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ENERGY STABILITY IN RECIRCULATING, ENERGY–RECOVERING LINACS IN THE PRESENCE OF AN FEL

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Abstract

Recirculating, energy–recovering linacs can be used as driver accelerators for high power FELs. Instabilities which arise from fluctuations of the cavity fields are investigated. Energy changes can cause beam loss on apertures, or, when coupled to M_{56}, phase oscillations. Both effects change the beam induced voltage in the cavities and can lead to unstable variations of the accelerating field. An analytical model which includes amplitude and phase feedback, has been developed to study the stability of the system for small perturbations from equilibrium. The interaction of the electron beam with the FEL is a major perturbation which affects both the stability of the system and the development of start–up and recovery scenarios. To simulate the system's response to such large parameter variations, a numerical model of the beam–cavity interaction has been developed which includes low level rf feedback, phase oscillations and beam loss instabilities and the FEL interaction. Agreement between the numerical model and the linear theory has been demonstrated in the limit of small perturbations. In addition, the model has been benchmarked against experimental data obtained during CEBAF's high current operation. Numerical simulations have been performed for the high power IR DEMO approved for construction at CEBAF.

1 LINEAR THEORY

The interaction of the beam with the cavity fields can be described, to a very good approximation, by the following first order differential equation,

$$\frac{d\tilde{V}_c}{dt} + \frac{\omega_0}{2Q_L}(1 - i\tan\Psi)\tilde{V}_c = \frac{\omega_0 R_L}{2Q_L}(\tilde{I}_g - \tilde{I}_b) \qquad (1)$$

where ω_0 is the cavity resonant frequency, Q_L is the loaded Q of the cavity and R_L is the loaded shunt impedance $R_L = (R/Q)Q_L$. The beam in the cavity is represented by a current generator. In arriving at (1) we assume that the cavity voltage, generator and beam current vary as $e^{i\omega t}$, where ω is the rf frequency, and $\tilde{V}_c$, $\tilde{I}_g$ and $\tilde{I}_b$ are the corresponding complex amplitudes (phasors) in the rotating frame of reference, varying slowly with time. In this equation I_b (absence of tilde denotes the magnitude of the corresponding quantity) is equal to the average beam current (in the limit of short bunches). Also Ψ is the tuning angle defined by $\tan\Psi = -2Q_L(\omega - \omega_0)/\omega_0$. In steady–state the generator power is given by

$$P_g = \frac{(1+\beta)}{4\beta} I_g^2 R_L \quad ,$$

where β is the cavity coupling coefficient, and can be calculated from $Q_L = Q_0/(1+\beta)$.

1.1 Open Loop Analysis

To carry out the analysis we assume that the accelerator consists of an injector, and a superconducting rf linac with a one–pass recirculation transport, which accelerates the beam, decelerates it for energy recovery, and transports it to a dump. Therefore, in this model, there are two beams in the linac cavities at any time (one accelerating and one decelerating). The generalization to multi–pass recirculation transport is straightforward [1], [2].

Two effects may trigger an unstable behavior of the system: a) Beam current loss which may originate from energy offset which shifts the beam centroid off its central trajectory and leads to beam scraping on apertures. b) Phase shift which may originate from an energy offset coupled to the finite compaction factor (M_{56}) in the arc.

In the open loop analysis, we assume that the generator current $\tilde{I}_g$ is constant and is expressed in the polar form $\tilde{I}_g = I_{g0}e^{i\Psi_{g0}}$. The cavity voltage is perturbed in amplitude and phase, by $\hat{v}(t)$ and $\hat{\phi}(t)$ respectively,

$$\tilde{V}_c = [V_{c0} + \hat{v}(t)]e^{i\hat{\phi}(t)} \quad ,$$

where V_{c0} is the steady–state cavity voltage. We measure all phases with respect to the phase of the steady–state cavity voltage. We assume that the accelerating beam remains unperturbed and express it in polar form as

$$\tilde{I}_1 = I_0 e^{i\Psi_1} \quad ,$$

where Ψ_1 is the beam phase. The FEL interaction however, which takes place downstream of the linac, greatly increases the energy spread of the beam, which is then transported through an non–isochronous arc back to the linac for energy recovery. Therefore the decelerating beam can, in principle, be perturbed both in magnitude and phase,

$$\tilde{I}_2 = [I_0 + \hat{i}_2(t)]e^{i[\Psi_2 + \phi_2(t)]}$$

where

$$\hat{i}_2 = -bI_0\epsilon_1$$

$$\phi_2 = -h\epsilon_1$$

and ϵ_1 is the energy error at the end of pass 1. The coefficient h is proportional to the compaction factor of the arc,

$$h = \frac{M_{56}\omega}{cE} \quad .$$

Similarly, b can be expressed as

$$b = -\frac{\eta_x}{LE}$$

where η_x is the horizontal dispersion of the arc, L is a loss coefficient which characterizes the amount of beam loss, and E is the beam energy.

