

**CP Conserving and CP Violating Asymmetries in $e^+e^- \rightarrow t\bar{t}\nu_e\bar{\nu}_e$
at the NLC: Application to Determining the Higgs Width**David Atwood^a and Amarjit Soni^b*a)* Theory Group, CEBAF, Newport News, VA 23606*b)* Theory Group, Brookhaven National Laboratory, Upton, NY 11973

Abstract: The polarization asymmetries of t, \bar{t} quarks can be used in the reaction $e^+e^- \rightarrow t\bar{t}\nu_e\bar{\nu}_e$ to measure the Higgs width, in the Standard Model or in its extensions, and to search for non-standard CP violating phases. As an application of the CPT theorem the Higgs width is monitored through a CP-even, T_N -odd, polarization asymmetry, B_y . CP violation manifests through interference at tree graph level with the resonant Higgs amplitude. Consequently, the asymmetries are all quite sizable and can be in the range of 10–50% for a wide choice of the Higgs mass.

A high energy e^+e^- collider, called the Next Linear Collider (NLC), with center of mass (cm) energies ranging from $\sqrt{s} = 0.5\text{--}1.5$ TeV has been receiving considerable attention in the last few years [1]–[3]. The clean environment that it should possess endows it with a unique ability to probe detailed issues pertaining to dynamics of symmetry breaking, flavor violations, CP nonconservation etc. Thus such a facility should nicely complement the physics reach of the Large Hadron Collider (LHC).

In this work we discuss how the reaction

$$e^+ + e^- \rightarrow t\bar{t}\nu_e\bar{\nu}_e \quad (1)$$

can be used for confronting two important issues, namely, extracting the width of the Higgs particle(s) and searching for non-standard CP violating phases. The key point is that due to its large mass the top quark does not bind into hadrons [4]. Decays of the top quark then act as very effective analyzer of its spin [5]. This ability to track the top spin allows us to construct CP-even and CP-odd observables utilizing the top polarization. CP even observables that are odd under naive time reversal [6] (T_N) have the important property, that follows from the CPT theorem of Quantum Field Theory, that they are driven by the absorptive part of Feynman amplitudes. In the process under consideration, i.e. reaction (1), the absorptive part of the amplitude is proportional to the width of the Higgs particle thus allowing the possibility of experimentally measuring the Higgs width. Since the Higgs width is an important characteristic of the nature of the electroweak symmetry breaking mechanism its measurement is clearly significant. In addition, of course, CP odd observables, utilizing the top spin can be readily constructed to enable us to probe the presence of non-standard CP-violating phases that reside in the neutral Higgs sector [7] in extensions of the Standard Model (SM), say in a two Higgs doublet model (2HDM).

A distinctive feature of reaction (1) that we exploit is that CP violation can manifest itself by interference of the Higgs resonance in tree amplitudes. The resulting asymmetries are substantial [8, 9]. The CP violating asymmetries that are of interest can be in the range of tens of percents, quite often 10–50%. The CP conserving, T_N -odd asymmetry can also be as big as 40%. This is especially striking as this asymmetry is a feature of the SM itself and does not require non-standard physics; it has a very important application to determining the Higgs width. Of course, measurements of the Higgs width will serve to elucidate whether the Higgs is standard or not.

For convenience we work in the analog of the equivalent photon approximation, i.e. the equivalent W -boson approximation. At large c.m. energies i.e. as s/M_W^2 becomes very large, s being the total energy squared in the e^+e^- c.m. frame, the cross section for reaction (1) is dominated by the collisions of the longitudinally polarized W 's [10]. In this approximation reaction (1) can be replaced by a simpler reaction, i.e. the W -boson fusion process:

$$W^+ + W^- \rightarrow t + \bar{t} \tag{2}$$

Indeed, the salient features of the underlying physics can be succinctly stated

in the context of reaction (2). Furthermore, although the expression for the cross section thus deduced is accurate only in the leading log (in s/M_W^2) approximation, the asymmetries that are of more central importance to this work hold to a much better accuracy as they result from the ratios of cross-sections.

