functions and the difference $F^a(x, y) - F^{\bar{a}}(x, y)$ which appears in the valence functions. Extending the DDs onto the whole $-1 \le x \le 1$ segment does not require extra dynamical information: one should only take into account that the singlet term $\tilde{f}_a^S(x, \alpha)$ must be odd in x (see Eq.(10)) while the valence term $\tilde{f}_a^V(x, \alpha)$ must be even in x (see Eq.(11)). As a result, the singlet contribution $\tilde{\mathcal{F}}_\zeta^{S,a}(X)$ is an odd function of $X - \zeta/2$ while the valence one $\tilde{\mathcal{F}}_\zeta^{V,a}(X)$ is an even function of $X - \zeta/2$.

In our approach, DDs are the starting point while SPDs are derived from them by integration. However, even if one starts directly with SPDs the latter possess a property which forces the use of double distributions. According to Eq.(16), the X^N moment of $\mathcal{F}_\zeta(X)$ *must be a polynomial* in ζ of a degree not larger than N. A similar statement holds for off-forward distributions $\tilde{H}(\tilde{x}, \xi; t)$: their $\tilde{x}^N$ moments are Nth order polynomials of ξ. As explained by X. Ji [10], this restriction on the interplay between $\tilde{x}$ and ξ dependences of $\tilde{H}(\tilde{x}, \xi; t)$ follows from a simple fact that the Lorentz indices $\mu_1 \ldots \mu_N$ of the nonforward matrix elements of a local operator $O^{\mu_1 \ldots \mu_N}$ can be carried either by P^{μ_i} or by r^{μ_i}. As a result,

$$\langle P - r/2 | \phi(0) (\overleftrightarrow{\partial}^+)^N \phi(0) | P + r/2 \rangle = \sum_{k=0}^{N} \binom{N}{k} (P^+)^{N-k} (r^+)^k A_{Nk} = (P^+)^N \sum_{k=0}^{N} \binom{N}{k} \xi^k A_{Nk} , \qquad (24)$$

where $\binom{N}{k} \equiv N!/(N-k)!k!$ is the combinatorial coefficient. Our derivation (17) of OFPDs from α-DDs automatically satisfies the polynomiality condition (24), since

$$\int_{-1}^{1} \tilde{H}(\tilde{x}, \xi; t)\, \tilde{x}^N d\tilde{x} = \sum_{k=0}^{N} \xi^k \binom{N}{k} \int_{-1}^{1} dx \int_{-1+|x|}^{1-|x|} \tilde{f}(x, \alpha)\, x^{N-k} \alpha^k\, d\alpha . \qquad (25)$$

Hence, the coefficients A_{Nk} in (24) are given by double moments of α-DDs. This means that modeling SPDs one cannot choose the coefficients A_{Nk} arbitrarily: symmetry and support properties of DDs dictate a nontrivial interplay between N and k dependences of A_{Nk}'s. After this observation, the use of DDs is an unavoidable step in building consistent parametrizations of SPDs.

