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## Unified Description of Kaon Electroweak Form Factors

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# Unified Description of Kaon Electroweak Form Factors

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## I. ABSTRACT

A calculation of the semi-leptonic decays of the kaon ( $K_{l3}$ ) is presented. The results are direct predictions of a covariant model of the pion and kaon introduced earlier by Ito, Buck, Gross. The weak form factors for  $K_{l3}$  are predicted with absolutely no parameter adjustments of the model. We obtained for the form factor parameters:  $f_-(q^2 = m_l^2)/f_+(q^2 = m_l^2) = -0.28$  and  $\lambda_+ = 0.028$ , both within experimental error bars. Connections of this approach to heavy quark symmetry will also be discussed.

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## II. INTRODUCTION

The success of the Ito-Buck-Gross (IBG) [1] model in the description of many properties of both the  $\pi$  and  $K$  mesons motivated the calculation of the  $K_{l3}$  decays reported on here. It is the  $K_{l3}$  decays that combine both the pion and kaon wave functions generated previously [1-3]. A successful  $K_{l3}$  calculation that is coupled to other observables, constrains further the physics described by the model.

The work reported on here is predictive and employs no new parameters and no parameter adjustment. The results, found below, are in good agreement with the data and are as good as Chiral Perturbation Theory (CPT) approaches [4] and [5], even though a low energy parameter is involved in them, and better, at least in the light-quark sector, than the Quark Potential Model (ISGW2), with a hyperfine interaction, predictions [6]. It is noted that an older version of the Quark Potential Model without hyperfine interaction (ISGW) gives results similar to ours [7]. These comparisons and the details of our calculation are presented below. Nonetheless, with the success of CPT and this work, the question still remains for nuclear physics as to singling out quark structure from hadronic. That is, where do hadrons leave off and quarks begin? The Quark Model has been very successful at reproducing hadronic static properties such as the mass spectrum and moments. But it is the dynamic properties, we feel, that will delineate the differences between hadronic physics and quark physics. For this reason, we take the position that not only is the near, or at  $q^2 = 0$  physics important but the  $q^2 \neq 0$  domain will delineate theoretical approaches. Thus, our predictions for the non-perturbative weak transition form factors as a function of  $q^2$  are also presented here in an attempt to draw both theoretical and empirical interests.

A detailed review of the theoretical and experimental status of semileptonic

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kaon decays is given in Ref. [8].

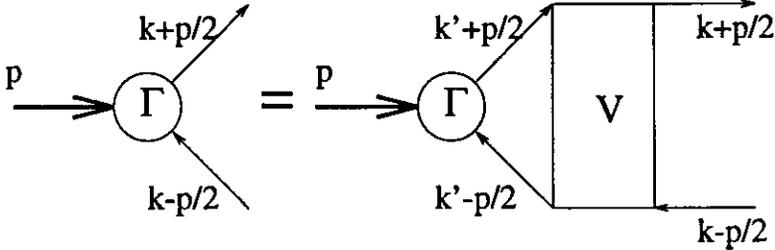
### III. THE MODEL

The theoretical model employed is an extension of the Nambu–Jona–Lasinio (NJL) model but with a definite momentum distribution generated by a separable interaction:

$$V(p, k) = g f(p^2) f(k^2) [I \otimes I - (\gamma^5 \tau) \otimes (\gamma^5 \tau)], \quad (1)$$

where  $f(p^2) = (\Lambda^2 - p^2)^{-1}$  with  $\Lambda$  being the interaction cutoff parameter for a given meson state. With this choice of the  $q\bar{q}$  interaction, one can integrate all momentum integrals to infinity; there is no need for an integral cutoff as employed in the NJL model.

The IBG model requires that the Bethe–Salpeter equation be solved for the vertex function,  $\Gamma$ , for each meson considered,



where  $p$  is the 4-momentum of the meson.

The self energy of each flavour quark is treated by solving the Schwinger–Dyson equation

$$\Sigma(k^2) = 4in_f g f(k^2) \int \frac{d^4 p}{(2\pi)^4} f(p^2) \frac{m_0 + \Sigma(p^2)}{p^2 - [m_0 + \Sigma(p^2)]^2} \quad (2)$$

where  $n_f$  is the number of quark flavours (equal to 3 in our  $q\bar{q}$  system) in a coupled sense (coupled via quark masses and interaction strength) to the Bethe–Salpeter equation. Though, in the case of the strange quark mass, the self energy is assumed to be the (constituent) quark mass and is treated as a parameter to be fixed. [3]

Electromagnetic gauge invariance is imposed on the electromagnetic current of a pseudoscalar meson

$$J^\mu = F(q^2)(p + p')^\mu, \quad (3)$$

where  $F(q^2)$  is the meson charge form factor and  $p(p')$  is the 4-momentum of initial (final) meson. The work of Buck, Williams, and Ito (BWI) [3] has shown that both the pion and kaon (charged and neutral) charge form factors can be predicted and it is the pion and kaon vertices from this work that are employed in the calculation of the weak form factors.