To lowest order there are four Feynman graphs, shown in Fig. 1, relevant to reaction (2). The blob in Fig. 1(a) indicates that the propagator of the Higgs resonance is highly unstable i.e. it possesses a non-negligible width which can in fact be a substantial fraction of its mass depending on what the mass is. It is this Breit-Wigner nature of the scalar propagator in Fig. 1 that endows the Feynman amplitudes for reactions (1) or (2) to have an absorptive part that is of special interest to us [11].

Consider now the t, \bar{t} polarization asymmetries. In the rest frame of the t let us define the basis vectors: $-e_z \propto (\vec{p}_{W^+} + \vec{p}_{W^-})$; $e_y \propto \vec{p}_{W^+} \times \vec{p}_{W^-}$ and $e_x = e_y \times e_z$. Let P_j (for $j = x, y$ or z) be the polarization of the t along e_x, e_y, e_z . For the anti-top we use a similar set of definitions in the \bar{t} frame related by charge conjugation: $-\bar{e}_z \propto (\vec{p}_{W^-} + \vec{p}_{W^+})$; $\bar{e}_y \propto \vec{p}_{W^-} \times \vec{p}_{W^+}$ and $\bar{e}_x = \bar{e}_y \times \bar{e}_z$. Similarly \bar{P}_j the polarization of the \bar{t} is in the $\bar{e}_x, \bar{e}_y, \bar{e}_z$ direction. Combining the information from t, \bar{t} system we may define the following asymmetries:

$$\begin{aligned}
 A_x &= \frac{1}{2}(P_x + \bar{P}_x) \quad ; \quad B_x = \frac{1}{2}(P_x - \bar{P}_x) \\
 A_y &= \frac{1}{2}(P_y - \bar{P}_y) \quad ; \quad B_y = \frac{1}{2}(P_y + \bar{P}_y) \\
 A_z &= \frac{1}{2}(P_z + \bar{P}_z) \quad ; \quad B_z = \frac{1}{2}(P_z - \bar{P}_z)
 \end{aligned}
 \tag{3}$$

Here A 's are CP-odd and B 's are CP-even, also $\{A_x, B_y, A_z\}$ are CPT_N -odd whereas $\{B_x, A_y, B_z\}$ are CPT_N -even. Thus $\{A_x, B_y, A_z\}$ are proportional to the absorptive part of the Feynman amplitude which receives dominant contribution from the Higgs exchange graph of Fig. 1a, particularly if m_H is large. The other two CP even observables, B_x and B_z , are not relevant to our discussion as they do not receive contributions from an absorptive phase. Of course, all the CP odd observables are important; they all require CP violating phase(s) in the underlying Lagrangian. A_y is driven by the real part of Feynman amplitudes and A_x and A_z again, by the imaginary part and thus proportional to the Higgs width.

It is important to stress that since B_y is CP conserving it can be used for determining the width of Higgs particle(s) in the SM as well as its extensions. Indeed it should be useful for scalar as well as pseudoscalar Higgs particles.

Let us now consider, in some generality, models of CP violation based on the use of an extended Higgs sector. As is well known such models require at least two Higgs doublets [7]. A feature of many such models which has important bearing on the phenomenological implications of CP violation is that when the masses of all the Higgs states are degenerate then CP violation effects due to the Higgs sector must vanish. This means that in high energy processes, such as $W^+ + W^- \rightarrow t\bar{t}$, as the c.m. energy becomes much larger than the masses of all the Higgs particles, then the total contribution to CP violation from all the Higgs must necessarily vanish. For instance the 2HDM in [7] has this feature.

