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NEW METHODS FOR EXTRACTING THE CKM ANGLE γ
USING $B^\pm \rightarrow D^0 K^-; \bar{D}^0 K^-$

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Abstract:

In this talk I will discuss the extraction of the CKM angle γ at B -factories through the interference of the subprocesses $B^- \rightarrow K^- D^0$ and $B^- \rightarrow K^- \bar{D}^0$. This seemingly impossible interference may be accomplished by allowing both D^0 and \bar{D}^0 to decay to a common final state. If only CP eigenstate decay modes of the D are considered, the branching ratio for $B^- \rightarrow \bar{D}^0 K^-$ must be experimentally determined in order to extract γ . I describe why this determination is likely to be experimentally impossible. On the other hand, if more general D decays are considered, the angle γ may then be determined. In fact, it is possible that a reasonable determination of γ may be made with $O(10^8)$ B 's.

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In this talk I will discuss the extraction of the CKM angle γ at B -factories through the interference of the subprocesses $B^- \rightarrow K^- D^0$ and $B^- \rightarrow K^- \bar{D}^0$. This seemingly impossible interference may be accomplished by allowing both D^0 and \bar{D}^0 to decay to a common final state. If only CP eigenstate decay modes of the D are considered, the branching ratio for $B^- \rightarrow \bar{D}^0 K^-$ must be experimentally determined in order to extract γ . I describe why this determination is likely to be experimentally impossible. On the other hand, if more general D decays are considered, the angle γ may then be determined. In fact, it is possible that a reasonable determination of γ may be made with $O(10^6)$ B 's.

1 Introduction

The Unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix is one of the fundamental predictions of the Standard Model¹. The experimental activity at the various B factories will therefore concern itself largely with the accurate determination of the various elements of this matrix. While the magnitude of CKM elements may be determined by the rates of appropriately chosen processes, direct measurement of the phases of these elements require the observation of CP violation. Indeed, with the exception of strong CP violation, the phase in the CKM matrix is the only place the Standard Model admits CP violation. Indeed there are good reasons to believe that CP violation will be present at an observable level in the B system since the CKM matrix in three generations will in general have a complex phase and CP violation has been known for 30 years in the K_L system which may be explained though such a phase.

This being the case, the standard model makes definite prediction regarding CP violation that will be present in B physics. This may be summarised with the usual "Unitarity Triangle"² which shows how the CP violation from the CKM matrix will be distributed among different B decay channels. In this talk I will focus on the determination of the angle γ which is the phase of the element V_{ub} in the CKM parameterisation of [3].

2 Determining γ with $B^- \rightarrow D^0 K^-$

On the quark level, the determination of γ which I will discuss is based on the interference of the tree level decays $\bar{b} \rightarrow c\bar{u}s$ with $\bar{b} \rightarrow u\bar{c}s$ (and their charge conjugates). The CKM phase difference between these two amplitudes is readily seen to be γ however it is not obvious how these channels, leading to seemingly different final states, can have any quantum mechanical interference.

It is only by considering certain specific hadronic final states common to both sub-processes that the desired interference may be obtained. First we specify that $\bar{b} \rightarrow c\bar{u}s$ hadronise as $B^- \rightarrow D^0 K^-$ and that $\bar{b} \rightarrow u\bar{c}s$ hadronise as $B^- \rightarrow \bar{D}^0 K^-$. These may interfere only if both the D^0 and \bar{D}^0 decay to a common final state X . In what follows, we will consider what choices of X can lead to a practical method for the determination of γ .

This clever method of extracting γ was first proposed in 1990⁴ and has since been studied extensively in the case where X is a CP eigenstate⁵ (which we will refer to as the Gronau-London-Wyler (GLW) method) and also for more general values of X ^{6,7}. Recently it has been realised⁷ that the recipe for extracting γ as originally proposed using CP eigenstate modes require the knowledge of $Br(B^- \rightarrow \bar{D}^0 K^-)$ and that this is virtually impossible to obtain experimentally. In this talk I will emphasize that considering more general values of X which are not CP eigenstates allows one to get around this difficulty and develop a practical method for the determination of γ .

Indeed, the only other method for determining γ is through oscillation effects in the B_s ². Methods based on the interference of $\bar{b} \rightarrow u\bar{c}s$ with $\bar{b} \rightarrow c\bar{u}s$ are thus of great importance since they may be used at $T(4s)$ B -factories which do not produce B_s mesons

3 Using CP-eigenstate D^0 Decays

Let us first consider the case where X is a CP eigenstate. Some examples of such a decay are $D^0 \rightarrow \pi^+ \pi^-$, $D^0 \rightarrow K_s \pi^0$, $D^0 \rightarrow K_s \eta$ etc. For instance if $X = K_s \pi^0$ the $\bar{b} \rightarrow c$ channel hadronises as $B^- \rightarrow K^- (D^0 \rightarrow K_s \pi^0)$ while the $\bar{b} \rightarrow u$ transition hadronises as $B^- \rightarrow K^- (\bar{D}^0 \rightarrow K_s \pi^0)$. Overall both processes lead to a common final state (ie. $K^- K_s \pi^0$ with $M_{K_s \pi^0} = M_{D^0}$) and they will interfere. In general, two phases will enter into the interference process, the CP odd phase γ which we wish to measure and the rescattering phase ξ resulting from the fact that the final state $K^- D^0$ rescattering phase is different from that of $K^- \bar{D}^0$.