### IV. WEAK FORM FACTORS

In the Standard Model, the weak current for  $K_{l3}$  decays has the following structure

$$J^\mu = \frac{G_F}{\sqrt{2}} V_{us} [f_+(q^2)(P_K + P_\pi)^\mu + f_-(q^2)(P_K - P_\pi)^\mu] \quad (4)$$

where  $P_K$  and  $P_\pi$  are the kaon and pion 4-momenta,  $q = P_K - P_\pi$ , and  $f_\pm$  are dimensionless form factors.

The semi-leptonic decays studied are:

$$\begin{aligned} (K_{e3}) K^\pm &= \pi^0 e^\pm \nu_e \\ K_L^0 &= \pi^\pm e^\mp \nu_e \\ (K_{\mu 3}) K^\pm &= \pi^0 \mu^\pm \nu_\mu \\ K_L^0 &= \pi^\pm \mu^\mp \nu_\mu \end{aligned} \quad (5)$$

In the limit of exact isospin symmetry,  $m_u = m_d$ , form factors of charged and neutral kaon decays are related:

$$f_\pm^\pm / f_\pm^0 = 1/\sqrt{2},$$

and in the limit of exact SU(3) symmetry, the form factor  $f_-$  is zero. For the decay channel, the transferred 4-momentum  $q$  is time-like, and the physical region is limited to  $m_l^2 \leq q^2 \leq (m_K - m_\pi)^2$ . The vertices appearing in this weak current and the ones employed in this work are the kaon and pion vertices (wave functions) previously obtained by BWI; namely

$$\Gamma_{K,\pi}(k) = \frac{N_{K,\pi} \gamma^5}{\Lambda_{K,\pi}^2 - k^2}, \quad (6)$$

with  $N_{K,\pi}$  being the normalization.

From equation 4, one can uncouple  $f_{\pm}$ :

$$f_{\pm} = \frac{(P_K \mp P_{\pi})^2 J^{\mu} (P_K \pm P_{\pi})^{\mu} - (m_K^2 - m_{\pi}^2) J^{\mu} (P_K \mp P_{\pi})^{\mu}}{(P_K - P_{\pi})^2 (P_K + P_{\pi})^2 - (m_K^2 - m_{\pi}^2)^2} \quad (7)$$

To compare to available experimental data, the following low- $q^2$  expansion is used for the form factors:

$$f_{\pm}(q^2) = f_{\pm}(q^2 = m_i^2) \left(1 + \lambda_{\pm} \frac{q^2}{m_{\pi+}^2}\right) \quad (8)$$

where  $\lambda_{\pm}$  is the slope of  $f_{\pm}$  evaluated at  $q^2 = m_i^2$  and  $f_{\pm}(q^2 = m_i^2)$  corresponds to the normalization. Note that it is the charged, not the neutral, pion mass that enters the above expansion.

Another set of the form factor parameters commonly used in the literature is  $\lambda_+$ ,  $\lambda_0$ , arising as coefficients of linear expansion of the form factors  $f_+$  and  $f_0$ , with  $f_0$  defined as

$$f_0 = f_+ + \frac{q^2}{m_K^2 - m_{\pi}^2} f_-, \quad (9)$$

The form factors  $f_+$  and  $f_0$  describe, respectively,  $P$ -wave and  $S$ -wave projections of weak current matrix elements in the crossed channel.

To obtain the values of  $\lambda_{\pm}$ , a calculation of  $J^{\mu}$  must be performed. In this work,  $J^{\mu}$  is the direct result of a triangle diagram (Fig.1) with a flavour changing operator having  $V-A$  spin structure  $\gamma^{\mu}(1-\gamma^5)$ . In the Standard Model, the  $K_{l3}$  decay form factors are determined only by the vector part of the charged weak current operator.

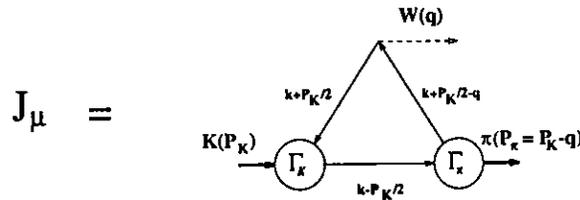


FIG. 1. Triangle diagram for the charged weak current of  $K_{l3}$  decay.