In order to produce sample numerical calculations for CP violating effects where this type of cancelation is built in on the one hand while on the other hand the number of free parameters to be considered is still small, we will assume that there are n neutral scalars and that k of them are degenerate with mass m_H while the remaining $n - k$ are degenerate with mass m'_H where $m_H < m'_H$. For perturbation theory to remain valid we must also require that $m_{H'} \lesssim 1$ TeV. Within the states which are degenerate at m_H one can in general perform an orthogonal rotation so that only one has a coupling to the top quark term proportional to $\bar{t}i\gamma_5 t$. We will denote this Higgs state H . Likewise one may perform an orthogonal rotation among the $n - k$ states with mass m'_H so that only one of these states has a coupling proportional to $\bar{t}i\gamma_5 t$ which we will denote H' . The remaining $n - 2$ Higgs states in the model, denoted by h_i for $i = 1 \rightarrow (n - 2)$, have CP conserving interactions. All of the CP violation relevant to $WW \rightarrow t\bar{t}$ is thus controlled by H and H' .

The Lagrangian terms involving the Higgs coupling to $t\bar{t}$ and WW can be written as:

$$\begin{aligned} \mathcal{L}_{Htt} &= -\frac{g_W}{2} \frac{m_t}{m_W} H \bar{t} (a_H + ib_H \gamma^5) t & \mathcal{L}_{HWW} &= g_W c_H m_W g^{\mu\nu} H W_\mu^+ W_\nu^- \\ \mathcal{L}'_{Htt} &= -\frac{g_W}{2} \frac{m_t}{m_W} H' \bar{t} (a'_H + ib'_H \gamma^5) t & \mathcal{L}'_{HWW} &= g_W c'_H m_W g^{\mu\nu} H' W_\mu^+ W_\nu^- \\ \mathcal{L}_{h(i)tt} &= -\frac{g_W}{2} \frac{m_t}{m_W} h_i \bar{t} a_{h(i)} t & \mathcal{L}_{h(i)WW} &= g_W c'_{h(i)} m_W g^{\mu\nu} h_i W_\mu^+ W_\nu^- \end{aligned} \quad (4)$$

For the cancelation to apply the above couplings are subject to the constraints

that $b_H c_H + b'_H c'_H = 0$. The corresponding Feynman rules can be easily derived and the amplitudes and expectation values for the observables of interest $\{A_x, A_y, A_z, B_y\}$ can be calculated in the standard manner. From Eqn. (4) we see, as is also well known, that the Htt vertex violates CP due to the simultaneous presence of the scalar and pseudoscalar interactions.

Specifically in the 2HDM in [7] the neutral Higgs states ϕ_j for $j = 1 - 3$ have couplings

$$a_j = d_{2j}/\sin\beta \quad b_j = -d_{3j}/\tan\beta \quad c_j = d_{1j}\cos\beta + d_{2j}\sin\beta \quad (5)$$

where d_{ij} forms an orthogonal 3×3 matrix. For our numerical examples with CP violation we will identify $H = \phi_1$, $H' = \phi_2$, $h_1 = \phi_3$ (where we let h_1 and H' be degenerate so $k = 1$) and $d_{31} = -d_{12} = \sin\beta'$; $d_{32} = d_{11} = -\cos\beta'$; $d_{23} = 1$ with the other components being 0. In addition we will use the value $\tan\beta = 1/2$ and set $\beta' = \beta$ [12].

We compute the asymmetries as a function of $\sqrt{\hat{s}}$ for various Γ_H, m_H . Note that, we assume that Γ_H, Γ'_H should be mostly given by decays to $t\bar{t}$, WW and ZZ :

$$\Gamma_H \simeq \Gamma \equiv \Gamma_{H \rightarrow t\bar{t}} + \Gamma_{H \rightarrow WW} + \Gamma_{H \rightarrow ZZ} \quad (6)$$

To allow for the presence of modes other than the above three we express the width as:

$$\Gamma_H = \lambda_H \Gamma \quad (7)$$

We now present some of the numerical results. Figs. 2 and 3 show the four asymmetries of interest to us. Fig. 2 shows the asymmetries as a function of $\sqrt{\hat{s}}$ for a fixed m_H whereas in Fig. 3 they are shown as a function of m_H for a fixed s . As an illustration the Higgs masses are held fixed in Fig. 2 at $m_H = 500\text{GeV}$ and $m'_H = 1000\text{GeV}$ and we use the couplings with $\tan\beta$ in the version of the model of [7] described above. We also show the CP conserving T_N -odd asymmetry that occurs in the SM, i.e. B_y^{SM} . For this purpose we, of course, use the SM couplings, $a_H = c_H = 1$, $b_H = 0$.