The formalism of DDs also allows one to easily establish some important properies of skewed distributions. Notice that the length of the integration line nonanalytically depends on X for $X = \zeta$. Hence, unless a double distribution identically vanishes in a finite region around the upper corner of the DD support triangle, the X-dependence of the relevant nonforward distribution *must be nonanalytic* at the border point $X = \zeta$. Furthermore, the length of the integration line vanishes when $X \to 0$. As a result, the components $\mathcal{F}_\zeta^{a,\bar{a}}(X)$ vanish at $X = 0$ if the relevant double distribution $F^{a,\bar{a}}(x, y)$ is not too singular for small x. The combined contribution of $\mathcal{F}_\zeta^a(X)$ and $\mathcal{F}_\zeta^{\bar{a}}(\zeta - X)$ into the total function $\tilde{\mathcal{F}}_\zeta^a(X)$ in this case is continuous at the nonanalyticity points $X = 0$ and $X = \zeta$. As emphasized in ref. [3], because of the $1/X$ and $1/(X - \zeta)$ factors ($1/(\tilde{x} \pm \xi)$ factors if OFPD formalism is used) contained in hard amplitudes, this property is crucial for pQCD factorization in DVCS and other hard electroproduction processes. Note, that there is also the exchange contribution $\mathcal{F}_\zeta^M(X)$. If it comes from a $\delta(x)\varphi(y)$ type term and $\varphi(y)$ vanishes at the end-points $y = 0, 1$ the relevant part of NFPD $\mathcal{F}_\zeta^M(X)$ vanishes at $X = 0$ and $X = \zeta$. The total function $\tilde{\mathcal{F}}_\zeta^a(X)$ is then continuous at these nonanalyticity points (OFPDs $\tilde{H}(\tilde{x}, \xi; t)$ in this case are continuous at $x = \pm\xi$). In the quark singlet case, the DDs should be odd in x, hence the singular term involves $\delta'(x)\varphi(y)$ (or even higher odd derivatives of $\delta(x)$). One can get a continuous SPD in this case only if $\varphi'(y)$ vanishes at the end points. Such a restriction might be too strong to be satisfied in all the cases. In particular, an essentially discontinuous behavior of singlet quark OFPDs for $\tilde{x} = \pm\xi$ was obtained in a nonperturbative (chiral soliton) model [12].

V. MODELS FOR SKEWED DISTRIBUTIONS

The properties discussed above can be illustrated by SPDs constructed using simple models of DDs specified in Section III. In particular, for the model $F^{(0)}(x,y) = \delta(y - \bar{x}/2)f(x)$ (equivalent to $f^{(0)}(x,\alpha) = \delta(\alpha)f(x)$), we get

$$\mathcal{F}_\zeta^{(0)}(X) = \frac{\theta(X \geq \zeta/2)}{1-\zeta/2} f\left(\frac{X-\zeta/2}{1-\zeta/2}\right), \tag{26}$$

i.e., NFPDs for non-zero ζ are obtained from the forward distribution $f(X) \equiv \mathcal{F}_{\zeta=0}(X)$ by shift and rescaling. This is an example of a peculiar case of a DD with an empty upper corner: it gives NFPDs with no explicit nonanalyticity at $X = \zeta$. "As a compensation", $\mathcal{F}_\zeta^{(0)}(X)$ vanishes not only for $X = 0$, but on the finite segment $0 \leq X \leq \zeta/2$. Using the relations [3]

$$\tilde{H}(\tilde{x},\xi;t)|_{\tilde{x}>\xi} = (1-\zeta/2)\,\mathcal{F}_\zeta(X;t)|_{X>\zeta} \;\; ; \;\; \tilde{x} = \frac{X-\zeta/2}{1-\zeta/2} \;\; ; \;\; \xi = \frac{\zeta}{2-\zeta} \tag{27}$$

between our nonforward distributions in the $X > \zeta$ region and Ji's off-forward parton distributions $H(\tilde{x},\xi;t)$ [1] in the $\tilde{x} > \xi$ region, one can see that the narrow $F^{(0)}(x,y)$ ansatz gives the simplest model $H^{(0)}(\tilde{x},\xi;t=0) = f(x)$ in which OFPDs at $t = 0$ have no ξ-dependence. This result can be obtained directly by using the model $f^{(0)}(x,\alpha) = \delta(\alpha)f(x)$ for the α-DDs. Another example is the model [13,14] in which NFPDs do not depend on ζ, i.e., $\mathcal{F}_\zeta(X) = f(X)$. Using the inversion formula (21) and Eq. (23), we obtain $F(x,y) = \delta(y)f(x)$, *i.e.*, the support of this DD is on the y-axis only, which violates the mandatory $y \leftrightarrow 1-x-y$ symmetry. Unlike the ξ-independent ansatz for OFPDs, the ζ-independent ansatz for NFPDs is forbidden.

