

Additional copies are available from the authors.

The Southeastern Universities Research Association (SURA) operates the Continuous Electron Beam Accelerator Facility for the United States Department of Energy under contract DE-AC05-84ER40150

## Scaling Limit of Deeply Virtual Compton Scattering

A.V. RADYUSHKIN\*

*Physics Department, Old Dominion University,  
Norfolk, VA 23529, USA*

*and*

*Continuous Electron Beam Accelerator Facility,  
Newport News, VA 23606, USA*

### Abstract

I outline a perturbative QCD approach to the analysis of the deeply virtual Compton scattering process  $\gamma^* p \rightarrow \gamma p'$  in the limit of vanishing momentum transfer  $t = (p' - p)^2$ . The DVCS amplitude in this limit exhibits a scaling behavior described by a two-argument distributions  $F(x, y)$  which specify the fractions of the initial momentum  $p$  and the momentum transfer  $r \equiv p' - p$  carried by the constituents of the nucleon. The kernel  $R(x, y; \xi, \eta)$  governing the evolution of the non-forward distributions  $F(x, y)$  has a remarkable property: it produces the GLAPD evolution kernel  $P(x/\xi)$  when integrated over  $y$  and reduces to the Brodsky-Lepage evolution kernel  $V(y, \eta)$  after the  $x$ -integration. This property is used to construct the solution of the one-loop evolution equation for the flavor non-singlet part of the non-forward quark distribution.

#### DISCLAIMER

This report was prepared as an account of work sponsored by the United States government. Neither the United States nor the United States Department of Energy, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, mark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States government or any agency thereof.

---

\*Also Laboratory of Theoretical Physics, JINR, Dubna, Russian Federation

1. *Introduction.* Recently, X. Ji [1] suggested to use the deeply virtual Compton scattering (DVCS) to get information about some parton distribution functions inaccessible in standard inclusive measurements. He considers the non-forward light-cone matrix elements

$$\langle P - r/2 | \bar{q}(-\lambda n/2) \{1, \gamma_5\} \gamma^\mu q(\lambda n/2) | P + r/2 \rangle, \quad (1)$$

which appear in the lowest-order pQCD contribution to the DVCS amplitude ( $r$  is the momentum transfer and  $n$  a light-like 4-vector) and parameterizes them using the functions  $H(x, r^2, r \cdot n)$ , etc., with  $x$  being the Fourier conjugate variable to  $\lambda$ . He observes that, in the  $r \rightarrow 0$  limit, the matrix element (1) defines the usual (forward) distribution functions like  $f(x)$ ,  $g_1(x)$  and proposes that the DVCS process can be used to get information about such functions. Since the kinematics of the DVCS requires that  $r \neq 0$ , the  $r^2 \equiv t \rightarrow 0$  limit can be accessed only by extrapolating the small- $t$  data to  $t = 0$ . The limit  $r \rightarrow 0$  looks even more tricky since  $(r \cdot n) = 0$  formally corresponds to vanishing of the Bjorken variable  $x_{Bj}$ . Anyway, as emphasized by Ji [1], the DVCS amplitude has a scaling behavior in the region of small  $t$  and fixed  $x_{Bj}$  which makes it a very interesting object on its own ground.

In this letter, I briefly describe an alternative pQCD formalism for the analysis of the DVCS amplitude in the limit when  $t \rightarrow 0$  and  $x_{Bj}$  is fixed. My main point is that, to construct a consistent pQCD picture for the scaling limit of DVCS, one should treat the initial momentum  $p$  and the momentum transfer  $r$  on equal footing by introducing non-forward distributions  $F(x, y)$ , which specify the fractions of  $p$  and  $r$ , *resp.*, carried by the constituents of the nucleon<sup>†</sup>. These distributions have hybrid properties: they look like distribution functions with respect to  $x$  and like distribution amplitudes with respect to  $y$ . Writing the matrix element of a composite operator in terms of the non-forward distributions is the first step in developing the pQCD parton picture for the DVCS. The next step is to take into account the logarithmic scaling violation. To this end, I write down the evolution equation for the simplest case of the flavor non-singlet part of the non-forward quark distribution  $F^{NS}(x, y; \mu)$ . As one may expect from the preceding description, the relevant evolution kernel  $R(x, y; \xi, \eta)$  has a remarkable property: it produces the GLAPD evolution kernel  $P^{NS}(x/\xi)$  [2–4] when integrated over  $y$ , while integrating  $R(x, y; \xi, \eta)$  over  $x$  gives the expression coinciding with the Brodsky-Lepage evolution kernel  $V(y, \eta)$  [5] for the pion distribution amplitude<sup>‡</sup>. Using these properties of the kernel, I construct the solution of the one-loop