Let us define

$$\begin{aligned} a(K) &= Br(B^- \rightarrow K^- D^0) & b(K) &= Br(B^- \rightarrow K^- \bar{D}^0) & c(X) &= Br(D^0 \rightarrow X) \\ c(\bar{X}) &= Br(D^0 \rightarrow \bar{X}) & d(K, X) &= Br(B^- \rightarrow K^- [X]) & \bar{d}(K, X) &= Br(B^+ \rightarrow K^+ [\bar{X}]) \end{aligned}$$

where $[X]$ indicates that it proceeds through the interfering D^0 and \bar{D}^0 channels. Then d and \bar{d} are given by:

$$\begin{aligned} d(K, X) &= a(K)c(X) + b(K)c(\bar{X}) + 2\sqrt{a(K)b(K)c(X)c(\bar{X})} \cos(\xi + \gamma) \\ \bar{d}(K, X) &= a(K)c(X) + b(K)c(\bar{X}) + 2\sqrt{a(K)b(K)c(X)c(\bar{X})} \cos(\xi - \gamma) \end{aligned}$$

Let us now assume that $a(K)$, $b(K)$, $c(X)$ and $c(\bar{X})$ are known experimentally (if X is CP eigenstate $c(X) = c(\bar{X})$). Then if $d(K, X)$ and $\bar{d}(K, X)$ are measured, the two equations above may be solved for the two unknown phases ξ and γ . This is the essence of the GLW method. The assumption that $b(K) = Br(B^- \rightarrow K^- \bar{D}^0)$ can be measured however requires careful scrutiny since it may be estimated⁷ that $b(K) \approx 3 \times 10^{-6}$ is rather small.

In order to measure $b(K)$ we need some way to tag the \bar{D}^0 and in particular to tell it from a D^0 . Logically, there are two possible ways one can accomplish this tagging, via a hadronic mode (which is the method considered in the literature to date⁸) or via a semi-leptonic mode.

Possible hadronic modes which tag \bar{D}^0 are decays of \bar{D}^0 which are Cabibbo allowed. For instance the decay $\bar{D}^0 \rightarrow K^+ \pi^-$ where $Br(\bar{D}^0 \rightarrow K^+ \pi^-) = 3 \times 10^{-2}$ [8]. The total decay rate for the chain $B^- \rightarrow K^- [\bar{D}^0 \rightarrow K^+ \pi^-]$ will thus be $\sim 10^{-7}$. Unfortunately D^0 may also decay into $K^+ \pi^-$ although this decay is doubly Cabibbo suppressed; in particular $Br(D^0 \rightarrow K^+ \pi^-) \approx 3 \times 10^{-4}$ [6]. The primary decay $B^- \rightarrow K^- D^0$ however has a branching ratio of $\sim 3 \times 10^{-2}$ so that chain $B^- \rightarrow K^- [\bar{D}^0 \rightarrow K^+ \pi^-]$ is also $\sim 10^{-7}$. Since both chains lead to the same final state, there will be $\sim 100\%$ interference effects between the two channels and so $b(K)$ cannot be determined in isolation. All possible hadronic tags of \bar{D}^0 will be likewise afflicted with these interference effects.

A semi-leptonic tag is any decay of the form $\bar{D}^0 \rightarrow e^- \bar{\nu} X_s$. This signature however is subject to a background from $B^- \rightarrow e^- \bar{\nu} X$ which is 10^6 times larger.

4 Using Non-CP Eigenstates

The key to extracting γ without the use of $b(K)$ is to take advantage of precisely the large interference effects which prevented the determination of $b(K)$ above.

Consider the case where X is not a CP eigenstate in particular where $D^0 \rightarrow X$ is doubly Cabibbo suppressed, for example $X = K^+ \pi^-$. Now the strong phase difference between the decay chain $B^- \rightarrow K^- [D^0 \rightarrow X]$ and $B^- \rightarrow K^- [\bar{D}^0 \rightarrow X]$ is $\zeta = \xi + \eta$ where ξ is the strong phase difference arising from the rescattering of $D^0 K^-$ versus $\bar{D}^0 K^-$ and η arises from the phase difference between $D^0 \rightarrow X$ versus $\bar{D}^0 \rightarrow X$.

