

**Meson-meson scattering:
 $K\bar{K}$ -thresholds and $f_0(980) - a_0(980)$ mixing**

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Abstract

We study the influence of mass splitting between the charged and neutral pions and kaons in the Jülich meson exchange model for $\pi\pi$ and $\pi\eta$ scattering. The calculations are performed in the particle basis, which permits the use of physical masses for the pseudoscalar mesons and a study of the distinct thresholds associated with the neutral and the charged kaons. Within this model we also investigate the isospin violation which arises from the mass splitting and an apparent violation of G -parity in $\pi\pi$ scattering which stems from the coupling to the $K\bar{K}$ channel. Nonvanishing cross sections for $\pi\pi \rightarrow \pi^0\eta$ indicate a mixing of the $f_0(980)$ and $a_0(980)$ states.

11.80.Gw, 13.75.Lb, 14.40.Cs, 13.40.Dk

I. INTRODUCTION

In strong interaction physics isospin is a nearly exact symmetry weakly broken by the slightly different masses within isospin multiplets - for example between proton and neutron, or between charged and neutral pions or kaons. For that reason it is in general a good approximation to describe baryon-baryon, meson-baryon and meson-meson scattering in an isospin basis with explicit account taken of Coulomb effects. In phase shift analyses of scattering processes isospin is taken to be a conserved quantum number.

The Jülich model for meson-meson interactions [1,2] is constructed in the same spirit, *i.e.* the mass splitting between neutral and charged pseudoscalar mesons is neglected and the scattering equation is solved in the isospin basis. The corresponding interaction kernel is based on an effective Lagrangian which includes the scalar, pseudoscalar and vector nonets. An important feature of the model is the coupling between $\pi\pi$ and $K\bar{K}$ which in the scalar-isoscalar channel ($JI=00$) gives rise to a bound $K\bar{K}$ state ($K\bar{K}$ -molecule [3]). This 'dynamical pole' is the $f_0(980)$ 'meson', which therefore strongly decays into the $K\bar{K}$ channel. The decay into $K\bar{K}$ is under experimental investigation at the cooler-synchrotron COSY [4]. Because of the high momentum resolution of COSY the thresholds from neutral and charged kaons are well separated and can be studied independently.

An appropriate theoretical description of the $K^0\bar{K}^0/K^+K^-$ threshold region has to account for their kinematical separation. In this article we therefore reformulate the Jülich $\pi\pi/K\bar{K}$ model in a basis specified by mass eigenstates which permits inclusion of the mass splitting between charged and neutral pseudoscalar mesons. These mass differences give rise to a violation of isospin. Furthermore, we also include the $\pi\eta$ channel in our calculations. It's coupling to the $K\bar{K}$ channel is crucial for the understanding of the $a_0(980)$ meson [1,3]. As it will turn out, the such constructed $\pi\pi - K\bar{K} - \pi\eta$ model not only accounts for isospin symmetry breaking effects but also leads to a mixing of states with different G -parity. This surprising result has a very simple physical origin: in the particle basis the $\pi\pi$ and $\pi\eta$ channels couple through the $K\bar{K}$ channel which does not have a definite G -parity. Intrinsically

this mixing is very small. However, as we will see, the effect is enhanced in the region of the $K\bar{K}$ -molecule by the resonance structure; this gives rise to quite a strong mixing of the $\pi\pi$ ($\pi^0\pi^0$ and $\pi^+\pi^-$) and $\pi\eta$ channel.

In the next section we briefly review the Jülich meson-meson model and give the pertinent formulas for the extension to the particle basis. The results of our calculations are presented in sect. III together with a discussion of the related physics issues. In sect. IV we summarize and address future applications and extensions.

II. THE JÜLICH MODEL OF MESON-MESON INTERACTIONS

Our starting point is the Jülich $\pi\pi$ and $\pi\eta$ model [1,2], which is constructed from an effective meson lagrangian. The corresponding potentials include t-/u-channel ρ exchange for $\pi\pi \rightarrow \pi\pi$, $K^*(892)$ exchange for $\pi\pi, \pi\eta \rightarrow K\bar{K}$ as well as ρ , ω and ϕ exchange for $K\bar{K} \rightarrow K\bar{K}$. Furthermore, s-channel polegraphs for ρ , $f_2(1270)$ and $\epsilon(1200-1400)$ formation are necessary to reproduce the resonance structures in the corresponding partial waves. The scalar-isoscalar particle ϵ is an effective description of the singlet and octet member of the scalar nonet as well as a possible glueball.