Substituting the above equations into the cavity equation (1), separating real and imaginary parts, performing the linearization, and taking the Laplace transform of the equations, we obtain two algebraic equations $MA = 0$, where M is a 2×2 matrix and A is the column vector with $\hat{v}(s)$ and $\hat{\phi}(s)$ as components.

The determinant of M is then set to zero and the two roots of s are examined. The real parts of the roots will provide the damping or growth rates of perturbations. The imaginary parts of the roots will give the oscillation frequencies relative to the driving rf frequency. If both roots have zero or negative real parts, the system is stable; otherwise the system is unstable. Taking this into account, the two roots of s are

$$s\tau = -1 + \frac{I_0 R_L}{2}(hS + bC) \pm \sqrt{\left[\frac{I_0 R_L}{2}(hS + bC)\right]^2 - \Lambda}$$

where $\tau = 2Q_L/\omega_0$ is the cavity's time constant, and

$$\Lambda = I_0 R_L (hC - bS)\tan\Psi + \tan^2\Psi$$

is a coupling term arising from the non-zero tuning angle Ψ, and S and C are defined as $S = \sin(\Psi_1 - \Psi_2)$ and $C = \cos(\Psi_1 - \Psi_2)$, where Ψ_i , i=1,2 are the steady–state phases of the beam for passes 1,2 with respect to the cavity voltage.

In the absence of coupling ($\Lambda = 0$) and $(hS + bC) \leq 0$ the system is stable for all values of the beam current. For $(hS + bC) > 0$ however, the system becomes unstable for currents above a threshold current $I_{\rm th}$ given by

$$I_{\rm th} = \frac{1}{R_L(hS + bC)} \quad .$$

In this case the growth rate of the instability increases linearly with the beam current. Coupling, in this parameter regime, can manifest itself as a frequency shift, and the system remains unstable.

For $(hS + bC) \leq 0$, if the coupling term is strong enough it can make the system unstable. The growth rate of this instability however, is slow and approaches asymptotically a constant value as the beam current increases.

1.2 Analysis with Feedback

In the presence of feedback, the generator current $\tilde{I}_g$ is no longer constant, but it assumes the form

$$\tilde{I}_g = [I_{g0} + \delta I_g(t)]e^{i\Psi_{g0}}[1 + i\delta t(t)]$$

where $\delta I_g(t)$ is the additional signal providing amplitude feedback, and $\delta t(t)$ is the additional signal providing phase feedback [3]. The transfer function in the feedback path is presently modeled as a low–pass filter with gain G and roll–off frequency $(2\pi T)^{-1}$. Therefore the Laplace transforms of δI_g and δt are

$$\frac{\delta I_g(s)}{I_{g0}} = -\frac{G}{1 + sT}\frac{\hat{v}(s)}{V_{c0}} \quad ,$$

and

$$\delta t(s) = -\frac{G}{1 + sT}\hat{\phi}(s) \quad ,$$

where $\hat{v}(s)$ and $\hat{\phi}(s)$ are the errors in the amplitude and phase of the cavity field.

The analysis is similar to the open loop case, only DetM=0 is now a quartic equation in s. The roots of DetM=0 determine the stability of the system. In Section 3 we present solutions to this equation for CEBAF's IR DEMO parameters.

2 NUMERICAL SIMULATIONS

2.1 The Model

To simulate the system's response to large parameter variations, we developed a model of the cavity and low level controls using SIMULINK, a MATLAB program for simulating dynamic systems. The model includes a realistic representation of the low level controls, modeled after CEBAF's rf control system. A detailed description of the model can be found in [4]. The model has the capability of correctly dealing with microphonic noise, transient effects and klystron saturation. The FEL turn–on is presently modeled as a linear change of the phase of the decelerating beam by 1.4^0 occuring over 4 μsecs. Two additional loops enable the two types of instabilities, caused by beam loss and phase oscillations.

2.2 Numerical Model Benchmarking

To benchmark the model, we compared the numerical results with the linear theory, as well as experimental data obtained during CEBAF's operation. In the limit of small perturbations, and amplitude and phase feedback with a single low–pass filter in the feedback path, the numerical model predicts the same gains and cross–over frequencies required for stability, as the linear theory.