In case of two other models, simple analytic results can be obtained only for some explicit forms of $f(x)$. For the "valence quark"-oriented ansatz $\tilde{f}^{(1)}(x,\alpha)$, the following choice

$$f^{(1)}(x) = A\,x^{-a}(1-x)^3 \tag{28}$$

is both close to phenomenological quark distributions and produces a simple expression for the double distribution since the denominator $(1-x)^3$ factor in Eq. (14) is canceled. As a result, the integral in Eq. (17) is easily performed and we get

$$\tilde{H}^{(1V)}(\tilde{x},\xi)|_{|\tilde{x}|\geq\xi} = \tilde{A}\left(1-\frac{a}{4}\right)\left\{\left[(2-a)\xi(1-\tilde{x})(x_1^{2-a}+x_2^{2-a}) + (\xi^2-\tilde{x})(x_1^{2-a}-x_2^{2-a})\right]\theta(\tilde{x}) + (\tilde{x}\to-\tilde{x})\right\} \tag{29}$$

for $|\tilde{x}| \geq \xi$, where $\tilde{A} = 6A\,\Gamma(1-a)/\Gamma(5-a)$, and

$$\tilde{H}^{(1V)}(\tilde{x},\xi)|_{|\tilde{x}|\leq\xi} = \tilde{A}\left(1-\frac{a}{4}\right)\left\{x_1^{2-a}[(2-a)\xi(1-\tilde{x}) + (\xi^2-x)] + (\tilde{x}\to-\tilde{x})\right\} \tag{30}$$

in the middle $-\xi \leq \tilde{x} \leq \xi$ region. We use here the notation $x_1 = (\tilde{x}+\xi)/(1+\xi)$ and $x_2 = (\tilde{x}-\xi)/(1-\xi)$ [10]. As expected, these expressions are explicitly non-analytic for $x = \pm\xi$. It is interesting to note that in a particular case $a = 0$, the $x > \xi$ part of OFPD has the same x-dependence as its forward limit, differing from it by an overall ξ-dependent factor only:

$$\tilde{H}^{(1V)}(\tilde{x},\xi)|_{a=0} = 4\tilde{A}\,\frac{(1-|\tilde{x}|)^3}{(1-\xi^2)^2}\,\theta(|\tilde{x}|\geq\xi) + 2\tilde{A}\,\frac{\xi+2-3\tilde{x}^2/\xi}{(1+\xi)^2}\,\theta(|\tilde{x}|\leq\xi). \tag{31}$$

To extend this expression onto negative values of ξ, one should substitute ξ by $|\xi|$. One can check, however, that no odd powers of $|\xi|$ would appear in the $\tilde{x}^N$ moments of $\tilde{H}^{(1V)}(\tilde{x},\xi)$.

For the singlet quark distribution, the α-DDs $\tilde{f}^S(x,\alpha)$ should be odd functions of x. Still, we can use the model like (28) for the $x > 0$ part, but take $\tilde{f}^{(1S)}(x,\alpha)|_{x\neq0} = f^{(1)}(|x|,\alpha)\,\mathrm{sign}(x)$. Note, that the integral (17) producing $\tilde{H}^S(\tilde{x},\xi)$ in the $|\tilde{x}| \leq \xi$ region would diverge for $\alpha \to \tilde{x}/\xi$ if $a \geq 1$, which is the usual case for standard parametrizations of singlet quark distributions for sufficiently large Q^2. However, due to the antisymmetry of $\tilde{f}^S(x,\alpha)$ wrt $x \to -x$ and its symmetry wrt $\alpha \to -\alpha$, the singularity at $\alpha = \tilde{x}/\xi$ can be integrated using the principal value prescription which in this case produces the $x \to -x$ antisymmetric version of Eqs.(29) and (30). As far as $a < 2$, the resulting functions are finite for all $\tilde{x}$ and continuous at $\tilde{x} = \pm\xi$. For $a = 0$, the middle part reduces to

$$\tilde{H}^{(1S)}(\tilde{x},\xi)|_{|\tilde{x}|\leq\xi,a=0} = 2\tilde{A}\,x\,\frac{3\xi^2 - 2x^2|\xi| - x^2}{|\xi|^3(1+|\xi|)^2}. \tag{32}$$