From Fig. 2 we see that the asymmetries tend to be fairly large ranging from about 10 to 50% for a wide range of values of $\sqrt{\hat{s}}$. The CP conserving, T_N -odd asymmetry, B_y , that is proportional to the Higgs width is also appreciable. For the SM it can be as big as about 40%.

In the leading log approximation that we are using the cross section for reaction (1) and the corresponding asymmetries can be readily calculated in the e^+e^- cm frame. Fig. 3 shows the four relevant asymmetries for $\sqrt{s} = 1.5$ TeV; we fix $m'_H = 1000\text{GeV}$. We see that for a wide range of Higgs masses the asymmetries are appreciable. Indeed A_z approaches 25% for $m_H \sim 500$ –700 GeV whereas A_y tends to be large, i.e. around 30%, when m_H is in the range 100–300 GeV. For the SM, the CP conserving asymmetry (B_y^{SM}) is around 10–30% for $m_H \gtrsim 100$ GeV.

Even with an ideal detector it is, of course, not possible to measure the polarization of a top quark with 100% precision. Let us here consider two possible modes (and their conjugates in the case of \bar{t}) useful for polarimetry:

1. The decay $t \rightarrow W^+b$ with $W^+ \rightarrow \ell^+\nu$, where $\ell = e, \mu$. In this case we will include only the hadronic decays of the \bar{t} to avoid problems in reconstruction.
2. The decay $t \rightarrow W^+b$ with $W^+ \rightarrow \text{hadrons}$. Now we exclude the decay of \bar{t} to a τ^- to avoid problems due to reconstruction.

Case (1) occurs with a branching ratio of about $B_1 = (2/9)(2/3) = 4/27$ where $(2/9)$ is the probability that the t decays to $b\ell\nu$ or $b\mu\nu$ and $(2/3)$ is the probability that the \bar{t} decays hadronically. If the top quark has polarization P in a given direction then, in general, the angular distribution of the lepton is $\propto (1 + R_1 P \cos \eta_l)$ where η_l is the angle between the polarization axis and the lepton momentum in the top frame. For top decays in the SM, $R_1 = 1$. The optimal (in the sense that it minimizes statistical error) method to obtain the value of P is to use $P^{(1)} = 3 \langle \cos \eta_l \rangle / R_1$ where $P^{(1)}$ therefore is the polarization extracted using this method.

Likewise in case (2) the distribution of the W momentum in the top frame is $\propto (1 + R_2 P \cos \eta_W)$ where, in the SM, $R_2 = (m_t^2 - 2m_W^2)/(m_t^2 + 2m_W^2)$ and in this case $B_2 = (2/3)(8/9) = 16/27$; where $(2/3)$ is the probability that the t decays hadronically and $(8/9)$ is the probability that the \bar{t} does not decay to a τ . Here η_W is the angle between the momentum of the W and the polarization axis. To obtain the polarization from this mode we can use $P^{(2)} = 3 \langle \cos \eta_W \rangle / R_2$.

The above two cases may be combined to obtain $P^{12} = (B_1 R_1^2 P^{(1)} + B_2 R_2^2 P^{(2)}) / (B_1 R_1^2 + B_2 R_2^2)$. Bearing in mind that the asymmetries A_i, B_i

are combinations of the t and \bar{t} polarizations and that in each event we can potentially measure the polarization of a t and a \bar{t} , the number of events needed to obtain a $3\text{-}\sigma$ signal is

$$N_{t\bar{t}}^{3\sigma} = \frac{27}{2}(R_1^2 B_1 + R_2^2 B_2)^{-1} a^{-2} \quad (8)$$

where a is the asymmetry in question (either A_i or B_i). Numerically then $N^{3\sigma} \approx 52a^{-2}$ requiring some 5200 events for an asymmetry of 10%.

