JLAB-THY-98-03*

Duality-Violating $1/m_Q$ Effects in Heavy Quark Decay

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Abstract

I identify a source of Λ_{QCD}/m_Q corrections to the assumption of quark-hadron duality in the application of heavy quark methods to inclusive heavy quark decays. These corrections could substantially affect the accuracy of such methods in practical applications and in particular compromise their utility for the extraction of the Cabibbo-Kobayashi-Maskawa matrix element V_{cb}.

*an abbreviated version of the original JLAB-THY-98-03 entitled "Duality in Inclusive Semileptonic Heavy Quark Decay"

Although the classic application of heavy quark symmetry is in the exclusive semileptonic decays of heavy quarks [1], there has also been substantial work on using heavy quark effective theory (HQET) [2] to systematically improve decay predictions for inclusive decays of heavy hadrons [3–5]. In these inclusive applications, decays are treated in an operator product expansion (OPE) which leads *via* HQET to a $1/m_Q$ expansion in which the leading term is free quark decay and $1/m_Q$ terms appear to be absent. Although these calculations have become very sophisticated [4,5], it is widely appreciated [4–7] that there remains a basic unproved hypothesis in their derivation: the assumption of quark-hadron duality. It is the accuracy of this assumption that I want to call into question here.

While supposedly of wide validity, recent applications have centered around the hope that this approach offers an alternative to the classic exclusive methods for determining V_{cb}, and I will accordingly focus most of my remarks on the case $b \to c\ell\bar{\nu}_\ell$ where both quarks are heavy. In inclusive $b \to c\ell\bar{\nu}_\ell$ decays, which materialize as $\bar{B} \to X_c\ell\bar{\nu}_\ell$, about 65% of the X_c spectrum is known to be due to the very narrow ground states D and D^*. The relatively narrow $s_\ell^{\pi_\ell} = \frac{3}{2}^+$ states [8] $D_2^*(2460)$ and $D_1(2420)$ account for perhaps another 5% of the rate, and it may be assumed that the remaining rate involves decays to higher mass resonances (quarkonia and hydrids) and continua [9]. The inclusive calculations predict continuous X_c spectra which are assumed to be dual to the true hadronic spectrum (see Fig. 1).

A picture like Fig. 1 might lead one to dismiss the duality approximation since the inclusive spectrum clearly does not meet the usual requirement that it be far above the resonance region [10]. *I.e.*, normally the accuracy of quark-hadron duality would be determined by a parameter Λ_{QCD}/E where the relevent energy scale E is the mean hadronic excitation energy $\Delta m_{X_c} \equiv \bar{m}_{X_c} - m_D$. However, as first explained by Shifman and Voloshin [11,12], this is *not* the expansion variable in this case: duality for heavy-to-heavy semileptonic decays sets in *at threshold* since even as $\delta m \equiv m_b - m_c$ (and therefore Δm_{X_c}) approaches zero, as $m_b \to \infty$ the heavy recoiling c quark has an energy much greater than Λ_{QCD} so that it *is* a free quark in leading order. In the small velocity (SV) limit, it *must* therefore hadronize with unit probability (up to potential Λ_{QCD}/m_Q corrections) as D and D^*. This "cannonball" approximation is in fact an essential part of the physical basis of the HQET expansion in $1/m_Q$. Thus the question is not whether duality holds in semileptonic heavy quark decays, but rather how accurately it holds.

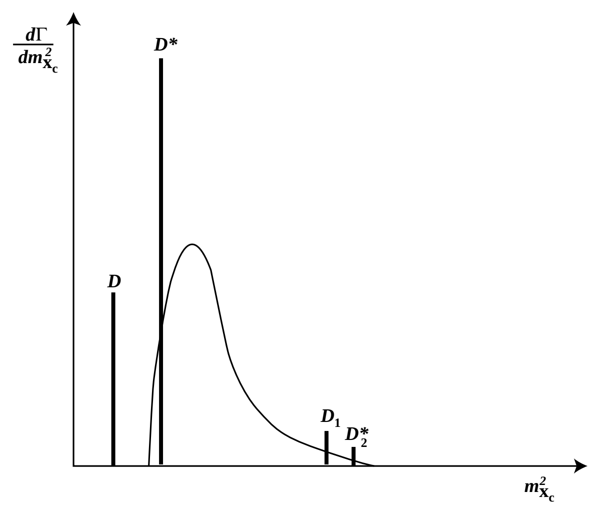

Fig. 1: A sketch for $b \to c$ semileptonic decay of the continuous inclusive recoil spectrum of the OPE calculations (smooth curve) compared to the known hadronic spectrum (shown as individual resonance lines).