<sup>†</sup>I found no advantage in introducing the average nucleon momentum  $P = (p + p')/2$  like in eq. (1)

<sup>‡</sup>Originally, the evolution equation for the pion distribution amplitude in QCD was

evolution equation for the flavor non-singlet part of the non-forward distribution. I also discuss the infrared sensitivity of the DVCS amplitude due to the presence of the light-like momentum  $q$  and its implications for the structure of the simplest radiative and higher-twist corrections to the leading-twist result.

2. *Non-forward distributions.* The kinematics of the amplitude of the process  $\gamma^* p \rightarrow \gamma p'$  can be most conveniently specified by the initial nucleon momentum  $p$ , the momentum transfer  $r = p - p'$  and the momentum  $q$  of the final photon. Since  $q^2 = 0$ , it is natural to use  $q$  as one of the basic light-cone (Sudakov) 4-vectors. In the scaling limit, the invariant momentum transfer  $t \equiv r^2$  and the square of the proton mass  $m_p^2 = p^2$  can be neglected compared to the virtuality  $-Q^2 \equiv (q - r)^2$  of the initial photon and the energy invariant  $p \cdot q \equiv m_p \nu$ . Thus, we set  $p^2 = 0$  and  $r^2 = 0$ , and use  $p$  as another basic light-cone 4-vector. Furthermore, in this limit, the requirement  $p'^2 \equiv (p + r)^2 = p^2$  reduces to the condition  $p \cdot r = 0$  which can be satisfied only if the two lightlike momenta  $p$  and  $r$  are proportional to each other:  $r = \zeta p$ , where  $\zeta$  coincides with the Bjorken variable  $\zeta = x_{Bj} \equiv Q^2/2(p \cdot q)$  which satisfies the constraint  $0 \leq x_{Bj} \leq 1$ .

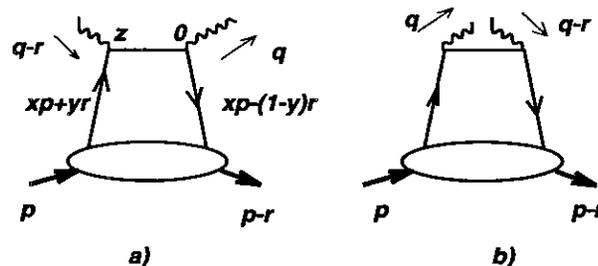


FIG. 1. Handbag diagrams contributing into the DVCS amplitude. The lower blob corresponds to non-forward quark distributions  $F(x, y)$ ,  $G(x, y)$ .

The leading contribution in the large- $Q^2$ , fixed- $x_{Bj}$ ,  $t = 0$  limit is given by the handbag diagrams shown on Fig.1, in which the long-distance dynamics is described by matrix elements like

$$\langle p - r | \bar{\psi}_a(0) \gamma_\mu E(0, z; A) \psi_a(z) | p \rangle \quad \text{and} \quad \langle p - r | \bar{\psi}_a(0) \gamma_5 \gamma_\mu E(0, z; A) \psi_a(z) | p \rangle, \quad (2)$$

where, for Fig.1a,  $z$  is the coordinate of the virtual photon vertex and  $E(0, z; A)$

derived and solved in ref. [6], where the anomalous dimension matrix  $Z_{nk}$  was used instead of  $V(x, y)$  (see also [7]).

is the usual  $P$ -exponential of the gluonic  $A$ -field along the straight line connecting 0 and  $z$ .