Fig. 4 shows the cross-section as a function of m_H for $\sqrt{s} = 1.5$ TeV. We see that the typical cross-section tends to be a few fb . For example, for $m_H = 500$ GeV, it is around $5fb$ for the SM and can be about $15fb$ in a 2HDM with the couplings described above. At $\sqrt{s} = 1.5$ TeV the projected luminosity is about $5 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ [1]–[2]. Thus a cross section of $10fb$ would yield about 5000 events rendering it feasible to detect asymmetries $\gtrsim 10\%$.

To summarize, the reaction $e^+e^- \rightarrow \nu_e \bar{\nu}_e t\bar{t}$ can be a very powerful probe of CP violation. In models with an extended Higgs sector appreciable asymmetries result through interference of the Higgs resonance with tree graph amplitudes. It can also be very useful for extracting the Higgs width in the Standard Model or in its extensions.

This research was supported in part by the U.S. DOE contracts DC-AC05-84ER40150 (CEBAF) and DE-AC-76CH00016 (BNL).

References

- [1] Proceedings of the Workshop on Physics and Experiments with Linear e^+e^- Colliders, eds. F. Harris, S. Olsen, S. Pakvasa and X. Tata, World Scientific, Singapore, 1996.
- [2] Proceedings of the Workshop on Physics and Experiments with Linear Colliders, eds. A. Miyamoto and Y. Fujii, World Scientific, Singapore, 1996.
- [3] H. Muryama and M. Peskin, preprint SLAC-PUB-7149, hep-ex/9606003.
- [4] I. Bigi, V. Dokshitzer, V. Khoze, J. Kühn and P.M. Zerwas, Phys. Lett. **B181**, 157 (1986).
- [5] C.R. Schmidt and M. Peskin; G.L. Lane, G.A. Ladinsky and C.P. Yuan, Phys. Rev. D**45**, 124 (1992); D. Atwood, A. Aeppli, and A. Soni, Phys. Rev. Lett. **69**, 2754 (1992).
- [6] For the purpose of self-containment we should mention that T_N (naive time reversal) differs from the true time reversal operator in that the latter requires switching of initial and final states which the former does not.
- [7] See, e.g. W. Bernreuther, T. Schröder, and T.N. Pham, Phys. Lett. **B279**, 389 (1992).
- [8] Another example of substantial CP asymmetries resulting from tree graphs is the process $e^+e^- \rightarrow t\bar{t}H^0$; see S. Bar-Shalom, D. Atwood, G. Eilam, R. Mendel and A. Soni, Phys. Rev. D**53**, 1162 (1996). See also J. Gunion, B. Grzadkowski and X-G. He, preprint UCD-96-14.
- [9] Throughout this paper CP violation effects due to the CKM phase of the SM will be neglected as they are expected to be extremely small.
- [10] R. Cahn and S. Dawson, Phys. Lett. **136B**, 196 (1984); **136B**, 464 (E) (1984); S. Dawson, Nucl. Phys. B**249**, 42 (1985); M. Chanowitz and M.K. Gaillard, Phys. Lett. **142B**, 85 (1984); G.L. Kane, W.W. Repko and W.B. Rolnick, *ibid* **148B**, 367 (1984); R. Kauffman, Phys. Rev. D**41**, 3343 (1989).

- [11] CP conserving absorptive parts can, of course, also arise through radiative corrections of Figs. 1b and 1c even in the absence of the Higgs exchange graph in Fig. 1a. For example, gluon exchanges will render the $WW \rightarrow t\bar{t}$ amplitude complex at one loop. We are taking the view that these provide a calculable, and most likely, subdominant correction to the dominant effect, namely due to the Higgs width, especially for $m_H \gtrsim 100$ GeV, that is our primary concern here.
- [12] There is little change in the asymmetries if $\tan \beta = 1$ is used instead.