Up to *caveats* regarding the unknown accuracy of the assumption of duality, the combined HQET and OPE methods indicate that inclusive calculations should in fact be accurate up to corrections of order Λ^2_{QCD}/m^2_Q. Here I will identify a source of duality-violation which leads to Λ_{QCD}/m_Q corrections for any finite final quark kinetic energy. The problems are revealed by considering a Bjorken sum rule [13] which may be viewed as an extension of Shifman-Voloshin duality to arbitrary recoils. Bjorken's sum rule guarantees that, as $m_b \to \infty$, duality will be enforced locally in the semileptonic decay Dalitz plot of rate versus $w-1$ and E_ℓ (where $w \equiv v \cdot v'$ is the usual heavy quark double-velocity variable and E_ℓ is the lepton energy). For regions of the Dalitz plot for which $w-1$ is not large (and in $b \to c$ decay nearly the whole Dalitz plot satisfies this condition), the Bjorken sum rule explicitly relates the loss of total rate from the "elastic" $s_\ell^{\pi_\ell} = \frac{1}{2}^-$ channels, as the Isgur-Wise function falls, to the turn-on of the production of $s_\ell^{\pi_\ell} = \frac{1}{2}^+$ and $\frac{3}{2}^+$ states [14].

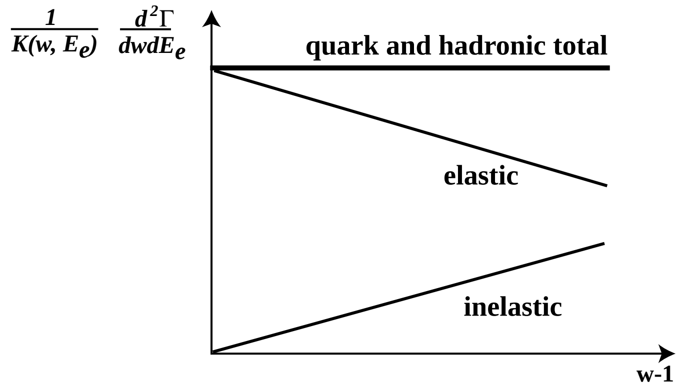

Fig. 2: The exact compensation by inelastic channels of the fall of the elastic rate in the linear region as $m_b \to \infty$.

In particular, in this region the Isgur-Wise function may be taken to be linear:

$$\xi(w) \simeq 1 - \rho^2(w-1) \equiv 1 - \left[\frac{1}{4} + \rho^2_{dyn}\right](w-1) \quad , \tag{1}$$

and if we define

$$\frac{d^2\Gamma^{inclusive}_{quark}}{dwdE_\ell} = K(w, E_\ell) \tag{2}$$

then [13,14], as $m_b \to \infty$,

$$\frac{d^2\Gamma^{inclusive}_{hadron}}{dwdE_\ell} = K(w, E_\ell)\left(\frac{w+1}{2}|\xi(w)|^2 + 2(w-1)\left[\sum_m |\tau^{(m)}_{\frac{1}{2}}(1)|^2 + 2\sum_p |\tau^{(p)}_{\frac{3}{2}}(1)|^2\right]\right) \tag{3}$$

up to corrections of order $(w-1)^2$. With

$$\left(\frac{w+1}{2}\right)|\xi(w)|^2 \simeq 1 - 2\rho^2_{dyn}(w-1) \quad , \tag{4}$$

the Bjorken sum rule guarantees that for fixed $r \equiv m_c/m_b$, as $m_b \to \infty$ inelastic $s_\ell^{\pi_\ell} = \frac{1}{2}^+$ and $\frac{3}{2}^+$ channels will open up to give a semileptonic rate that exactly and locally compensate in the Dalitz plot the loss of rate from the elastic channels due to ρ^2_{dyn}. This situation is sketched in Figure 2; if it were applicable to $b \to c$ decays, then quark-hadron duality would be exact.