The $\pi\pi\rho$ coupling constant $g_{\pi\pi\rho}$ for t-/u-channel ρ exchange is determined by the decay width of the ρ -meson. On the other hand, s-channel resonances are renormalized by solving the scattering eq. (1) (see below), so that the corresponding bare coupling constants and masses have to be determined by reproducing experimental information. All other coupling constants are given from SU(3) symmetry relations. For the regularization of the scattering eq. (1) the vertex functions are supplemented with hadronic form factors which we choose of dipole form:

$$F(\vec{k}, \vec{k}') = \left(\frac{2\Lambda^2 - M^2}{2\Lambda^2 + (\vec{k} - \vec{k}')^2} \right)^2 \quad \text{for t-,u-channel}$$

$$F(k) = \left(\frac{2\Lambda^2 + M^2}{2\Lambda^2 + 4\omega_k^2} \right)^2 \quad \text{for s-channel ,}$$

where the cutoffs Λ are adjusted to experimental data. The parameters of the model [1] are given in tables I, II.

The scattering amplitudes are calculated by iterating the potentials in a coupled channel scattering equation which we obtain from the Blankenbecler-Sugar (BbS) reduction [5] of the Bethe-Salpeter equation. After partial wave expansion, one has for given isospin I and angular momentum J :

$$T_{\mu\nu}^{IJ}(k, k', E) = V_{\mu\nu}^{IJ}(k, k', E) + \sum_{\lambda} \int dk'' k''^2 V_{\mu\lambda}^{IJ}(k, k'', E) G_{\lambda}(k'', E) T_{\lambda\nu}^{IJ}(k'', k', E), \quad (1)$$

where $\mu, \nu, \lambda = \pi\pi, K\bar{K}$ for the $\pi\pi$ model ($\mu, \nu, \lambda = \pi\eta, K\bar{K}$ for the $\pi\eta$ model) and $E = \sqrt{s}$ is the starting energy. The BbS two-meson propagator for channel λ reads

$$G_{\lambda}(k'', E) = \frac{\omega_1 + \omega_2}{(2\pi)^3 2\omega_1\omega_2} \frac{1}{E^2 - (\omega_1 + \omega_2)^2 + i\eta}, \quad (2)$$

with $\omega_{1/2} = \sqrt{k''^2 + m_{1/2}^2}$. The on-shell momentum k_0 is defined as

$$k_0 = \frac{\sqrt{[E^2 - (m_1 + m_2)^2][E^2 - (m_1 - m_2)^2]}}{2E}. \quad (3)$$

Let us now turn to the transformation from isospin basis to particle basis. The physical particles are represented by

$$\begin{aligned} \pi^+\pi^- &= -|11\rangle|1-1\rangle \\ \pi^0\pi^0 &= |10\rangle|10\rangle \\ K^+K^- &= -|\frac{1}{2}\frac{1}{2}\rangle|\frac{1}{2}-\frac{1}{2}\rangle \end{aligned}$$

and so on. The phase convention is taken from the SU(3) representation of the pseudoscalar octet, where $|\pi^+\rangle = -|11\rangle$ and $|K^-\rangle = -|\frac{1}{2}-\frac{1}{2}\rangle$; all other particles are identified with a plus sign. The potentials are then transformed according to

$$\langle\pi^+\pi^-|V^J|\pi^+\pi^-\rangle = \sum_{I=0}^2 \langle\pi^+\pi^-|II_z\rangle\langle II_z|V^J|II_z\rangle\langle II_z|\pi^+\pi^-\rangle, \quad (4)$$

where $\langle\pi^+\pi^-|II_z\rangle$ are Glebsch-Gordan coefficients. A factor of $\sqrt{2}$ must be multiplied to matrix elements transforming isospin states of identical particles (such as $|\pi\pi\rangle$) to distinguishable particle states (e.g. $|\pi^+\pi^-\rangle$) in order to compensate the factor of $\frac{1}{\sqrt{2}}$ arising from the normalization of states of identical particles, *e.g.*