Figure Captions

Figure 1: The Feynman diagrams that participate in the sub-process $W^+W^- \rightarrow t\bar{t}$. The blob in Fig. 1a represents the width of the Higgs resonance and the cut across the blob is to indicate the imaginary part.

Figure 2: A graph of the asymmetries A_x (solid); A_y (dashes); and A_z (dots) as a function of $\sqrt{\hat{s}}$ given $m_H = 500$ GeV $m'_H = 1000$ GeV and the coupling parameters for $\tan\beta = 1/2$ as described in the text. The dash-dot curve is the asymmetry B_y^{SM} for the standard model couplings $a_H = c_H = \lambda_H = 1$ and $b_H = 0$ with no H' present.

Figure 3: A graph of the asymmetries integrated over \hat{s} as a function of m_H for $\sqrt{s} = 1500$ GeV and $m'_H = 1000$ GeV. See Fig. 2 for notations.

Figure 4: Cross section (in picobarns) as a function of m_H for $\sqrt{s} = 1.5$ TeV. Solid is for SM with $a_H = 1 = c_H$, $b_H = 0$, dashed is for 2HDM with the couplings described in the text. The dotted line is for a 2HDM with $a_H = -1$, $c_H = 1$; $b_H = 0$ while $C'_H = 0$. For all three cases $\lambda_H, \lambda'_H = 1$ is assumed.