Though the momenta  $p$  and  $r$  are proportional to each other  $r = \zeta p$ , to construct an adequate QCD parton picture, one should make a clear distinction between them. The basic reason is that  $p$  and  $r$  specify the momentum flow in two different channels. For  $r = 0$ , the net momentum flows only in the  $s$ -channel and the total momentum entering into the composite operator vertex is zero. In this case, the matrix element coincides with the standard (forward) distribution function. The partons entering the composite vertex then carry the fractions  $x_i$  of the initial proton momentum ( $-1 < x_i < 1$ ). When  $x$  is negative, the parton is interpreted as belonging to the final state and  $x_i$  is redefined to secure that the integral always runs over the segment  $0 \leq x \leq 1$ . In this parton picture, the spectators take the remaining momentum  $(1-x)p$ . On the other hand, if the total momentum flowing through the composite vertex is  $r$ , the matrix element has the structure of the distribution amplitude in which the momentum  $r$  splits into the fractions  $y r$  and  $(1-y)r \equiv \bar{y} r$  carried by the quark fields attached to that vertex. In a combined situation, when both  $p$  and  $r$  are nonzero, the initial quark has the momentum  $x p + y r$ , while the final one carries the momentum  $x p - \bar{y} r$ . In more formal terms, this corresponds to the following parameterization of the light-cone matrix elements

$$\begin{aligned} & (p-r | \bar{\psi}_a(0) \hat{z} E(0, z; A) \psi_a(z) | p) |_{z^2=0} \\ &= \bar{u}(p-r) \hat{z} u(p) \int_0^1 \int_0^1 \left( e^{-ix(pz) - iy(rz)} F_a(x, y) \right. \\ & \left. - e^{ix(pz) + iy(rz)} F_{\bar{a}}(x, y) \right) \theta(x+y \leq 1) dy dx, \end{aligned} \quad (3)$$

$$\begin{aligned} & (p-r | \bar{\psi}_a(0) \gamma_5 \hat{z} E(0, z; A) \psi_a(z) | p) |_{z^2=0} \\ &= \bar{u}(p-r) \gamma_5 \hat{z} u(p) \int_0^1 \int_0^1 \left( e^{-ix(pz) - iy(rz)} G_a(x, y) \right. \\ & \left. + e^{ix(pz) + iy(rz)} G_{\bar{a}}(x, y) \right) \theta(x+y \leq 1) dy dx, \end{aligned} \quad (4)$$

where  $\hat{z} \equiv \gamma_\mu z^\mu$  and  $\bar{u}(p-r)$ ,  $u(p)$  are the Dirac spinors for the nucleon.

Though we arrived at the matrix elements (3), (4) in the context of the scaling limit of the DVCS amplitude, they accumulate a process-independent information and, hence, have a quite general nature. The coefficient of proportionality between  $p$  and  $r$  is then just the parameter characterizing the “non-forwardness” of the matrix elements. The fact that, in our case,  $\zeta$  coincides with the Bjorken

variable is specific for the DVCS amplitude. The most non-trivial feature implied by the representation (4) is the absence of the  $\zeta$ -dependence in the non-forward distributions  $F_a(x, y)$  and  $G_a(x, y)$ . Using the methods developed in ref. [8], one can prove that this property and the spectral constraints  $x \geq 0$ ,  $y \geq 0$ ,  $x+y \leq 1$  hold for any Feynman diagram. As a result, both the initial active quark and the spectators carry positive fractions of the light-cone “plus” momentum  $p$ :  $x + \zeta y$  for the active quark and  $\bar{x} - \zeta y = (1-x-y) + (1-\zeta)y$  for the spectators. Note, however, that the fraction of the initial momentum  $p$  carried by the quark going out of the composite vertex is given by  $x - \bar{y}\zeta$  and it may take both positive and negative values. At first sight, this result contradicts the intuition based on the infinite momentum frame picture. Recall, however, that all the fractions are positive only when the “plus” component of the momentum transfer  $r$  vanishes, which is not the case here, since  $r$  has *only* the “plus” component.