Having established conditions for its validity as $m_b \to \infty$, it is easy to see why one should be concerned about quark-hadron duality for $b \to c$ decays. For fixed r, $w-1$ lies in the fixed range from 0 to $(1-r)^2/2r$, and as $m_b \to \infty$ any given hadronic threshold collapses to the point $w = 1$. However, for finite m_b there is a gap in $w-1$ in which the rate to the elastic $\frac{1}{2}^-$ channels falls by Λ_{QCD}/m_Q terms but the potentially compensating excited state channels $\frac{1}{2}^+$ and $\frac{3}{2}^+$ are not yet kinematically allowed. More precisely, if $m_{D^{**}}$ is the mass of a generic charmed inelastic state, then $t^{**}_m = (m_B - m_{D^{**}})^2$ would be the threshold in t for this state, corresponding to a value of $w-1$ in the quark-decay Dalitz plot of

$$\frac{t_m - t^{**}_m}{2m_b m_c} \simeq (1-r)\frac{\Delta}{m_c} \tag{5}$$

where $t_m \equiv (m_B - m_D)^2 \simeq (m_b - m_c)^2$ and $\Delta \equiv m_{D^{**}} - m_D$. Since $\Delta \simeq 500$ MeV and $(w-1)_{max} \simeq 0.6$, this region covers more than one third of the Dalitz plot and the compensation is very substantially delayed: see Figure 3. Eqs. (5) and (1) show that this effect is of order Λ_{QCD}/m_Q, seemingly at odds with the OPE result.

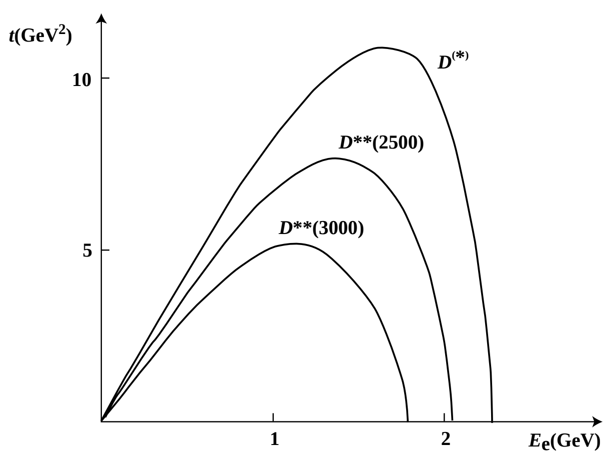

Fig. 3: An overlay of the Dalitz plots for $\bar{B} \to D^{(*)} e\bar{\nu}_e$, $\bar{B} \to D^{**}(2500)e\bar{\nu}_e$, and $\bar{B} \to D^{**}(3000)e\bar{\nu}_e$. The $D^{(*)}$ mass is taken as the hyperfine average of the D and D^* masses; the two D^{**} masses are chosen for illustrative purposes.

Despite this apparent contradiction, there is actually no inconsistency: the OPE result that the leading corrections to the inclusive rate are of order Λ^2_{QCD}/m^2_Q can still be valid as derived in the limit of large energy release in the $b \to c$ transition, while Λ_{QCD}/m_Q effects can arise for energy releases of the order of Λ_{QCD} due to a finite radius of convergence of the OPE. The main purpose of this paper is indeed to call attention to this effect.

The basic issues can be most easily exposed by considering [4] spinless quarks coupled to a scalar field ϕ of mass μ, and by studying the decay $b \to c\phi$ with weak coupling constant g. Differential semileptonic decay rates have a more complex spin structure, but otherwise correspond to the case $\mu = \sqrt{t}$; total semileptonic rates correspond to a weighted average over kinematically allowed μ but, as we shall see below, this averaging does not change the essentials of the problem. In our simplified case

$$\Gamma(b \to c\phi) = \frac{g^2 p_{cb}}{8\pi m_b^2} \tag{6}$$

where $p_{fi} \equiv [(m_i - m_f)^2 - \mu^2]^{1/2}[(m_i + m_f)^2 - \mu^2]^{1/2}/2m_i$ is the momentum of ϕ from the two-body decay of mass m_i into masses m_f and μ.