In general each possible instance of X will have a different value of ξ so that if two choices of X are used, the set of equations above are replaced by the system of four equations:

$$\begin{aligned} d(K, X_i) &= a(K)c(X_i) + b(K)c(\bar{X}_i) + 2\sqrt{a(K)b(K)c(X_i)c(\bar{X}_i)} \cos(\zeta_i + \gamma) \\ \bar{d}(K, X_i) &= a(K)c(X_i) + b(K)c(\bar{X}_i) + 2\sqrt{a(K)b(K)c(X_i)c(\bar{X}_i)} \cos(\zeta_i - \gamma) \end{aligned}$$

for $i = 1, 2$. Assuming that $a(K)$, $c(X_i)$ and $c(\bar{X}_i)$ are already known and $d(K, X_i)$ and $\bar{d}(K, X_i)$ are then measured, the system above provides four equations for the four unknowns $\{\gamma, \zeta_1, \zeta_2, b(K)\}$ which can in principle be solved (and as a by product we also get the value of $b(K)$). These equations will be non-degenerate if either $c(X_1)/\bar{c}(X_1) \neq c(X_2)/\bar{c}(X_2)$ or $\zeta_1 \neq \zeta_2$ which will occur if both X_1 and X_2 are not CP eigenstates or if X_1 is a CP eigenstate and X_2 is not.

5 Improvements

Note that the system of equations above are quartic in nature and so in addition to the ambiguity between γ and $-\gamma$, there is a four-fold ambiguity in the determination of γ . If, however, a third state is also used, the resulting system of six equations is over-determined and the four-fold ambiguity is resolved.

Indeed there are a number of possible non CP-eigenstate modes that may be used for X , in particular $K^+ \pi^-$, $K^+ \rho^-$, $K^+ a_1^-$, $K^{*+} \pi^-$ etc. One may further generalize to related B^- decays such as $B^- \rightarrow K^- D^0$ and $B^- \rightarrow K^- \bar{D}^0$ all of which tend to build up the statistics for the determination of γ .

The accuracy in determining γ through this method depends on the value of γ as well as the completely unknown values of the strong phase shifts involved. This error will typically⁷ be between 5° and 20° given the total number of $\Upsilon(4s)$ of $\sim 10^9$ (not including acceptance factors).

Finally, one can improve the determination of γ by considering 3-body decays of D^0 . In particular if $D^0 \rightarrow K^+ \pi^- \pi^0$ additional information may be obtained by considering the distribution as a function of the energy of the K^+ and π^- in the rest frame of the D^0 .

Let us define

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6 Conclusion

In conclusion, we have seen that the original method of GLW for determining γ has a problem due to the fact that interference prevents the determination of $Br(B^- \rightarrow \bar{D}^0 K^-)$ through hadronic decays of \bar{D}^0 . We can, however, exploit these effects to salvage a method for determining γ and, since these interferences are between two roughly equal amplitudes, CP violating effects will be $O(100\%)$. Assuming that modes such as $K^- + n\pi$ may be tagged with a reasonable efficiency, there is a prospect that the luminosities typical of B factories will give a determination of γ to a precision of $5-20^\circ$ where the exact precision obtainable depends on unknown strong rescattering phases. If this is achieved, it could have a significant impact on the determination of CKM parameters and more importantly, it is probably the only way of directly determining γ at $Y(4s)$ B-factory experiments.

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References

1. M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* 49, 652 (1973).
2. For a review see, e.g. the article of H. Quinn in *Particle Data Book*, R.M. Barnett et al., *Phys. Rev. D* 54, 507 (1996).
3. L. Wolfenstein, *Phys. Rev. Lett.* 51, 1945 (1984).
4. M. Gronau and D. London, *Phys. Lett.* B253, 483 (1991); M. Gronau and D. Wyler, *Phys. Lett.* B265, 172 (1991); see also I.I.Y. Bigi and A.I. Sanda, *Phys. Lett.* 211B, 213 (1988) where similar ideas are discussed.
5. Several feasibility studies have been conducted. See, for instance, M. Witherell, private communication; S. Stone, *Nucl. Instrum. Meth. A* 333, 15 (1993); A. Snyder, BaBar notes # 80, #84; I. Dunietz, *Z. Phys.* C56, 129 (1992); I. Dunietz, in *B Decays*, 2nd Edition, S. Stone ed. (World Scientific, Singapore, 1994), p. 550.
6. I. Dunietz, *Phys. Lett.* B270, 75 (1991).
7. D. Atwood, A. Soni and I. Dunietz, *Phys. Rev. Lett.* 78, 3257 (1997); D. Atwood, I. Dunietz, and A. Soni, in preparation.
8. R.M. Barnett et al. (Particle Data Group), *Phys. Rev. D* 54, 1 (1996).