Taking the limit  $r = 0$  gives the matrix element defining the parton distribution functions  $f_a(x)$ ,  $f_{\bar{a}}(x)$  (or  $g_a(x)$ ,  $g_{\bar{a}}(x)$ ). This observation results in the following reduction formulas for the non-forward distributions  $F(x, y)$ ,  $G(x, y)$ :

$$\int_0^{1-x} F_a(x, y) dy = f_a(x) \quad , \quad \int_0^{1-x} G_a(x, y) dy = g_a(x). \quad (5)$$

*3. Leading-order contribution.* Using the parameterization for the matrix elements given above, we get a parton-type representation for the handbag contributions to the DVCS amplitude:

$$\begin{aligned} T^{\mu\nu}(p, q, r) &= \left( g^{\mu\nu} - \frac{1}{p \cdot q} (p^\mu q^\nu + p^\nu q^\mu) \right) \sum_a e_a^2 \sqrt{1-\zeta} (T_V^a(\zeta) + T_{\bar{V}}^a(\zeta)) \\ & \quad + i\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta \frac{\bar{u}(p') \gamma_5 \hat{q} u(p)}{2 p \cdot q} \sum_a e_a^2 (T_A^a(\zeta) + T_{\bar{A}}^a(\zeta)), \end{aligned} \quad (6)$$

where  $\hat{q} \equiv \gamma_\mu q^\mu$ , the factor  $\sqrt{1-\zeta}$  comes from  $\bar{u}(p') = \sqrt{1-\zeta} \bar{u}(p)$  and  $T_V^a(\zeta)$ ,  $T_{\bar{V}}^a(\zeta)$  are the invariant amplitudes depending on the scaling variable  $\zeta$ :

$$T_V^a(\zeta) = \int_0^1 dx \int_0^{1-x} \left( \frac{1}{x - \zeta \bar{y} + i\epsilon} + \frac{1}{x + \zeta y} \right) F_a(x, y) dy, \quad (7)$$

$$T_{\bar{V}}^a(\zeta) = \int_0^1 dx \int_0^{1-x} \left( \frac{1}{x - \zeta \bar{y} + i\epsilon} - \frac{1}{x + \zeta y} \right) G_a(x, y) dy. \quad (8)$$

The terms containing  $1/(x - \zeta \bar{y} + i\epsilon)$  generate the imaginary part:

$$\frac{1}{\pi} \text{Im} T_V^a(\zeta) = \int_0^1 \int_0^1 \delta(x - \zeta \bar{y}) F_a(x, y) \theta(x+y \leq 1) dx dy$$

$$= \frac{1}{\zeta} \int_0^\zeta F_a(x, 1-x/\zeta) dx = \int_0^1 F_a(\bar{y}\zeta, y) dy, \quad (9)$$

with a similar expression for  $\text{Im} T_A^a(\zeta)$ . Because of the integration remaining in eq.(9), the relation between  $\text{Im} T(\zeta)$  and the non-forward distributions  $F_a(x, y)$  is not as direct as in the case of forward virtual Compton amplitude, the imaginary part of which is just given by distribution functions  $f_a(\zeta)$ . Note, that the  $y$ -integral in eq.(9) is different from that in the reduction formula (5), *i.e.*, though

$$\Phi_a(\zeta) \equiv \int_0^1 F_a(\bar{y}\zeta, y) dy \quad (10)$$

is a function of the Bjorken variable  $\zeta$ , it does not coincide with  $f_a(\zeta)$ .

To get the real part of the  $1/(x - \zeta\bar{y} + ic)$  terms, one should use the principal value prescription, *i.e.*,  $\text{Re} T(\zeta)$  is related to  $F_a(x, y)$  through two integrations.

4. *Evolution equation.* In QCD, the limit  $z^2 \rightarrow 0$  for the matrix elements in eq.(2) is singular. As a result, in perturbation theory, the distribution  $F(x, y)$  contains logarithmic ultraviolet divergences which can be removed in a standard way by applying the  $R$ -operation characterized by some subtraction scale  $\mu$ :  $F(x, y) \rightarrow F(x, y; \mu)$ . The  $\mu$ -dependence of  $F(x, y; \mu)$  is governed by the evolution equation

$$\left( \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right) F(x, y; \mu) = \int_0^1 d\xi \int_0^1 R(x, y; \xi, \eta; g) F(\xi, \eta; \mu) d\eta. \quad (11)$$