$$\begin{aligned}
\langle \pi^+ \pi^- | V^J | \pi^+ \pi^- \rangle &= \frac{2}{3} V_{\pi\pi \rightarrow \pi\pi}^{I=0,J} + V_{\pi\pi \rightarrow \pi\pi}^{I=1,J} + \frac{1}{3} V_{\pi\pi \rightarrow \pi\pi}^{I=2,J} \\
\langle \pi^+ \pi^- | V^J | \pi^0 \pi^0 \rangle &= \frac{\sqrt{2}}{3} V_{\pi\pi \rightarrow \pi\pi}^{I=0,J} - \frac{\sqrt{2}}{3} V_{\pi\pi \rightarrow \pi\pi}^{I=2,J}
\end{aligned} \tag{5}$$

and so on. As a consequence of Bose symmetry, the u-channel graphs cancel the corresponding t-channel graphs for $(-1)^{I+J} = -1$, but give the same contribution if $(-1)^{I+J} = 1$. So we include u-channel graphs by multiplying t-channel graphs with a factor of 2 and neglecting the whole process if $(-1)^{I+J} = -1$. *E.g.* for S-waves, $V_{\pi\pi \rightarrow \pi\pi}^{I=1,J=0}$ does not contribute. The transformation is completed by employing the physical masses in the coupled channel formalism.

Since isospin is no longer conserved we additionally include $\pi\eta$ states in the scattering equation. Restricting ourselves to charge neutral initial and final states leads to five channels $\mu, \nu, \lambda = \pi^0\pi^0, \pi^+\pi^-, \pi^0\eta, K^+K^-$ and $K^0\bar{K}^0$ in eq. (1).

To determine the poles of the scattering amplitude $T_{\mu\nu}$ in the complex energy plain we follow the approach of Janssen et. al. [1]. Due to the mass splitting of pions and kaons and the inclusion of $\pi\eta$ states we now have 5 right hand cuts with distinct thresholds. Thus, we extend the sheet notation using a 5 character string composed of the two letters b and t to denote whether the energy lies on the bottom or top sheet of the corresponding cut. The cuts are ordered with increasing energy, so the first, second, third, fourth and fifth character corresponds to the $\pi^+\pi^-, \pi^0\pi^0, \pi^0\eta, K^+K^-$ and $K^0\bar{K}^0$ channel, respectively.

III. NUMERICAL RESULTS AND PHYSICS ISSUES

The numerical calculations are performed in a similar way as reported in ref. [1]. The main difference is the larger number of coupled channels which have to be considered. We concentrate in our present investigation on the $K\bar{K}$ threshold region around 1 GeV where the scalar mesons $f_0(980)$ and $a_0(980)$ are located. Here the influence of the pion mass differences is negligible small. All the effects we are discussing here stem from the mass splitting between the neutral and charged kaons. In fig. 1 we show the various $\pi\pi$ scattering

cross sections for $J=0$ in the charge-neutral channels in the vicinity of the $K\bar{K}$ threshold. All three cross sections exhibit the well known interference pattern related to the $f_0(980)$ meson which, however, in the present model is a $K\bar{K}$ molecule and not a genuine $q\bar{q}$ state. The full and dashed lines represent the results obtained with and without mass splitting between the charged and neutral kaons. If the mass splitting is considered we obtain two kinks in the cross sections rather than one. These (predicted) discontinuities arise from the K^+K^- and $K^0\bar{K}^0$ thresholds and will be investigated with the cooler synchrotron COSY in the near future. This prominent two-kink structure is directly connected with a strong interaction in the $K\bar{K}$ channel which gives rise to a $K\bar{K}$ molecule. Therefore an experimental verification of this structure will strongly support the $K\bar{K}$ molecule character of the $f_0(980)$.

In fig. 2 we show the threshold behavior of the cross sections for the transitions $\pi\pi \rightarrow K^0\bar{K}^0, K^+K^-$. The full and dotted lines are the results of the full model whereas the dashed and dotted-dashed lines arise from calculations where the $K\bar{K}$ interaction is neglected. The energy dependence of these cross sections near threshold is again directly linked to the $K\bar{K}$ interaction. Therefore the analyses of experiments as *e.g.* presently performed at COSY [4] give a quite model-independent measure of the $K\bar{K}$ interaction and, correspondingly, insight into the structure of the $f_0(980)$ meson.