To compare Eq. (6) with a hadronic world (I initially consider a large N_c world of narrow resonances, but will generalize below), define

$$\Gamma(B \to D^{(n)}\phi) = \frac{g^2 p_{D^{(n)}B}}{8\pi m_B^2}\Big(\frac{m_{D^{(n)}} m_B}{m_c m_b}\Big)|\xi^{(n)}(\vec{v}_{D^{(n)}B})|^2 \tag{7}$$

where the generalized Isgur-Wise functions $\xi^{(n)}$ depend on

$$\vec{v}_{D^{(n)}B} = \vec{p}_{D^{(n)}B}/m_{D^{(n)}} \quad , \tag{8}$$

the recoil velocity of the n^{th} excited state $D^{(n)}$ of the D meson system, and I have introduced some conventional mass factors to explicitly reflect hadronic normalizations. I next introduce a "scaled energy release" variable

$$T^* \equiv \frac{m_b - m_c - \mu}{\Delta} \quad , \tag{9}$$

where $\Delta \equiv m_{D^{**}} - m_D$ is the mass gap to the first $s_\ell^{\pi_\ell} = 1^-$ excited state of the D meson system (corresponding to $s_\ell^{\pi_\ell} = \frac{1}{2}^+$ and $\frac{3}{2}^+$ in the physical case where the $\bar{d}$ has $j^p = \frac{1}{2}^-$), and an order Λ_{QCD}/m_Q expansion parameter ϵ defined by the expansion

$$|\xi_{DB}|^2 = 1 - \epsilon T^* + O(\epsilon^2) \tag{10}$$

for small T^* (*i.e.*, small charm quark velocities). In the quark model [16,17] one would have

$$\epsilon = \frac{m_d(m_b - m_c)}{m_b m_c} \tag{11}$$

where m_d is the mass of the light spectator antiquark $\bar{d}$ (or, more generally, of the "brown muck"). Defining $|\xi_{D^{**}B}|^2 \equiv \sum_{m_\ell} |\xi^{m_\ell}_{D^{**}B}|^2$ (where $\xi^{m_\ell}_{D^{**}B}$ is the analog of ξ_{DB} for transitions into the lowest $s_\ell^{\pi_\ell} = 1^-$ excited state with magnetic quantum number m_ℓ), we would have

$$|\xi_{D^{**}B}|^2 = \epsilon(T^* - 1) + O(\epsilon^2) \tag{12}$$

from the (spinless) Bjorken sum rule in the limit that it is saturated by the first D^{**}. Since in this limit

$$\frac{p_{D^{(n)}B}}{p_{cb}} = \Big[\frac{m_{D^{(n)}} m_b}{m_c m_B}\Big]^{1/2} \Big(\frac{T^* - 1}{T^*}\Big)^{1/2} \quad , \tag{13}$$

we can obtain a model [4] for

$$R \equiv \frac{\sum_n \Gamma(B \to D^{(n)}\phi)}{\Gamma(b \to c\phi)} \quad . \tag{14}$$

by truncating the sum over n after the first D^{**}:

$$R_1^{D^{**}} \equiv \frac{\Gamma(B \to D\phi) + \Gamma(B \to D^{**}\phi)}{\Gamma(b \to c\phi)} \tag{15}$$

$$= [1 + \frac{3}{2}\epsilon - \epsilon T^*]\theta(T^*) + \epsilon \frac{(T^* - 1)^{3/2}}{{T^*}^{1/2}} \theta(T^* - 1) \quad , \tag{16}$$

wherein I have shown explicitly the two thresholds at $T^* = 0$ and $T^* = 1$. It is interesting to observe that the quark model of Refs. [16,17] gives exactly Eq. (16), including the $+\frac{3}{2}\epsilon$ term [18], as expected [7]. I also note that

1. At $T^* \to \infty$, Eq. (16) is of the form $1 + O(\epsilon^2) + O(\epsilon/T^*)$ as required by the OPE.

2. There are no other terms of order 1, ϵ, or ϵT^* possible beyond those shown: a more accurate treatment of $\Gamma(B \to D\phi)$ could only generate ϵ^2, $\epsilon^2 T^*$, $\epsilon^2 T^{*2}$, ... terms; a more

accurate treatment of $\Gamma(B \to D^{**}\phi)$ could only generate $\epsilon^2 T^*$, $\epsilon^2 T^{*2}$, ... terms; and all higher states first make a contribution at order $\epsilon^2 T^{*2}$ or higher. Conversely, we note that if, for example, $\epsilon^2 T^{*2}$ terms are retained, they must all cancel exactly or the requirements of the OPE would be violated as $T^* \to \infty$.