Since the integration over  $y$  converts  $F(x, y)$  into the parton distribution function  $f(x)$ , whose evolution is governed by the GLAP equation

$$\left( \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right) f(x; \mu) = \int_x^1 \frac{d\xi}{\xi} P(x/\xi; g) f(\xi; \mu) d\xi, \quad (12)$$

the kernel  $R(x, y; \xi, \eta; g)$  must have the property

$$\int_0^{1-x} R(x, y; \xi, \eta; g) dy = \frac{1}{\xi} P(x/\xi). \quad (13)$$

For a similar reason, integrating  $R(x, y; \xi, \eta; g)$  over  $x$  one should get the Brodsky-Lepage kernel:

$$\int_0^{1-y} R(x, y; \xi, \eta; g) dx = V(y, \eta; g). \quad (14)$$

Explicit one-loop calculations give the following result for the  $qq$ -component of the kernel:

$$R_{qq}(x, y; \xi, \eta; g) = \frac{\alpha_s}{\pi} C_F \frac{1}{\xi} \left\{ \theta(0 \leq x/\xi \leq \min\{y/\eta, \bar{y}/\bar{\eta}\}) - \frac{1}{2} \delta(1-x/\xi) \delta(y-\eta) + \frac{\theta(0 \leq x/\xi \leq 1) x/\xi}{(1-x/\xi)} \left[ \frac{1}{\eta} \delta(x/\xi - y/\eta) + \frac{1}{\bar{\eta}} \delta(x/\xi - \bar{y}/\bar{\eta}) \right] - 2\delta(1-x/\xi) \delta(y-\eta) \int_0^1 \frac{z}{1-z} dz \right\}. \quad (15)$$

Here the last (formally divergent) term, as usual, provides the regularization for the singularities of the kernel for  $x = \xi$  (or  $y = \eta$ ). Note that, in the Feynman gauge, the first two lines of eq.(15) correspond to operators with ordinary derivatives  $\partial^\nu$  while the last two lines result from the  $\partial^\nu \rightarrow D^\nu = \partial^\nu - igA^\nu$  change. It is easy to verify that the kernel  $R_{qq}(x, y; \xi, \eta; g)$  has the property that  $x + y \leq 1$  if  $\xi + \eta \leq 1$ . Using our expression for  $R_{qq}(x, y; \xi, \eta; g)$  and explicit forms of the  $P_{qq}(x/\xi)$  and  $V(y, \eta)$  kernels

$$P_{qq}(z) = \frac{\alpha_s}{\pi} C_F \left( \frac{1+z^2}{1-z} \right)_+, \quad (16)$$

$$V(y, \eta) = \frac{\alpha_s}{\pi} C_F \left\{ \left( \frac{y}{\eta} \right) \left[ 1 + \frac{1}{\eta-y} \right] \theta(y \leq \eta) + \left( \frac{\bar{y}}{\bar{\eta}} \right) \left[ 1 + \frac{1}{y-\eta} \right] \theta(y \geq \eta) \right\}_+ \quad (17)$$

(where “+” denotes the standard “plus” regularization [3]), one can check that  $R_{qq}(x, y; \xi, \eta; g)$  satisfies the reduction formulas (13) and (14).

Note also that the flavor non-singlet light-cone operator  $\mathcal{O}^{NS}(z, 0) \equiv \bar{\psi}(z) \lambda^a \hat{z} \psi(0)$  satisfies the Balitsky-Braun evolution equation [9]

$$\left( \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right) \mathcal{O}^{NS}(z, 0) = \int_0^1 du \int_0^u K(u, v) \mathcal{O}^{NS}(uz, vz) dv. \quad (18)$$

This means that our kernel  $R_{qq}(x, y; \xi, \eta; g)$  should be related to the  $K$ -kernel by

$$R_{qq}(x, y; \xi, \eta; g) = \frac{1}{\xi} K(\bar{y} + \eta x/\xi, \bar{y} - \eta x/\xi). \quad (19)$$

Indeed, taking the explicit form of  $K(u, v)$  from ref. [9]<sup>§</sup>

<sup>§</sup>In the definition adopted in ref. [9]  $K(u, v)$  has the opposite sign.

$$K(u, v) = \frac{\alpha_s}{\pi} C_F \left( 1 + \delta(\bar{u})[\bar{v}/v]_+ + \delta(v)[u/\bar{u}]_+ - \frac{1}{2} \delta(\bar{u})\delta(v) \right) \quad (20)$$

and combining it with eq.(19), one immediately obtains our expression (15) for  $R_{qq}(x, y; \xi, \eta; g)$ .