Integrals with respect to loop momentum were evaluated in the following way: In the expression for the weak current given by a Feynman diagram Fig.1, the spin trace was calculated and the terms dependent on loop momentum in the numerator were divided out by corresponding terms in the denominator. This procedure reduces the expression for 'impulse' current Fig.1 to the sum of scalar integrals of products of three to five denominator factors (three of them coming from quark propagators and two from meson- $q\bar{q}$  vertex form factors). Each denominator factor is a polynomial quadratic in the loop momentum. The terms involving three denominators, are in fact scalar 3-point functions which may be expressed analytically in terms of Spence functions [9]. In this work, to calculate the 3-point functions, we parametrized them in terms of two Feynman parameters. Integration with respect to one Feynman parameter was done analytically, and the other was a numerical Gauss integration. We did not use a low  $q^2$  expansion to evaluate loop integrals but we do extract from our results the low  $q^2$  behavior and find it is consistent with the low  $q^2$  expansion of equation 8 above, employed by all researchers.

Our task of computing 4-dimensional loop integrals of products of more than three denominators is greatly simplified when taking advantage of the fact that only two external momenta in the integrand are linearly independent. As a result, products of four and five denominators are reduced to the sum of products of three denominators with redefined masses  $M_i$ . This procedure is described by the following identity

$$\frac{1}{(k^2 - m_0^2)((k + q_1)^2 - m_1^2)((k + q_2)^2 - m_2^2)((k + q_3)^2 - m_3^2)((k + q_4)^2 - m_4^2)} = \sum_{i,j,n} \frac{a_i}{(k^2 - M_i^2)((k + q_j)^2 - M_j^2)((k + q_n)^2 - M_n^2)}, \quad (10)$$

if

$$\begin{pmatrix} q_3 \\ q_4 \end{pmatrix} = \mathbf{A} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}, \quad \det \mathbf{A} \neq 0,$$

where the sum is taken over different combinations of the external 4-momenta  $q_i$  involved in the reaction,  $m_i$  are quark masses and mass parameters in meson- $q\bar{q}$  vertex form factors,  $a_i$  are coefficients independent of loop momentum  $k$ , and  $\mathbf{A}$  is a  $2 \times 2$  matrix setting relations between external momenta in the integrand. After this reduction, scalar 3-point integrals are computed within the technique described above.

## V. RESULTS

In the physical region of  $K_{e3}$  decays,  $q^2$  may be as low as the square of electron mass,  $25 \cdot 10^{-8} \text{ GeV}^2/c^2$ , and as high as the square of the mass difference between the kaon and the pion,  $0.123 \text{ GeV}^2/c^2$ . The form factors  $f_{\pm}$  in this region with a good precision appear to be linear functions of  $q^2$  thereby justifying a linear parametrization of equation 8 usually employed in analyses of experimental data [10]. To compare our results with experiment, we extracted the slopes and ratios of the form factors  $f_{\pm}$  at  $q^2 = m_l^2$  via numerical differentiation. Numerical values for the parameters in this calculation were taken to be the same as in Ref. [3], viz.  $m_{u,d} = 250 \text{ MeV}$ ,  $m_s = 430 \text{ MeV}$ ,  $\Lambda_{\pi} = 600 \text{ MeV}$ , and  $\Lambda_K = 690 \text{ MeV}$ .

The direct predictions of our approach for  $\lambda_+$  and  $\frac{f_-}{f_+}|_{q^2=m_l^2}$  are 0.028 and  $-0.28$ , respectively. These results are to be compared to the experimental values of

$$\lambda_+ = 0.0286 \pm 0.0022 \text{ and } \xi_A = f_-/f_+ = -0.35 \pm 0.15.$$

We obtain  $\lambda_- = 0.029$ , i.e., in our model both  $f_-$  and  $f_+$  have approximately the same slopes, in agreement with early Quark Model results [12]. Our calculation for  $K_{e3}$  and  $K_{\mu 3}$  yields equal results, within the quoted precision, since the  $\lambda_{\pm}$  are almost constant in the range  $m_e^2 \leq q^2 \leq m_{\mu}^2$ . Naturally, the decay rates should be different due to phase space factors; they can be calculated by known formulas in terms for form factor slopes, see, e.g. Ref. [4]; however, we have yet to perform the calculation of these rates.

Table 1 illustrates the comparison between our work, that of CPT, vector meson dominance (VMD), and the ISGW2 model. One sees that the work reported on here compares very favorably to experiment and to CPT, except for the prediction for  $\lambda_0$ .