3. As $\Delta m \equiv m_b - m_c \to 0$, $R_1^{D^{**}} \to 1 + O(\frac{\Lambda_{QCD}\Delta m}{m_b^2})$ as required [15].

4. Near $T^*_{max} \equiv m_b - m_c$, ϵT^*_{max} is in general large. This observation corresponds in the usual language of heavy quark symmetry to the statement that the natural scale of the slope ρ^2 of the Isgur-Wise function is of order unity. It is also consistent with the experimental observation that $|\xi_{DB}|^2$ has dropped to less than half its value between zero and maximum recoil. Given this, the extension of Eq. (16) to higher orders in T^* will require a "conspiracy" of the entire spectrum of possible hadronic final states. We may nevertheless use Eq. (16) across the full range of T^* as an indicator of the Λ_{QCD}/m_Q effects arising from the order 1 and order T^* terms in the expansion of R. This corresponds to a "best case" assumption that duality is locally perfect for the terms T^{*n} with $n > 1$.

Thus, while extreme, this truncation has all the properties required by the OPE and so stands as a simple explicit example of the existence of the claimed duality-violating Λ_{QCD}/m_Q effects for finite T^*.

It is straightforward to introduce a number of simple variants of this prototypical model. The first corresponds to the more realistic case where the Bjorken sum rule is only saturated by the full tower of $s_\ell^{\pi_\ell} = 1^-$ resonances so that the second term in Eq. (16) becomes

$$\frac{\epsilon}{T^{*1/2}} \sum_n f_n (T^* - t_n^*)^{3/2} \theta(T^* - t_n^*) \tag{17}$$

with $\sum_n f_n = 1$ and t_n^* being the threshold for channel n. As $T^* \to \infty$, these contributions automatically cancel the $-\epsilon T^*$ term from the elastic form factor, and constrain the $O(\epsilon)$ correction to give

$$R^{D^{**}}_{1+2+...} = [1 + \frac{3}{2}\epsilon \bar{t}^* - \epsilon T^*]\theta(T^*) + \frac{\epsilon}{T^{*1/2}} \sum_n f_n (T^* - t_n^*)^{3/2} \theta(T^* - t_n^*) \quad , \tag{18}$$

where

$$\bar{t}^* = \sum_n f_n t_n^* \tag{19}$$

is the weighted average threshold position. Note that since some T_n^* exceed T_{max}^*, $R_{1+2+...}^{D^{**}}$ cannot heal to unity in the physical decay region.

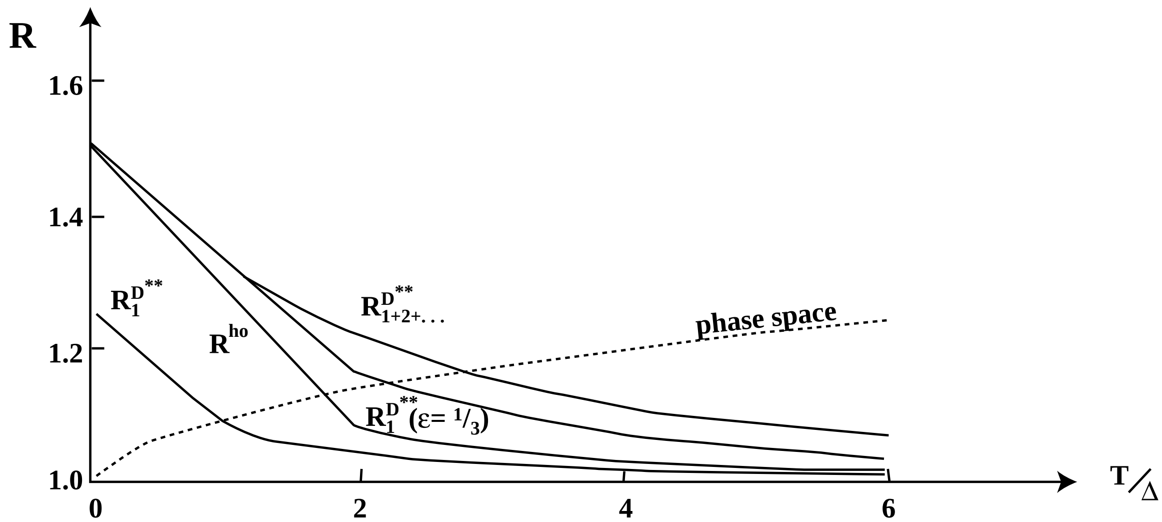

Fig. 4: Four resonance models of the approach to duality: (a) $R_1^{D^{**}}$ (with the baseline value $\epsilon = 1/6$), (b) $R_1^{D^{**}}$ (with $\epsilon = 1/3$), (c) $R_{1+2+...}^{D^{**}}$ (with $t_n^* = n$ and $f_n = (\frac{1}{2})^n$ so that $\bar{t}^* = 2$), and (d) R^{ho}. The "baseline" parameters follow from the observations that $\rho^2 \sim 1$, $(w-1)_{max} \sim 1/2$ and $(m_b - m_c)/\Delta \sim 6$. The alternative $\epsilon = 1/3$ corresponds to the case $T_{max}^* = 3$, *i.e.*, to using $\Delta_{eff} = 2\Delta$ for the mean location of the $s_\ell^{\pi_\ell} = 1^-$ strength.