To solve the evolution equation, I propose to combine the standard methods used to find solutions of the underlying GLAP and Brodsky-Lepage evolution equations. Here, we will consider only the simplest case, *i.e.*, the evolution equation for the flavor-nonsinglet component in which  $qg$ ,  $gq$  and  $gg$  kernels do not contribute. Recall first that, to solve the GLAP equation, one should consider the moments with respect to  $x$ . Integrating  $x^n R_{qq}(x, y; \xi, \eta; g)$  over  $x$  and utilizing the property  $R_{qq}(x, y; \xi, \eta; g) = R_{qq}(x/\xi, y; 1, \eta; g)/\xi$ , we get

$$\left( \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right) F_n(y; \mu) = \int_0^1 R_n(y, \eta; g) F_n(\eta; \mu) d\eta, \quad (21)$$

where  $F_n(y; \mu)$  is the  $n$ th  $x$ -moment of  $F(x, y; \mu)$

$$F_n(y; \mu) = \int_0^1 x^n F(x, y; \mu) dx \quad (22)$$

and the kernel  $R_n(y, \eta; g)$  is given by

$$R_n(y, \eta; g) = \frac{\alpha_s}{\pi} C_F \left\{ \left( \frac{y}{\eta} \right)^{n+1} \left[ \frac{1}{n+1} + \frac{1}{\eta-y} \right] \theta(y \leq \eta) + \left( \frac{\bar{y}}{\bar{\eta}} \right)^{n+1} \left[ \frac{1}{n+1} + \frac{1}{y-\eta} \right] \theta(y \geq \eta) - \frac{1}{2} \delta(y-\eta) - \delta(y-\eta) \int_0^1 \frac{z}{1-z} dz \right\}. \quad (23)$$

It is straightforward to check that  $R_n(y, \eta; g)$  has the property

$$R_n(y, \eta; g) w_n(\eta) = R_n(\eta, y; g) w_n(y),$$

where  $w_n(y) = (y\bar{y})^{n+1}$ . Hence, the eigenfunctions of  $R_n(y, \eta; g)$  are orthogonal with the weight  $w_n(y) = (y\bar{y})^{n+1}$ , *i.e.*, they are proportional to the Gegenbauer polynomials  $C_k^{n+3/2}(y-\bar{y})$  (cf. [5,10]). Now, we can write the general solution of the evolution equation

$$F_n(y; \mu) = (y\bar{y})^{n+1} \sum_{k=0}^{\infty} A_{nk} C_k^{n+3/2}(y-\bar{y}) [\log(\mu/\Lambda)]^{-\gamma_k^{(n)}/\beta_0}, \quad (24)$$

where  $\beta_0 = 11 - \frac{2}{3}N_f$  is the lowest coefficient of the QCD  $\beta$ -function and the anomalous dimensions  $\gamma_k^{(n)}$  are related to the eigenvalues of the kernel  $R_n(y, \eta; g)$ :

$$\gamma_k^{(n)} = C_F \left[ \frac{1}{2} - \frac{1}{(n+k+1)(n+k+2)} + 2 \sum_{j=2}^{k+n+1} \frac{1}{j} \right]. \quad (25)$$

They coincide with the standard non-singlet anomalous dimensions  $\gamma_N$  [11,12]:  $\gamma_k^{(n)} = \gamma_{n+k+1}$ . Note, that  $\gamma_0^{(0)} = 0$ , while all other anomalous dimensions are positive. Hence, in the formal  $\mu \rightarrow \infty$  limit, we have  $F_0(y, \mu \rightarrow \infty) \sim y\bar{y}$  and  $F_n(y, \mu \rightarrow \infty) = 0$  for all  $n \geq 1$ . This means that

$$F(x, y; \mu \rightarrow \infty) \sim \delta(x)y\bar{y},$$

*i.e.*, in each of its variables, the limiting function  $F(x, y; \mu \rightarrow \infty)$  acquires the characteristic asymptotic form dictated by the nature of the variable:  $\delta(x)$  is specific for the distribution functions [11,12], while the  $y\bar{y}$ -form is the asymptotic shape for the lowest-twist two-body distribution amplitudes [6,5].