A quite unexpected result is shown in fig. 3 where the transition cross sections $\pi\pi \rightarrow \pi\eta$ are displayed. Such a transition is only possible if not only isospin but also G -parity is no longer a conserved quantum number. The isospin violation comes from the mass splitting of the charged and neutral kaons. The apparent violation of G -parity in $\pi\pi$ scattering actually stems from the coupling to the $K\bar{K}$ channel. Whereas pions possess $G = -1$, kaons do not have a definite G -parity because K^+, K^0 and K^-, \bar{K}^0 belong to two different isospin doublets (the G -parity transformation is defined as a rotation of 180 degrees about the y -axis in isospin space followed by charge conjugation). From fig. 3 we clearly see that the G -parity mixing (violation) is again a phenomenon which is directly connected with the resonance structure at the $K\bar{K}$ threshold. Only in the vicinity of the threshold the transition cross

section is relatively large but it rapidly ceases when moving away from the $K\bar{K}$ resonance. If we neglect the $K\bar{K}$ interaction this resonance enhancement does not show up. In fig. 4 we show the dependence of the transition cross section on the magnitude of the kaon mass splitting which is a measure for the isospin violation. As one can see, the effect depends very strongly on the magnitude of the mass splitting. As mentioned before, the isospin violation due to the mass splitting of the pions does not have any impact on this energy region.

The origin of the $K\bar{K}$ molecule can be seen most clearly by determining the pole positions of the scattering amplitude $T_{\mu\nu}$ in the complex energy plane. Here a bound state pole is a single pole on a sheet near the physical region, which moves to the real axis as the coupling to the other channels decreases. At zero coupling the bound state pole is located on the top sheet below threshold on the real axis. In our model we find two single poles at energies $[bbtt](1014.1, \pm 14.1)$ MeV and $[bbttt](1014.2, \pm 13.9)$ MeV. When neglecting the $\pi^0\eta$ channel these two poles move to the same energy $(1014.2, \pm 13.8)$ MeV on both sheets $[bbtt]$ and $[bbttt]$, *i.e.* the $\pi^0\eta$ channel has only a very minor effect on the $f_0(980)$ bound state leaving two (almost) degenerate poles. Next we remove the ϵ s-channel contribution: the two poles are mildly affected moving to $[bbtt](1020.0, \pm 59.4)$ MeV (no $\pi^0\eta$ channel included). The last step is the decoupling of the $\pi\pi$ channel, which can be achieved by continuously decreasing the $\pi\pi \rightarrow K\bar{K}$ coupling. By doing so, the pole moves below threshold onto the real axis and ends up on all $\pi\pi$ sheets at $[xytt](986.8, \pm 0.0)$ MeV ($x, y = b, t$) indicating a $K\bar{K}$ molecule.

This pole analysis in particle basis fully confirms the findings in isospin basis. The same is true for the $a_0(980)$ where we also find that it originates from the $\pi\eta \rightarrow K\bar{K}$ transition (cusp effect, see ref. [1]).

Our theoretical finding that the $K\bar{K}$ molecule can decay into $\pi\pi$ as well as into $\pi\eta$ sheds new light on the structure of the $a_0(980)$ meson. In all previous analyses the $\pi\eta$ decay was considered as an unambiguous signal for the a_0 meson. In the light of our present result this assumption seems to be no longer justified. It can well be that at least in some of the experiments the $\pi^0\eta$ decay may have had its origin from the $f_0(980)$ -'meson'. Thus, the fact

that in some experiments the width of the a_0 and f_0 appeared to be very similar could be naturally explained by our findings and would mean that in both cases the width of the $K\bar{K}$ molecule has been measured. In this respect it is interesting to note that in the calculations of ref. [1] only the $f_0(980)$ turns out to be a bound $K\bar{K}$ molecule whereas the $a_0(980)$ does not: even though the latter is strongly affected by the $K\bar{K}$ threshold, the t-channel vector meson exchanges in the $I=1$ are not attractive enough to create a bound state. Therefore the enhancement of the $K\bar{K}$ state in the isovector channel ($I=1$) is much less pronounced than in the isoscalar one ($I=0$). This means that the $\pi^0\eta$ decay of the $f_0(980)$ can simulate an $a_0(980)$ 'meson', but the $\pi\pi$ decay of the $a_0(980)$ will hardly influence the appearance of $f_0(980)$.