As described above, both $R_1^{D^{**}}$ and $R_{1+2+...}^{D^{**}}$ are still "best case" truncations which assume exact cancellations of $\epsilon^2 T^*$, $\epsilon^2 T^{*2}$, ... terms. While sufficient for the purposes of this study, this limitation is easily removed: it is straightforward to recursively "construct duality" to create models to the required order in ϵ to any finite order in T^*. Consider, for example,

$$
\begin{aligned}
R^{ho} = \frac{exp(-\epsilon T^*)}{T^{*1/2}} \Big(& [1+\frac{3}{2}\epsilon] T^{*1/2}\theta(T^*) \\
& +\epsilon[1+\frac{5}{2}\epsilon+\frac{35}{16}\epsilon^2+\frac{35}{32}\epsilon^3+\frac{385}{1024}\epsilon^4+...](T^*-1)^{3/2}\theta(T^*-1) \\
& +\frac{1}{2!}\epsilon^2[1+\frac{7}{2}\epsilon+\frac{21}{4}\epsilon^2+\frac{77}{16}\epsilon^3+...](T^*-2)^{5/2}\theta(T^*-2) \\
& +\frac{1}{3!}\epsilon^3[1+\frac{9}{2}\epsilon+\frac{297}{32}\epsilon^2+...](T^*-3)^{7/2}\theta(T^*-3) \\
& +\frac{1}{4!}\epsilon^4[1+\frac{11}{2}\epsilon+...](T^*-4)^{9/2}\theta(T^*-4) \\
& +\frac{1}{5!}\epsilon^5[1+...](T^*-5)^{11/2}\theta(T^*-5)+...\Big) \quad ,
\end{aligned}
$$

where the ellipses denote terms of order $\epsilon^6 T^{*n}$ with $1 \leq n \leq 5$ and all terms of order $\epsilon^m T^{*m}$ and higher with $m > 5$. This harmonic-oscillator-like expansion is accurate even at $T^* = 5$ up to corrections of order Λ^2_{QCD}/m^2_Q.

The three models just introduced are all based on the duality of $b \to c\phi$ to a simple tower of $c\bar{d}$ resonances controlled by the single scale Δ. Figure 4 shows that the thresholds associated with such towers could easily be a source of duality-violating Λ_{QCD}/m_Q corrections of order 10% in $b \to c$ decays. This must be a cause for concern in comparing inclusive calculations with experiment.

I am even more alarmed by processes which could give a high-mass nonperturbative tail to the recoil mass distribution. The hadronization of $b \to c\phi$ will not be saturated by ordinary quark model $c\bar{d}$ states even in the large N_c limit: hybrid mesons (*i.e.,* states with a $c\bar{d}$ valence structure but with internal gluonic excitation) will also contribute. Such states are expected at substantially higher masses than the ordinary quark model states. Moreover, their production will not be exhausted until the constituent $\bar{d}$ antiquark in the D meson has been fully resolved into a current quark at high recoil momentum $p_c >> 1$ GeV. For a crude estimate of the effects of the delayed onset of these states, I take a simple two-component resonance model consisting of "normal" $c\bar{d}$ resonances with $\bar{t}^*_{c\bar{d}}$ and $c\bar{d}$ hybrids with $\bar{t}^*_{hybrid}$ substantially larger. If we assume that the latter are responsible for a fraction κ of ρ^2_{dyn}, then we would have

$$R^{hybrid} = [1 + \frac{3}{2}\epsilon\bar{t}^* - \epsilon T^*]\theta(T^*)$$
$$+(1-\kappa)\epsilon\frac{(T^* - \bar{t}^*_{c\bar{d}})^{3/2}}{T^{*1/2}}\theta(T^* - \bar{t}^*_{c\bar{d}})$$
$$+\kappa\epsilon\frac{(T^* - \bar{t}^*_{hybrid})^{3/2}}{T^{*1/2}}\theta(T^* - \bar{t}^*_{hybrid}) \quad , \tag{20}$$

with $\bar{t}^* = (1-\kappa)\bar{t}^*_{c\bar{d}} + \kappa\bar{t}^*_{hybrid}$.