5. *Infrared sensitivity and hard gluon exchange corrections.* The structure of the leading term of the DVCS amplitude  $T(\zeta, Q^2)$  is very similar to that of the forward virtual Compton scattering amplitude  $T_f(\omega, Q^2)$  which is the starting point of the DIS analysis. The major difference between the two amplitudes is that one of the photons in the DVCS amplitude is real. As a result, in higher orders, the coefficient functions  $C(\zeta, Q^2, q^2)$  of the formal operator product expansion of two electromagnetic currents may be singular (non-analytic) in the limit  $q^2 \rightarrow 0$ . These singularities are related to the possibility of a long-distance propagation in the  $q$ -channel. In fact, the long-distance sensitivity simply means that the relevant contribution is non-calculable in perturbation theory, and one should describe/parameterize it by introducing the distribution amplitude for the real photon.

Two simplest contributions of this type are shown in fig.2. They are analogous to the hard gluon exchange diagrams for the pion form factor. The basic common feature is that the large virtuality flow bypasses the real photon vertex, and the relevant contribution factorizes into the product of the hard scattering (short-distance) amplitude formed by quark and gluon propagators and two long-distance parts. In our case, one long-distance part is given by the non-forward distribution  $F(x, y)$  (or  $G(x, y)$ ) and the other by the photon distribution amplitude  $\varphi_\gamma(u)$ . The latter can be understood as the probability amplitude to obtain the photon with momentum  $q$  from a collinear  $\bar{q}q$  pair, with quarks carrying the momenta  $uq$  and  $(1-u)q$ . The  $u^n$ -moments of the lowest-twist function  $\varphi_\gamma(u)$

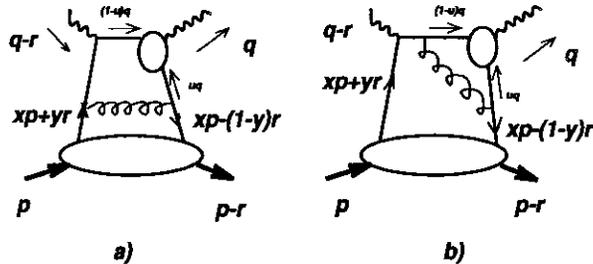


FIG. 2. Simplest hard gluon exchange contributions to the DVCS amplitude. The upper blob corresponds to the photon distribution amplitude  $\varphi_\gamma(u)$  and the lower one to non-forward quark distributions  $F(x, y), G(x, y)$ .

are related to the difference between the “exact” and perturbative versions of the correlator

$$\Pi_{\mu\nu\nu_1\dots\nu_n}(q) \equiv \int e^{iqz} \langle 0 | T \{ J_\mu(z) \bar{\psi}_a(0) \gamma_\nu D_{\nu_1} \dots D_{\nu_n} \psi_a(0) \} | 0 \rangle d^4z \quad (26)$$

of the electromagnetic current  $J_\mu(z)$  with a composite operator containing  $n$  covariant derivatives. The perturbative amplitude for the  $\bar{q}q \rightarrow \gamma$  transition is taken into account in the pQCD radiative correction to the coefficient function (*cf.* [13]). By a simple counting of the propagators, one can easily find out that the contribution of Fig.2 behaves like  $\alpha_s/Q^2$  in the large- $Q^2$  limit. Furthermore, due to the EM current conservation, this correlator is proportional to the tensor structure  $g_{\mu\nu}q^2 - q_\mu q_\nu$ . Since, for a real photon,  $q^2 = 0$  and  $(q\epsilon) = 0$  ( $\epsilon^\mu$  being the photon’s polarization), the relevant contribution vanishes. Hence, the hard gluon exchange corrections to the DVCS amplitude are rather strongly suppressed.

At the leading-power level  $O(Q^0)$ , the absence of the non-analytic  $\ln(q^2)$ -terms in the one-loop coefficient function contribution for the  $A$ -part of the amplitude can be read off from existing results for the  $\alpha_s$  correction to the  $\gamma\gamma^* \rightarrow \pi^0$  amplitude [14–16].