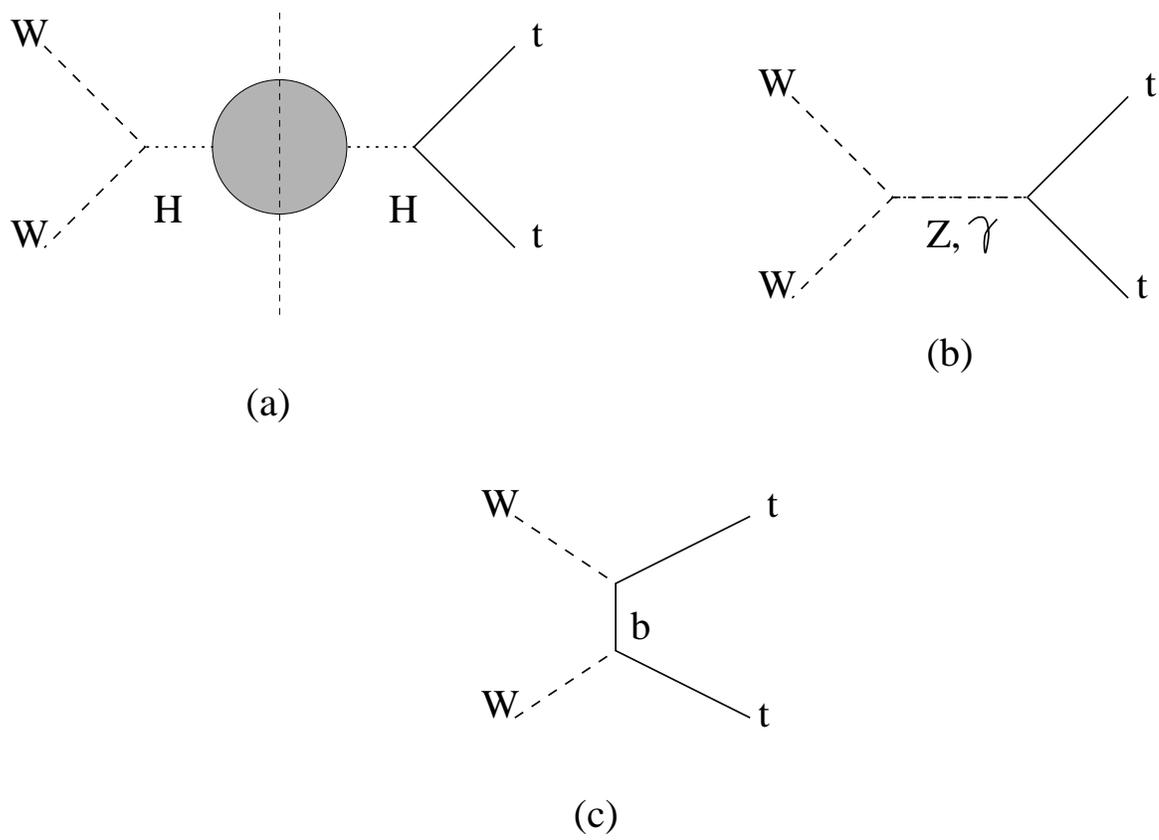


Figure 1

Figure 2

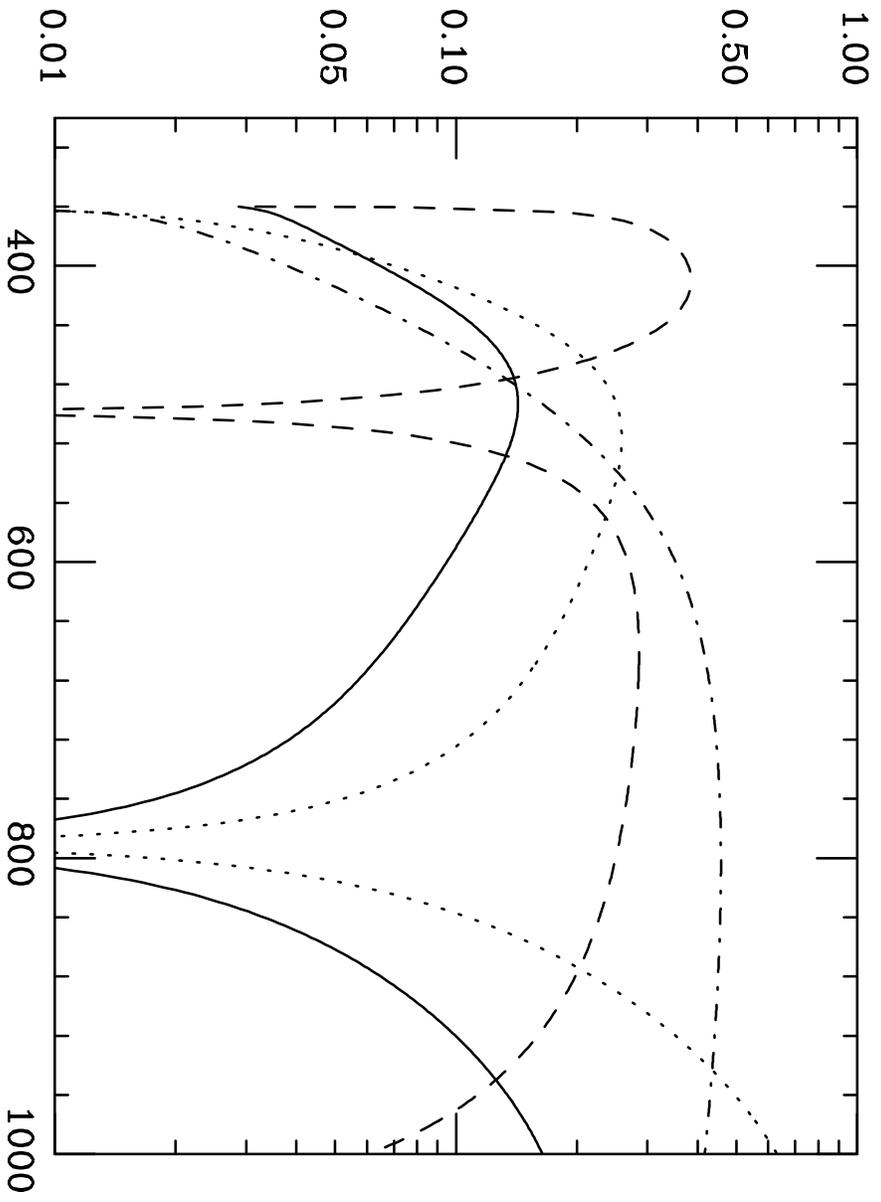


Figure 3.