Clearly, the use of the principal value prescription is equivalent to imposing a subtraction procedure for the divergent second integral in Eq. (19) defining the untilded functions $H^{a,\bar{a}}(\tilde{x},\xi)$.

In general case, to study the deviation of skewed distributions from their forward counterparts for small ξ (or ζ), let us consider the integral producing the $x \geq \xi$ part of $H(x,\xi)$ [see Eq.(19)] and expand it in powers of ξ:

$$H(\tilde{x};\xi)|_{\tilde{x}\geq\xi} = f(\tilde{x}) + \xi^2\left[\frac{1}{2}\int_{-(1-\tilde{x})}^{(1-\tilde{x})} \frac{\partial^2 f(\tilde{x},\alpha)}{\partial\tilde{x}^2}\,\alpha^2\,d\alpha + (1-\tilde{x})^2\left(\frac{\partial f(\tilde{x},\alpha)}{\partial\alpha} - 2\frac{\partial f(\tilde{x},\alpha)}{\partial\tilde{x}}\right)\Big|_{\alpha=1-\tilde{x}}\right] + \ldots, \tag{33}$$

where $f(\tilde{x})$ is the forward distribution. For small ξ, the corrections are formally $O(\xi^2)$, i.e., they look very small. However, if $f(x,\alpha)$ has a singular behavior like x^{-a}, then

$$\frac{\partial^2 f(\tilde{x},\alpha)}{\partial\tilde{x}^2} \sim \frac{a(1+a)}{\tilde{x}^2} f(\tilde{x},\alpha),$$

and the relative suppression of the first correction is $O(\xi^2/\tilde{x}^2)$. The corrections are tiny for all $\tilde{x}$ except for the region $\tilde{x} \sim \xi$ where the correction has no parametric smallness. Nevertheless, even in this region it is suppressed numerically, because the α^2 moment is rather small for a distribution concentrated in the small-α region. It is easy to write expicitly all the terms which are not suppressed in the $\tilde{x} \sim \xi \to 0$ limit

$$H(\tilde{x};\xi) = \sum_{k=0} \frac{\xi^{2k}}{(2k)!}\int_{-1}^{1} \frac{\partial^{2k} f(\tilde{x},\alpha)}{\partial\tilde{x}^{2k}}\,\alpha^{2k}\,d\alpha + \ldots, \tag{34}$$

where the ellipses denote the terms vanishing in this limit. Due to strong numerical suppression of higher terms, the series converges rather fast. For small x, we can neglect the x-dependence of the profile function $h(x,\alpha)$ in Eq. (12) and take the model $f(x,\alpha) = f(x)\rho(\alpha)$ with $\rho(\alpha)$ being a symmetric weight function on $-1 \leq \alpha \leq 1$ whose integral over α equals 1. In the region where both $\tilde{x}$ and ξ are small, we can approximate Eq. (17) by

$$\tilde{H}(\tilde{x};\xi) = \int_{-1}^{1} \tilde{f}(\tilde{x}-\xi\alpha)\rho(\alpha)\,d\alpha + \ldots, \tag{35}$$

i.e., the OFPD $H(\tilde{x};\xi)$ is obtained in this case by averaging the usual (forward) parton density $\tilde{f}(x)$ (extended onto $-1 \leq x \leq 1$) over the region $\tilde{x} - \xi \leq x \leq \tilde{x} + \xi$ with the weight $\rho(\alpha)$. In terms of NFPDs, the relation is

$$\tilde{\mathcal{F}}_\zeta(X) = \int_{-1}^{1} \tilde{f}(X - \zeta(1+\alpha)/2)\rho(\alpha)\,d\alpha + \ldots, \tag{36}$$

i.e., the average is taken over the region $X - \zeta \leq x \leq X$.