**6. Conclusions.** In this paper, I formulated the basics of the pQCD approach for studying the deeply virtual Compton scattering amplitude in the scaling limit. The essential point is that information about the long-distance dynamics in this case is accumulated in the non-forward distributions  $F(x, y)$ . I studied the evolution properties of  $F(x, y)$  and also showed that  $F(x, y)$ ’s are related to the standard (forward) distribution functions  $f(x)$  through the reduction formulas (5). An interesting problem for the future studies is to understand a possible interplay between the  $x$ - and  $y$ -dependencies of the non-forward distributions and

construct phenomenological models for  $F(x, y)$ . The simplest idea is to try a factorized ansatz  $F(x, y) = f(x)g(y)$ . However, the explicit expression for the evolution kernel  $R(x, y; \xi, \eta)$  (which satisfies a similar reduction formula (13)) suggests that the structure of  $F(x, y)$  is not necessarily as simple as that.

In the experimental aspect, as emphasized by Ji [1], a continuous electron beam accelerator with an energy 15 - 30 GeV (like proposed ELFE) may be an ideal place to study the scaling limit of the DVCS. It is also worth studying the possibility of observing the first signatures of the scaling behavior of DVCS at CEBAF, especially at upgraded energies.

In a forthcoming paper [17], I discuss the gluonic non-forward distributions  $F_g(x, y)$  which play a crucial role in the perturbative QCD approach to hard diffractive electroproduction processes like  $\gamma^* p \rightarrow p' \rho$ .

*Acknowledgments.* I thank Xiangdong Ji for the discussion which stimulated this investigation and Charles Hyde-Wright for persisting efforts to make me an enthusiast of the virtual Compton scattering studies. My special gratitude is to Nathan Isgur for continued encouragement and advice. This work is supported by the US Department of Energy under contract DE-AC05-84ER40150. I thank the DOE’s Institute for Nuclear Theory at the University of Washington for its hospitality and support during the workshop “Quark and Gluon Structure of Nucleons and Nuclei” where this work has been started.

## REFERENCES

- [1] X. Ji, preprint MIT-CTP-2517, Cambridge (1996); hep-ph/9603249.
- [2] V.N. Gribov and L.N. Lipatov, *Sov. J. Nucl. Phys.* **15** (1972) 78;  
L.N. Lipatov, *Sov. J. Nucl. Phys.* **20** (1975) 94.
- [3] G. Altarelli and G. Parisi, *Nucl. Phys.* **B126** (1977) 298.
- [4] Yu. L. Dokshitzer, *Sov.Phys. JETP*, **46** (1977) 641.
- [5] S.J. Brodsky and G.P. Lepage, *Phys.Lett.* **87B** (1979) 359.
- [6] A.V. Efremov and A.V. Radyushkin, JINR report E2-11983, Dubna (October 1978),  
published in *Theor. Math. Phys.* **42** (1980) 97.
- [7] A.V. Efremov and A.V. Radyushkin, *Phys.Lett.* **B94** (1980) 245.
- [8] A.V. Radyushkin, *Phys.Lett.* **B131** (1983) 179.
- [9] I.I.Balitsky and V.M.Braun, *Nucl.Phys.* **B311** (1988/89) 541.
- [10] S.V. Mikhailov and A.V. Radyushkin, *Nucl. Phys.* **B273** (1986) 297.
- [11] D.J. Gross and F. Wilczek, *Phys. Rev.* **D9** (1974) 980.
- [12] H. Georgi and H.D. Politzer, *Phys. Rev.* **D9** (1974) 416.
- [13] A.V. Radyushkin and R. Ruskov, preprint CEBAF-TH-95-17, hep/ph 9511270 (to appear in *Phys.Lett. B*).
- [14] F.Del Aguila and M.K.Chase, *Nucl.Phys.* **B193** (1981) 517.
- [15] E. Braaten, *Phys. Rev.* **D28** (1983) 524.
- [16] E.P.Kadantseva, S.V.Mikhailov and A.V.Radyushkin, *Sov.J. Nucl.Phys.* **44** (1986) 326.
- [17] A.V. Radyushkin (in preparation).