Table1: Model predictions for the parameters of  $K_{l3}$  decay form factors.

\* From the corresponding values of  $\lambda_+$  and  $\lambda_0$  [4]; \*\* From the corresponding values of  $\lambda_+$  and  $\xi_A(0)$ .

	CPT: Refs. [4] and [5], respectively	VMD [8]	ISGW2 [6]	This work	Experiment [10]
$\lambda_+$	0.031; 0.0328	0.0245	0.019	0.028	$0.0286 \pm 0.0022 (K_{e3})$
$\xi_A(0)$	$-0.164 \pm 0.047^*$ ; $-0.235$	-0.28	-0.28	-0.28	$-0.35 \pm 0.15 (K_{\mu 3})$
$\lambda_0$	$0.017 \pm 0.004$ ; $0.0128$	0.0	$-0.005^{**}$	0.0026	$0.004 \pm 0.007 (K_{\mu 3}^+)$ $0.025 \pm 0.006 (K_{\mu 3}^0)$

A prediction for the slope parameter  $\lambda_0$  obtained within our model is 0.0026, which is consistent with experiments on charged kaon decays ( $\lambda_0^{\pm} = 0.004 \pm 0.007$ )

and inconsistent with neutral kaon decay measurements ( $\lambda_0^0 = 0.025 \pm 0.006$ ). Since the experimental results for this slope parameter are not firm, it is hard to draw any positive conclusions about agreement or disagreement of our result for  $\lambda_0$  with experiment. However, we can compare it with predictions from other models. It can be seen from the Table 1 that the quark model in general gives much smaller numbers for  $\lambda_0$  than CPT.

To test the sensitivity of our results, an arbitrary change in the  $\Lambda_{\pi}$  cutoff from 600 MeV to 450 MeV (a 25% change), results in a  $\lambda_+ = 0.028$  to 0.031 (an almost 10% change),  $\xi_A(0)$  from  $-0.28$  to  $-0.27$  ( $\approx 5\%$  change), and  $\lambda_0$  from 0.0026 to 0.0058 ( $\approx 100\%$  change), respectively. One is reminded that changing  $\Lambda_{\pi}$  changes the pion charge radius as well as the pion decay constant. In fact, the value  $\Lambda_{\pi} = 450 \text{ MeV}$  was used in Refs.[1,2] as the best fit to pion decay constant and charge radius alone, and this parameter was adjusted to 600 MeV in Ref. [3] to be able to treat *both* pion *and* kaon in a coupled approach.

It should be noted that our model gives a stable, with respect to variation of the model parameter, prediction for  $\xi_A(0)$ , and appears to give a highly parameter-dependent result for  $\lambda_0$ . This model dependence is due to cancellation between two large terms on the r.h.s. of equation (9). The situation is different in CPT, where uncertainties due to higher-order loop corrections give rise to about 30% uncertainty for  $\xi_A(0)$ :  $-0.164 \pm 0.047$  and about 25% uncertainty for  $\lambda_0$  [4].

Finally, CPT suggests that once the ratio of the weak form factors is known, then an estimate of the mass of the strange sigma ( $m_{\sigma_K}$ ), a meson with  $J^P = 0^+$ , can be made. The relationship referred to is [5]

$$\xi_A(0) = (M_K^2 - M_{\pi}^2)(M_{\rho}^{-2} - M_{\sigma_K}^{-2}). \quad (11)$$

Taking our result for  $\xi_A(0)$  and assuming duality between our model predictions and the model with effective exchanges of vector and scalar mesons at low  $q^2$ , we have  $m_{\sigma_K} = 1.5 \text{ GeV}$ , which compares favorably to the mass of  $K_0^*(1430)$ . A test of this value could be made through the hypernuclear spectroscopy measurements (CEBAF E89-009; CEBAF PR-95-002) [13] inferring the interaction that contains this strange sigma.

Another feature of our approach is revealed in the limit as  $\Lambda_K$ ,  $m_K$ , and  $m_s$  become infinitely large. The ratio  $\xi_A(0)$  is calculated in this limit and its asymptotic value is  $-1$ ; Figure 2 illustrates the mass dependence. In the limit of Heavy Quark Symmetry (HQS), the  $q^2$  dependence of the semileptonic decay form factors is factorized out in the form of Isgur-Wise function [11], and  $\xi_A$  is given by the combination of the initial ( $M$ ) and final ( $M'$ ) meson masses,

$$\xi_A = -\frac{M - M'}{M + M'}. \quad (12)$$

As a result,  $\xi_A|_{HQS} = -1$  if the initial meson is much heavier than the final. Note that for HQS to be applied, both initial and final mesons should be heavy, whereas assuming  $m_s$  to be large in our model, we keep the final meson light. This implies that this particular result of HQS appears to be more generally applicable. Of course, the ratio is zero for mesons of equal mass.