IV. SUMMARY

In the present article we have studied the influence of the mass splitting between charged and neutral pions and kaons in $\pi\pi$, $K\bar{K}$ and $\pi\eta$ scattering. For this purpose we have generalized the Jülich meson exchange model for meson-meson scattering: the calculations have been performed in particle basis rather than in isospin basis which permits the use of physical masses for the pseudoscalar mesons. Our interest was focused on the distinct thresholds associated with the neutral and charged kaons and on the $f_0(980)$ and $a_0(980)$ mesons which, at least in our model, are directly related to them. The two thresholds can be clearly separated in the $J=0$ $\pi\pi$ cross sections where they give rise to the well known interference pattern around 1 GeV. The structure of the corresponding pole, which is responsible for this interference and which one identifies with the $f_0(980)$ meson, is essentially unchanged compared to the calculations in isospin basis of ref. [1]: it remains a $K\bar{K}$ bound state (molecule). We have also calculated the transition cross sections $\pi\pi \rightarrow K^+K^-$ and $\pi\pi \rightarrow K^0\bar{K}^0$. The slope of these cross sections depends sensitively on the direct $K\bar{K}$ interaction. The analysis of the forthcoming data [4] will give important information on the magnitude of this interaction and, connected with it, on the structure of the scalar mesons

in this region.

Within our model we have furthermore investigated the isospin violation which arises from the mass splitting and, most interestingly, an apparent violation of G -parity in $\pi\pi$ scattering which stems from the coupling to the $K\bar{K}$ channel. The simultaneous violation of isospin and G -parity gives rise to nonvanishing cross sections for $\pi\pi \rightarrow \pi\eta$ indicating a mixing of $f_0(980)$ and $a_0(980)$ states. This has important consequences for the analysis and interpretation of the $a_0(980)$ data and it may well be that most of the $\pi^0\eta$ decay observed in this region originates from the $f_0(980)$ rather than from the $a_0(980)$ meson. In this context we are currently investigating pion production in high-energy π^-p reactions where the $f_0(980)$ and $a_0(980)$ can be seen. The results will be published elsewhere.

ACKNOWLEDGMENTS

It is a pleasure to thank Alex Dzierba for several discussions on the experimental situation which stimulated some of the present investigations. Two of us (O.K. and J.S.) wish to thank Nathan Isgur for discussions and the hospitality they enjoyed during their visits at Jefferson Lab. This work was supported in part by the US Department of Energy under contract DE-AC05-84ER40150

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FIGURES

FIG. 1. S-wave cross sections for $\pi\pi$ scattering. The solid lines are calculated using physical masses, whereas for the dotted lines average masses ($m_{\pi^0} = m_{\pi^+} = 137.273$ MeV, $m_{K^+} = m_{K^0} = 495.675$ MeV) are employed.

FIG. 2. S-wave cross sections for $\pi\pi \rightarrow K\bar{K}$. The solid/dotted line corresponds to $\pi^+\pi^- \rightarrow K^+K^-/K^0\bar{K}^0$, respectively. The dashed/dotted-dashed line shows the cross section neglecting the $K\bar{K}$ interaction. The cross sections for $\pi^0\pi^0 \rightarrow K\bar{K}$ can be obtained by multiplying $\sigma_{\pi^+\pi^- \rightarrow K\bar{K}}$ with $\frac{1}{2}$.

FIG. 3. Our prediction for the transition cross sections $\pi\pi \rightarrow \pi^0\eta$ for $J = 0$

FIG. 4. The $\pi\pi \rightarrow \pi^0\eta$ cross sections when decreasing $K\bar{K}$ interaction. We achieve the reduction of the $K\bar{K}$ interaction by multiplying the $K\bar{K} \rightarrow K\bar{K}$ potentials with a factor given in the legend.