Furthermore the model has been used to predict the magnitude of induced phase and amplitude transients when 250μsec beam pulses enter CEBAF's superconducting cavities. Both the shape and the magnitude of the transients as predicted by the model, are in very good agreement with the experimental data [4].

3 CEBAF'S IR DEMO: AN EXAMPLE

As a concrete example, we take the energy–recovering driver accelerator design of the CEBAF IR DEMO [5]. The accelerator consists of a 10 MeV injector, a superconducting rf linac with one–pass recirculation transport, which accelerates the beam to 42 MeV, decelerates it for energy recovery to about 10 MeV and transports it to a dump. Longitudinal dynamics imposes off–crest operation for the two beams (accelerating and decelerating), and that in turn implies that the cavities must be operated off resonance to minimize the required generator power. The optimum detuning is approximately -25 Hz. When the FEL is turned on, the phases of the two beams with respect to the rf crest are $\Psi_1 = 12.5^0$ and $\Psi_2 = -170.0^0$. With 4 kW (unsaturated) klystrons and energy recovery, an optimum Q_L of 4×10^6 allows operation at 8 MV/m in the presence of microphonics of 370 Hz p–p, for 5 mA of average current. For an $M_{56} = -0.15$m, and assuming that 1 mm offset produces 10^{-3} losses, $(hS + bC) > 0$ and the system is unstable at $I_0 = 5$ mA. The instability threshold is 1.6 mA and the growth rate of the instability is 3.3 kHz at $I_0 = 5$ mA. The threshold current for the longitudinal instability alone ($b = 0$) is greater than 5 mA, and for the beam loss instability alone ($h = 0$) is 1.4 mA and the growth rate at 5 mA is 3 kHz. Therefore, when both instabilities are present the threshold is dominated by the beam loss instability.

With both longitudinal and beam loss effects present, a gain of 8 with roll–off frequency greater or equal to 520 Hz is sufficient to bring the system to the stability boundary, for small perturbations around the equilibrium. For the scraping instability alone, the required gain is also 8 and the bandwidth is 500 Hz. In conclusion, for small perturbations from equilibrium, modest gains at reasonable frequencies (well within the range of CEBAF's rf control system) are required to stabilize the system.

To evaluate the system's performance and stability under large parameter variations, we performed simulations using the numerical model. For microphonic noise typical for the CEBAF accelerator, the system appears to be stable for a range of operating conditions. Figure 1 displays simulation results of the effect of the FEL turn–on on the cavity gradient and phase. The FEL turn–on causes transients on the cavity gradient of magnitude equal to 1.5×10^{-4}, and transients on the cavity phase of 3 mrads. The low frequency modulation of the cavity phase is due to microphonic noise of amplitude 100 Hz p–p.

4 CONCLUSIONS

We have developed an analytical model to study instabilities arising from cavity field fluctuations in recirculating linacs, used as drivers for high power FELs. To study realistic perturbations, we have also developed a numerical model, using SIMULINK, that includes realistic low level controls based on CEBAF's rf control system, beam loss and phase oscillations instability loops, a simplified model of the FEL start–up, and capability of including microphonic noise, cavity detuning and nonlinear effects arising from klystron saturation and transient phenomena. The numerical model has been benchmarked against the linear theory as well as experimental data obtained during CEBAF's operation.

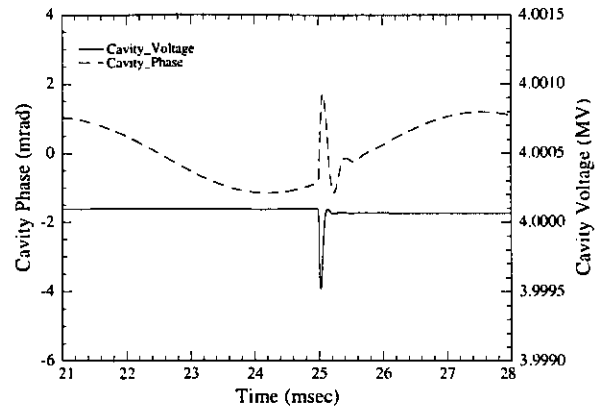

Figure 1: Cavity voltage and phase during FEL turn–on, in the presence of microphonics.

Numerical simulations have been performed with CEBAF's high power IR DEMO parameters. For microphonic noise of amplitude typical for the CEBAF accelerator, CEBAF's rf control system appears adequate to ensure stable and robust operation.

In the future we plan to develop a more complete start–up scenario for the FEL and its interaction with the rf system.

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6 REFERENCES

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