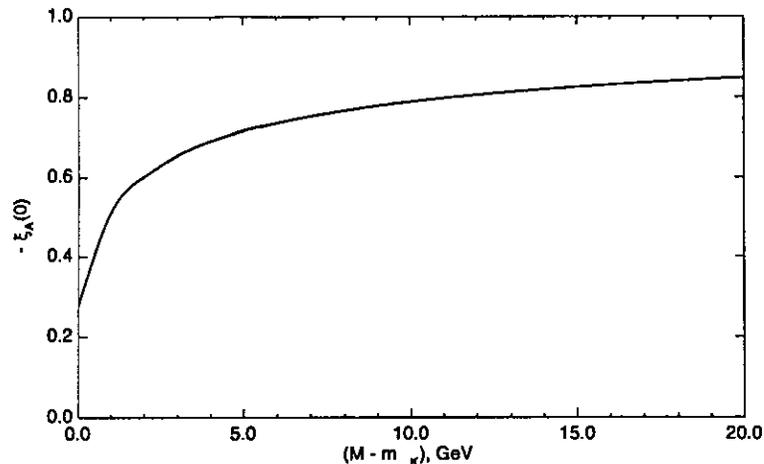


FIG. 2. Ratio  $\xi_A(0)$  as a function of initial meson mass.

Though, it is tempting to make exuberant statements in regard to the identical results, one is cautioned by the manner in which the limits are taken, and to the nature of the physics examined, respectively.

Furthermore, Figure 3 illuminates our predictions for  $f_+(q^2)$  at space-like momentum transfers describing the neutrino production processes  $\nu\pi \rightarrow lK$ ,  $\nu\pi^0 \rightarrow l\pi^+$  and corresponding weak lepton capture. We stress that no low  $q^2$  expansion was assumed in our calculations, so that present results have the same validity range in terms of  $q^2$  as results of Refs.[1-3] for electromagnetic form factors of pion and kaon. The form factor  $f_+$  at large  $q^2$  behaves as  $1/q^4$  (up to a logarithmic correction) indicating that our model effectively describes soft, nonperturbative reaction mechanism, and does not include perturbative QCD contributions.

The weak  $K \rightarrow \pi$  transition form factors in the space-like region could be

possibly accessed experimentally in the production of kaons on a hadronic target induced by neutrinos or lepton weak capture. The latter possibility is being studied for a CEBAF experiment [14].

It would be instructive to see if the earlier success of the IBG model, that include the pion and kaon observables as well as the results of this present work, can be reproduced with other interactions and/or with other wave equations; by this, it is meant the predictive characteristics associated with the low energy axial anomaly, such as the pion transition and elastic charge form factors, the kaon charge form factors, and the  $K_{l3}$  decays.

In brief summary, weak form factors and slope parameters have been calculated for  $K_{l3}$  decays. The results compare very favourably to available experimental data. The model employed was that of IBG Refs.[1-3] and there were no parameter adjustments, thus, rendering this calculation predictive.

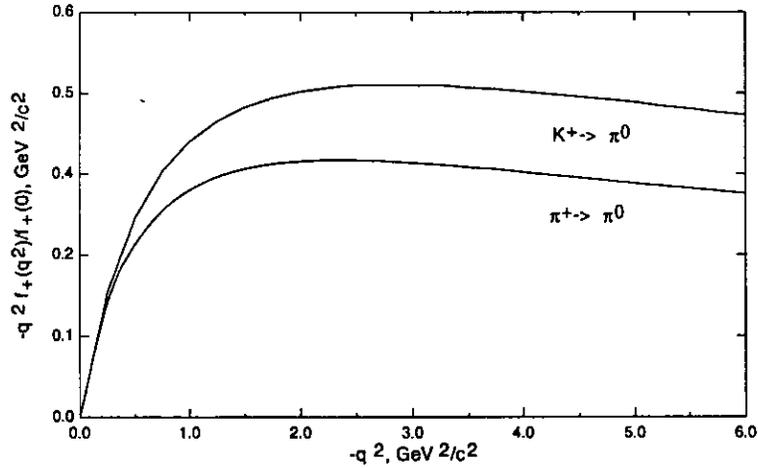


FIG. 3. Form factors of weak transitions  $K^+ \rightarrow \pi^0$  and  $\pi^+ \rightarrow \pi^0$  at space-like transferred momenta.

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