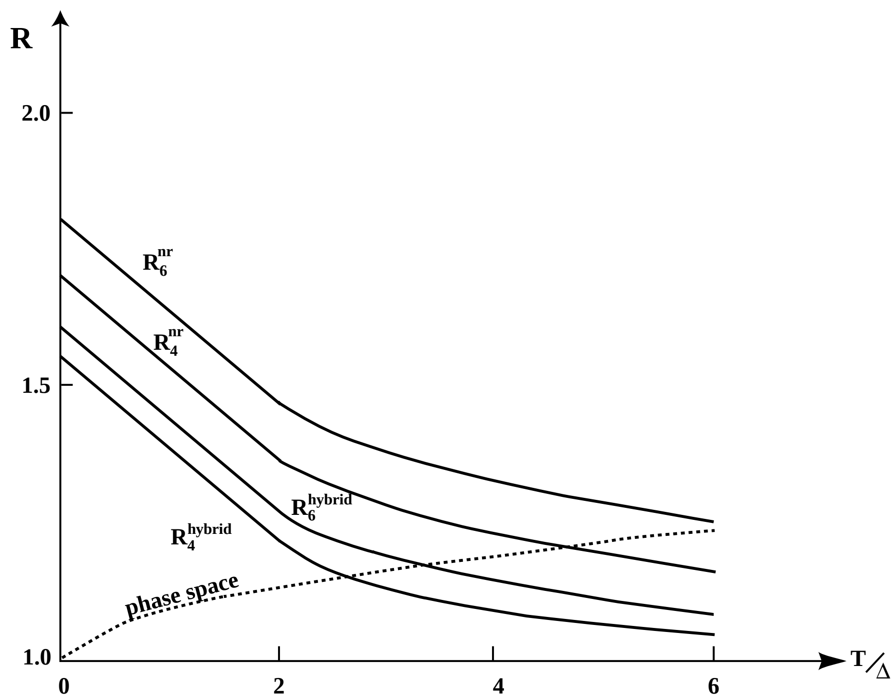

Fig. 5: Four examples of the effects of a nonperturbative high mass tail on the recoil mass spectrum: (a) R_4^{hybrid} (with $\kappa = 1/10$, $\bar{t}^*_{c\bar{d}} = 2$, and $\bar{t}^*_{hybrid} = 4$), (b) R_6^{hybrid} (as in (a), but with $\bar{t}^*_{hybrid} = 6$), (c) R_4^{nr} (with $\lambda = 1/5$, $\bar{t}^*_{c\bar{d}} = 2$, $T^*_{min} = 2$, and $s = 4$), and (d) R_6^{nr} (as in (c) but with $s = 6$). The values of κ and λ are based on the model-dependent estimates of Ref. [9], but are certainly reasonable (*e.g.,* λ is a $1/N_c$ effect). The illustrative values for $\bar{t}^*_{hybrid}$ are based on the high threshold for hybrids and their presumed "hard" production mechanism. The nonresonant spectrum is assumed to have the form $\rho(t^*) = \frac{1}{s}exp(\frac{T^*_{min} - t^*}{s})\theta(t^* - T^*_{min})$, with the choices for s reflecting the assumed persistence of nonresonant contributions to invariant masses of order 2 GeV above threshold.

I suspect that $1/N_c$-suppressed nonresonant contributions are an even more serious source of delayed compensation of duality. An appropriate model for such continua would be

$$R^{nr} = [1 + \frac{3}{2}\epsilon\bar{t}^* - \epsilon T^*]\theta(T^*) \\ +(1-\lambda)\epsilon\frac{(T^* - \bar{t}^*_{c\bar{d}})^{3/2}}{T^{*1/2}}\theta(T^* - \bar{t}^*_{c\bar{d}}) \\ +\lambda\epsilon\int_{T^*_{min}}^{T^*} dt^*\rho(t^*)\frac{(T^* - \bar{t}^*)^{3/2}}{T^{*1/2}} \quad , \tag{21}$$

where λ is the fraction of ρ^2_{dyn} due to nonresonant states and $\rho(t^*)$ is the appropriate normalized spectral function ($\int_{T^*_{min}}^{\infty} dt^*\rho(t^*) = 1$) which begins at T^*_{min}. In this situation, $\bar{t}^* = (1-\lambda)\bar{t}^*_{c\bar{d}} + \lambda\int_{T^*_{min}}^{\infty} dt^*\rho(t^*)t^*$. Nonperturbative quark pair creation leading to $\bar{B} \to X_c Y\phi$ may be expected [19] to persist up to 2 GeV above threshold.