The imaginary part of hard exclusive meson electroproduction amplitude is determined by the skewed distributions at the border point. For this reason, the magnitude of $\mathcal{F}_\zeta(\zeta)$ [or $H(\xi,\xi)$], and its relation to the forward densities $f(x)$ has a practical interest. Assuming the infinitely narrow weight $\rho(\alpha) = \delta(\alpha)$, we have $\mathcal{F}_\zeta(X) = f(X - \zeta/2) + \ldots$ and $H(x,\xi) = f(x)$. Hence, both $\mathcal{F}_\zeta(\zeta)$ and $H(\xi,\xi)$ are given by $f(x_{Bj}/2)$ since $\zeta = x_{Bj}$ and $\xi = x_{Bj}/2 + \ldots$. Since the argument of $f(x)$ is twice smaller than in deep inelastic scattering, this results in an enhancement factor. In particular, if $f(x) \sim x^{-a}$ for small x, the ratio $\mathcal{F}_\zeta(\zeta)/f(\zeta)$ is 2^a. The use of a wider weight function $\rho(\alpha)$ produces further enhancement. For example, taking $\rho(\alpha) = \frac{3}{4}(1-\alpha^2)$ and $f(x) \sim x^{-a}$ we get

$$\frac{\mathcal{F}_\zeta(\zeta)}{f(\zeta)} = \frac{1}{(1-a/2)(1-a/3)} \tag{37}$$

which is larger than 2^a for $0 < a < 2$. Due to evolution, the effective parameter a is an increasing function of Q^2. As a result, the above ratio slowly increases with Q^2.

Finally, I want to point out that possible profiles of $f(x,\alpha)$ in the α-direction are restricted by inequalities (see [14,10,15,16]) relating skewed and forward distributions. For quark OFPDs, I obtained [15]

$$H^q(\tilde{x},\xi) \leq \sqrt{\frac{1}{1-\xi^2}\,f\left(\frac{\tilde{x}+\xi}{1+\xi}\right)f\left(\frac{\tilde{x}-\xi}{1-\xi}\right)} = \frac{1}{\sqrt{1-\xi^2}}\sqrt{f(x_1)f(x_2)}. \tag{38}$$

If one uses the infinitely narrow model $f^{(0)}(x,\alpha) = f(x)\,\delta(\alpha)$ [corresponding to $H^{(0)}(\tilde{x},\xi) = f(\tilde{x})$], the inequality (38) is satisfied for any function $f(x)$ of $x^{-a}(1-x)^{b}$ type with $a \geq 0$, $b > 0$. For the model (31) which has a wider $\frac{3}{4}(\bar{x}^2 - \alpha^2)$ profile and $f(x) = 4A\,(1-x)^3$, the inequality (38) is exactly saturated. Finally, if one takes the model $f^{(4)}(x,\alpha) = \bar{x}\,f(x)\,\delta(\bar{x}^2 - \alpha^2)$ with an extremely wide profile, one obtains the result $H^{(4)}(\tilde{x},\xi) = \frac{1}{2}\{f(x_1)/(1+\xi) + f(x_2)/(1-\xi)\}$ which violates (38).

VI. SUMMARY

In this paper, we treated double distributions as the basic objects for parametrizing nonforward matrix elements. An alternative description in terms of skewed distributions was obtained by an appropriate integration of relevant DDs. The use of DDs helps to establish important features of SPDs such as their nonanalyticity at the border points $X = \zeta$ and $\tilde{x} = \pm\xi$. DDs are crucial for securing the property that the moments of SPDs should be polynomial in the skewedness parameter. For these reasons, the use of DDs is unavoidable in constructing consistent models of SPDs.

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