TABLES

TABLE I. Vertex parameters for t- and s-channel (index (0)) graphs.

Vertex	g	Λ [MeV]
$\pi\pi\rho$	6.04	1355
πKK^*	$-\frac{1}{2}g_{\pi\pi\rho}$	1900
$KK\rho$	$\frac{1}{2}g_{\pi\pi\rho}$	1850
$KK\omega$	$\frac{1}{2}g_{\pi\pi\rho}$	2800
$KK\phi$	$\frac{1}{\sqrt{2}}g_{\pi\pi\rho}$	2800
ηKK^*	$-\frac{\sqrt{3}}{2}g_{\pi\pi\rho}$	3290
$\pi\pi\epsilon^{(0)}$	0.286	1850
$KK\epsilon^{(0)}$	-0.286	2500
$\pi\pi\rho^{(0)}$	5.32	3300
$KK\rho^{(0)}$	$\frac{1}{2}g_{\pi\pi\rho^{(0)}}$	2000
$\pi\pi f_2^{(0)}$	1.002	2320
$KK f_2^{(0)}$	$\frac{2}{3}g_{\pi\pi f_2^{(0)}}$	2800

TABLE II. Bare masses m_0 used in s-channel graphs (in MeV)

$\epsilon^{(0)}$	$\rho^{(0)}$	$f_2^{(0)}$
1520	1125	1665

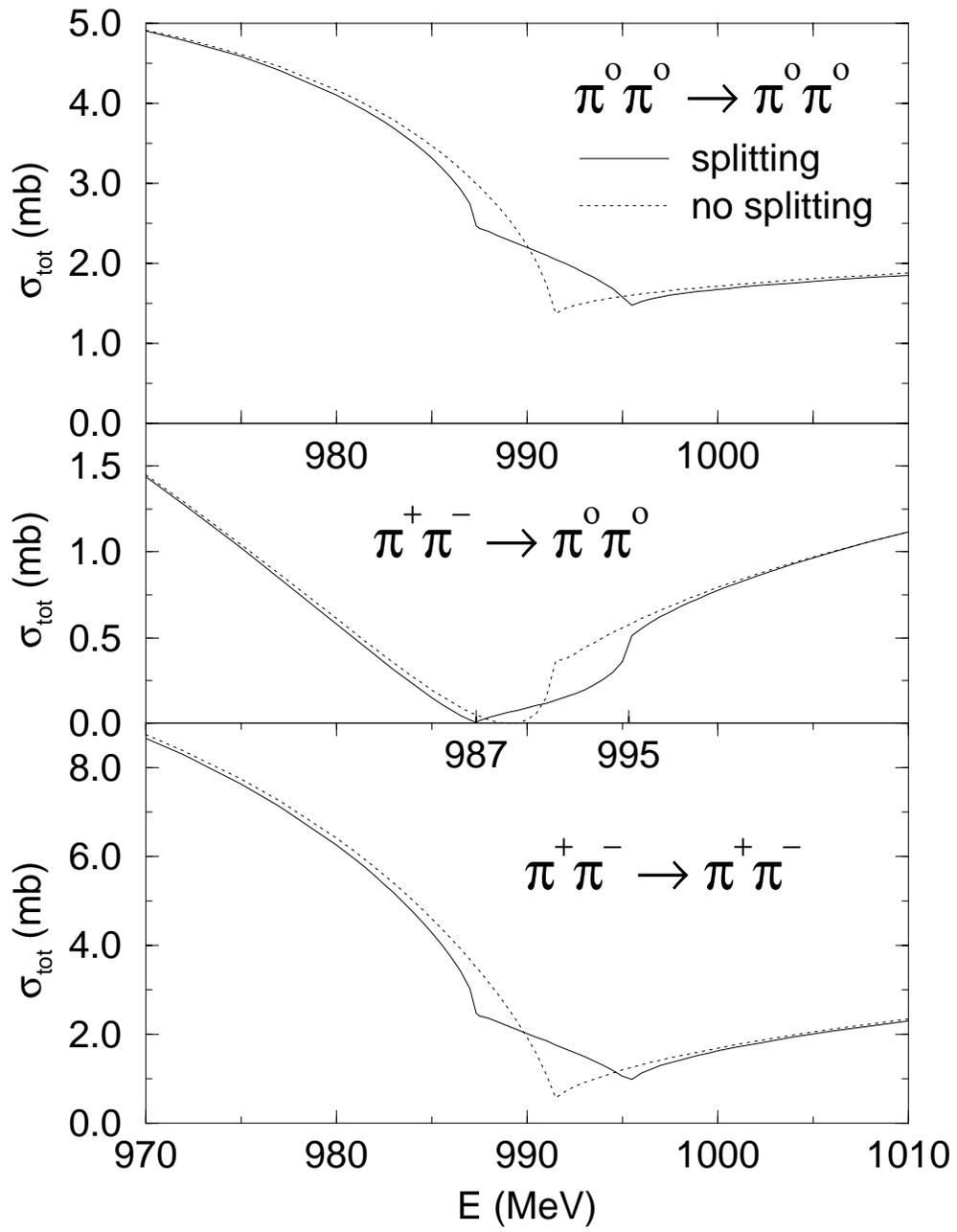


Figure 1

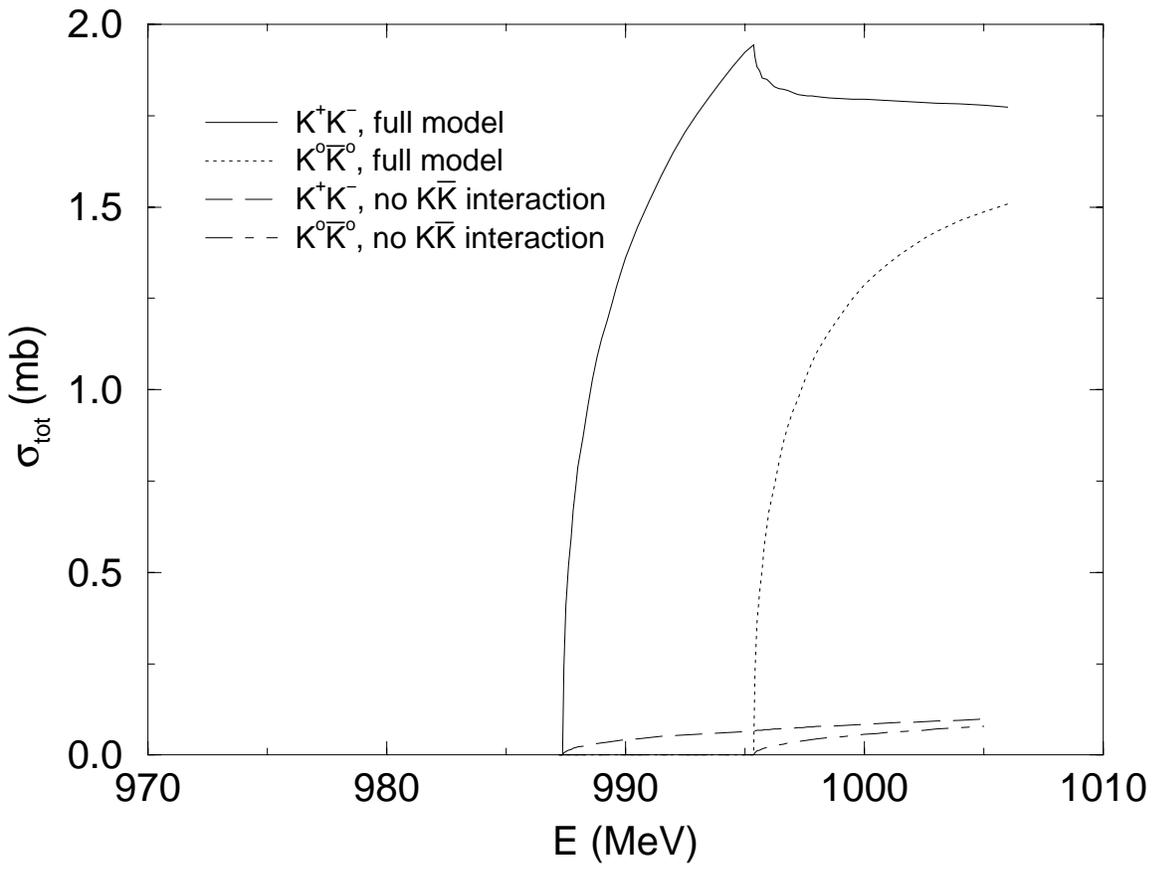


Figure 2

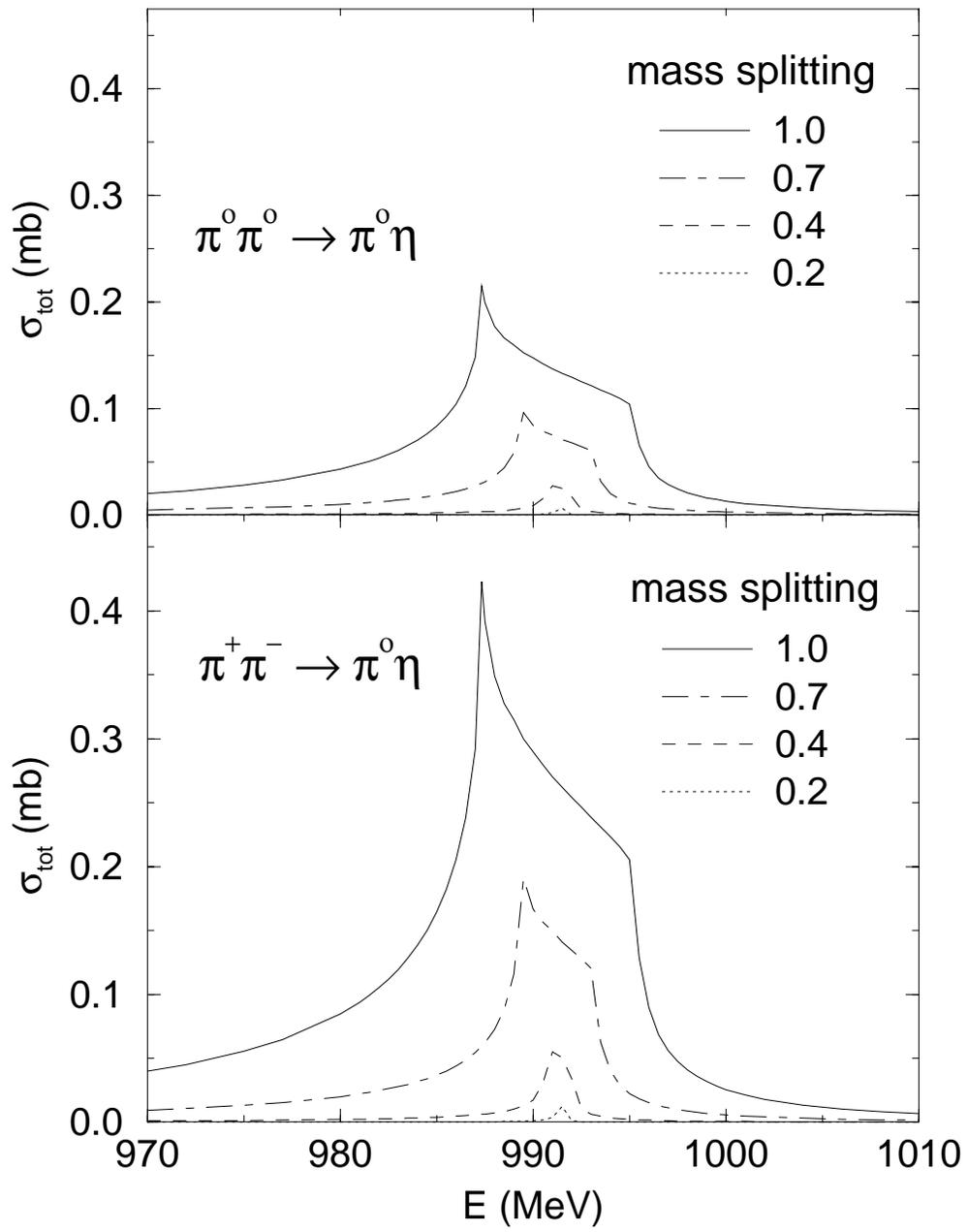


Figure 3