Figure 5 shows that modest couplings to either hybrids or high mass continua could lead to even more substantial duality violations than those associated with the delayed onset of the normal $c\bar{d}$ resonances.

Although my main focus has been on heavy-to-heavy transitions, the physics issues raised here (if not their explicit forms) are also relevant for heavy-to-light transitions. Before concluding, let me therefore point out a simple application of the OPE to inclusive heavy-to-light transitions where it seems certain to me that they will fail: Cabibbo-forbidden charm decays. (Even though such decays might be an unimportant application of the inclusive calculations in practice, they provide a valid theoretical testing ground for their accuracy.) In particular, consider the $c \to d\bar{\ell}\nu_\ell$ decays of the D^0 and D^+. They will be dominated by the channels $D^0 \to \pi^-\bar{\ell}\nu_\ell$ and $\rho^-\bar{\ell}\nu_\ell$ and by $D^+ \to \pi^0\bar{\ell}\nu_\ell$, $\eta\bar{\ell}\nu_\ell$, $\eta'\bar{\ell}\nu_\ell$, $\rho^0\bar{\ell}\nu_\ell$, and $\omega\bar{\ell}\nu_\ell$. Since the OPE corrections in the D^0 and D^+ are *identical*, their Cabibbo-forbidden semileptonic partial widths and spectral distributions are predicted to be identical. However, simple isospin symmetry implies that $\Gamma(D^+ \to \pi^0\bar{\ell}\nu_\ell) = \frac{1}{2}\Gamma(D^0 \to \pi^-\bar{\ell}\nu_\ell)$, so the inclusive Cabibbo-forbidden rates can only be equal if $\Gamma(D^+ \to \eta\bar{\ell}\nu_\ell) + \Gamma(D^+ \to \eta'\bar{\ell}\nu_\ell) = \Gamma(D^+ \to \pi^0\bar{\ell}\nu_\ell)$. In many models this latter relation would be true if $m_\eta = m_{\eta'} = m_\pi$, since it is rather natural for the squares of matrix elements to satisfy its analogue. However, with real phase space factors, this relation is typically badly broken. Since Cabibbo-forbidden decays, like their Cabibbo-allowed counterparts, will receive little excited state compensation given the available phase space, I expect this prediction to fail.

Finally, I note that the duality-violating effects I have highlighted here will have an effect on the long-standing $\bar{B}$ semileptonic branching ratio puzzle [20]. Since the hadronic mass distribution in $b \to c\bar{u}d$ is weighted toward higher masses than the leptonic mass distribution in $b \to c\ell\bar{\nu}_\ell$, the ratio of these two rates will be changed.

In summary, I have shown here that hadronic thresholds lead to Λ_{QCD}/m_Q violations of duality in $b \to c$ decays which do not explicitly appear in the operator product expansion. Since such violations cannot appear as the $b \to c$ energy release $T \to \infty$, there are "conspiracies" (*i.e.*, sum rules) which relate hadronic thresholds and transition form factors. As emphasized by Bigi, Uraltsev, Shifman, Vainshtein, and others [4,5,7], these relations tend to compensate the otherwise extremely large Λ_{QCD}/m_Q effects even at small T. In this paper I have displayed several models of such hadronic compensation mechanisms which indicate that these duality-violating Λ_{QCD}/m_Q effects could nevertheless be very substantial. While the examples I have selected are perhaps pessimistic, they indicate that these effects must be better understood before inclusive methods can be applied with confidence to heavy quark semileptonic decays.

ACKNOWLEDGEMENTS

I am very grateful to number of colleagues who examined and commented upon a draft version of this paper, including Adam Falk, Howard Georgi, Richard Lebed, Zoltan Ligeti, Matthias Neubert, Iain Stewart, and Mark Wise.

I am particularly indebted to Ikaros Bigi, Misha Shifman, Nikolai Uraltsev, and Arkady Vainshtein who replied with a detailed and very pedagogical explanation of their understanding of the effects discussed in this paper. I certainly learned more from this exchange than they did, and substantial errors in the draft version were corrected as a result.

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