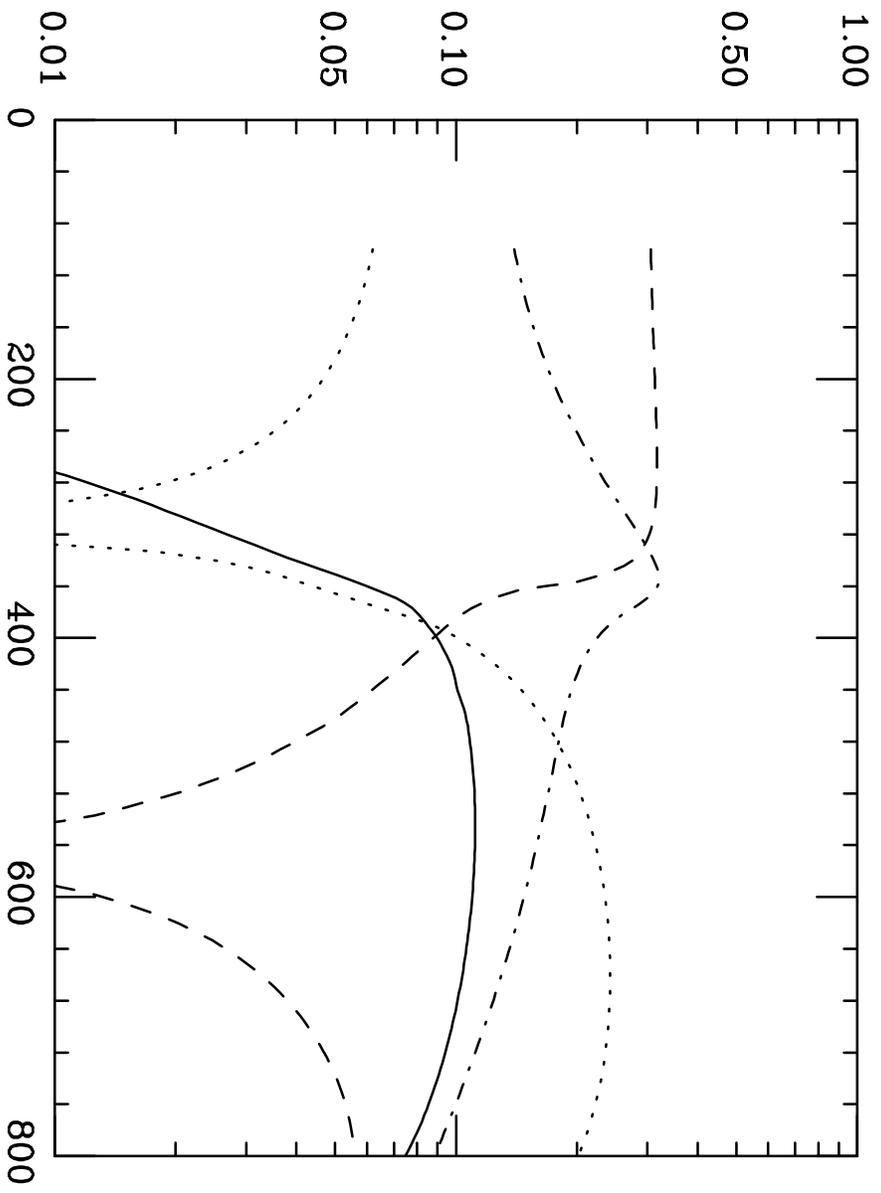


Figure 4

